Non-Linear Vibration

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Module - 4
Stability and Bifurcation Analysis of Nonlinear Responses
Lecture - 5
Stability and Bifurcation Analysis of Periodic Responses

Welcome to today class of non-linear vibration. So, today class, we are going to study about the stability and bifurcation analysis of periodic responses. Previous class, we have studied the stability and bifurcation of fixed point responses, where we have studied about the stability and bifurcation of static and dynamic bifurcations where we have studied the static bifurcation, which includes these pitch fork bifurcation, a saddle node bifurcation, and transcritical bifurcation, and in case of Hopf bifurcation, that is the dynamic bifurcation, we have (( )) the response to be periodic, when there is a Hopf bifurcation of the system.

So, today class, we are going to study about the periodic responses, already we have studied several types of governing equations. For example: So, we have studied Duffing equation, vander pol equation, hill's equation and Mathieu equation, so in while solving these equations. So, in addition to the fixed point response, we obtained this periodic response also.

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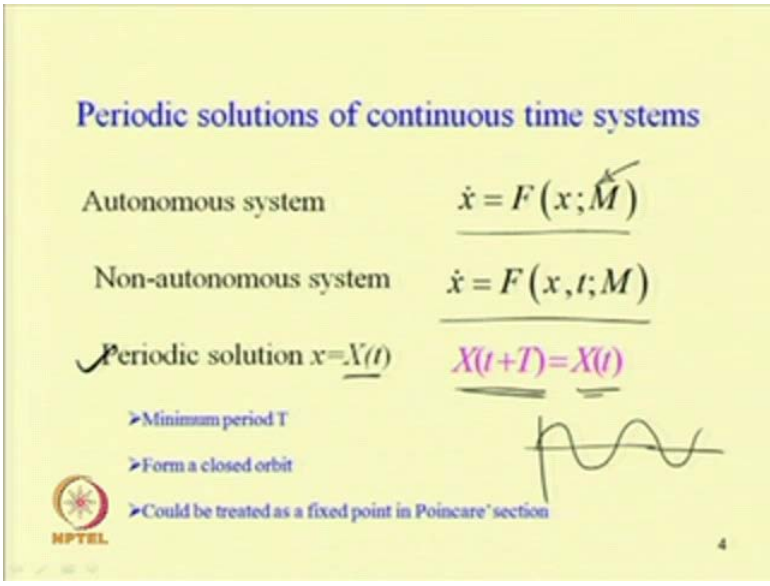

And the periodic a response is said to be periodic or the solution of a response is said to be periodic, when the solution at a, so let this us take a autonomous system, that is x dot equal to F x M or non autonomous system, where time is explicitly present, that is x dot equal to F x M. So, were control parameter is M and the response is x.

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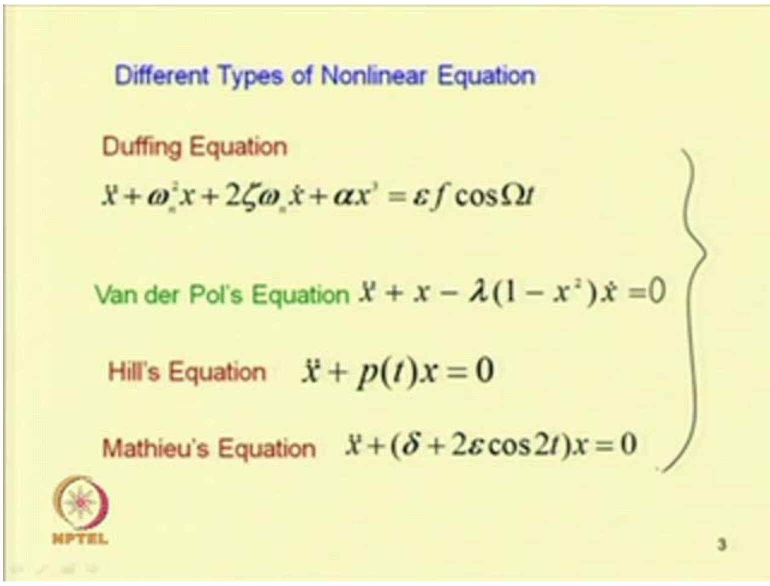

So, in this case, the response or the solution x t. So, let a time t equal to t. So, if we write it in this way, this x t then, if we have a minimum period t, then x t plus t should be equal to x t. So, if x t plus t equal to x t, then it will be said to be periodic. So, in case of equilibrium position or fixed point response. So, we have found those fixed point response by substituting this x dot equal to 0 but, in this case as the response is periodic.

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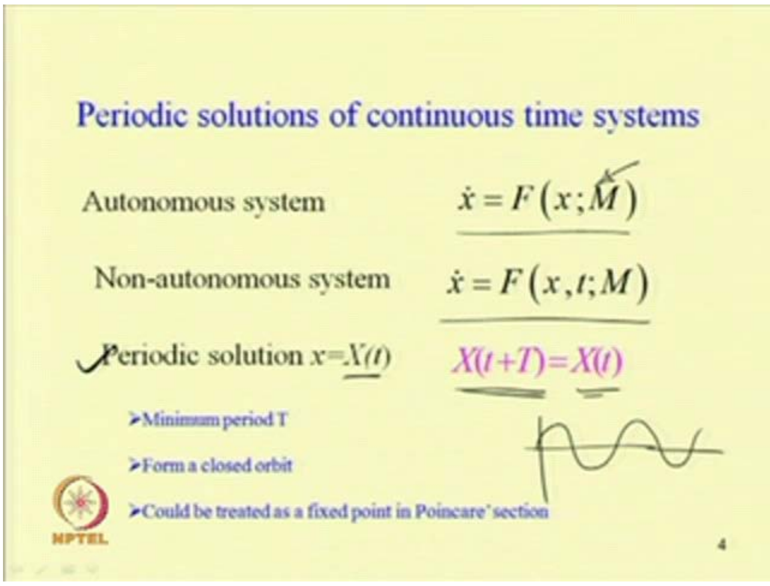

So, for example, the response may be of sin or cosign type. So, this is periodic response. So, we need not have to put this x dot equal to 0, to find the solution. So, in this case, already we have developed several methods, to find the solution either we can use this numerical method to solve these equations numerical methods by using this.

Runge Kutta method or we can use several perturbation methods, what we have studied in module 3. So, we have studied perturbation methods like method of multiple scale lingsted poincare method, modified lingster poincare method, averaging method and also , we have studied some other types of methods like harmonic balance method to solve these type of equations. So, in those cases, we have found the response to be periodic; So, in periodic response, For example: Let us take some of the example. So, in case of the periodic system so, we. So, it will form a close orbit. So, if you plot the phase portrait of the system, then it will form a close orbit either in phase portrait or in state space and could be treated as a fixed point. So, if you take the poincare section in the introduction class, we know about the poincare section. So, if you find the poincare section of this periodic response then It will be a fixed point response.

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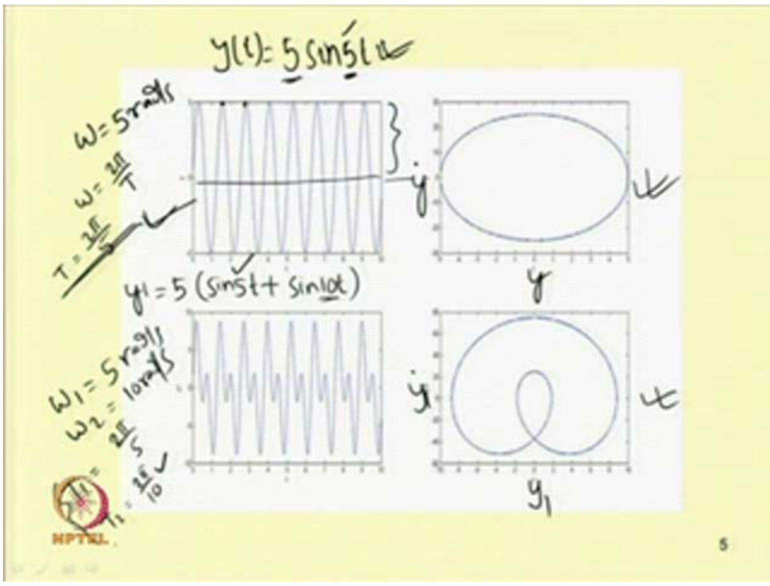

So, in case of. So, for example:. So, these are some of the periodic response in time response and phase portrait is plotted. So, for example, in the, this represent the first curve this one this curve, that is . This represent y t so, it is plotted y t equal to 5 sin. So, this is y t equal to 5 sin t 5 sin t ,so here the amplitude is 5 and. So, this is 5 sin 5 t and the frequency is 5. So, omega equal to 5 radian per second in this case and so, the amplitude equal to 5 unit. So, the amplitude is shown here. So, this is the this part is the amplitude of the response amplitude of the periodic response and by finding. So, let us this is the 0 line.

So, by taking one cycle. So, let us take one cycle. So, this will represent one cycle, this to this represent one cycle. So, one can find the time period. So, this is the minimum time period or from this to this. So, this is the time period. So, one can find the time period and. So, time period can be obtained. So, this is one can find the time period and the corresponding response and also one can plot the phase portrait by plotting this x versus or this is plotted y. So, this is y versus y dot by plotting y versus y dot. So, one can plot the phase portrait and by plotting this, So, this is for a single periodic response. So, instead of taking a single frequency response, if you take 2 frequency for example: let me take y 1 y 1 equal to. So, same 5 sin.

In addition to this let me take another term that is sin 10 t. Here the frequency the first frequency. So, in this case this is. So, this one is 5 and the second one is 10. So, in the previous example omega equal to 5. So, the time period t will be equal to so, we know omega equal to 2 pi by. So, omega equal to 2 pi by t. So, that one can find this t equal to 2 pi by omega. So, in this case, 2 pi by 5. So, this is the time period in the previous first case. So, in the second case. So, one can observe. So, we have 2 frequency . So, frequency one equal to 5 radian per second and frequency 2 that is omega 2 equal to 10 radian per second. So, in this case. So, i can find t one. So, t one will be equal to 2 pi by 5 and t 2 will be equal to 2 pi by 10.

So, by finding these time periods. So, one can see that this is the least time period and. So, with these 2 time period. So, this clearly the phase portrait. So, the phase portrait which is plotted y one versus y one dot. So, clearly show 2 loops. So, one corresponding to one period and other corresponding to the other period. So, in the previous case. So, we have a close loop in both the cases as the response is or the solution is periodic. So, we have close orbits in state space or phase portrait. So, here we have a single loop with single frequency and when our frequency. So, we have 2 frequency. So, we have 2 loops. So, with increase in number of frequencies. So, we can have several loops.

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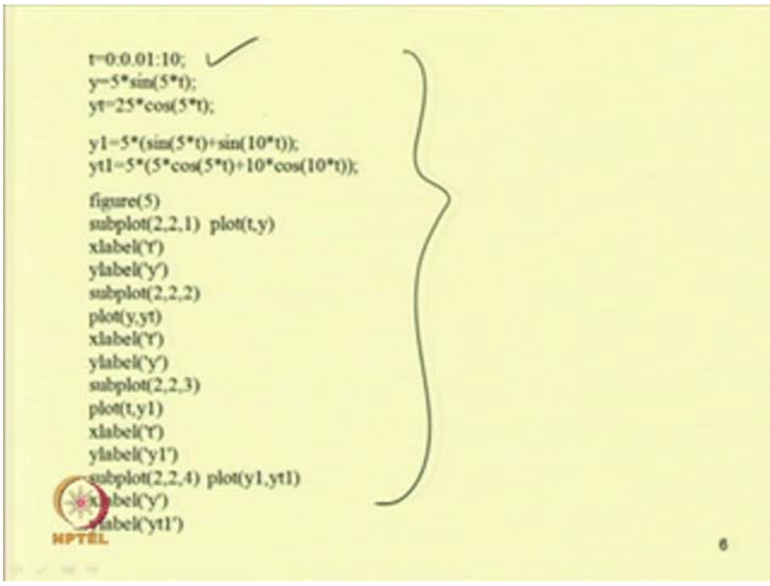

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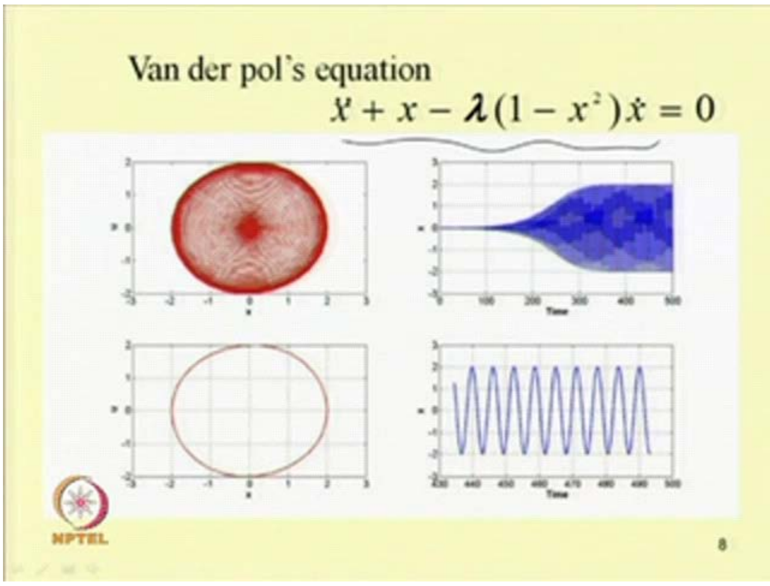

Present in the system ,so this way one can visualize a periodic response. So, here a simple sin term is given, similarly one can take different other types of periodic responses, which may or may not be harmonic or we know that, if we have a periodic response, we can convert it to that of a harmonic system. So, this is the mat lab code written for plotting the previous curves. So, here by simply taking different time one can write the plot to write the or find this displacement and velocities and plot the response curve. So, let us see some other equations for example, let us take this van der pol equation. So, in case of the van der pol equation. So, the equation can be written in this form x double dot plus x.

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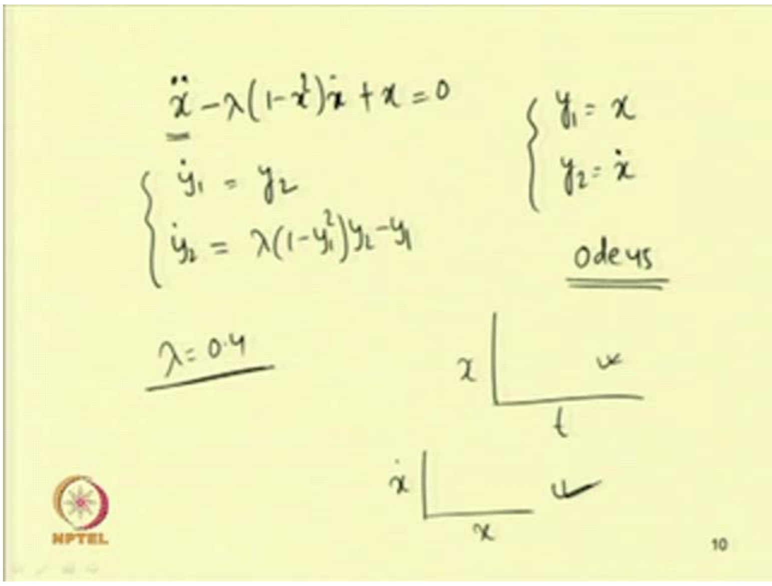

Minus lambda into one minus x square x dot equal to 0. So, if. So, to plot the second order differential equation. So, we can take this x dot equal to. So, we can take. So, first we have to reduce this second order differential equation to a set of first order equation. So, our equation x double dot minus lambda into one minus x square into x dot plus x equal to 0. So, this way one can write this equation. So, x dot plus x double dot plus x minus lambda into one minus x square x dot equan to 0. So, we have to use the numerical methods. So, we can reduce this equation to a set of first order equation. So, to reduce this thing to a set of first order equation. Let us write y one equal to we can write y one equal to x and y 2 equal to x dot . So, y 1 equal to x and y 2 equal to x dot. So, our first order equation will reduce to.

So, first order equation, that is our y one dot, that is x dot y one dot equal to y 2 and second equation y 2 dot. So, y 2 dot y 2 equal to x dot ,so y 2 dot equal to x double dot. So, this x double dot can be written as lambda into 1 minus. So, lambda into 1 minus y 1 square into y 2 plus or by taking to right hand side this becomes minus. So, minus x means. So, this becomes y one. So, these are the set of first order equation, So, now solving the set of first order equation by using Runge Kutta method. So, in mat lab one can use this o d e 4 5 o d e 4 5 function o d e 2 3 function to solve this equation. So, by solving this equation we can plot the x versus time, that is the phase that is the time response. So, we can plot.

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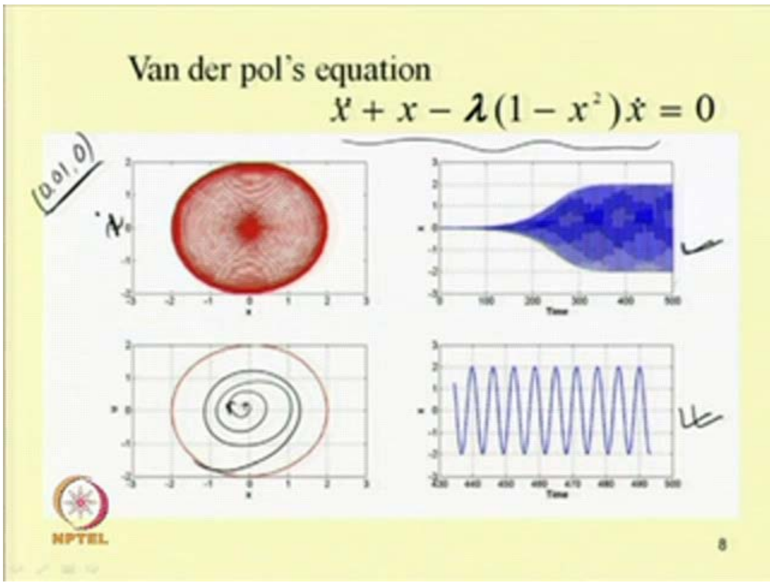

X t and we may plot x versus x dot. So, this is the phase portrait. So, we can plot the phase portrait and time response. So, in this case taking lambda equal to point 4. So, lambda equal to point 4 and taking different initial conditions. So, the response has been plotted as shown here. So, taking the initial point. So, initial point 0 point 0 one and 0. So, this this is x and u equal to x dot. So, x and x dot. So, in this is the phase portrait. So, taking this thing. So, one can obtain. So, with time. So, taking this initial condition we can see that. So, this is the time response.

From 0 to 500 only the steady state part is plotted in this figure. So, this figure shows only the steady state part ,so this figure shows the steady state part along with the transient and clearly it is observed. That the steady state response is periodic and by taking the initial condition this. So, it finally, as t tends to infinity. So, it reaches this cycle. So, this cycle or this periodic response. So, this periodic response or this periodic orbit is known as a limit cycle similarly if you take another boundary another initial condition. So, taking another initial condition outside this limit cycle. So, outside this if you take this as the initial point. So, one can observe that with increase in time it approaches. So, it approaches the same periodic orbit. So, in case of the…

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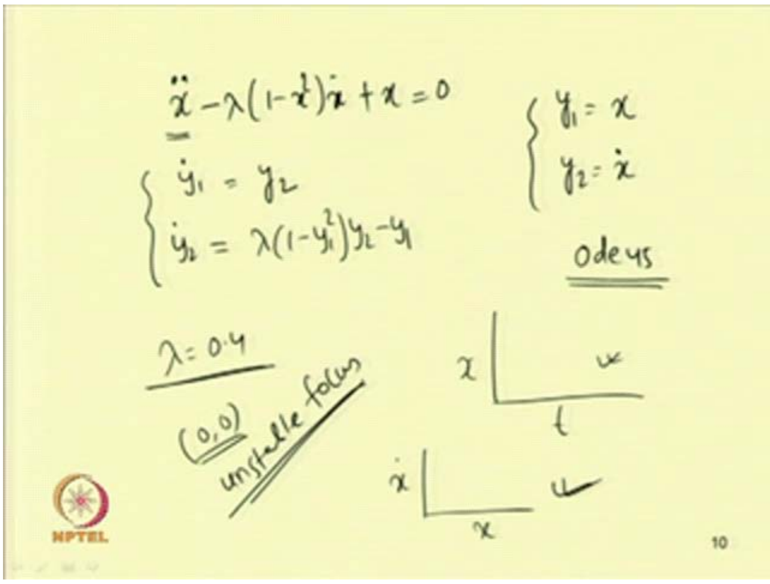

So, these are the examples of limit cycle. So, in case of the van der pol equation. So, if you see this equation. So, we have. So, in this case by putting this y dot and y 2 dot equal to 0. So, 0 0. So, we can see this 0 0 is an unstable focus. So, one can check that this the stationary point or the fixed point response is a unstable focus. So, as the origin is an unstable focus. So, it will repel on the solutions towards the limit cycle. So, in this case one can observe that by taking any initial condition, if you take a initial condition inside this periodic orbit. So, it reaches this periodic orbit or if you take another point which is outside this periodic orbit it also reaches the same circle.

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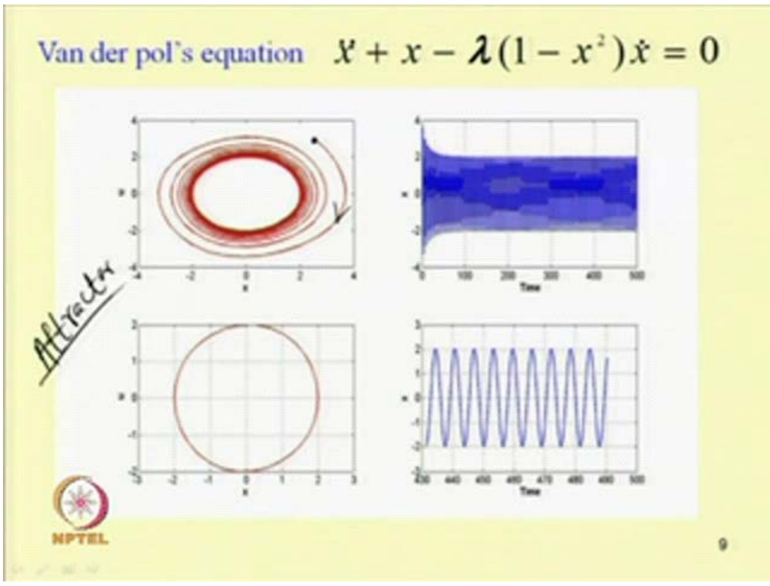

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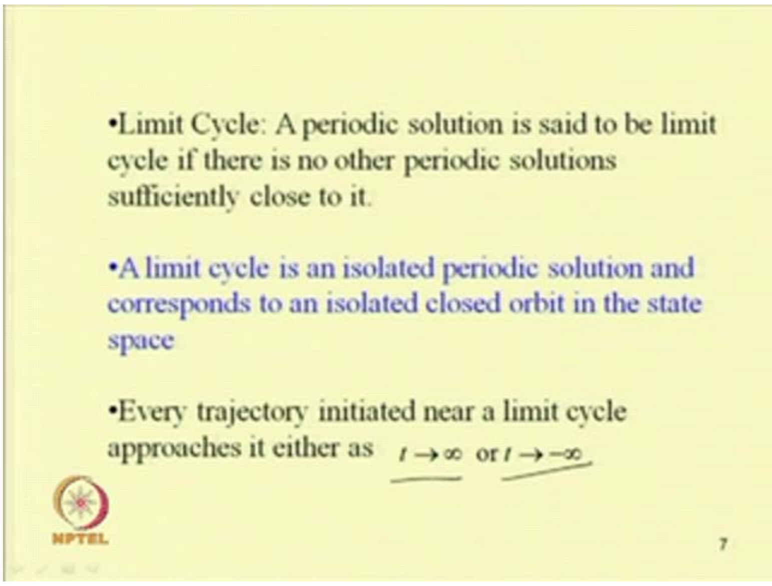

So, by taking either inside or outside this point. So, always it come to the same periodic orbit, that is why this is a limit cycle or it is also known a known as an attractor. So, this is also known as attractor or limit cycle, So, we can define a limit cycle as a periodic solution or a periodic solution is said to be limit cycle. If there is no other periodic solution sufficiently close to it a limit cycle is an isolated periodic solution and correspond to an isolated close orbit in the state space. So, for every trajectory initiated near a limit cycle approaches it either as t tends to infinity or t tends to minus infinity. So, in case of this van der pol equation. So, we have seen. So, we are getting some limit cycle ,so periodic response which is limit cycle.

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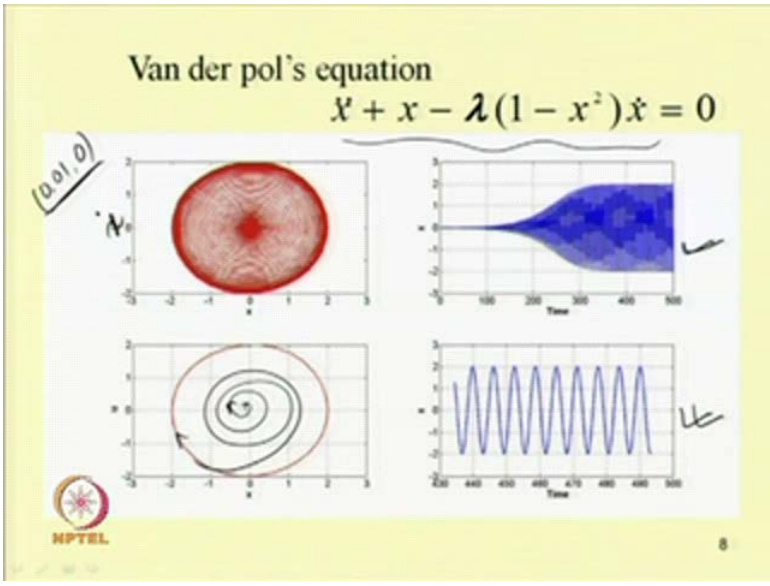

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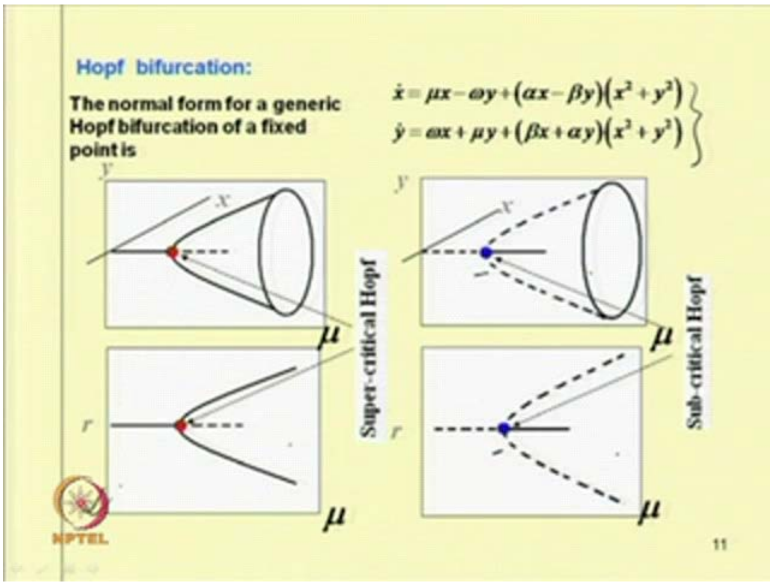

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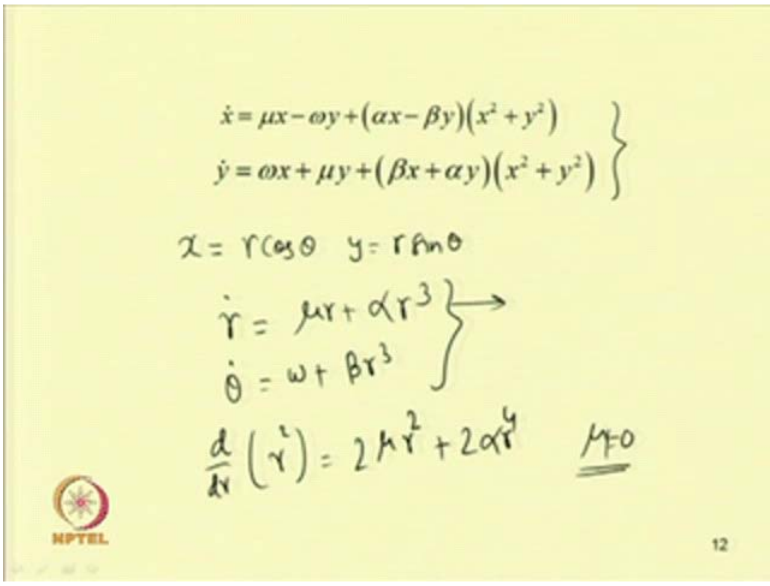

So, this sometimes this limit cycle may or may not be stable. So, in this case this is a stable periodic solution. So, later we will know how to study the stability of this periodic solution also let us take another example. So, we have seen in case of the Hopf bifurcation of the system. So, this is the generic form of the Hopf bifurcation equation. So, in this case we have seen that these are the unstable point. So, this point and this point correspond to unstable fixed point response where we can observe or where we can find the initiation of a periodic orbit.

So, these are the super critical Hopf bifurcation similarly we have seen. So, at these 2 points. So, this point and this point. So, we have unstable periodic response. So, the same equation here it is plotted in x y mu and in the below curve it is plotted in r mu space. So, this equation if we write this equation x dot equal to ,so these 2 equation x dot equal to mu x minus omega y plus alpha y minus alpha x minus beta y into x square plus y square and y dot equal to omega x plus mu y plus beta x plus alpha y into x square plus y square by substituting x equal to. So, already we have seen this thing by substituting x equal to r cos theta and y equal to r sin theta.

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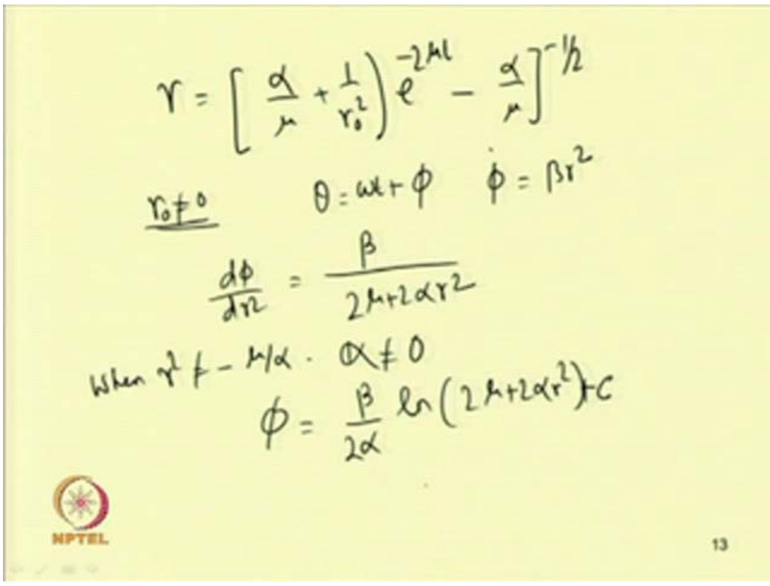

So, this equation can be reduced to this r dot equal to. So, this is r dot equal to mu r plus alpha r cube and theta dot equal to omega plus beta r cubed. So, already we have seen while discussing the Hopf bifurcation. So, these 2 equations reduce to that of r dot band theta dot equation in this form. So, now multiplying two r in this equation so, we can write this multiplying 2 r in this equation. So, we can write this d by d r of r square. So, this will be equal to 2 mu r square plus 2 alpha r fourth. So, assuming mu not equal to 0 and using the separation of variable. So, we can write or we can find this r. So, which can be written in this form. So, r is written in this form alpha by mu plus one by r 0 square e to the power minus 2 mu t minus alpha by mu to the power minus half and were r 0 not equal to 0.So, if r 0 not equal to 0. So, at t equal to 0.

So, we can have. So, from this equation we can write this theta equal to we can write theta equal to omega t plus phi. So, that this phi dot theta equal to omega t plus phi. So, we can write this phi dot equal to beta r square then we can write this d phi by d r square equal to beta by 2 mu plus 2 alpha r square. So, when r square not equal to minus mu by alpha. So, this equation we obtain. So, hence for alpha not equal to. So, alpha not equal to 0. So, we can write phi equal to beta by. So, from this equation we can write. So, this becomes beta by 2 alpha 1 n 2 mu plus 2 alpha r square plus c.

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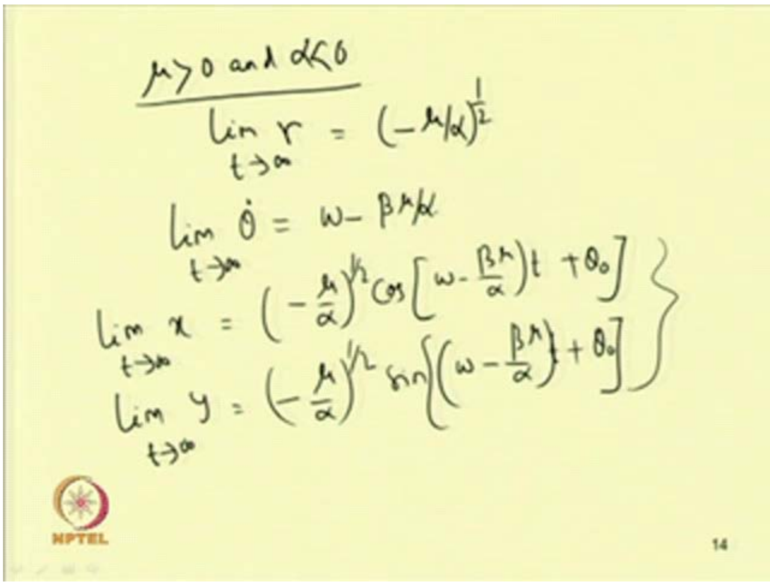

So, where c is a constant. So, substituting this value of r and phi. So, we can obtain a close form solution and we can obtain a close form solution, where we can write. So, when mu greater than 0 and alpha less than 0. So, we can write for this condition limit e tends to infinity. So, r will be equal to minus mu by alpha to the power 1 by 2. So, as t tends to infinity. So, we got a radius that is equal to minus mu by alpha. Here you are taking mu greater than 0 and alpha less than 0. So, this inside term will be positive.

So, we can have a radius. So, in this case one can observe that; it reduces to that of a periodic response similarly, limit t tends to infinity theta dot for theta dot equal to omega. So, this becomes omega minus beta mu by alpha. So, we can write this equation, in this form. So, t tends to infinity. So, x will be equal to minus mu by alpha to the power 1 by 2 cos omega minus beta mu by alpha into t plus theta 0 and similarly limit t tends to infinity. So, this y will becomes minus mu by alpha to the power 1 by 2 sin. So, this is omega minus beta mu by alpha into t plus theta 0. So, from this expression, one can see that we have a periodic solution and this periodic solution for different value of alpha and mu, if one plot then one can obtain the response.

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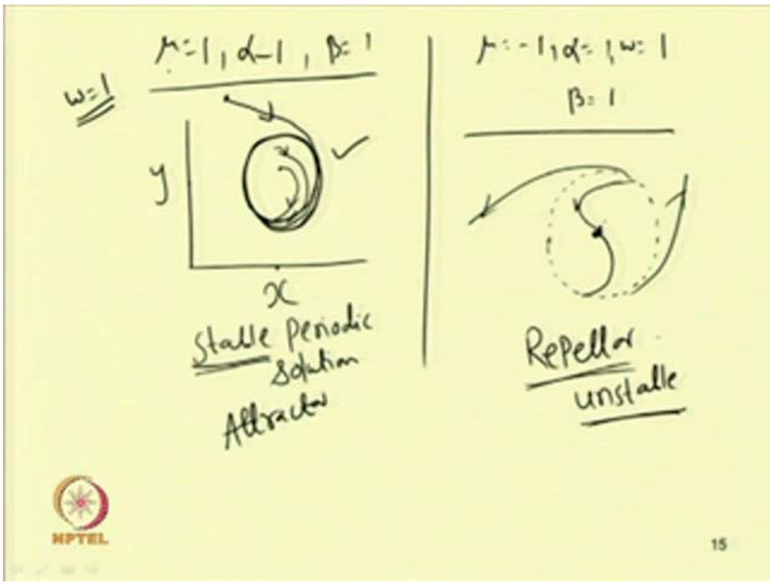

So, for example:. So, in case of, mu equal to taking mu equal to 1 alpha equal to minus 1 and beta equal to 1. So, beta equal to 1. So, one can obtain. So, mu equal to one alpha equal to. So, mu equal to 1 alpha equal to minus 1. So, this becomes 1 and then this beta equal to also taking 1 beta equal to 1 and mu equal to minus 1, this becomes minus. So, minus 1. So, this is omega plus 1 into t plus theta 0 similarly, one can find this thing. So, if one plot by taking different initial condition. So, one can obtain the response in x y plot if one plot. So, for this case in x y, so this is 0. So, one can obtain a periodic solution. So, one can see that by taking different initial conditions, let one take this initial condition.

Similarly, to this Duffing equation. So, if take one this initial condition. So, it will reach this point similarly by taking another initial condition also, this will come to this and taking some other initial condition. So, one can see that it is also reaching the same orbit. So, this orbit , which is obtained for different initial conditions is known as the limit cycle. So, as for all cases different from different initial condition as t tends to infinity, we are getting the same cycle so, this is or same orbit. So, this is a stable periodic solution. So, we can take so, this is a stable periodic solution. So, later, we will study it is stability and we will check, whether the system is stable or not. Similarly, for mu equal to taking mu equal to minus 1 alpha equal to 1 and omega equal to 1 and beta equal to 1. So, in this case also, we have taken omega equal to 1.

If we find the orbit. So, from this one can find this orbit but, in this one can observe that. So, one can observe as. So, one can observe that, this is one unstable periodic response as with increase. So, if we will take a initial condition, this always it will come to this position or if i will take an initial condition, this it will come to this origin that means, it will this point this initial condition repels it from this periodic solution. Similarly, if we will take a point outside, this one can see that it is coming out or it will refill. So, this is known as a repeller or this is known as attractor. So, this periodic response is a attractor, while this periodic response is an attractor, this is a repeller in this case. So, this is a repeller and that is, why this periodic solution is unstable. So, in the previous case, we have seen this is stable periodic solution or attractor and in this case, it is unstable periodic solution or a repeller. So, in this example, we started from this Hopf bifurcation or generic form of the Hopf bifurcation.

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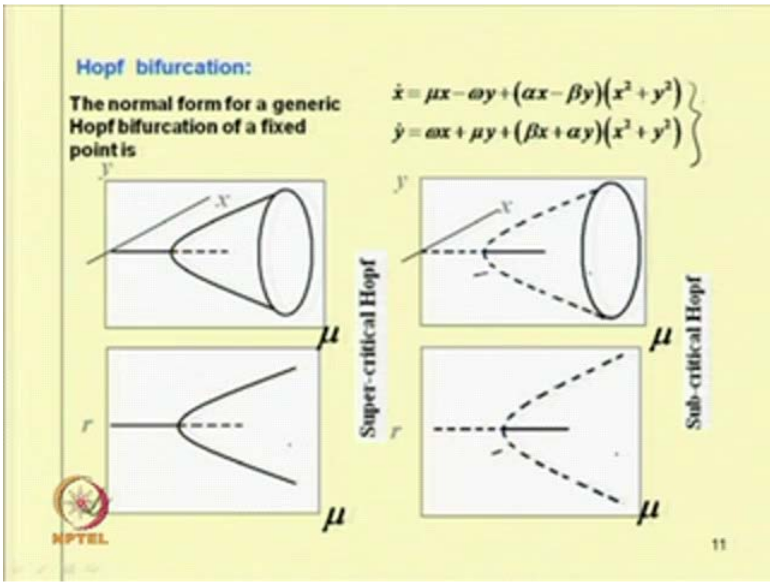

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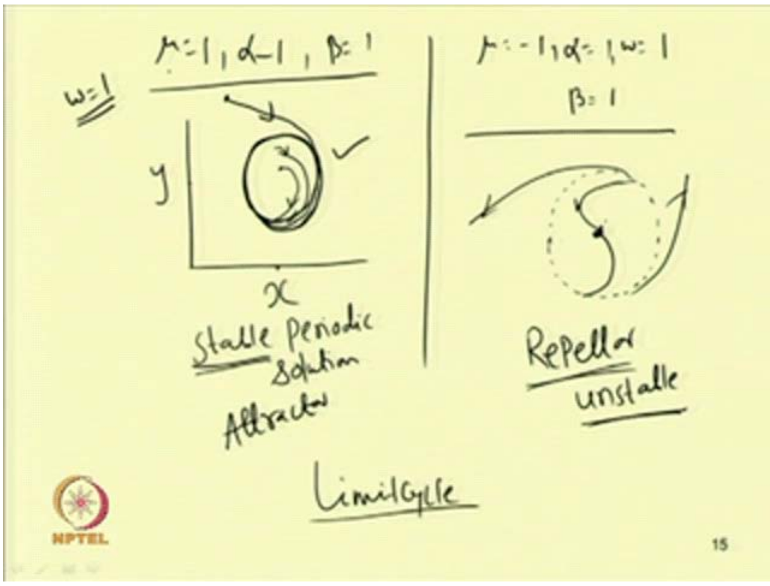

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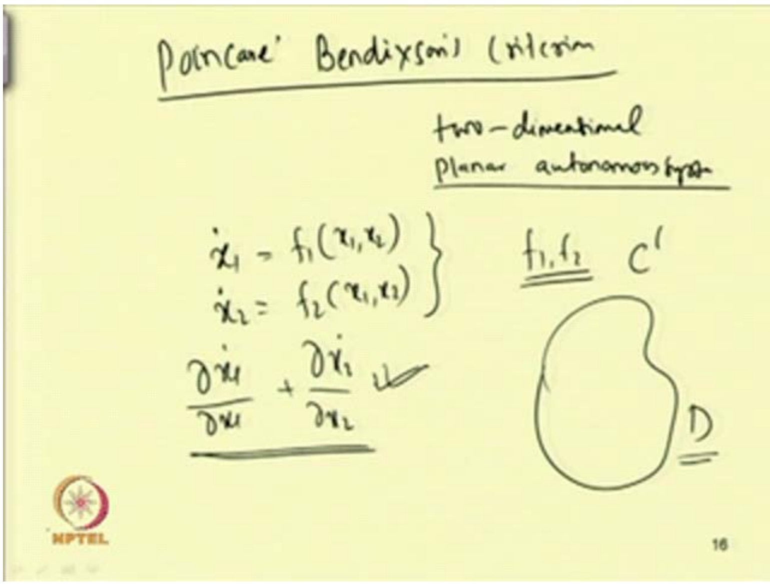

So, we have seen. So, by taking or taking different initial condition, we can see that one can obtain, one attractor or a stable periodic solution or one repeller or unstable periodic solution. So, we have taken we have seen in two examples;. So, these are the two example for example, is the vander pol equation and in this case, this is the in this equation also in both the equation, we have seen the limit cycle as no other periodic solution exist nearby. So, these solutions are limit cycles so, the existence of the limit cycle can be studied by poincare Bendixson criteria poincare and Bendixson's criterion.

So, according to this criteria. So, if we have. So, this criteria is applicable only for this 2 dimensional planner autonomous systems, 2 dimensional planner autonomous systems. So, in this case, if we have two equation that is for example, x 1 dot equal to f 1 x 1 x 2 and x 2 dot equal to f 2 x 1 x 2. So, then according to this criteria. So, in this case we have taken in this f 1 and f 2 are continuous function and it can be differentiated once at least can be differentiated once. So, they belongs to c 1 and let us take a region. So, let us take a domain d and in this domain, we want to search whether a periodic response is existed or not.

So, to check, whether a periodic response exist or not in this domain. So, if we apply the Bendixson's criteria. So, we can write this del x 1 dot by del x 1 plus del x 2 dot by del x 2. So, it show the either 0 or it would change the sin. So, for. So, if it will not change the sin or it becomes does not becomes 0 then, there will be no possibility of existence of the periodic solution so, for existence of a limit cycle in this region for the existence of the limit cycle in this region. So, either this thing should 1 is or it should be 0.

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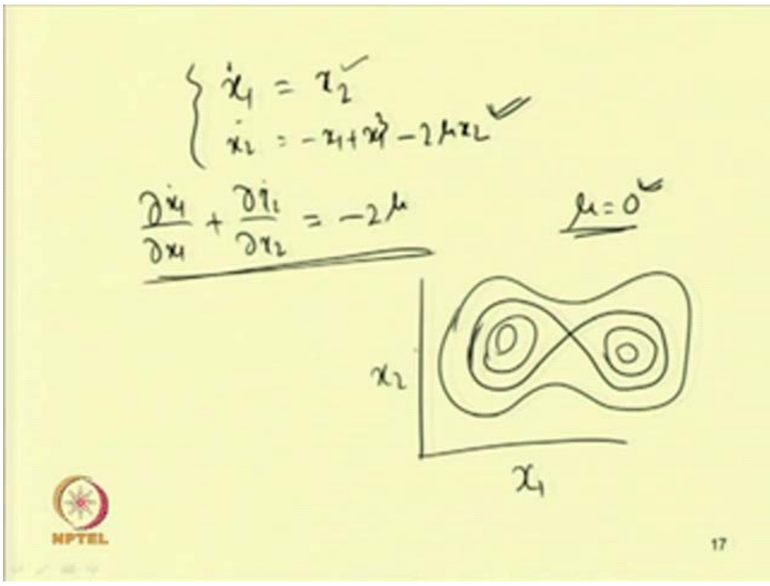

So, for example, let us take this equation, let us take one. So, the same equation, we can take and check whether it is existed or not. To just take one example. So, example let us take x 1 dot equal to x 2 and x 2 dot equal to minus x 1 plus x 1 cube minus 2 mu x 2. So, if you take this two equation, then our del x 1 dot by del x 1 plus del x 2 dot by del x 2. So, this becomes. So, this becomes minus. So, let us differentiate this with respect to x

1. So, this becomes 0 and if we differentiate the second equation with respect to x 2. So, at this positive term or sum of x 1 where o and this becomes minus 2 mu . So, it will when is only this mu equal to 0. So, incase of mu equal to 0. So, this thing will, when is. So, a periodic solution is exist if mu equal to 0, if one plot this for one to one can. So, this equation is similar to that of a simple pendulum or or this doffing equation; and in this case equal plot that response that this x versus x dot or x 1 versus x 2. So, the response are will. So, the this is the one can obtain this hetero clinic and homo clinic orbit. So, one can plot this thing and in between we will have a set of periodic solutions. So, this is the saddle point and this are the homo clinic orbit.

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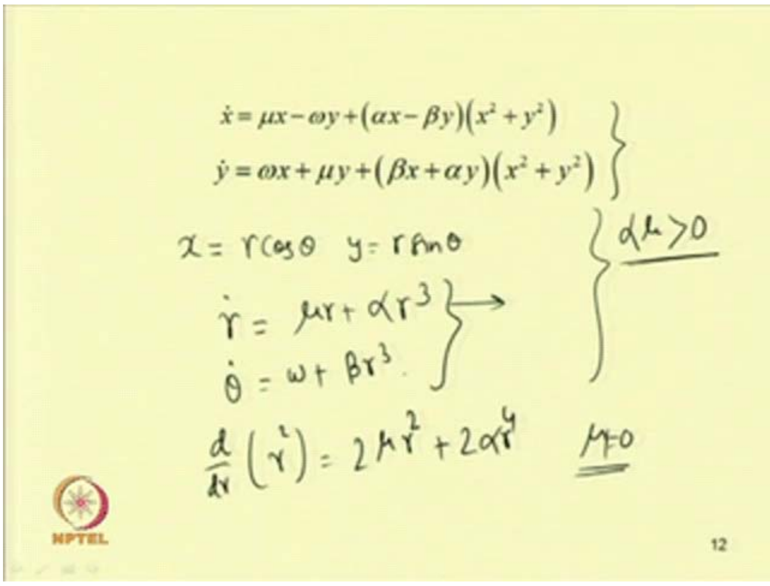

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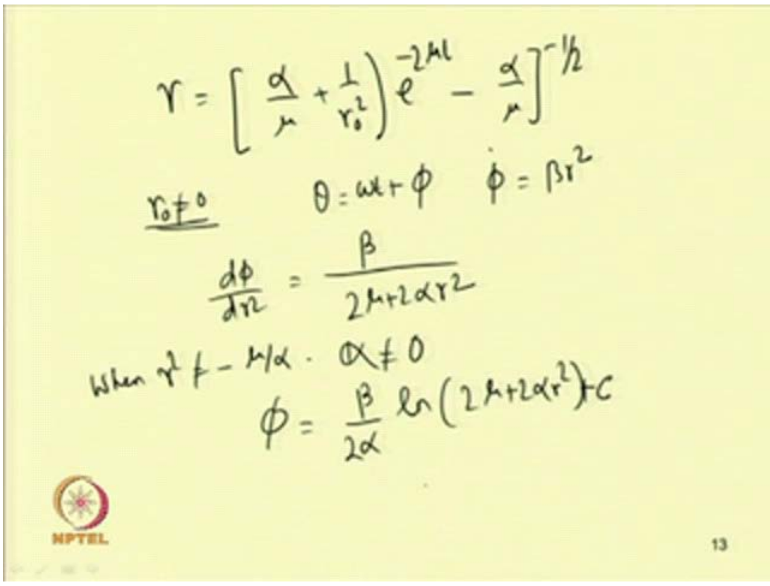

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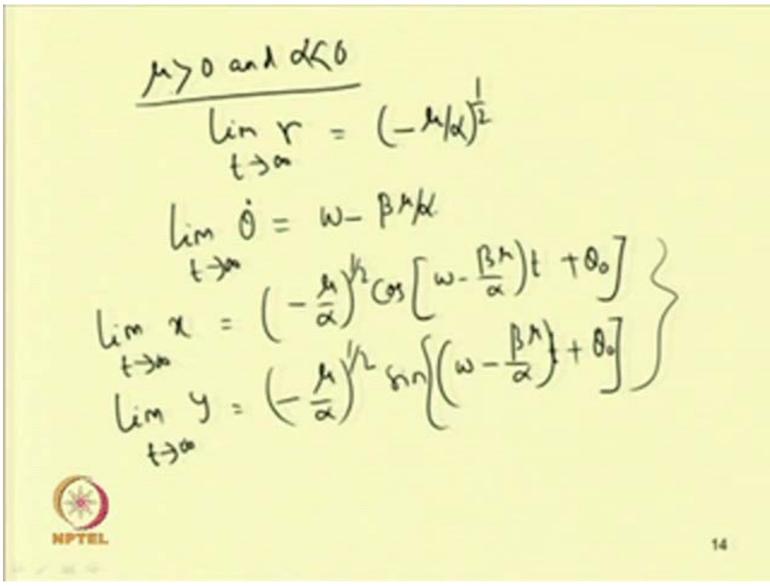

So, and in between, you can have periodic solutions. So, existence of the periodic solution. So, according to, our Bendixson’s theory so, the periodic solution will exist only if mu equal to 0 similarly in the previous example. So, in the previous example if we differentiate these past equation with respective x, and second equation with respective y, then we can find a periodic solution will exist. If only this alpha into mu. So, alpha into mu greater than 0. So, we can find this thing in case of. So, let us, find and check. So, if it greater than 0 or less than 0. So, if we differentiate this x dot and. So, as if differentiating with x.

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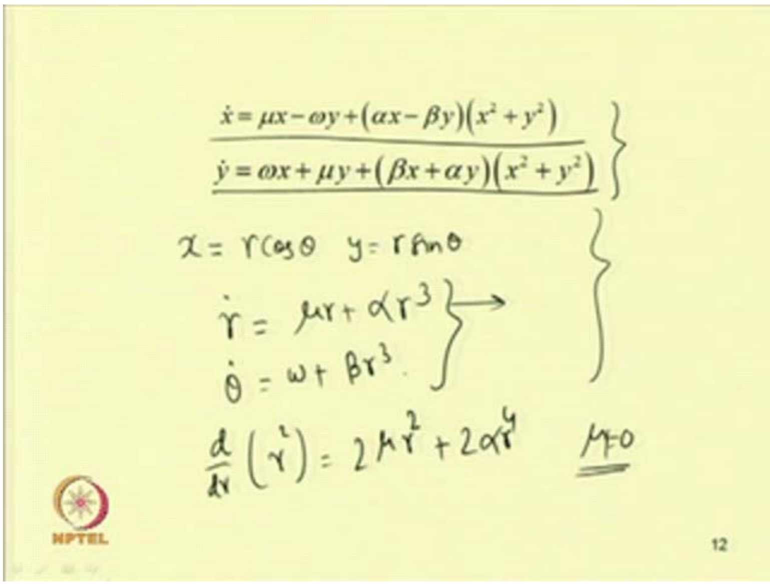

So, this becomes mu and then from this part you have this alpha into alpha x cube and if can differentiate this part, by with respective x and differentiate this part by with respective y to obtain this condition. So, in this while finding this condition, we can check whether there exist a periodic solution or not. So, in this case, you can see the solution will exist. If alpha into mu is less than 0. So, or if it is less than 0; that means, if it is negative. So, for that case. So, we have taken this example mu greater than 0 and alpha less than 0 .So, if we multiply this terms then this becomes negative. So, in this case already we have seen there is existed periodic solution.

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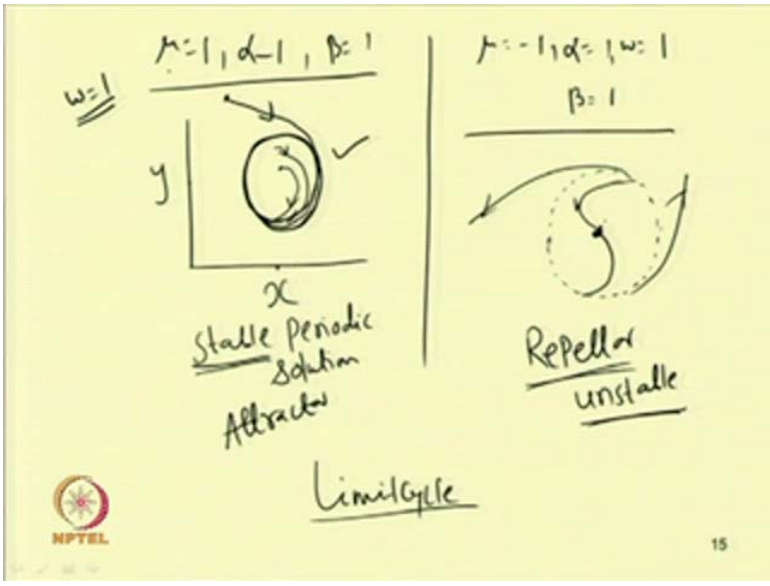

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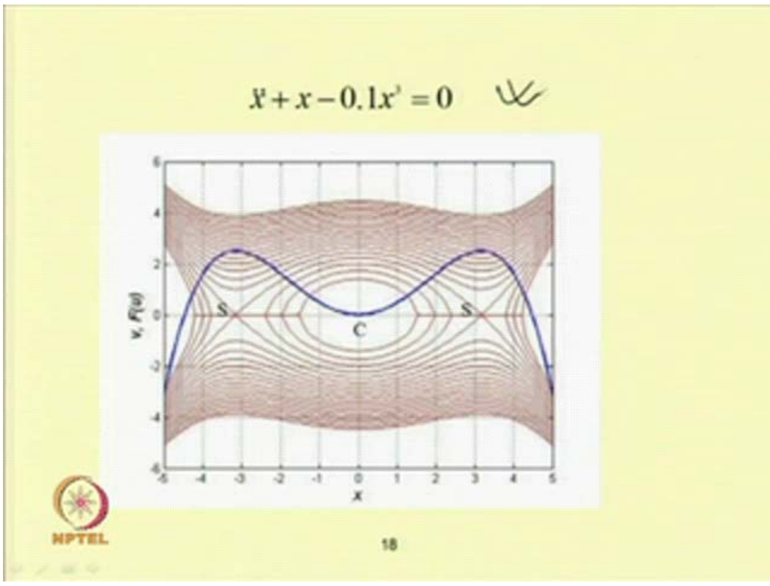

Similarly, in the next example also, we have taken this alpha equal to alpha equal to minus 1 and mu equal to 1. So, we choose alpha into mu it becomes minus1, which is negative and in this case also, we have seen alpha into mu also negative. So, in this case we are found the solution there existed solution similarly. One can take this assignment for this Vander pol's equation so, by taking this equation from let us take these are the equations. So, in this case. So, by taking del or d y one y d y one dot y d y one plus d y 2 dot y d y 2, one can find the condition are who is a periodic solution will exist. So, this is the bendixson's criteria. So, let us now see some other examples, how the periodic responses exist in case of doffing oscillator. So, in case of the doffing oscillator. So, we have taken this example previously.

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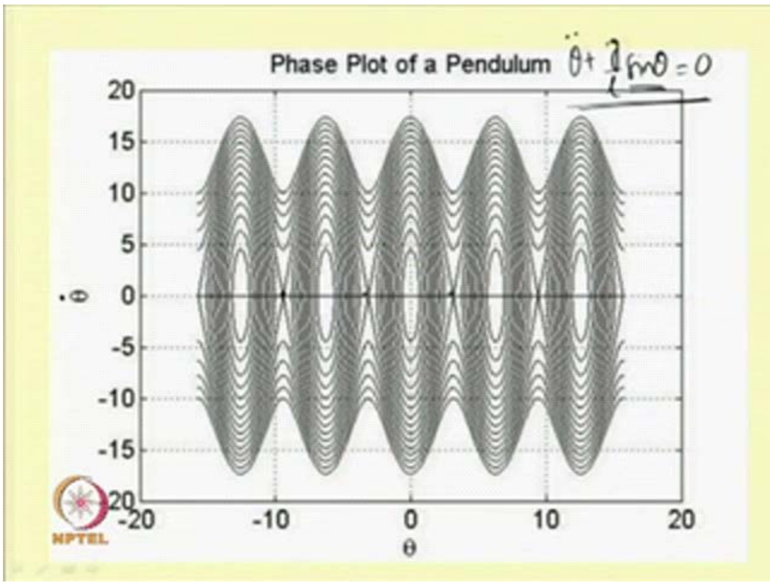

So, we have seen, there exist a set of periodic orbit but, here in between this two saddle points or in between this homo clinic orbit. So, if you take any initial condition. So, one can find that infinite number of periodic solution is exist. So, these are not limit cycle. So, these example. So, these are not limit cycle there exist a periodic solution in between this 2 homo clinic orbit in case of the doffing equation. Similarly, if you take another example of simple pendulum. So, in case of the simple pendulum the equation can be written in this form. So, theta double dot plus g by l sin theta. So, in this case one can expand sin theta in terms of higher adopt terms taking higher adopt terms in account. So, if is this is which is equal to 0. So, by plotting the phase portrait one can see or one can clearly observe in between this saddle point. So, these are the saddle point in between this saddle point there exist infinite number of periodic solutions.

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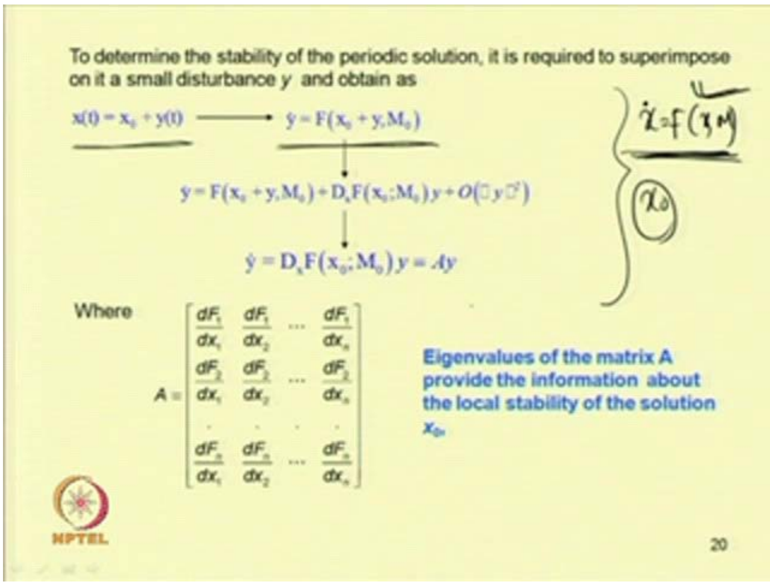

So, these periodic solutions defined on the initial conditions, by changing the initial condition, one can 1 by 2 different periodic solutions. So, we are seen two different types of existence of periodic solutions. So, one. So, one case we have seen the limit cycle and in another case there exist in a number of periodic solution depending on the initial condition, now out of these solutions. So, some of them may stable or some of them may be unstable. So, to study the stability of this periodic solutions, similar to analysis what we did in case of the fixed point response. So, we are also will proceed in similar way. So, we can. So, let our equation x equal to f x dot equal f x M.

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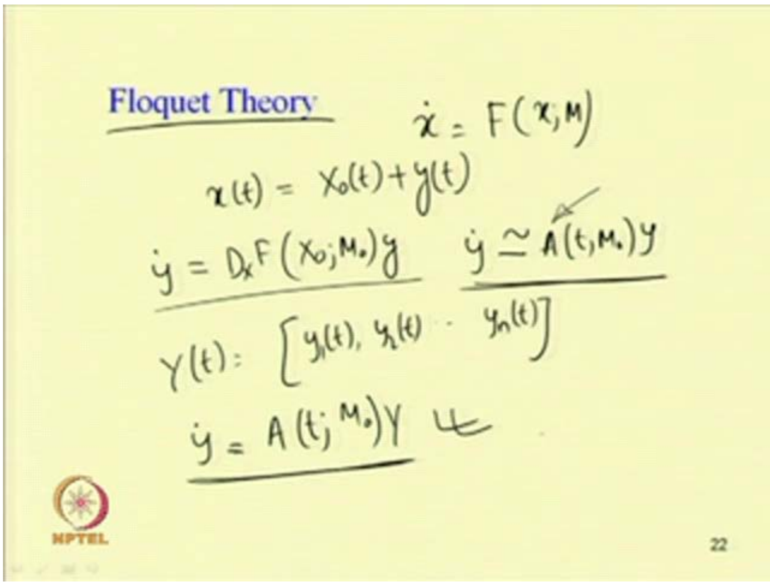

So, in this case. So, for the is equation for the autonomous system. So, in this autonomous system. So, if x 0 is the periodic solution. So, then it can part of this periodic solution and we can find. So, quartering by this periodic solution, let us take x t equal to x 0 plus y t and substituted in the governing equation that is x dot equal to f x m, then we can obtain this y dot. So, this equation you can obtain that is y dot will be equal to. So, we can obtain this y dot equal to f x 0 y m 0 or we can write this equation in this form, that is y dot equal to d x f x 0 m 0 and. So, from this equation.

So, we can find whether this system is stable or not. So, let us. So, we have to use this Floquet theory you studied this fability system. So, to find the Floquet theory or to use this Floquet theory. So, let us take this system x dot equal to f x m. So, in this case if the solution x t. So, if we have a periodic solution that this x 0 t now we can portrait using this y t, then we can obtain this equation linear space.

We have shown y dot will be equal to d x f x 0 m 0 y, and we can neglect the higher adopt terms. So, by neglecting the higher adopt terms, we can write this equation in this form that is equal to a t m 0 y. So, in case of this stationary solution, also or stationary orbits or fixed point response, also we obtain similar equation in this a is constant matrix, but in this case one can find as this x is a function of time to this matrix, a is also a periodic function time. So, one may obtain that if x is a periodic function with least period t. So, a may be function with a period half t. So, we can find. So, this y or x may be n dimensional vector.

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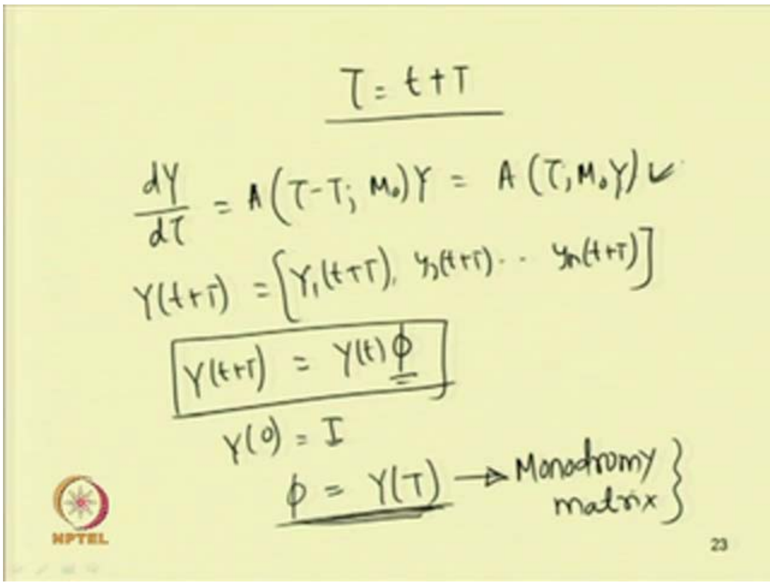

So, we can take this x or y as n dimensional vector. So, if this n dimensional vector this y t can be written as y one t y 2 t and y n t. So, these are known as the fundamental set up solution. So, we can have the fundamental set up solution for this thing. So, we can write this y dot equal to. So, y dot will be equal to. So, a is a function of time. So, previously it was constant,, but now it is a function of time and. So, y dot equal to. So, these equation is similar to the equation.

So, now one can write as t tends to by taking this t equal to now by changing this time. So, time, let us use this time tau another time tau equal to t plus t that is where t is the time period. So, we can write this equation d y by d tau is equal to a tau minus t m 0 y or it can we written as a function of tau as t is the constant. So, we can write this. So, why are this y t is y 1 y 2 and y n if it is n dimensional. So, depending on the order of the equation governing equation.

So, we can find this. So, as to from this one can write this y t plus t y t plus t, then we written as y one t plus t similarly, t 2 t plus t and y n t plus t. So, we can write this y t plus t as y t in to phi. So, this phi depends. So, phi we can write this y y t plus t as y this periodic. So, we can write this y t plus t as y t into phi here phi depends on the initial. So, we can choose this matrix phi and this is not unique. So, depending on the initial conditions. So, we can half different phi. So, to find this thing let us we can up to obtain this high matrix.

So, we can take this y 0 initial condition as i and then we can find. So, by substituting y 0 equal to i matrix, then we can write this phi will be equal. So, after one. So, t equal to 0 as t equal to 0, we are taking this equal to i then phi will be written or can be obtain by this thing y t. So, this phi equal to y t.

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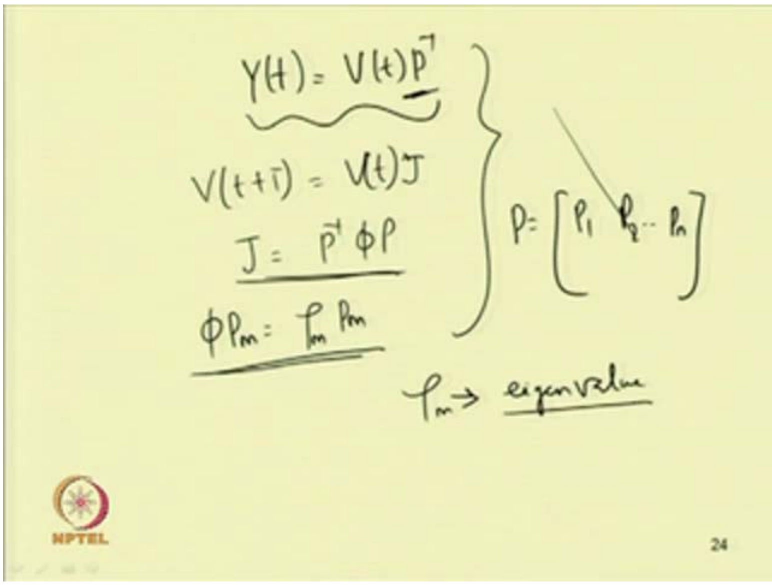

So, this is known as the monodromy matrix. So, by finding the Eigen value of this monodromy matrix. One can one can study the study this stability of this system. So, in this case. So, now, we can use a transform form. So, using a transform y t equal to using a transform y t equal to v t into p inverse. So, we can find a p for we can half this v t plus t equal to v t. So, always we can choose a value p at a matrix p such that we can find these v t plus t equal to v t into j. So, or we can write this j equal to p inverse phi p. So, this is similar to that used in case of the model analysis.

We are we have taken this model matrix to decoupled the equation. So, here by taking this v t plus t equal to v t into j. So, we can write j equal to p transpose p inverse phi p. So, we choose p in such a way that j will be a diagonal matrix. So, we choose in that way then we can write this phi. So, by finding this solution or we can we can write the p matrix p matrix can be written.

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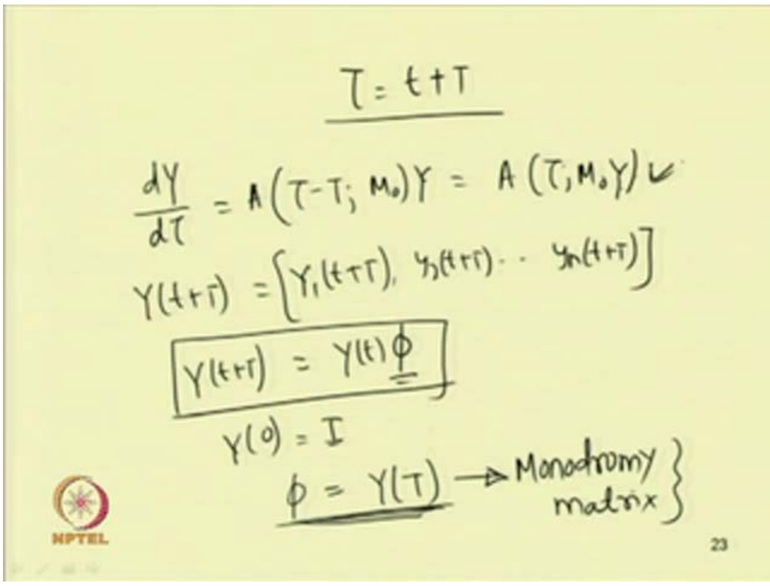

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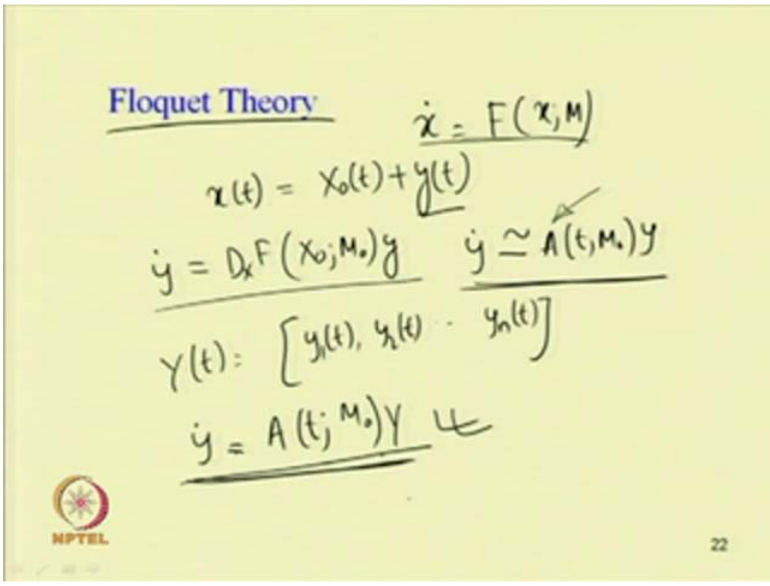

So, we have this p 1, p 2 and p n. So, are the Eigen values of this p matrix. So, we can write phi p m. So, this form. So, phi p m is equal to row m into p m. So, here this row m is the Eigen value. So, row m is the Eigen value of monodromy matrix. So, here instead of taking or we can have this monodromy matrix. So, let us start. So, we have a differential sell equation given this equation. So, past we have porter this equation by y t. So, by pottering this equation. We have obtain this equation y dot equal to a t m 0 y. So, now, by taking a initial condition y 0 y 0 equal to i. So, we can find this y.

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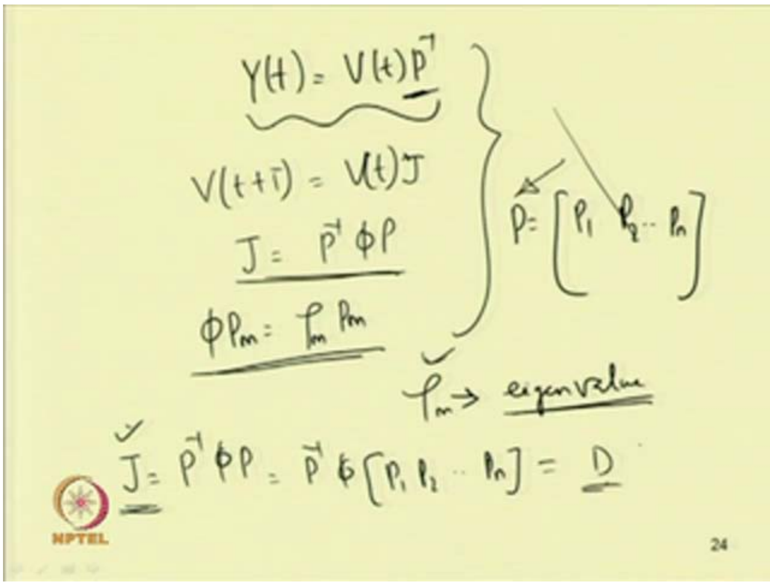

After one time period. So, taking this least time period equal to t. So, now, by solving this equation, we can find value of y at a time period t. So, after finding that thing. So, we can write this or we can find the monodromy matrix now while finding the monodromy matrix, finding the Eigen value of the monodromy matrix, which are written in terms of row m. So, we can write this equation in this form that this phi into row m phi into p m will be equal to row m into p m. So, here p equal to p 1, p 2, p n. So, if you take in this way. So, we can write this j equal to p inverse phi p or this is equal to p inverse phi p 1, p 2, p n. So, one can find. So, this will be equal to a diagonal matrix.

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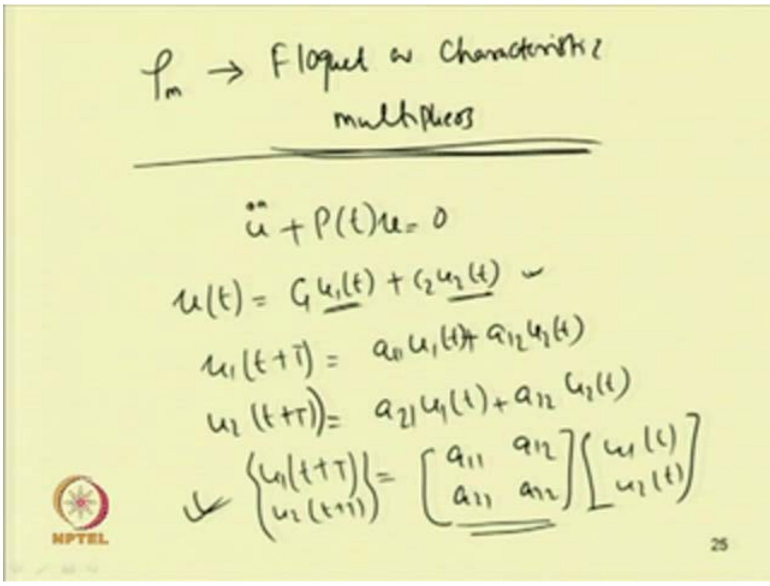

So, one can observe that by taking or by taking a p. So, it depends on how you are taking this p. So, by taking a proper a properly choosing this value of t. So, one can obtain the Jacobean matrix, and. So, we choose diagonal. So, by finding the Eigen value of the Jacobean matrix one can find the characteristic constant. So, this row m are known as Floquet or characteristic multiplier of the system. So, these are known as Floquet or characteristic multiplier of the system. So, by finding this row m one can study the stability of the system.

So, let us take another example and find the Floquet multiplier or discuss about the Floquet multiplier for example, let us take the hill's equation or mathew's hill equation, it is can be written in this form x double dot plus p t x equal to 0. So, this is the this is the Mathew Hill type of equation. So, in this equation this p t is a periodic function of a time and it is solution can be written;. So, it will half periodic solution, which can be written in this form u t equal to c one u 1 t plus c 2 u t 2 t, and we can write this. So, from this by substituting this equation in the original equation. So, we can let us write this equation mu. So, this mu double dot plus u t mu equal to 0.

So, we can assume this solution u t in this form c one u t plus c 2 u t. So, as this second order differential equation will have two set of fundamental solution, let this u 1 t and u 2 t are the fundamental set of solution. So, as these are the periodic. So, we can write this u 1 t plus t will be equal to a 1 1u 1 t plus a 1 2 u 2 t, similarly we can write this u 2 plus t equal to a. So, this can be written as a t 1 a 2 1 u 1 t plus a 2 2 u 2 t, or we can write this thing in matrix form, also. So, here we have this u t plus t we can write this u plus t using this that is u 1 u 2 t plus t.

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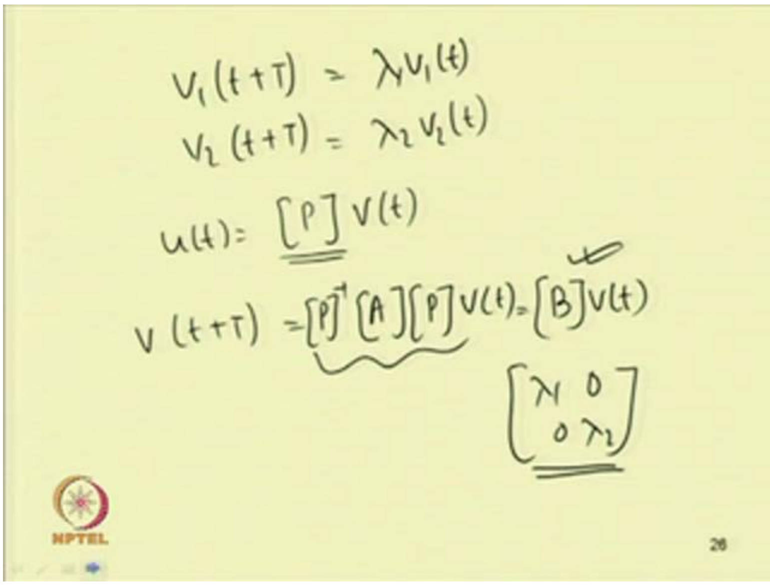

So, this is equal to a 1 1 a 1 2 a 2 1 a 2 2 u 1 t u 2 t. So, one can write this u 1 t plus t u 2 t plus t by using this matrix. So, we can find by making some transformation like what we have discussed in previous case. So, we can one this mu in terms of b, or you can find a set of solution.

So, the solution we can find a set of solution; it is can be written this v one t plus t. So, that is equal to lambda one v 1 t and v 2 t plus t. So, this will be equal to lambda 2 v 2 t. So, which will uncouple the set of equation we can find a set of solution from this. So, which will uncouple this. So, this thing we can find by substituting these u t equal to a p into v t. So, by taking a function p by taking matrix t we can write this u t equal to v t. So, now, we can obtain this. So, v t plus t we can write this v t plus t is equal to p inverse into. So, this is p inverse write this matrix term. So, this is p inverse into a into p into v t or we can write this is equal to. So, this equal to b into v t. So, by finding the Eigen value. So, this b now this b will be or this is diagonal. So, this will be 0 lambda one and lambda 2. So, this lambda will be same as these Eigen values will be same as that of the Eigen values here.

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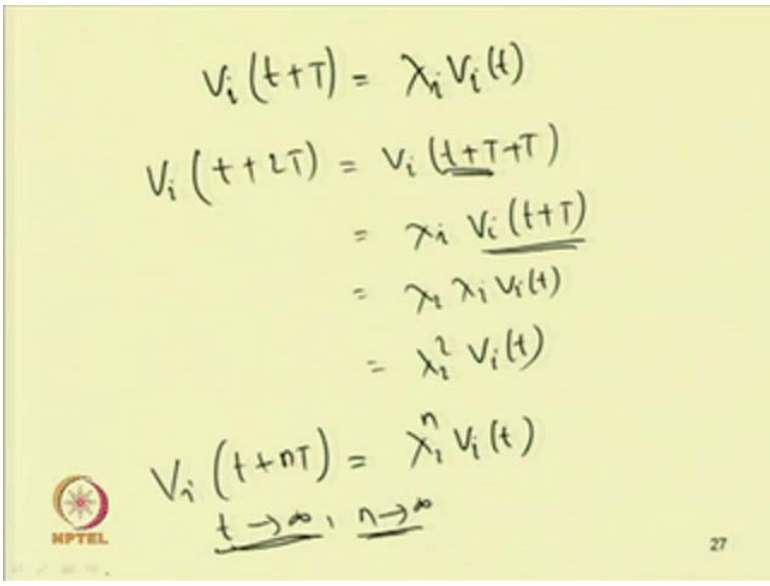

So, by finding the Eigen values either of this matrix a or from this matrix b. So, we can find the this thing Eigen value. So, if you take in this way. So, we can write our v one t plus t equal to r for we can write this n dimensional thing. So, V i t plus t we can write this is equal to lambda i V i t. So, for example. So, let me find what will V i t plus 2 t.

So, in case of V i t plus 2 t. So, this will equal to V i t plus t plus another t. So, this way one can write. So, this can be written as lambda i V i t plus t. So, again this V i t plus t can be written as another lambda i V i t. So, this becomes lambda i square V; it. So, similarly one can write this V i t plus n t will be equal to lambda i to the power n V i t. So, now, as t tends to infinity if we are interested for finding the solution. So, as t tends to infinity; that means, n tends also infinity. So, as t tends infinity n tends to infinity.

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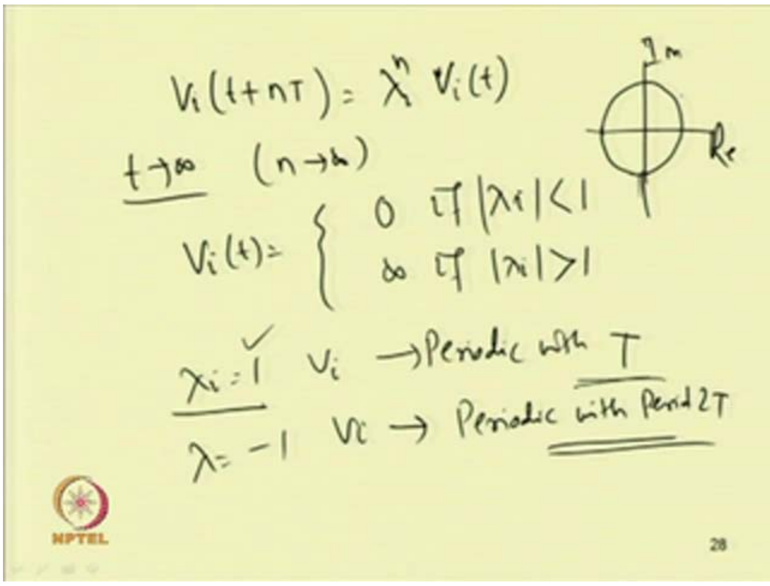

So, we can write this equation as n tends to infinity. So, it will be e i t plus n t or as t tends to infinity, this will be equal to lambda a to the power infinity into V i t. So, in this case. So, we can write. So, V i t plus n t equal to lambda i to the power n into V i t. So, as t tends to infinity or. So, or this n tends to infinity. So, we can write this V i t equal to. So, this will be equal to 0 if this lambda i mode lambda i will be less than one. Similarly, it will be infinity if lambda i will be greater than one. So, what we have observe the response will be periodic. So, if lambda i equal to one.

So, V i is periodic. So, it will be periodic with period with a period t and if lambda i equal to minus one. So, V i will be periodic with period 2 t. So, by finding this Eigen values or finding this characteristic multiplier lambda. So, if we plot this in real and imaginary terms. So, if it remains within the limit cycle. So, what we have seen. So, if we have a periodic orbit lambda will be equal to one. So, for a periodic orbit we have lambda equal to one. So, if it lies within this limit cycle then the periodic response become stable and if it lies outside this limit cycle.

So, this is the real part and this imaginary part. So, it will lies outside this imaginary. So, it lies outside the limit cycle then this becomes unstable. So, further bifurcation of the periodic system we have to study in the next class and will see different examples of this bifurcation of the periodic system similar to that we have discussed in case of the fixed point response. Thank you.