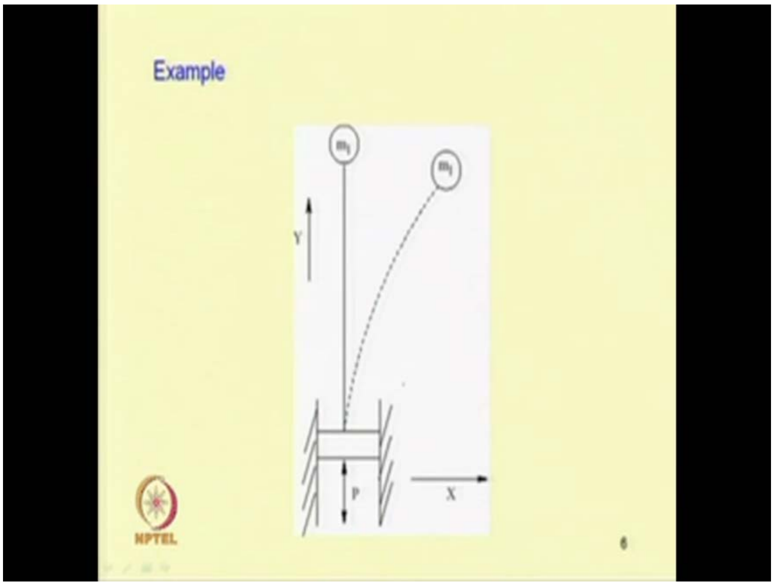
Non-Linear Vibration
Prof. S. K. Dwivedy
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 5
Numerical Techniques
Lecture - 1
Time Response FFT Frequency Response Curve

Welcome to today class of non-linear vibration. So, today class, we will start the module 5, where we will discuss about the non-linear techniques used for analysis of vibration, non-linear vibration systems. So in this module, so basically, we will discuss three things, one time modulation, how to find the time response of the system, how to find the frequency response of the system and the basin of attraction of the system? So, in time response to find the time response of the system, let us take one example, so and how we are proceed to solve this non-linear systems, we will discuss. And then different numerical techniques used in these systems also we are going to discuss.

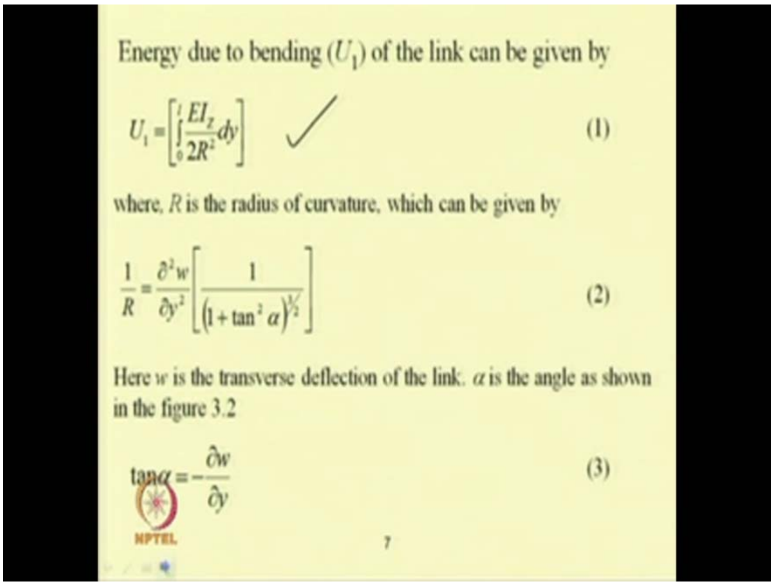
(Refer Slide Time: 01:09)



So, let us take the system, so in which, so we have a slender cantilever beam with one tip mass, and the base is excited and the base is moving up and down. Already, we know how to derive the equation motion of the system, the point of discussion in this case is, how we can numerically determine the system parameters. And then what other numerical techniques we are going to use in this case to solve or to find the response of the system. So, particularly today, we will discuss about this time response of the system, how to characterize those time response.

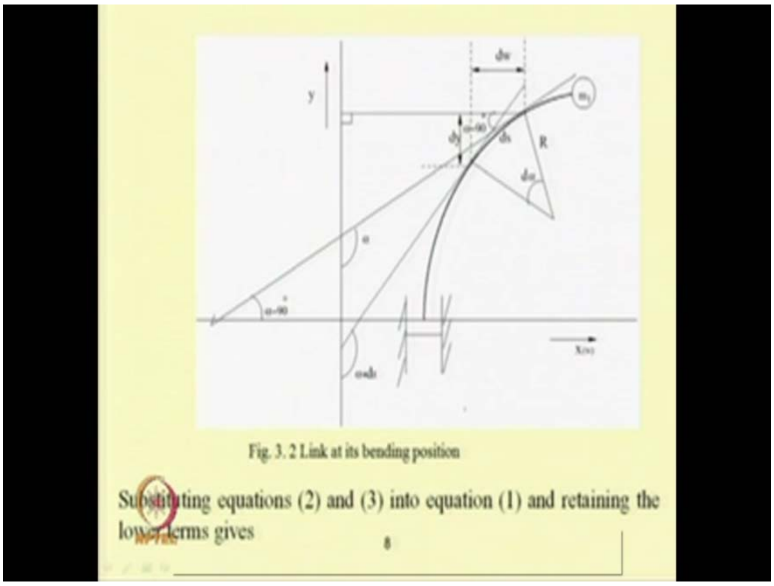
So, in case of time response, to characterize those time response, we may use this poincare section, we may use this spectra or we may go for this Lyapunov exponent. So, today class, we will discuss about how to determine this coefficient of the governing equation, then how to determine the time response. And after getting the time response, how to characterize them by using this poincare section and the Lyapunov exponent. So, in this case, to derive this equation briefly, let us discuss to derive this equation of the system.

(Refer Slide Time: 02:27)

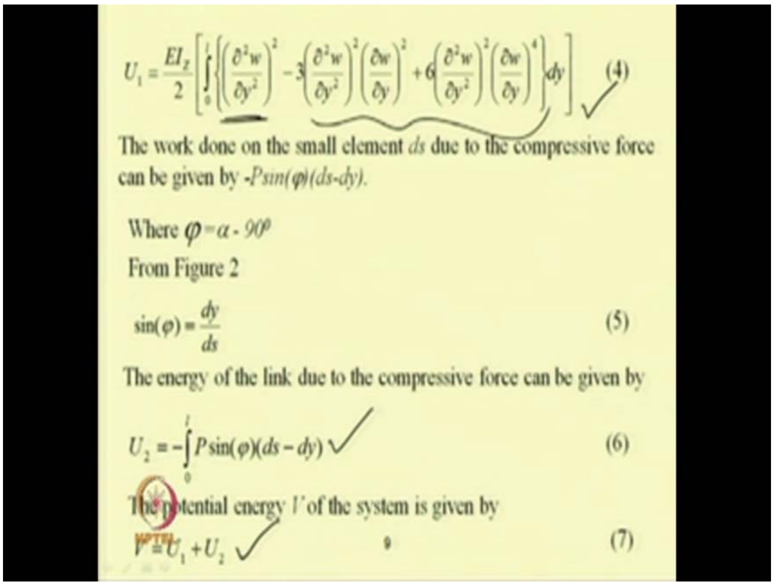


So, we can write the potential energy of the systems, the potential energy of the system can be retain in this way, and to make the system non-linear by assuming, by assuming moderately large, moderately large deflection of the system, moderately large transverse deflection of the system. We can write this curvature equal to 1 by R equal to del square w by del y square, into 1 by 1 plus tan square alpha to the power 3 by 2, where we can have this alpha is this angle. So, we can write this equation, so we can write this tan alpha equal to minus del w by del y, and substituting this equation, in this equation.

(Refer Slide Time: 03:13)



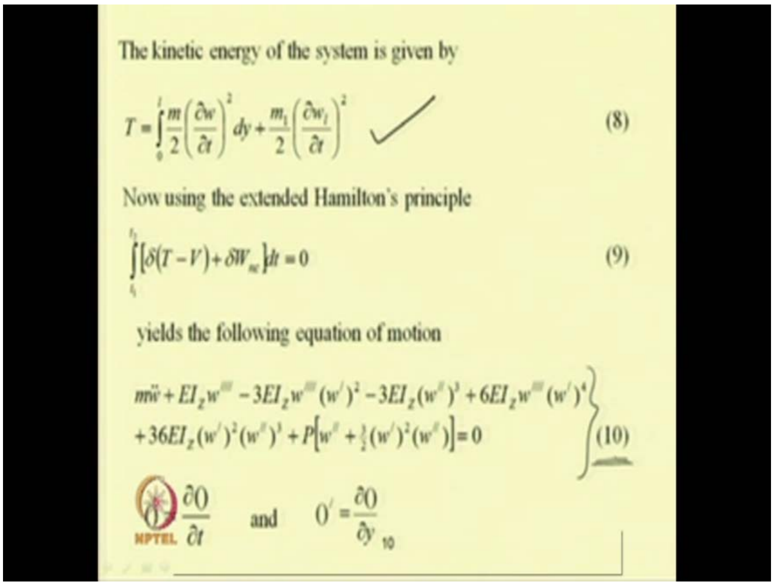
(Refer Slide Time: 03:31)



So, where so this is the, a small portion of the curve is shown here. So, this angle shows this is alpha and this is the d alpha. And from this one can find a detail study of this will give you the equation motion or we can write the potential energy using this equation where w is the transverse deflection, y is the coordinate in y direction, so. And then, we can have, we can write this E equal to young’s modulus, i is the moment of inertia of the system, and one can write the potential energy of the system using this equation.

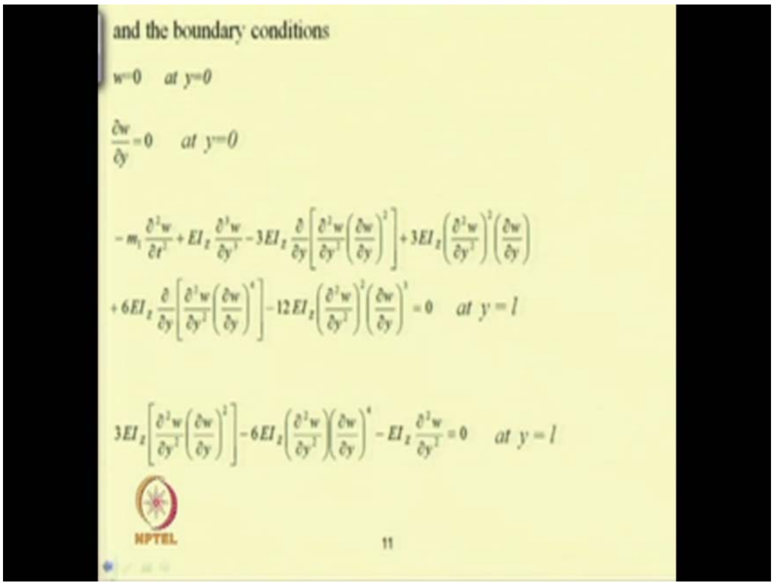
Now, one can note that this, equation contain not only the term, this term which is particularly useful for the linear system, but also contain these additional terms. So these additional terms will make the system non-linear, so now, one can find the energy of the link, due to compressive force. So, as a force is applied to the system, so due to considering a compressive force, so one can write the potential energy of the system or energy of the system using this equation, then the potential energy of the system, total potential energy will be U 1 plus U 2.

(Refer Slide Time: 04:40)



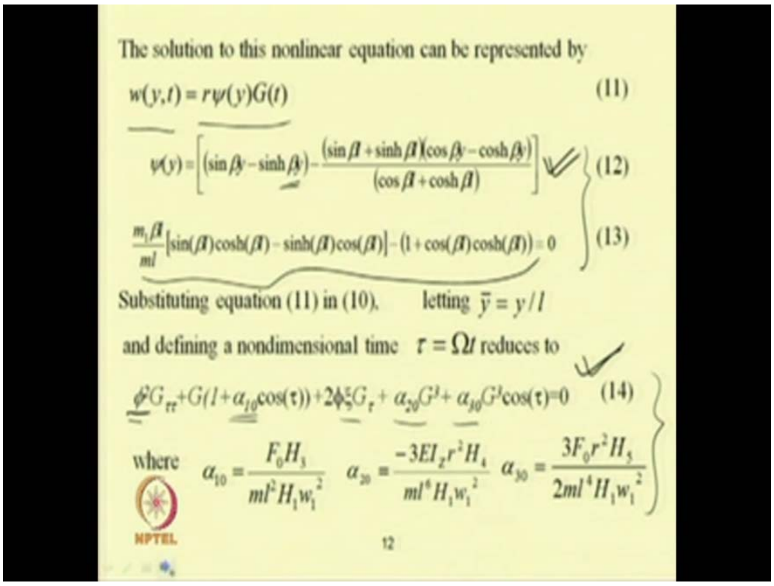
And the kinetic energy of the system can be written in this way. So now, using extended Hamilton principle, so one can find the equation motion. The purpose of today lecture is not to derive this equation motion, but to see how we can obtain the coefficient of these equation, and how we can obtain the time response of the system, when will be given a equation motion. So, for example, in this case the equation motion, in spacio temporal form is given by this equation 10. So, after getting this equation, now we have to convert this spacio temporal equation to its temporal form.

(Refer Slide Time: 05:24)



So, temporal forms by applying this Galerkin method, so to apply Galerkin method, we should consider one, we should consider or we should find the Eigen function, or one safe function of the system, and the time modulation. So, in this case, this is the equation and the corresponding boundary conditions are given by this, so these are the corresponding boundary conditions.

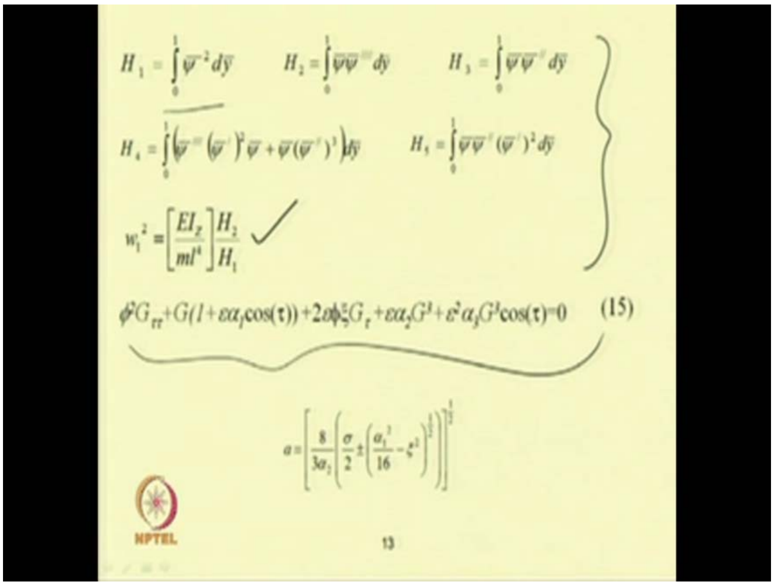
(Refer Slide Time: 05:50)



And now, to solve this equation, we have taken this w x y w y t equal to r psi y and G t, where r is the scaling factor, psi is the, psi is the safe function and G t is the time modulation. And psi y 1 can take this, psi y as the safe functions of or the Eigen function of either a cantilever beam or a cantilever beam, with tip mass. Here it is taken as that of a system with tip mass, so this is the psi y, and one can obtain this characteristic constant beta by solving this equation. So, by solving this equation, one can obtain the characteristic constant beta, and by using this equation 12 and 13.

So, one can and applying this Galerkin method, so one can find the temporal equation. So this is the temporal equation, now the purpose is to find, how we can find this coefficient phi square, coefficient phi square here phi and then alpha 1 0 then zeta alpha 2 0, alpha 3 0, and how to solve this equation to obtain the solution of the system. So, to obtain this equation, now first one has to find this beta l, how to find this beta l? So, to find this beta l, one has to solve this characteristic equation, so there are several methods, to find the solution of the characteristic equation.

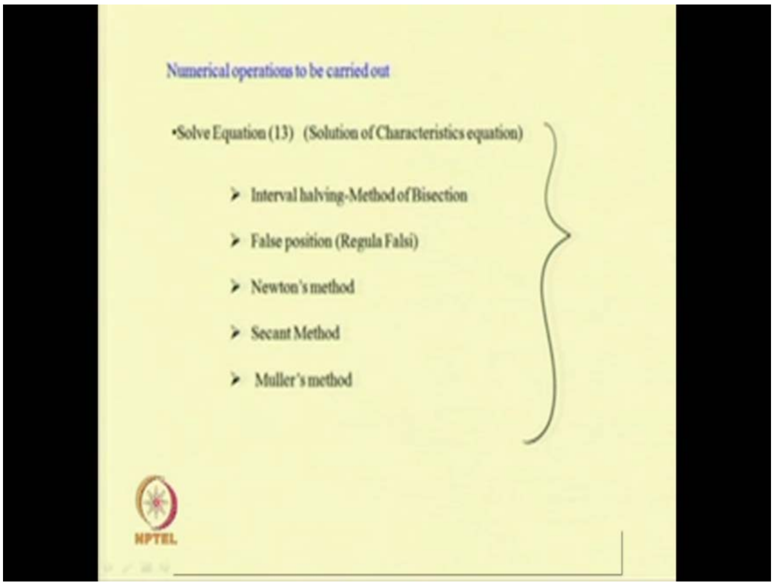
(Refer Slide Time: 07:28)



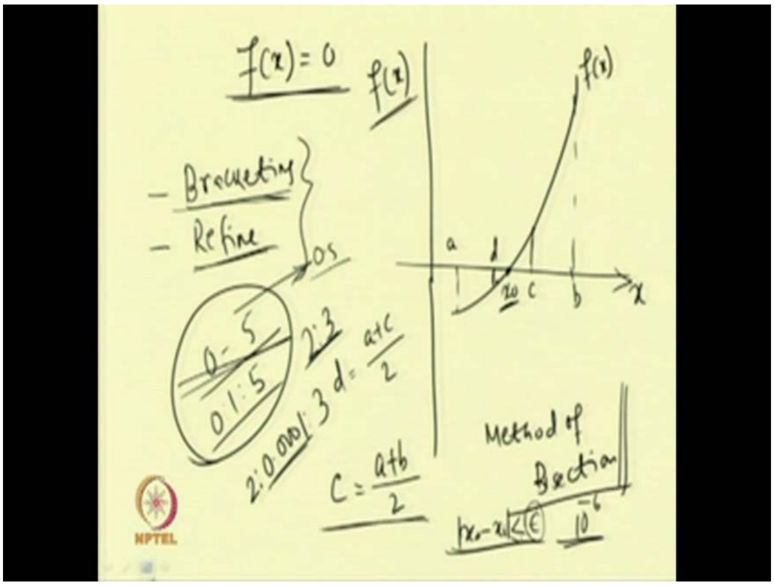
So, some of the solution methods, before going for the solution method, let us see what are the other coefficients. So, here to obtain these coefficients, one has to integrate these things to find for example, H 1 equal to integration of psi square, H 2 equal to psi into psi 4 dash. And similarly, H 3 H 4 H 5 and many other coefficients, one may get if one take the system to be more and more non-linear. So, here the first mode, frequency can be written in this term in this equation, E I z y m l fourth into H 2 by H 1. So to obtain the coefficient one has to integrate these psi functions also to find the psi functions, one has to solve the characteristic equation, so as the characteristic equation is not simple, and this is a transcendental equation.

so one has to solve this equation, numerically to find the value of beta l, so to solve this equation numerically, so one has to go for this numerical technique, and now after getting this equation getting this temporal equation, one has to solve this equation either qualitatively, by using either qualitatively or quantitatively. One can also solve this equation by using different numerical techniques, to find the response for the given system parameters, or one can use this perturbation method. To find the equation, to find the reduced equation, which are then solved to obtain the solution?

(Refer Slide Time: 09:09)



(Refer Slide Time: 09:47)



Now, let us go step by step, so first we have to find, the solution of the characteristic equation to find the solution of the characteristic equation, so either one can use this interval halving or method of bisection or false position or a regular falsi method or one can go for this Newton's method. One may use this secant method; one may use this Muller method. So, let us consider these methods in detail, so how one can use this method of bisection. So, let us take let us we have given a function f x equal to 0, so we have to solve this equation numerically to find the solution. So, for example, let us so while solving this equation, so one can use two different approach, first one can take this bracketing method.

So, one can take this bracketing, or and then, fine refine refining. So first one has to bracket the roots, so one first has to find roots where the roots lag, so to bracket this root one can use many different type of methods. One such method is the method in which one can take a course, let we have to find, let the, let we assume that the solution lies between 0 and 5, then taking these intervals 0 to 1 to 5. So we can find we can check whether the solution lies in between or in some range, then if you find that thing then we can let we think that or we found that it is lying between 2 and 3, then we can use the refined value.

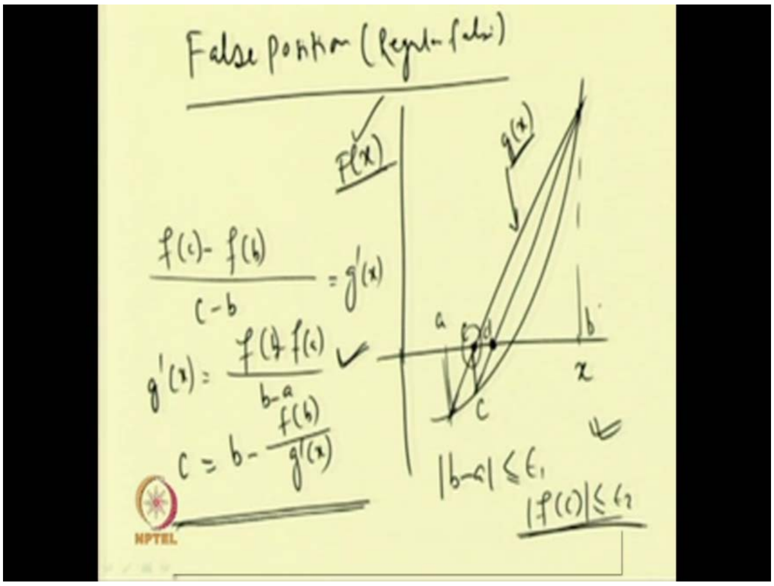
Now, we will increment this thing with vary small value, let for example, 0.001 to 3. So, initially we will to bracket this thing we will use a course increment, course increment for example, here in this case we have taken an increment of 1, or we may take for this course case, we may take it is equal to 0.5. So, we can increase this thing from 0 to 5 with increment of 0.5 and we can check, whether the solution lies in that range. So, after bracketing this thing, then to find the actual solution or to find the accurate solution, we can refine the range so we can refine this range with a very precision increment. So for example, we can take an increment of 0.001 in this case so in case of this interval halving or method of bisection. So, let us discuss what is method of bisection? So in case of method of bisection, so let this is the function, so this function affects after we have bracketed it, so we know it lies between a and b.

So, let it lies, it so this is x co-ordinate this is F x, so we know that this f a is negative and f b is positive, so the solution lie between this f region a and b or the solution lie within a and b. So after you know this thing, now the next step to find will be so let us take a point c. So in case of method of bisection, we will take a point c, such that c will be equal to a plus b by 2, so we will take this point c equal to a plus b by 2, and now let, let this is the point c. So, let this is the point c, now what you observe, so we can find this f c. So, we can see that f c is positive now then we will take the point between a and c. And then we can find this e dash or next time, we can find this again this c point we can find by taking the c dash will be equal to a plus c by 2. So, we will take another point, let c dash is this, so this is c dash, or let me take this is d instead of taking c dash, I can take this point d. So, this is point d now the solution will lie between d and c.

So, we will repeat this iteration by taking this points or the average value of the previously determine points, and in that way we can finally, obtain the solution x 0. So, this is the solution x 0, which we required, so in case of method of bisection, first we have to bracket by finding 2 points in which 1 we yield a positive value, and other will have a negative value. So, after getting these 2 points the next point we can or the solution, we can tell that the solution will be approached if you take a point which is half of this point a and b.

So, we will take C equal to a plus b by 2, then we will check the value of f x. So, we will repeat this iteration till we have achieved a accuracy, or we have achieved this thing. So if we can let finally we got this x 0. So, in this case we have to find this x 0 minus the previous value, let it is x i so this mod, should be less than epsilon, epsilon. So, where epsilon is a very small number, so or this is a precision number 10, we can take this number to be 10 to the power minus 6, or very small number. So, then we can tell that this point converge so in this way, we can use this method of bisection to find the find the characteristic root of this equation.

(Refer Slide Time: 15:49)



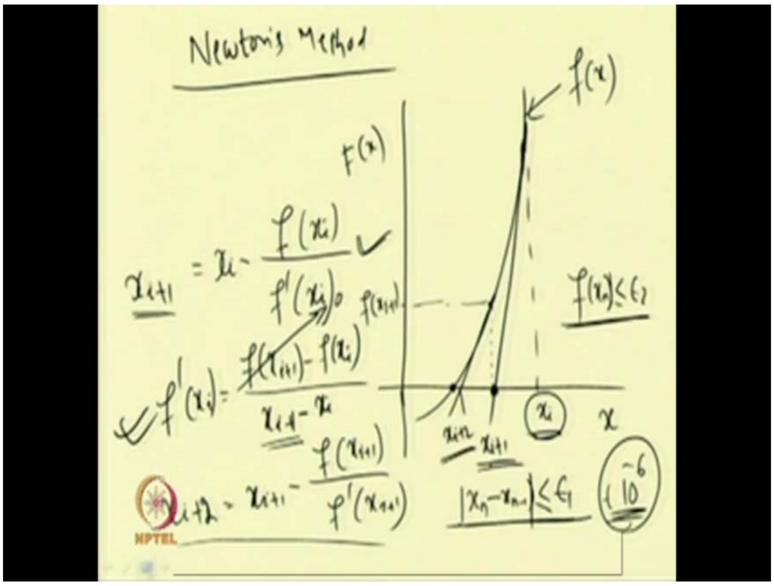
One can use the another method, that is the false position method, so in case of false position method, false position or regular falsi. So in this case, we can assume so let, let us take the same problem, this is our F x and x, so let us take, so this is the function. So we will take initially, we will take 2 positions, so this is position a, this is b, so which yield positive and negative value.

So the next point, we can assume by taking or we can assume that we will fit a linear curve between a and b, so let us join this a and b, so by linear line or straight line, so by joining this curve or approximating this thing by another function g x, where we can get this g x by taking this point a and b or joining this point a and b by a straight line. So, we can get this g x from this equation. So, then we can obtain this point c, so our f c, now this f c minus f b by c minus b, so this is equal to g dash x.

So, you can find the slope of this thing, where f c equal to 0. So, this point as it is intersecting this 0 line so f c equal to 0, so from this we can obtain this g dash x. Now g dash x will be equal to f b minus f a by b minus a, then so we can find the point c for which it will be 0 from this equation, so c will be equal to b minus f b by g dash x. So, in case of this regular falsi or regular falsi or false position method, so we are initially taking 2 points a and b for which we getting this f x value positive and negative. The next point we can find by using this equation that is c equal to b minus f b by g dash x where this g dash that is the slope, we can obtain by using this formula, that is f b minus f, f b minus f a by b minus a.

So, we will continue this iteration till we get this b minus a is very, very small that less than epsilon, and this f c value is less than epsilon that is very, very closer to 0. So, when we are considering to find the solution of this f x that means we are finding this value of x for which this f x equal to 0. So, in that case, we can use this false position method, so in this false position method, we have to find this point c, so this point c we can obtain by using this method that is c equal to b minus f b by g dash x, and where g dash equal to f b minus f a by b minus a.

(Refer Slide Time: 19:49)



Also, we can use this Newton’s method. So this is a versatile method to find the solution of this equation, so in case of Newton’s method. So let us take, so this is our equation that is F x, so let us f x versus x so this is our equation f x. Now to find so first we have bracketed this point, or so here we have to use. So we need not have to bracket this ting but, we can take an initial point a, and the next point we can find by finding let us take a point here.

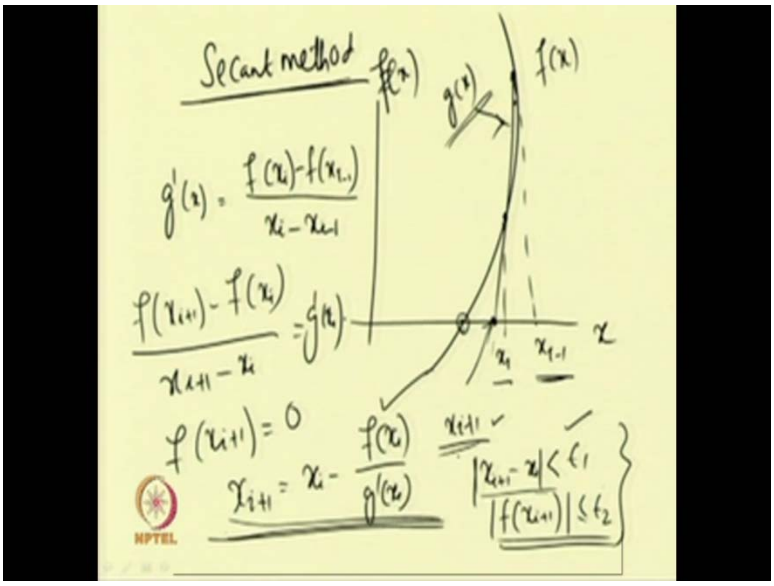
Now, find the find a tangent to this at this point, draw a tangent to this, so the next point, so if this is the ith point, let this is the ith point x i; the next point x i plus 1. So this point will be x i plus 1, and we can find which you can see is much closer to the solution point, so this is the solution point so this is mass closer to the solution point. So, initially in case of Newton's method, initially we just take a point x i this is the initial point, and the next point we can find that is x i plus 1.

So, we can find this x i plus 1 equal to x i minus f x i by f dash x i, so this thing we can obtain, so this is this equation is true, because we can we can write this f dash x i, it can be defined as, this is slope of x i. So, this is equal to f x i plus 1 minus f x i by x i plus 1 minus x i, but as we are assuming this f x i plus 1 equal to 0, so as this is equal to 0. So, where the slope caught this, this line the tangent line caught this x axis, so as x i plus 1 equal to 0, so we can find this x i plus 1 from this equation, so x i plus 1 will be equal to x i minus f x i by f dash x i. So, in this way we can find the next point which is closer to this. And now after finding this, this point, let us 5 so how to do it, so we got this point so at this point let us again draw one more tangent.

So, one can see so this is the i x i plus 2 so, we have started with x i, now we have plotted 1 tangent here, so it cots this x axis at this x i plus 1 so this x i plus 1 we obtained by using this expression. now at this x i plus 1 again let us take this point that is f. So, this is our f x i plus 1, now at this point let us again draw one more tangent so this is intersecting this x axis at x i plus 2. So, this x i plus 2 can obtain by substituting this i equal to i plus 1 so this x i plus 2 for example, in this case will be equal to x i plus 1 minus f x i plus 1, by f dash x i plus 1. So, here we can find what is this f x i plus 1? So this is f x i plus 1 and the slope.

So, we can find the slope by using this again this equation, so in this way we can go on finding the solution till we obtain the convergence. So, we can get the convergence by using the previous method by, or you can find this convergence from this. So, let us find let x n at, let x n is the, converge solution, so then x n minus x n minus 1 mod of this thing should be very, very less than epsilon. And this functional value also this functional value f x n should be very, very closer to very, very closer to the solution. So it should be very, very closer to epsilon 2, so here we have written this is epsilon 1. So, I can take this value this epsilon 1 and epsilon 2 of the order of 10 to the power minus 6 or less than this, to find 1 accurate solution. So, one can either use this method of bisection, method of false position, Newton's method or one may use the secant method also, so in case of secant method, so one can use so instead of using this straight line. So, one can use a quadratic polynomial here. So, we can use a secant here that is.

(Refer Slide Time: 25:35)



Let us use the secant method, and then we go for the Muller method. So in case of the secant method, so we can use a secant, so a secant to a curve is this straight line, which passes through 2 points on the curve. So, we have this is the curve, so now let us take 2 points on this, so if we take 2 points on this and draw a line, let this is our f x, now we will take 2 points on this and draw a line. So, this line is passing through this point and this point. So, we can approximate, the solution by using this equation, that is our g dash x equal to f x i minus, f x i minus 1 by x i minus x i minus 1.

So, we have taken this point, so this first point that is this 1 so this is x i minus 1, so this is x i so this is our x i this is x i minus 1 now our objective is to find this point. So, this is x i plus 1 so in on this curve we have taken 2 points, that is x i and x i minus 1, so these 2 points can be joined by a straight line. So that is the secant so we have joined this by a straight line, and were it is intersecting this x axis this is x axis and this is our f x.

So, it is intersecting at this point that is our x i plus, so this will be x i plus 1. So, but actually, we have to find this point, so this is the solution, so to find this point. So, we have started with x i minus 1 x i 2 points we have taken then we have made a straight line which is intersecting this x axis at i plus 1. So, our objective is to find this i plus 1, so to find that thing, so we can proceed this way. So, let us first find so we have approximated this by this line g x with these 2 points, then this g dash x slope of that straight line can be obtained from this equation, that is g dash equal to f x i minus f x i minus 1 by x i minus x i minus 1. So, the equation of the second line can be given by so we can we know that thing it can be given by x i plus 1 minus f x i by x i plus 1 minus x i so this is equal to g dash x i, and then we can write the solution of this equation.

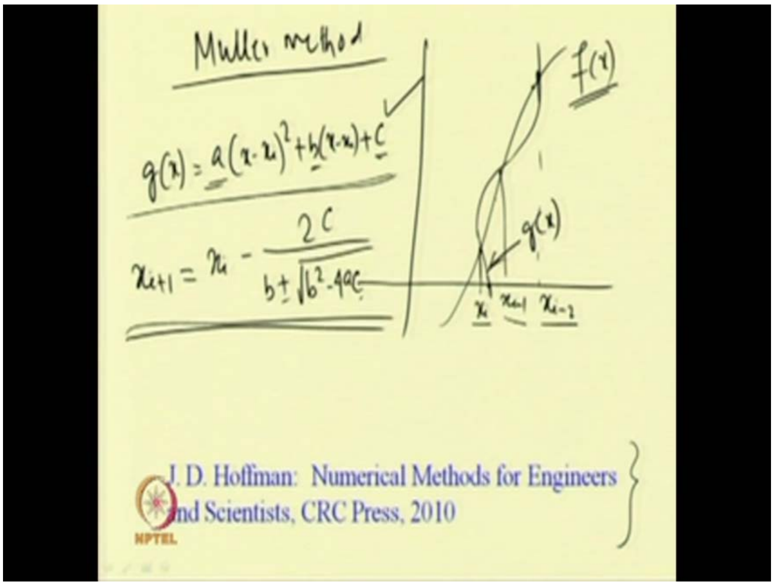
So, in this case, our f x i plus 1 as it is intersecting this line, so this will be equal to 0. Now, we can solve this equation, so by solving this equation, we can write our x i plus 1 will be equal to x i minus f x i by g dash x i. So, we can apply this equation repeatedly to find the solution which is closer to this equation, so that thing can be obtained by checking this x i plus 1 minus x i the norm of this thing should be less than this, or this functional value that is f x i plus 1 should be less than equal to epsilon 2. So, we will continue to find this x i plus 1 till this, objective is achieved, that is it should converge the solution should converge to a value, or it should be and the functional value should be closer to the 0.

So, the functional value that is f x should be closer to 0, or this x i plus 1 minus x i should be very, very small. So, either this or this one can take either this, criteria or this criteria or one can take both the criteria, to check the convergence. Also, now we have seen 4 methods, that is method of bisection, so in which the accurate point we have obtained or the iteration point c we have obtained by making this section by making the bisection or taking initially taking two points and dividing that thing so you obtain this c value.

And in case of regular falsi position, so we have taken these points joined them by a straight line. And then we obtained this false position c, after getting the false position we check whether the solution is converge or not or again we can take so that value, that c as the next point. And then by taking this a b and c again we join these two by another straight line. And c where it is intersecting the x axis then that c dash point or d point we can obtain, and this point we can iterate till we obtain the result.

So, in case of Newton’s method, so here we found the approximate solution x i plus 1, by using this equation so here we have to find the derivative of the point, initially we take a point x I. Then find the derivative of this function at that point and after finding the functional value and its derivative we can obtain the next point. And then we have to use the secant method, in case of secant method we will 2 points to straight line. So, we approximate the straight line, and we obtain the solution or we obtain the closer approximation or approximate value by taking the slope of this line, and the functional value at ith iteration. So, here we have to take 2 points 1 x i minus 1 other 1 is x i then find the functional value at x i, and then the slope to find the slope we can use this information about these two points. Then after finding the slope at x i and the function value of at x i and taking the previous value x i, then we can obtain the x i plus 1.

(Refer Slide Time: 33:03)



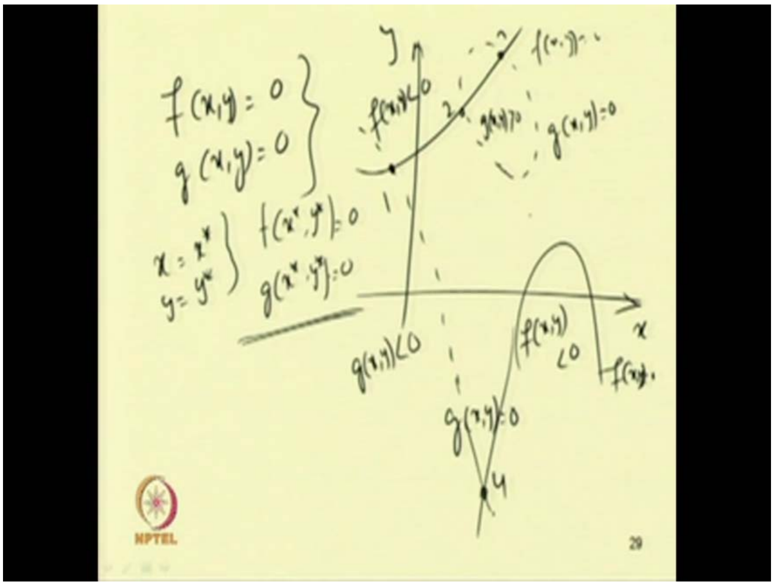
So, we can take one more method, so that is by Muller method. So, in case of Muller method, so instead of taking the straight line so we can fit a curve quadratic curve, so by fitting a quadratic curve so and using the similar procedure, what we have used in case of this Newton’s method, and the secant method. We can find the solution, let this is the equation or this is or function f x. Now, we will take a quadratic curve, so this quadratic curve we intersect this or function at 3 position. So, we will obtain this 3 position, so this 3 position, let us so we have approximated this line our f x by using this g x and this g x can be now it can be fitted by this quadratic polynomial.

So, one can write this g x equal to a x minus x i so we have taken three points; one is x i then this is our x i this point is x i minus 1 or 3 points we have taken this is x i i minus 1 and this is x i minus 2. So, this g x can be written by using this formula a into x i whole square plus b into x minus i plus c; so as we have three points, then we can obtain this a b and c, and after obtaining this value g x. Then we can proceed in a similar way as we did in case of the Newton's method or secant method to find this project solution that is we can write this x i plus 1 equal to x i minus 2 c by b, plus minus root over b square minus 4 a c.

So, the next iterative point, one can obtain by using this expression that is x i plus 1 equal to x i minus 2 c by root over b plus root over b square minus 4 a c, where a b c are can we obtained by solving this equation from these points. So, from this known points we have taken initially we have selected 3 points x i, x i minus 1 and x i minus 2 and so at these 3 points we know the functional value. So, after knowing this functional value, so we can fit this polynomial g x and from this g x as we know this functional value, we can solve these 3 equations to get a b c and then.

So, from this we can find at which this becomes 0 so this x i plus 1 by putting this g x equal to 0. So, we can find this x i plus 1 so x i plus 1 can be obtained from this equation, that is x i plus 1 equal to x i minus 2 c by root over 2 c by b plus minus root over b square minus 4 a c. So, a detailed of these methods one can find from this book by J D Hoffmann Numerical Methods for Engineer and Scientists, C R c press. So, so in this way by using this numerical techniques, one can find the roots of these equations, so these are for when we have a single equation f x, so in case we have a set of equations. So, when we have a set of equation in that case, in case of a set of equation, let us see a set of equation.

(Refer Slide Time: 37:16)

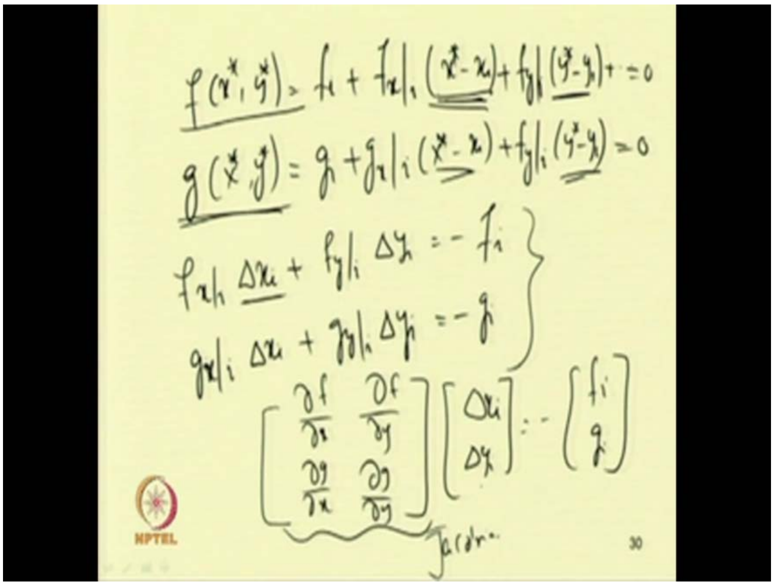


So, we can use this Newton’s method effectively to find the solution. So, for example, let us we have 2 equations that is, f x y equal to 0 and then g x y equal to 0. So, we have to for example, let us take 2 equations, so in this case this is x and y. So, we have 2 function that is f x y equal to 0 g x y equal to 0, so even these 2 functions our objective is to find x equal to x star, and y equal to y star for which, so for which we will have this f x star y star will be equal to 0.

And similarly, g x star y star will be equal to 0, so for example, let us take so let us see 2 equation, so this is 1 so let this is our f x f x y equal to 0 this is also f this represent f x y equal to 0. Similarly, let us take another equation this is g x y equal to 0 and let this is also g x y equal to 0, so this f x y equal to 0 for separate this plane, that is x y plane into 2 region that is for which for example, in this case so this region in this f x y less than 0, so outside of this thing f x y greater than 0. Similarly, here I can write this f x y less than 0 similarly, for g 1 can write this is g x y less than 0, and this side it is g x y greater than 0. Similarly, I can write here g x y greater than 0 outside it may be g x y less than 0.

So, we have to find these points, this is one point, this is the second point, this is the third point, in this particular case, so at these points, so let me extend this thing and extend so this is also 1 point, so we have these 4 points 1, 2, 3, 4. So, here we have seen, so we have 4 solutions for which so we can or we have to find 4 sets of solution for which this f x y will be equal to 0 and g x y will be equal to 0. So, to find the solution, so as we have discussed in case of method of bisection, so method of bisection, method of this regular falsi will, will not be suitable, and one can extend the Newton's method in this case, to find the solution.

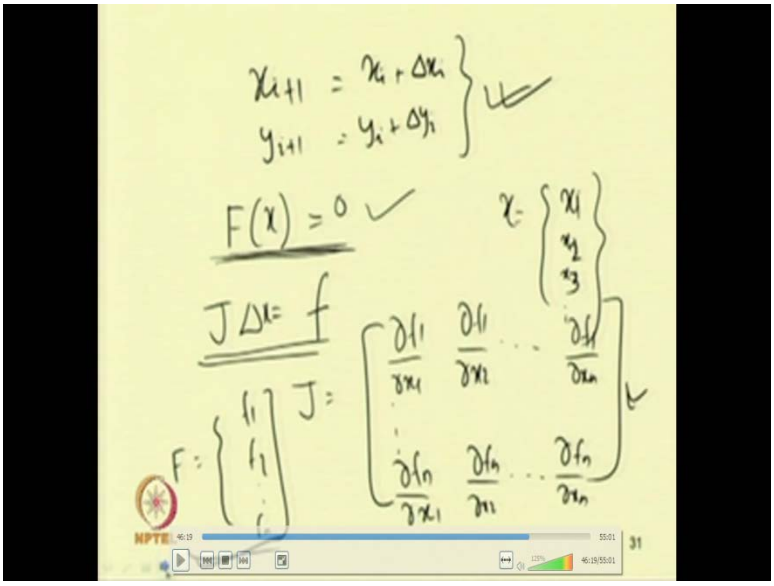
(Refer Slide Time: 40:18)



So, here to find the solution, we can write this f x y, we can write our f so we can expand this x star y star equal to f i plus f x or let we write simply f x i into x star minus x i plus f y i so y star minus y i. Similarly, I can write this g x star and y star expand this using this Taylor series, by this so it will be g i plus g x i, so this will be x star minus x i plus f y i y star minus y i. So, I have taken only one term, so this is equal to as this f x star y star equal to 0, and g x star y star equal to 0, our objective is to make this 0. So, from this we can write, so this equation can be written we can write now this, f x i. So, let me write this as delta x i so I can write this as delta x i plus f y at i delta y i, so this will be equal to minus f i. Similarly, I can write this g x i delta x i plus g y i delta y i so this will be equal to minus g i, so where this delta x this is delta x i and this is delta y i ,these and these are delta y i.

So, one can write these two equation in this form, or in matrix form 1 can write also this equation in this way so this will be equal to del f by del x del f by del y. Similarly, this is del g by del x and del g by del y into delta x i delta y I, so this will be equal to minus f i and g i. So, taking and initial conditions, x i so one can obtain this f i g i and also one can obtain this matrix this is known as the Jacobean matrix. So, one can obtain this Jacobean matrix J so which is the matrix of the fast derivative, and their 1 can obtain this delta x i, delta y i from this equation. So, after getting this delta x i and delta y i, one can write this x i plus 1 equal to x i plus delta x i.

(Refer Slide Time: 42:56)



Similarly, y i plus 1 is equal to y i plus delta y i, so for 2 equations one can obtain the next iteration value numerically, by using this x i and delta x i y i and delta y i by using this formula. But to find, but to find this Jacobean matrix, sometimes it is equal to numerically differentiate this equation, numerically differentiate the equation to find the differential and after finding this Jacobean matrix one can find the solution.

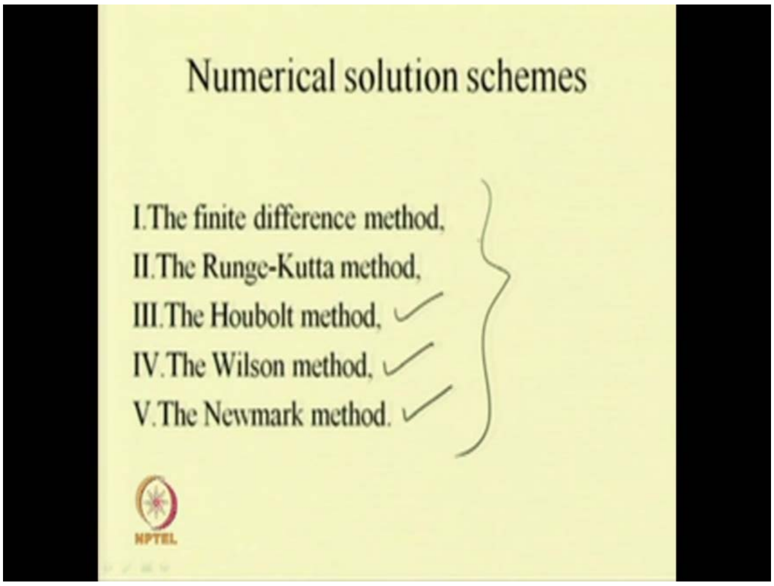
So, let us take, an n degree of freedom n degree of freedom system, so in case of n degree of freedom system. So, one can write the general equation in this form that is f x equal to 0 f x equal to 0. So, where so where one has to find this x value so x may be so x is a x 1 x 2 x 3 vector x is the vector so f x equal to 0, where to find by solving let our original differentiate. So, we have to solve this equation system, we have a system of equation f x equal to 0, and we have to solve this equation to find this x 1 x 2 x 3 and all the roots.

So, to do that thing, this is x 1 x 2 x 3 so to obtain we can follows this similar procedure as we have discussed in case of this 2 equation. So, in this case, we have to find this f x y equal to 0, so g x y so this 2 equation, we have taken so instead of 2 equation, if we have more equation we can follow the similar procedure. Now, we have to find this we can find this J delta equal to f, so here J is the Jacobean matrix. So, this Jacobean matrix, we can find this way so this will be equal to del f 1 by del x 1 del f 1 by del x 2 where this f can be written. So, we are writing, so this way this f is nothing but, our f 1 f 2 f n so if f equal to f 1 f 2 f n then, this Jacobean matrix can be found in this way, that is del f 1 by del x 1 del f 1 by del x 2 del f 1 by del x n and proceeding similarly, this similarly. So, we can del f n by x 1 del f n by del x 2 and finally, del f n by del x n.

So, after getting this Jacobean matrix, now by solving this equation Jacobean matrix into delta x, delta x equal to f so from this we can find this delta x and after getting this delta x our next iteration that this x i plus 1 will be equal to so x i plus 1 will be equal to x i plus del x i. So, in this way, we can find the roots, if we have a set of equations present in this case, so after finding this characteristic equations, now our next step is to find the so numerically integrate.

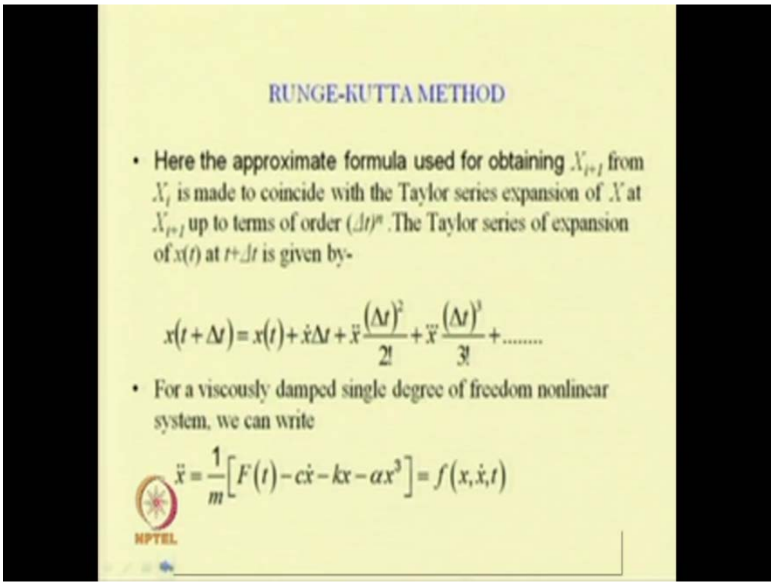
So, after finding this characteristic equation, then we can substitute this beta l value in this equation to find the safe functions, so this beta value by putting this beta value we can obtain the safe functions, so after obtaining this safe functions. Now our objective is to integrate this so one can use different numerical integration methods to find integration. For example, one can use this Simpson rule, or the trapezoidal rule, to find the integration of this to integrate function after integrating. So, one can obtain these coefficients, and after obtaining these coefficients, now after obtaining these coefficients, one can solve these equations either by perturbation method, or by using this numerical solution method. To solve this numerical solution method 1 may use this, the Runge-Kutta method.

(Refer Slide Time: 47:40)

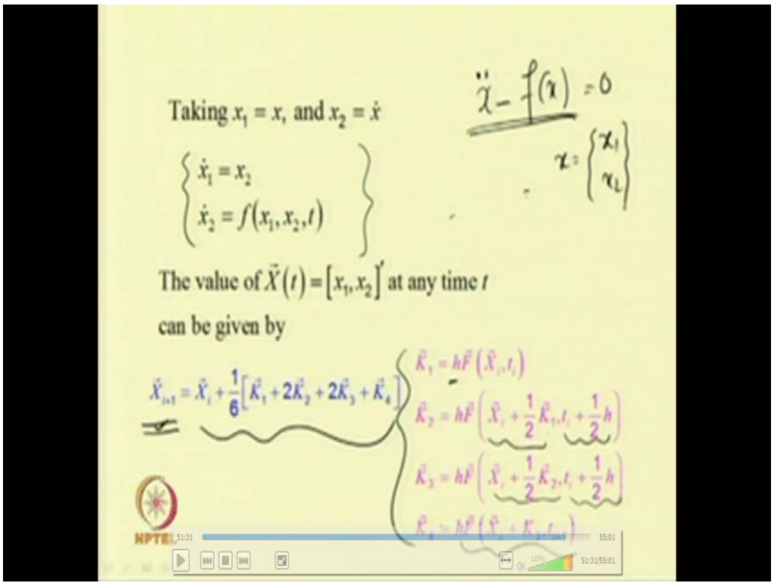


So, this let us see this Runge-Kutta method or one may other numerical solutions schemes. For example, one can use this finite difference method, Runge-Kutta method this Houbolt method, Wilson method, and Newmark method. So, these methods can be used to obtain the numerical solutions of these equations, so one can solve this equation numerically to find this thing. So, let discuss about this Runge-Kutta method to obtain the solution of the system.

(Refer Slide Time: 48:13)



(Refer Slide Time: 48:17)

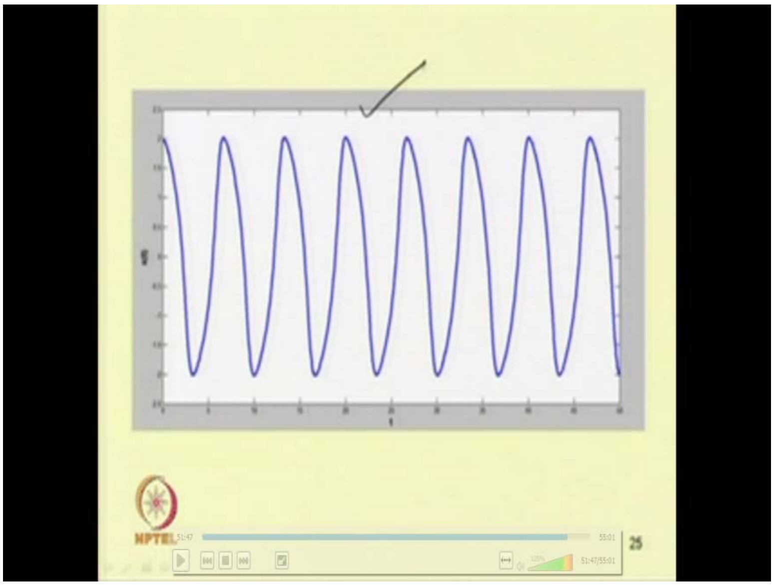


So, in case of Runge-Kutta method, so this fourth or fifth order of Runge-Kutta method, first we have to write the governing equation, using a set of first order differential equation. So, if we have equation, let we have a equation in this form x at x double dot plus f x equal to 0 so where this x can be written let us take only 2 terms, x 1 and x 2. So, our equation first equation is x 1 dot plus f x 1 x 2 equal to 0, then second equation x 2 dot equal to f x 1 x 2 equal to 0, so we can have a set of equation. So, in this equations now our objective is to find the solution, so to find that thing we have to write this equation using the, using a set of first order equation.

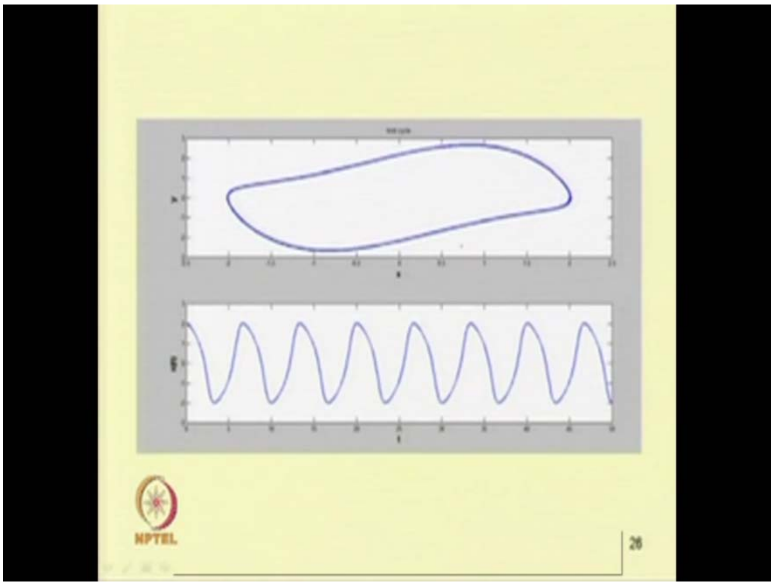
So, to write the set of order equation so first we can assume let x 1 dot equal to x 2 then, this x 2 dot which is equal to x double dot. We can write equal so one can write this equal to minus in this case, let me put this equation in this form minus this thing it will be x plus so f x 1 x 2 t. Now, we have reduce this set of first second order equation, to a set of equation so after getting the set of first order equation, and taking an initial condition X i then, this i plus 1 can be obtained by using this method. So, in this case, so this X i plus 1 can be obtained from this i eth iteration, that is X i equal to 1 by X i plus 1 equal to X i plus 1 by 6 into K 1 plus 2 K 2 plus 2 K 3 plus K 4 where this K 1 K 2 K 3 K 4 are obtained by using this equation, so where K 1 equal to h F X i t i, k 2 equal to h so h is the increment we are taking here.

So, K 2 h into F, so here the function at evaluated at their point at X i plus half K 1, and t i time equal to t i plus half h similarly, K 3 is evaluated at this point. That this X i plus half K 2, and here the time equal to t i plus half h and K 4 is obtained from by using this expression where the x as taken X i plus k 3 and time t equal to t i plus 1. So, by evaluating this constant K 1 K 2 K3 k 4 and substituting in this expression, so we can obtain this i 1 plus th iteration value. And in this way we can go on finding the value of X i plus 1 numerically till we converse to a solution.

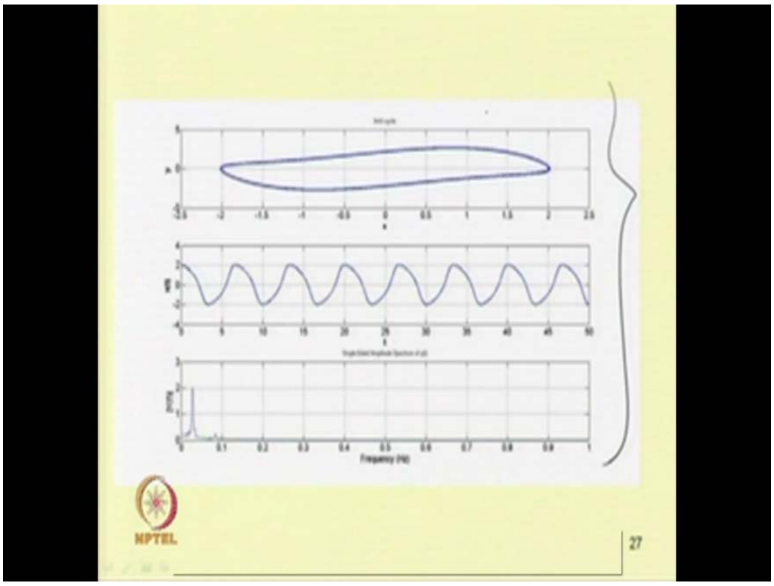
(Refer Slide Time: 51:39)



(Refer Slide Time: 51:50)



(Refer Slide Time: 51:54)

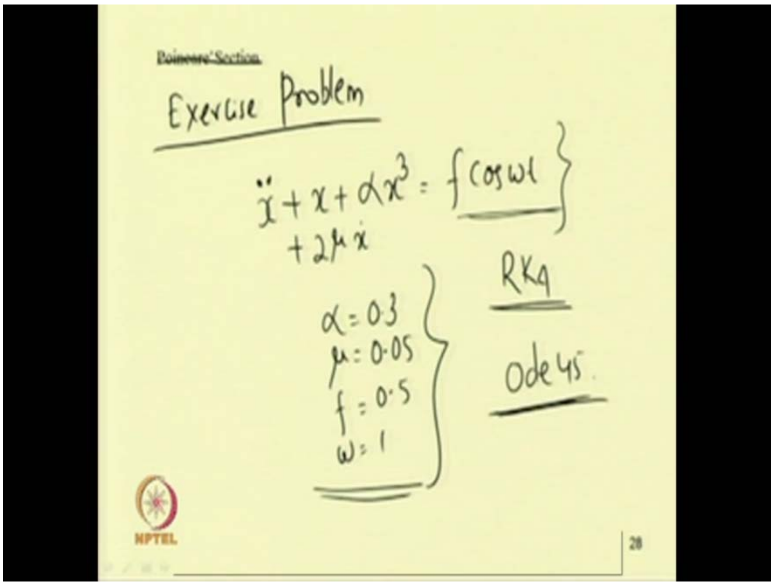


So, for example, let us take this Dufffing take this Vanderpol equation, so in case of the Vanderpol equation, so the obtained result the time response is, obtain like this. So, this is the pace portrait obtained, time response pace portrait and one can obtain the spectra also or spectra also one can obtain. So, in this way by using this R K 4 method, one can find the time response of the system. So, today class, we have studied how to find the roots of the characteristic equation by using different numerical techniques.

So, this numerical techniques include method of bisection, regular falsi method, Newton's method, secant method and Muller method. And then we have studied how to find this integration of numerical integration, to find the coefficients after getting this equation or after getting this temporal equations. Now, we know there are several numerical methods available, but we have studied this Runge-Kutta method to find the solution of the system.

So, in this way we can find the time response of the system. So, next class, we will discuss how to characterize the time response what we have obtained, then will discuss about the frequency response of the system. And then will study about how to obtain the basin of attraction when there are multiple solutions exist in this system. So, one can take some exercise problem or one can solve so take this exercise problem.

(Refer Slide Time: 53:19)



So, solve this Dufffing equation using Runge-Kutta method that is solve this equation x double dot plus, x plus, alpha x cube equal to f cos omega t take different value of this alpha. Let us add this damping also to 2 mu x dot so, let us take this alpha equal to 0.3 mu equal to 0.05 and f equal to 0.5, and omega equal to 1. So, taking this use R K 4 method to find, Runge-Kutta fourth order Runge-Kutta method to find the solution of this equation and so in Matlab, one can easily solve this equation by using the function o d e 4 5 function in the Matlab. So, next class will study or will find or will characterize this time response what we have observed, and will show the solution of this equation.

Thank you.