Non-Linear Vibration
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Module - 1
Introduction
Lecture - 2
Conservative and Non Conservative System

Welcome to the second class on non-linear vibration. Last class we have studied different types of systems. So, in that, we have studied or we have reviewed the liner single degree of freedom systems. Also I told you different elementary parts of vibrating systems for example, Simple pendulum.

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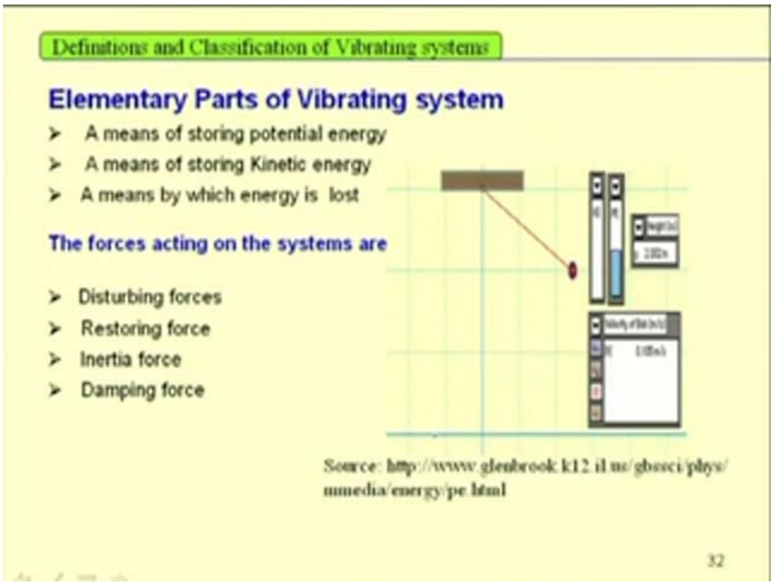

So, we required disturbing force, then restoring force, inertia force and damping force to have the analysis of vibration. For a vibrating system, a main means of storing potential energy, a means of storing kinetic energy and a means of dissipation energy is required. So, as the total energy of the system is constant, that means at this end, all the energy are potential energy, then this energy will be converted to kinetic energy. As it moves from one position to other position, taking this potential energy and kinetic energy of the system, so one can derive the equation of motion.

Already I told you about two different types of approach to find the equation of motion one is inertia based approach and other one is this energy based approach. So in case of inertia based approach, one can use Newton's second law or d'Alembert principle to find the equation of motion. So this is a vector approach hence it is difficult to apply for continuous or multi degree of freedom systems with very high degrees. One can apply this inertia based or inertia based approach for single or two degree of freedom systems. Energy based approach for example Lagrangian principal or Hamiltonian principle can be used conveniently for multi degree of freedom system generally, for continuous systems one can use Hamilton principal or extended Hamilton principal. So, last class we have reviewed, some part of the liner system so this is a spring mass damper system.

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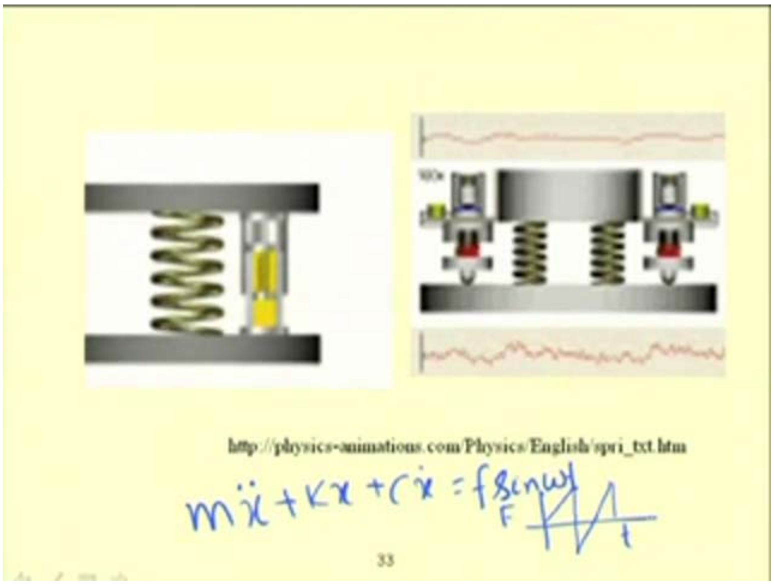

So, in the spring mass damper system, one can find the equation of motion by finding the potential energy of the spring, then this inertia of the mass and the damping energy. So, this inertia force equal to mass into acceleration; and it acts in a direction opposite to the direction of acceleration. And then the spring force will be equal to the stiffness of the spring and the displacement and this damping force, as one can consider this as a viscous dumping, where the dumping force is proportional to velocity, so one can find this damping force equal to c x dot. So the total equation motion for the system will be m x double dot plus k x plus c x dot equal to 0. If it is free vibration a force is continuously applied to the system, then that equation will be m x double dot. So in this equation one can write in this form m x double dot plus k x plus c x dot equal to f and if this force is periodic, then one can write this force equal to f sin omega t where f is the amplitude of the force and omega is the frequency of this force. Instead of a harmonic force in systems some other type of force may be applied, so the force may be a impulse type of force it may be ram type force or it may be some other periodic force, like triangular force so one can use a socket or triangular type of force so this is force versus time or one may use a bang type of force also, in all this periodic cases one can convert this forces into its corresponding harmonic force by using fore year series, so these single degree of freedom system force vibration one can studied and in this case so it will the response will be will contain two parts one is the transient part and other one is the study state response.

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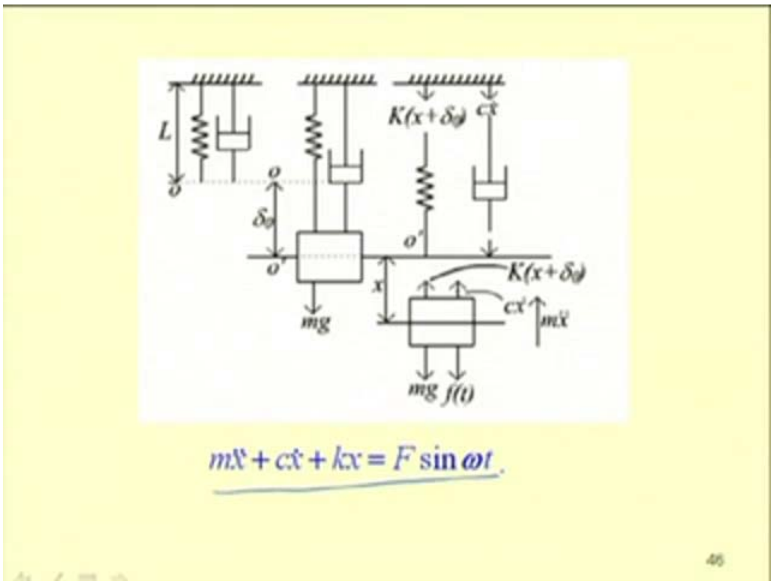

So, in this case so we have seen for example, for this spring mass damper system the equation of motion already, we know so that is m x double dot plus k x plus c x dot equal to f sine omega t.

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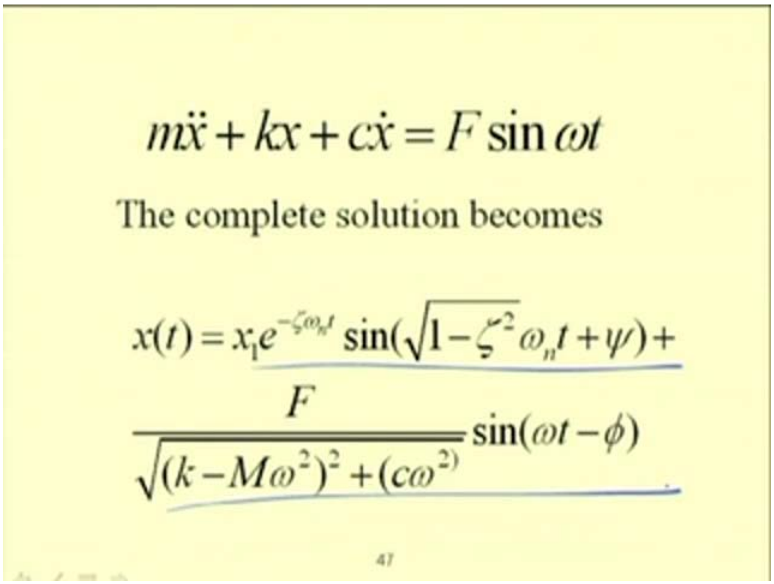

And the complete solution becomes this is the transient part, and this is study state response and last class. We have reviewed this rotating unbalance whirling of shafts, then the support motion and the principle of vibration isolation and also I told you the sharpness of resonance and the vibration measuring instrument. So, one can use the same instrument as an accelerometer or as a seismometer depending on the natural frequency of the system, and the measuring frequency. So let us review about 2 degrees of freedom systems, multi degrees of freedom system and continuous system briefly, then I will tell you about the super position rule for the non-linear systems. So, in case of 2 degrees of freedom unlike in single degree of freedom system, where we have only single equation of motion so here we will have two equation of motion.

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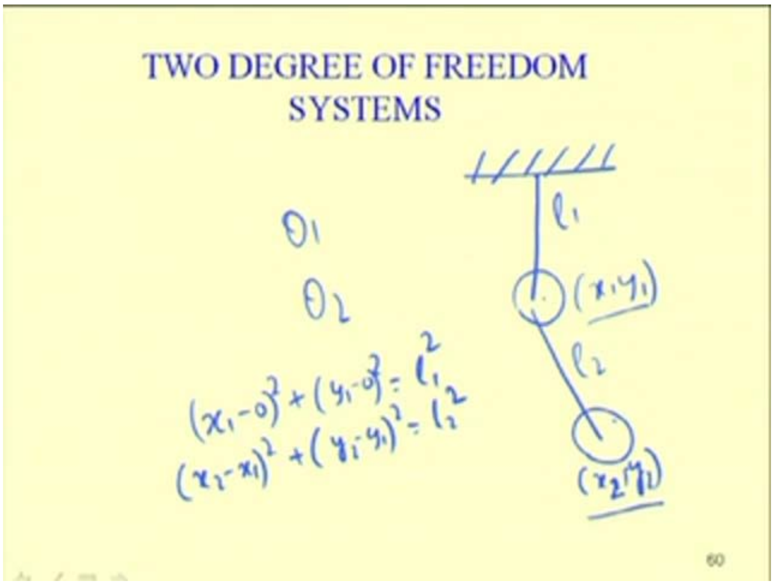

So for example, let me take a double pendulum so to derive the equation of motion. Let this is a double pendulum so in this case of double pendulum to derive the equation of motion I can use the generalize coordinates or the physical coordinates. So, one can have physical coordinate x 1 y 1 here x 1 y 1 and x 2 y 2 but, one may see that this is a 2 degrees of freedom system only, so one can use only two variables that is theta 1 and theta 2 to completely represent the motion of this mass and this mass. So instead of using x 1 y 1 and x 2 y 2 one may use coordinates generalize co-ordinates.

Theta 1 and theta 2 it may be noted that this x 1 y 1 and x 2 y 2 so out of these four parameter we can reduce to 2 parameter by using the constant equation, so the constant equations are so this length is l one so this length is l two, so one can have this constant equation x 1 minus 0 whole square plus y 1 minus 0 whole square equal to l one square or x 1 square plus y 1 square equal to l one square and similarly, here x 2 minus x 1 whole square plus y 2 minus y 1 whole square equal to l two square. So, these two constant equations can be used to reduce these four parameter x 1 y 1 x 2 y 2 to two parameters.

So, using this physical parameters or these two parameter theta 1 and theta 2 one can find the equation, so to find the equation of motion so either one can use the this Newton's second law or d'Alembert principle. Find the acceleration at this point and then equate or find the equation of motion using Newton's principal otherwise one can use this

Lagrange principle or Hamilton principle to derive this equation of motion. So in this case one can see the equation of motion for mass one and mass two, one will have two equation of motion.

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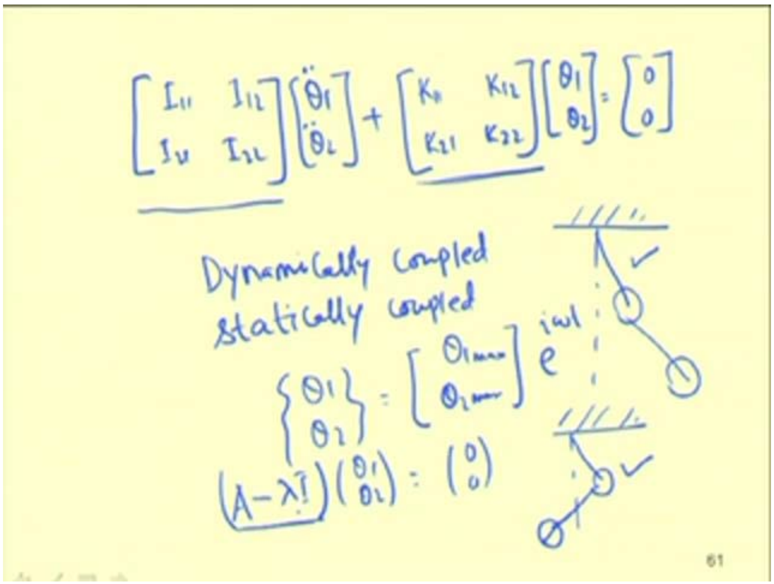

So these two equation of motion one can write in this form so it will be two is to two so I 1 1, I 1 2, I 2 1, I 2 2, theta 1 double dot, theta 2 double dot, so this is the general expression written for the system. So k 1 1, k 1 2, k 2 1 ,k 2 2, theta 1, theta 2, equal to 00 as no external force is acting on the system then one can write the right hand to be 0. So, in this case this is the mass matrix or inertial matrix and this is the stiffness matrix so if the mass matrix is coupled that means this non diagonal elements are present. Then, the system is known to be dynamically coupled system. But if the stiffness matrix is coupled that is the off diagonal terms are present then the system is known to be statically coupled.

So, if the off diagonal terms are present then the systems are coupled, so for this case it is both, if this off diagonal terms are non 0 then the system is both dynamically and statically coupled. So, to find the solution of the system so one can assume this theta 1 theta 2 theta 1 theta 2 to be theta 1 max theta 2 max in to e to the power I omega t and substitute it in this equation then this equation will reduce to a Eigen value problem that is a minus lambda, I theta 1 theta 2 equal to 0 solving this Eigen value problem a minus lambda I equal to 0. One can find the frequency of the system where this lambda equal to omega square so n this case one can get to Eigen values corresponding to two corresponding to two mode of vibrations so in one mode the response if one plot so for the first mode or for one mode the system may look like this and for the second mode the system may look like this. So, this was two mode of vibration. So, this corresponding to the first mode and this correspond to the second mode. In this way for any two degrees of freedom system one can find two natural frequencies by finding the Eigen value of this matrix.

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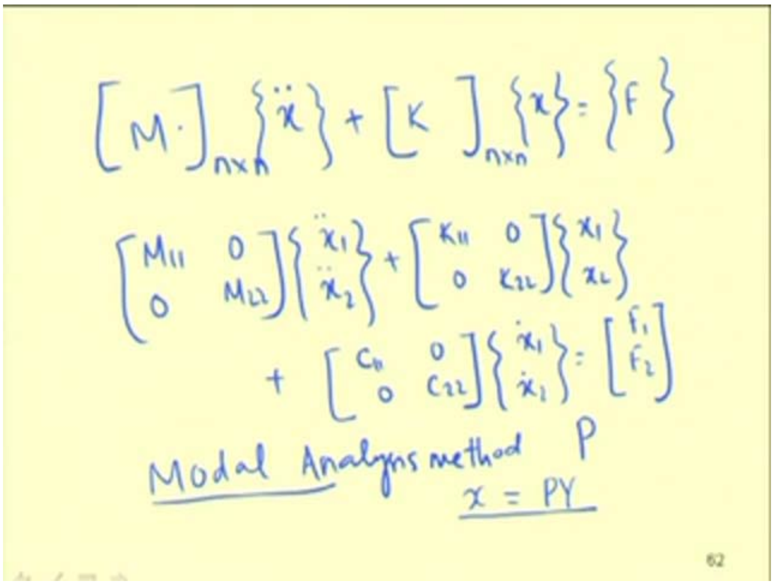

So, for multi degree of freedom system similarly, one can have a n cross n mass matrix. so if it is m degrees of freedom system then it will be n cross n mass matrix x. so it may be written as x double dot plus similarly, k so k also will be n cross n into x so this is equal to if this is a force force system then this will be replace by f so this mass matrix is n cross n so for example, for a ten storey building this will be ten cross ten matrix and the stiffness matrix also will be ten cross ten and corresponding to each floor one can have one deflection so x will be ten cross one. So to make this uncoupled if the off diagonal terms are not present so for example, if I will write the system like this that it is m 1 1 0 0 m 2 2 x 1 double dot x 2 double dot plus k 1 1 0 zero k 2 2 x 1 x 2 plus let me add the damping matrix also c 1 1 0 c 22 into x 1 dot x 2 dot.

So, let it equal to let me write this is f one and f two so in this case the equation can be reduce to two single degree of freedom system equation. For example, in this case the equation will be m 1 1 x 1 double dot plus k 1 1 x 1 plus c 1 1 x 1 dot equal to f one. Similarly, m 2 2 x 2 double dot plus k 2 2 x 2 plus c 2 2 x 2 dot equal to f two. So, we can solve this two equation as that of a signal degree of freedom system. So, in this case one can find the transient response and the study state response of the system. But, how to reduce this coupled equation to uncoupled equation? So for that purpose one can use this modal analysis method. One can use modal analysis method to reduce the set of coupled equation to a set of uncoupled equation. To find the modal analysis or to carry out the modal analysis first one can find the modal matrix p and substitute this x equal to p y, y is another coordinate system if one can one substitute this in this equation then it will reduce. So, in this case it will reduce so this equation will reduce to that of a signal degree of freedom system, so let us see so by substituting the first equation by substituting this.

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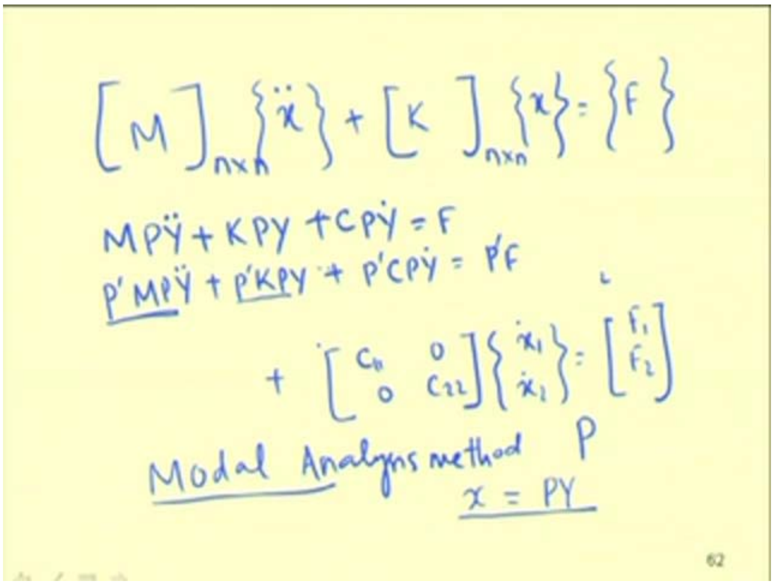

Now, I can write this equation as m so this will be p y double dot plus k p y plus c p y dot equal to f. Now by p multiplying p transpose so this equation will reduce to p transpose m p y double dot plus p transpose k p y plus p transpose c p y dot equal to p transpose f, so in this case by using the Orthogonality property of the modes so one can find this p transpose m p is a diagonal matrix. Similarly, this p transpose k p is a diagonal matrix for many cases this damping matrix can be replaced by the relay damping, so in that case this damping matrix can be written as a function, or this damping matrix can be written in terms of the mass matrix and the stiffness matrix.

So, the whole system can be reduced to that of a diagonal mass, matrix diagonal stiffness matrix and diagonal damping matrix. So, the equations will be uncoupled and this uncoupled equations can be use to find the solution of the system.

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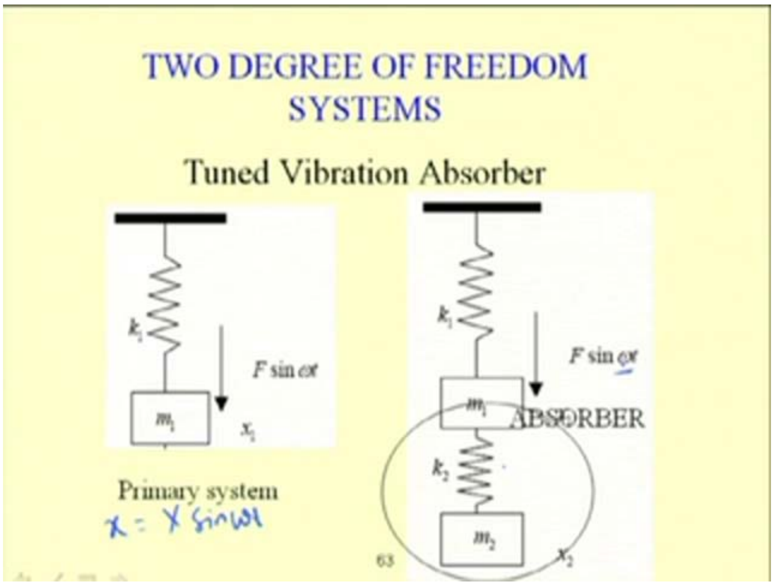

So, after this brief preview about this two and multi degree of freedom systems, so let us see how this two degree of freedom system analysis is helpful for us to reduce the vibration. So for example, consider a system for consider a pump or consider any internal combustion engine which are inherently unbalanced, so these system can be modal as a signal degree of freedom systems signal spring and mass system with a force f sin omega t applied to the system. That means the system is vibrating now it is vibrating with the response x equal to x sin omega t at study state. So to reduce this vibration one can use another secondary mass and spring, so this is the secondary spring and secondary mass which has a frequency which has a natural frequency equal to omega. So if one uses a spring and mass in such a way that this omega equal to root over k two by m two then it can be shown that this amplitude of vibration of the primary system can be reduced to 0, and the second at the same time the secondary spring and mass system will vibrate with a frequency equal to that of the applied force, so to absorb the vibration completely of a Primary system. One can use a tuned vibration absorber so this tune vibration absorber will work for a particular frequency that is omega but, for the system when resonance occur at many frequencies then one can go for centrifugal type of vibration absorber.

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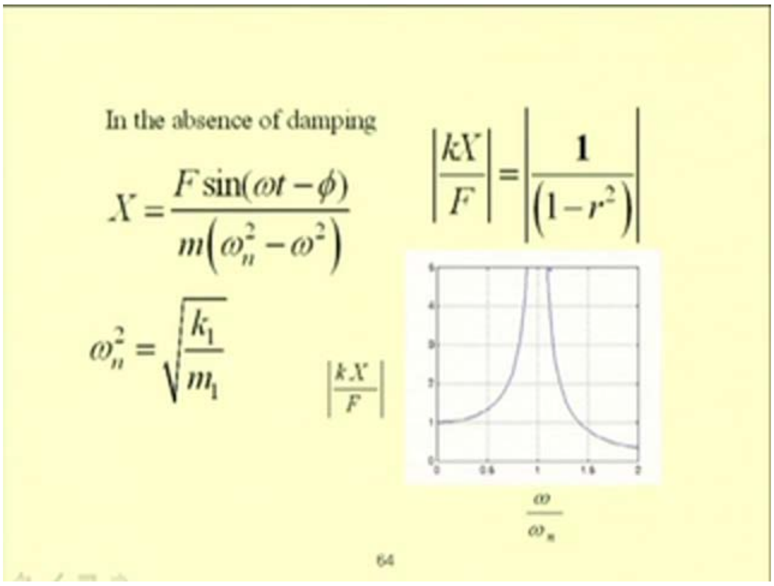

So, in case of this tuned vibration observer when one use the secondary spring and mass then the regencies point shifts to the left and to the right. So, previously the resonance occur at a frequency omega equal to omega n as there is no damping in the system. But, if one considers damping in the system then the resonance will occurs slightly left to this omega by omega n equal to one. So in this case the expression for x equal to f sin omega t minus phi by m into omega n square minus omega square, where omega n is the natural frequency of the primary system which is equal to k one by m one root over.

So, this is the external frequency, so this amplitude as the damping is not there so when omega tends to omega n this tends to infinity and to reduce this vibration or to absorb this vibration completely. One can use the secondary spring and mass system; in that case this resonance point is sifted to right.

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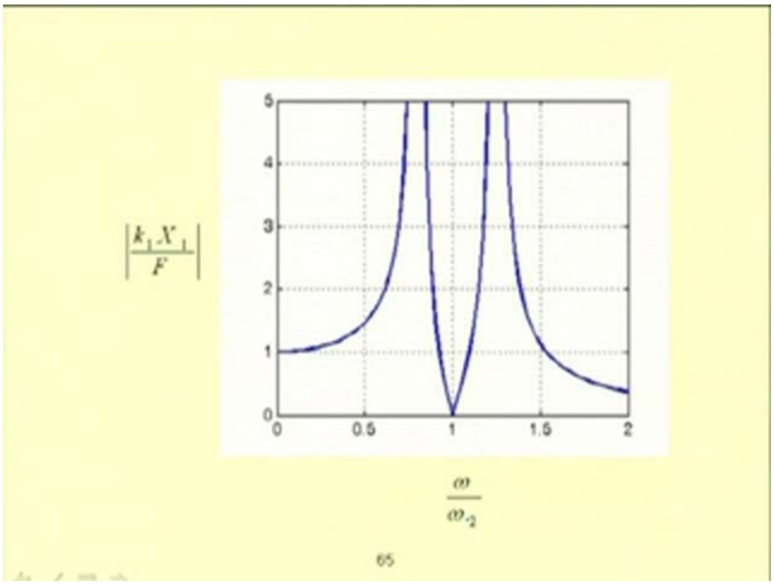

And left and it has a amplitude of 0 so this k one x 1 by f become 0 at omega equal to omega two, so when omega equal to omega two then this becomes zero.

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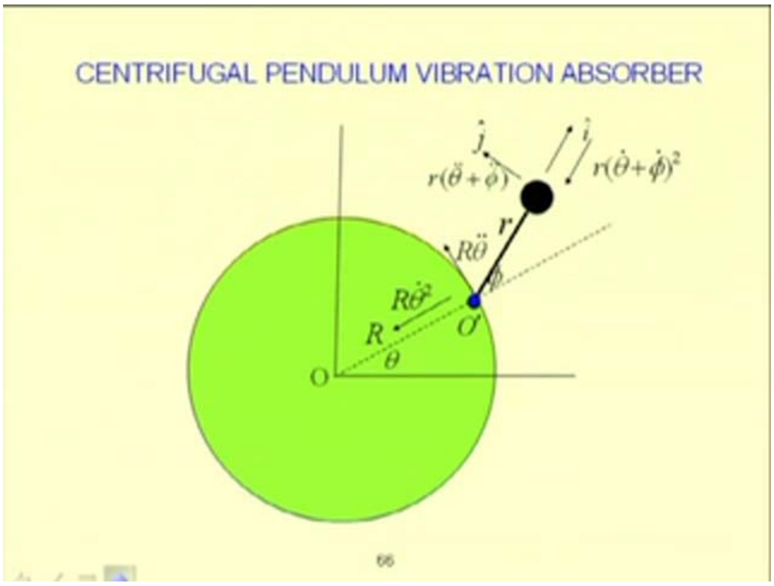

So, one may use this centrifugal type of pendulum vibration absorber for absorbing the vibration at different frequency. Similarly, one may use vibration damper to damp out the vibrations of the system.

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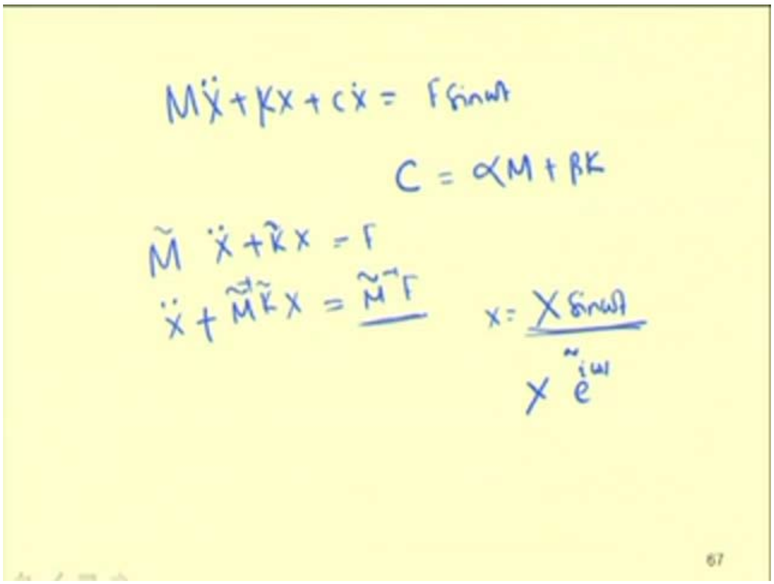

So, for multi degree of freedom system already we know we can write the equation in this form m x double dot plus k x plus c x dot equal to f, so to find the force response of the system one can write, if dumping is not present in the system or if one use relay dumping that is c one can write equal to alpha m plus beta k then this equation reduce to some m.

So, let me write m x double dot or simply m one can write so m x double dot plus k x equal to f, now one can write this expression like this x double dot by multiplying m inverse so it will be plus inverse into x will be equal to m inverse f, so now this equation can be by substituting x equal to x sin omega t or x sin either x sin omega t or x equal to x e to the power I omega t. So, let this function f, you just write it as sin omega t or it may be constant also so in that case first you convert that thing into a set of single equation of a single degree of freedom systems and then you just find the response of the system. Otherwise one can find the free vibration first one find the free vibration response of the system then for the study state one can consider the force part of the system.

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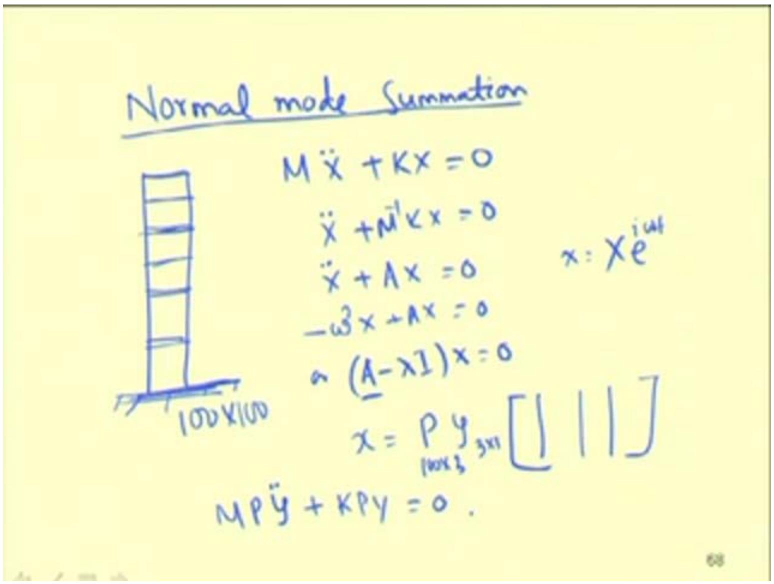

So, one can use this normal mode summation method if one want to reduce a very high degree of freedom system to a lower degree of freedom system. For example, in case of a multistoried building subjected to earthquake excitation, so this is a multistoried building subjected to earthquake excitation, so as only some lower frequencies will be affecting the motion of the system. So instead of considering all the degrees of freedom system one may take a lower order degrees of freedom to find the equation of motion or to solve the or to find the response of the system. For example, in this case let it is hundred is to hundred if it is a hundred storey building then the equation of motion will be m x double dot plus k x equal to for free vibration. So, it will be equal to 0 after earth quake the force no force is acting on the system so it can be considered as a if one consider the free vibration then one can take this equal to 0 then in that case one can only find the hundred natural frequency of the system by finding the Eigen value of the a minus.

So, let me write this equation in this form so it will be x double dot plus m inverse k x equal to 0 so or one can write this x double dot plus a x equal to 0 by substituting x equal to x e to the power I omega t, this will reduce to minus so this will be minus omega square x plus a x equal to 0 or one can write this is equal to a minus lambda I a minus lambda I x equal to 0, were this lambda equal to omega square.

So, by finding the Eigen value of the system one can find the hundred natural frequency of the system. But, to find the response of the system one may not required all this hundred modes so only the first few modes may be useful for finding the response of the system. So, to find the response let one take only first three modes so in that case one can write this x equal to. So, this is p y where this p will contain p is the modal matrix that will content only three columns so, it will contain this p will contain only three columns this correspond to the first mode the Second column will correspond to the second mode and third column will correspond to the third mode of the system. Then this will reduce to this p is reduced to hundred so it will reduce to hundred into three and y will be equal to three into one, if one substitute this in original equation so this is so one can write this as m p so m p y so m is hundred is to hundred and p is hundred is to three and y equal to three is to one and pre-multiplying. So, this will be this equation will be m p y double dot plus k p y equal to 0 now pre-multiplying that p transpose.

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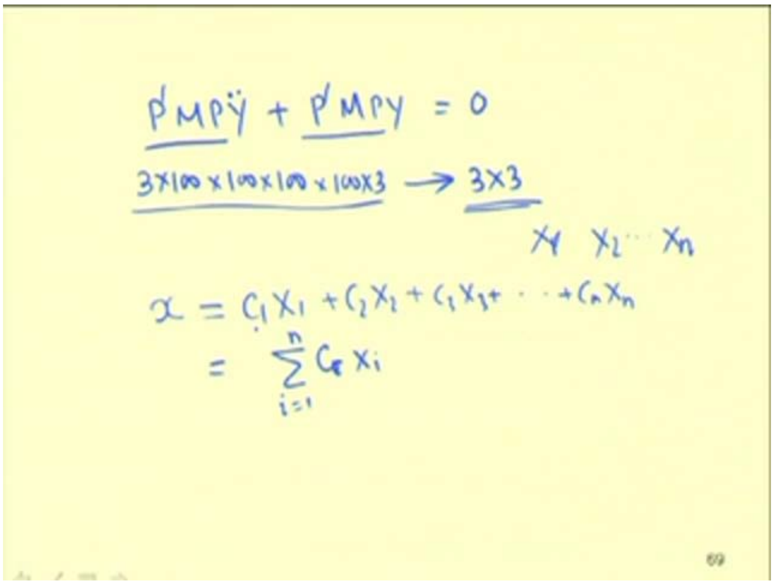

So, one can write this p transpose m p y double dot plus p transpose m p y equal to 0. But, this p transpose is p is 100 is to 3 so it will be 3 into 100 and aim is 100 is to 100 and then this p is 100 is to 3 so this reduces. Finally, it reduces to a 3 is to 3 matrix.

So, instead of solving this hundred is to hundred matrix one can solve this three is to three matrix. So, all these matrix, so this will be three is to three matrix and this will be also three is to three matrix so one can solve these three equation to find the solution or response of the system. So, this method is known as normal mode summation of the system, so one may note for multi degree of freedom system, the free vibration of the system, let the free vibration of the system is written as x so it will be the summation of different normal modes. So, let for first mode the modal vector is written as x 1 for second mode it is x 2 for third mode it is x three and for x n mode the modal vector is x n. One can find this modal vector correspond to the Eigen vector of the systems, so one can write the free vibration of the system as a combination of different normal modes. So, this x will be equal to c one x 1 plus c two x 2 plus c three x three plus c n x n. If you are considering n number of modes then the resulting free vibration of the system can be written as the summation I equal to one to n c I c I x i. Where c I will be the modal participation of that particular mode, so one can find the c I by using that is the modal participation, one can find this modal participation by using the Eigen property of the system.

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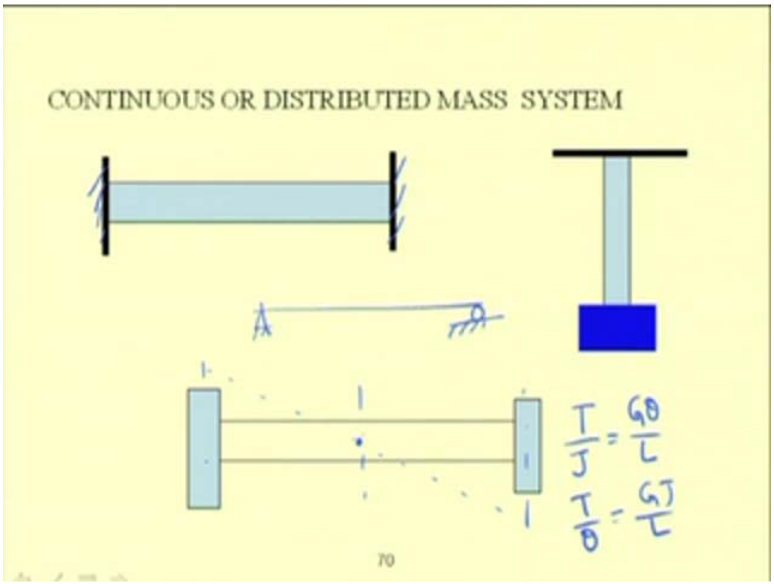

Now you can consider the continuous system, so in case of continuous system or distributed mass system unlike in case of discrete system what we have studied before the one can have infinite number of natural frequency of the system. So, as these continuous or distributed mass systems can be represent as infinite number of spring and mass systems, then one can have infinite number of natural frequency to the system. So for this infinite number of natural frequency one can find infinite number of mode shapes of the system.

So, mode shapes represent the variation of the point's variation of different points or vibration of different points along this length of the beam at a particular instant of time.

So, one can have different boundary conditions also in this case. For example, in this case the beam can be considered as a fixed fixed beam it may be considered as a simply supported beam also, it may be considered as a simply supported beam or it may be considered as a cantilever beam. So, this is the example of a two rotor system so two rotor's are mounted at the end of this case one can observe the body will have infinite number of natural frequency if the mass of the system is taking into account otherwise one may write or one may find two natural frequency of the system by considering only this two mass, so one frequency will be the one will be rigid body motion as this is a semi definite system. So, one can have a rigid body motion where the frequency is 0 and other motion where one can find the frequency by find the node points. So, in this case the vibrations so, if one rotate this left side in one direction and right side in opposite direction then one mode can be observed. So, this mode so in this mode one can write or one can find a node at this point.

So, for this point as a node is observed at this point, so one can assume that the soft is fixed at this point and rotating have a torsional vibration about this point. So, one can find the two natural frequency of the system by taking the stiffness of the system so, the stiffness of the system can be found. If the system parameters are known to us so by using this torsional relation t by j equal to g theta by l so one can find this t by theta that is the stiffness torsional, stiffness equal to g j by l. So, these two modes can be considered if one neglect the mass of the soft but, when mass of the soft is considered then it will have infinite number of natural frequency of the system.

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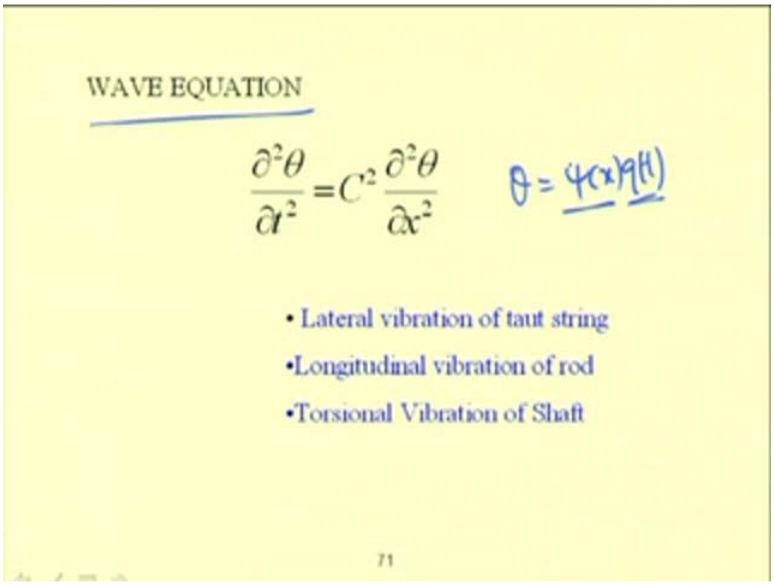

So, one can dived or one can write the equation of motion of the longitudinal vibration of the system or the rotational vibration of the system or the lateral vibration of a taut string by using wave equation. So, by using wave equation one can find the lateral vibration of taut string longitudinal vibration of rod or torsional vibration of shaft. So, this equation can be solved so this represent the inertia force and this is the stiffness part, so one can find the equation of motion or the response of the system by solving this wave equation so, unlike in case of discrete system where the equation of motions are written in terms of ordinary differential equations. So, here the equation of motions are written in terms of partial differential equation, so this partial differential equation can be reduced to a set of ordinary differential equation by using variable separation method and one can find so, by substituting this theta equal to psi x and q t. So, one can write the one can find the equation motion and one can find the mode shapes so the psi x represent the mode shape and q t represent the time modulation. One can find the response of the system.

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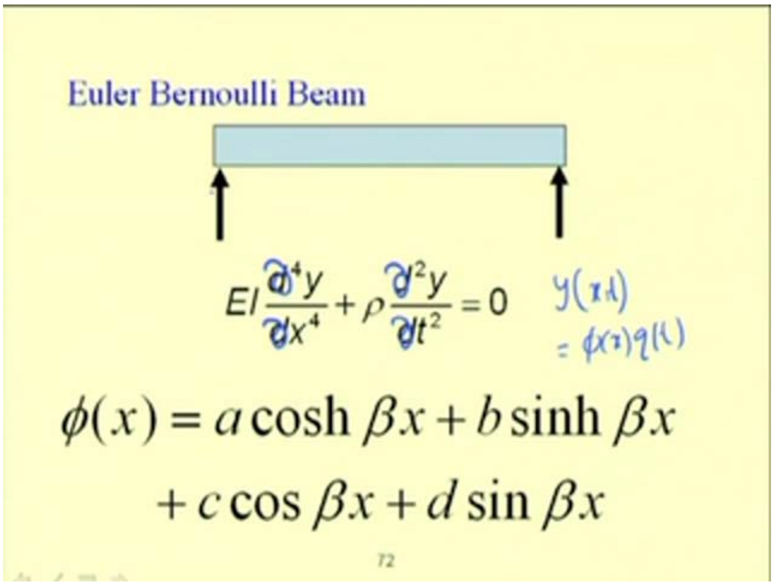

Similarly, for a beam beam in pure bending so in case of a beam in pure bending so one can use this Euler Bernoulli beam equation. So, in that case the equation can be written in this form e I del forth e I del forth. It will be replaced by partial differential equation del forth y by del x forth rho del square y by del t square equal to 0 because y is a function of both x and t.

Now, substituting this y x t equal to phi x and q t. Where phi is the time modulation phi is the mode, mode shapes and q is the time modulation, so one can find a general expression for the mode shape as phi x equal to a cos hyperbolic beta x plus b sin hyperbolic beta x plus c beta x plus d sin beta x. So, by using different boundary conditions one can find the actual response of the system.

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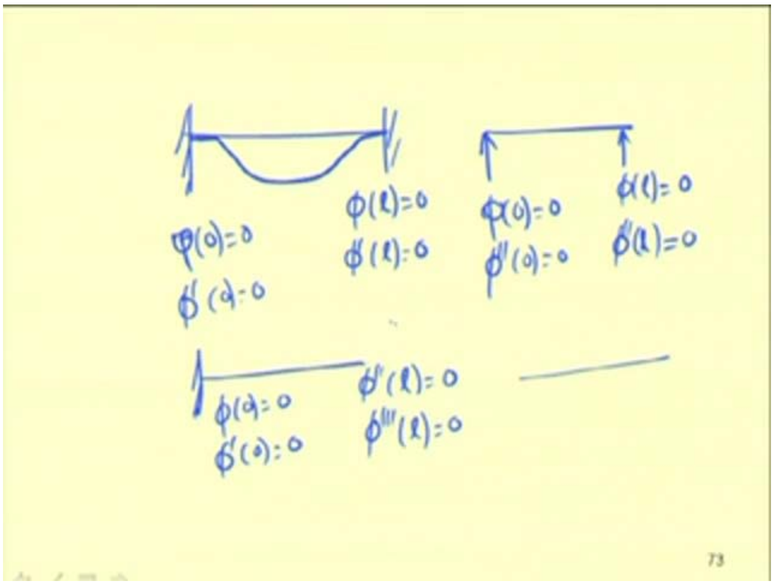

So, the different boundary conditions includes so, for a fixed, fixed beam so let us consider a fixed fixed beam so, in case of this fixed fixed beam the boundary conditions. So, displacement and slopes are 0 so, one can have both displacement and slope equal to 0 at both the ends. So, here displacement equal to 0 and slope equal to 0 at both the ends. So, one can get four boundary condition two for this left end and two for the right end that is phi x equal to phi 0 equal to 0 and phi dash 0 equal to 0. Similarly, phi l equal to 0 and phi dash l equal to 0 similarly, one can get the equation of motion for a cantilever beam. So, in this case of cantilever beam at the fix end the boundary conditions are same as that of the fixed beam. But, at the right end as it is free so, one can have natural boundary condition or forced boundary conditions.

So, those are sear force equal to 0 and bending movement equal to 0 so, for a simply supported beam which can be represented by this the boundary conditions are displacement equal to 0 and bending movement equal to 0. So, displacement correspond to your phi x so phi 0 will be equal to 0 and for bending movement it will be phi double dash 0 equal to 0 phi double dash is del square phi by del x d square phi by d x square so similarly, at this end phi l equal to 0 and phi dash double dash l equal to phi dash l equal to 0. So, in this case phi 0 equal to 0 phi dash 0 equal to 0 bending movement correspond to phi double dash l equal to 0 and sear force correspond to phi triple dash l equal to 0. Similarly, for a free beam, one can find the boundary conditions similar to that of the right side of the cantilever beam.

So, using these boundary conditions in the generalized equation one can find the frequency equation and the mode shapes of the system. So, here different mode shapes and frequency equations so, these are different mode shapes so one can find different frequencies corresponding to beta l.

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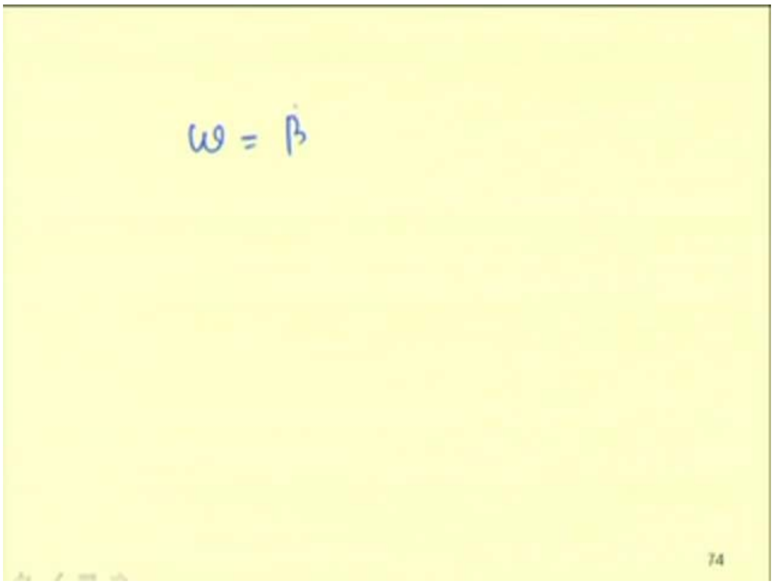

One can write the frequency equation omega square or omega will be equal to beta square l square root over e I by rho l forth so for different boundary conditions. So, the first three modes have been calculated. So for example, in case of simply supported it will be pi square beta square l square will be pi square and second mode is four times the first mode and third mode is nine times the first mode. Similarly, for a different other different boundary conditions they have been found.

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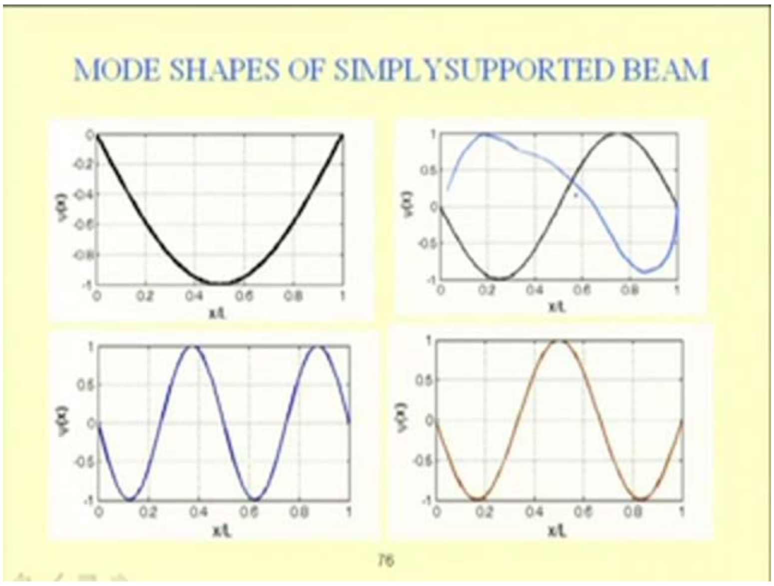

Like this and the mode shapes for a simply supported. For example, in case of simply supported the shape will be like this either it will be this or it may move upward or downward. So, this is downward it is shown one can have the upward motion also similarly, in this case one can have another motion also.

Similarly, so here one can observe a single node so, a single node can be observed at the middle point single node is observed at this middle point. Similarly, one can see there are number of modes will goes on increasing with increase in the modes of the system. So, this is for simply supported beam similarly, one can find for fixed-fixed so, both displacement and slopes are 0 so, for this is for first mode so this is for second mode third mode and like that so for the cantilever beam so, either it can move upward or downward in the first mode so, one has a node in the second mode similarly, for third and higher. So, this way one can study the single degree of freedom system, two degrees of freedom system, multi degrees of freedom system or continuous systems by finding the Eigen values of the systems. One can find the accurate response of the system frequency and response of the system, also one may use different approximate methods to study or to find the natural frequency of the system. For example, one may use this relay method so, in which one will equate the maximum potential energy with that of the maximum kinetic energy to find the frequency.

In case of a rotor system with multiple mass or multiple disc one can find approximately the natural frequency by using drunkerley procedure. So, in case of Dunkerley procedure so, let this is a sharp with multiple mass.

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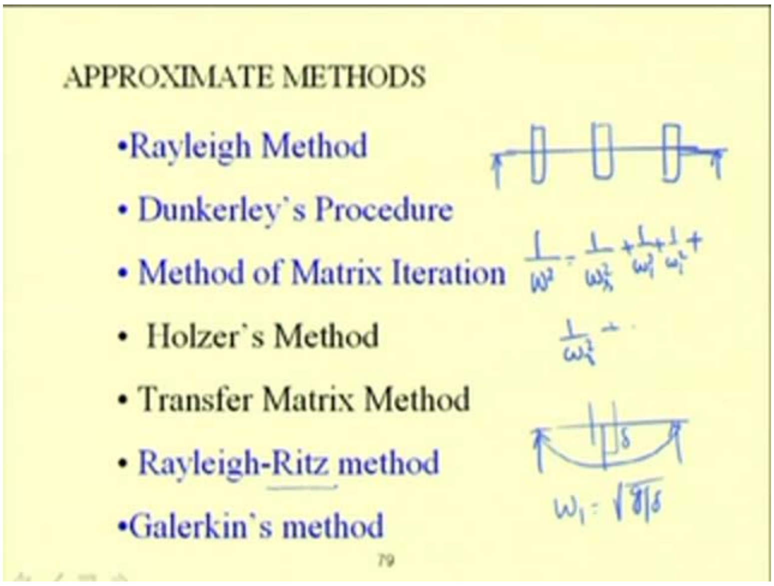

So, let three masses are there and let it is simple supported without mass if one know the natural frequency of the system. Let it is omega s then the natural frequency of the system omega square will be equal to one by omega a square plus one by omega one square plus one by omega two square plus one by omega three square and if there are n masses then it will go up to that. So, where omega one omega two omega three are the natural frequency of the system, when the shaft is considered to be massless so, this omega s is the natural frequency of the system.

When one consider the mass of the shaft in the absence of this disc so, as already we have seen this is a continuous system with infinite number of natural frequency one can get infinite number of natural frequency for this omega s and by taking care this omega one omega two and omega three the total response or the natural frequency of the system can be determined approximately. Here to determine for example, omega one so, one can find it in this way. So, this is the mass. Due to this mass one can find what is the deflection. Let the deflection is delta. So, if the deflection is delta then one can find the natural frequency omega one will be equal to root over g by delta.

So, by knowing this delta that is the deflection of the system due to this mass so, one can find the natural frequency of the system. So, this is the Dunkerley procedure similarly, one may use matrix iteration method Holzer method, transfer matrix method, Rayleigh Ritz method and Galerkin method to approximately, find the natural frequencies of the systems. So, this can be used for example, this matrix it method of matrix iteration Holzer method, transfer matrix method can be used for multi degree of freedom systems and one can easily find the natural frequency of the system by using this Raleigh Ritz method where a assumed function can be used so, this assumed function may not be the Eigen function, assumed functions are a determined by taking only the geometric boundary conditions of the system.

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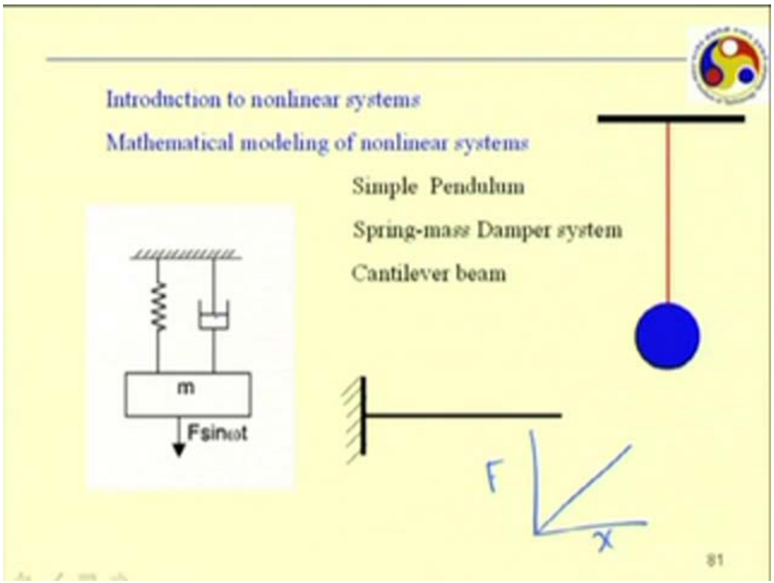

So, let us so, this is a brief review about the linear systems. Now we can study about the non-linear systems because the non-linear analysis required the knowledge of a linear system these two classes, we have used for find briefly reviewing the linear systems let us see how a linear system and a non-linear systems different.

So, in this case the spring mass damper system, we have assumed the spring to be linear so assuming the spring to be linear that means if you apply a force f to the spring then the displacement is force and displacement curve is linear. So, if one plot this force versus displacement so, this is force of the spring and this is displacement then the curve is like this but, in actual case the curve will not be linear the curve may be different. Similarly, for this damper we are assuming the damping force f to be linearly proportional to the velocity that is why we have written this force f equal to c x dot. Similarly, one can have this non-linear inertia force also.

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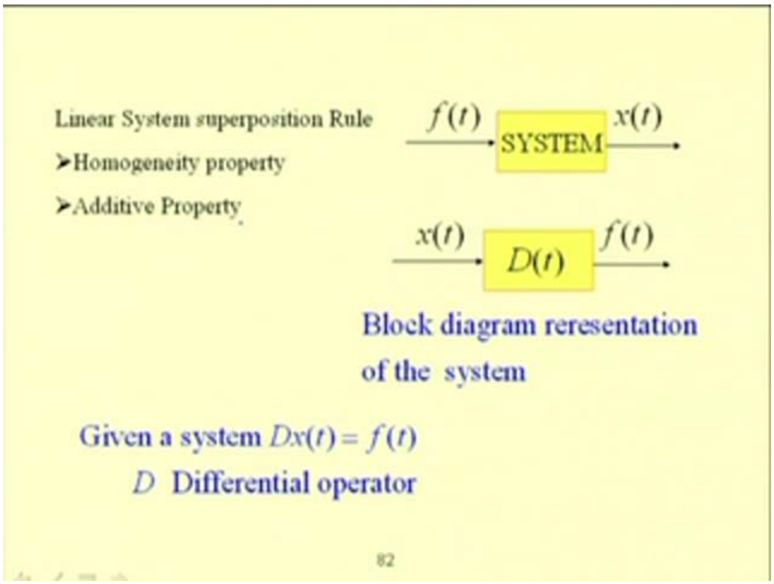

So, let us take a system so, the system incase of a linear system. If when we are applying a force f we are getting the response x so, in block diagram representation we can write x t equal to x t when applied here. So, this is d t and this is f t where d x t is the d is the differential operator.

So, in block diagram representation one can write this is the output when a input f t is applied to a system one can have, this is f t the input to the system so, one can have the output of the system in this case this is the response of the system and this is the forcing of the system. So, in case of linear system it satisfy both homogeneity property and additive property, in case of homogeneity property if we are applying a force f t the response is x t if for a given force f t the response is x t if we apply a force alpha f t where alpha is inconstant if you are applying a force alpha f f t then the response would be alpha x t so, this is homogeneity property of the system.

Similarly, for a force f one is the response is x 1 and for another force f 2 is the response is x 2 then if these two forces are applied simultaneously that means if a force f 1 plus f 2 applied to the system then the response should be x 1 plus x 2 so, that is additive property for any linear system. We have seen in these cases both homogeneity and additive properties have been additive and homogeneity property so, it should obey both homogeneity and additive property.

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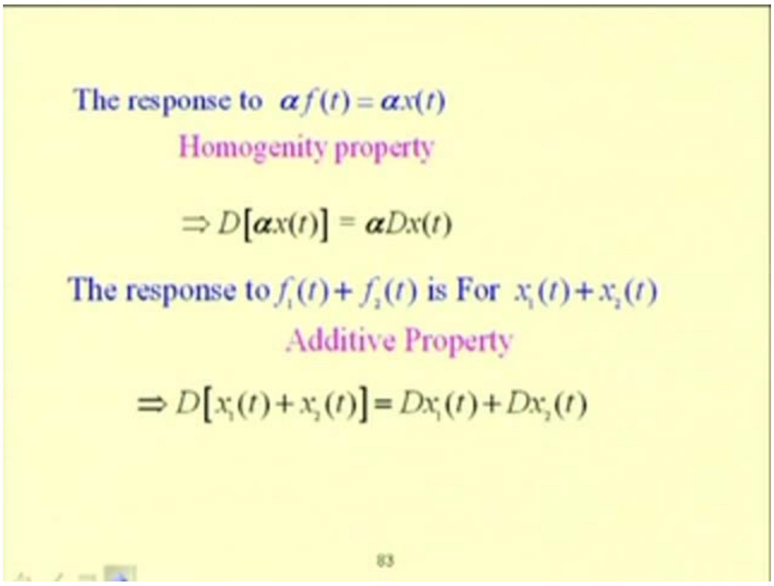

But, let us see in case of homogeneity property for a given alpha for a force alpha f t the response should be alpha x t that means in mathematical form if you can write. So, d of alpha x t should be alpha d x t. Similarly, for additive property when we are applying this force f 1 t plus f 2 t is simultaneously, then the response should be x 1 t plus x 2 t in mathematical form using this operator. So, if you can replace this x t by x 1 t plus x 2 t so, this equation should be d x 1 t plus x 2 should be d x 1 t plus d x 2 t. If these rules are satisfied then, the system is called to be linear and if these properties are not satisfied then the system is non-linear system.

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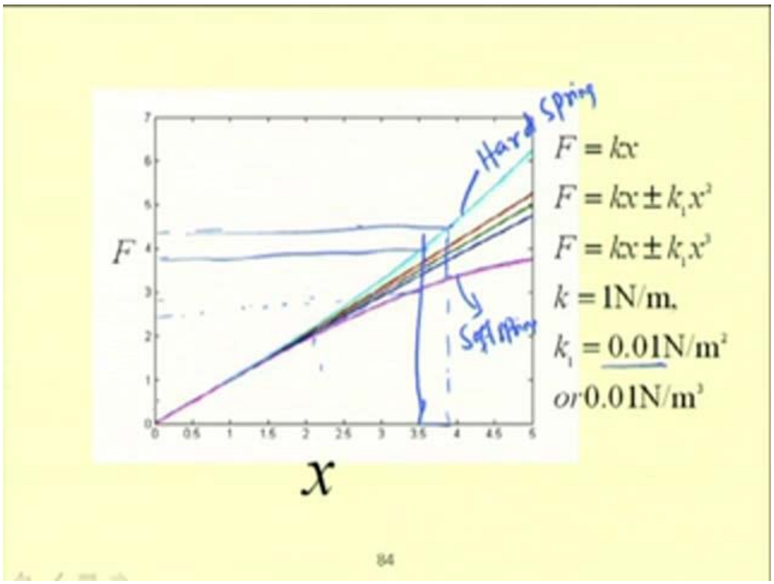

So, for example, in case of a linear spring so, this is linear and one can see if we are adding some term to this. So for example, in this case let us k equal to 1 Newton meter. So, this is curve is f equal to k x this is linear curve and then. So, if you are adding plus minus k 1 x square where I have taken k 1 equal to 0 point 0 one so, there is slight variation this linear so one can see up to certain position the response is. Or the force versus displacement response is linear and after that the response is non-linear. So, this is for k 1 plus minus k 1 x square and this one is plus minus k 1 x cube.

So, in this case if one can see a particular value of so value for a particular of x let us see what is happening for a particular value of x so, in this case when it is linear f equal to k x and if you compare for these. So, let us compare this curve with this curve so in this case, or this displacement, or to have this displacement one can apply a force. So, let us take two points and this. So, one has to apply this amount of force to have a displacement. In this case it may be noted for the same displacement less amount of force is applied but, in this case more force is applied for the same amount of displacement. So, this spring can be called as a hard spring and the other spring is a soft spring. So, we may have a hard spring or a soft spring so due to the presence of these types of springs the force displacement relationship cannot be linear.

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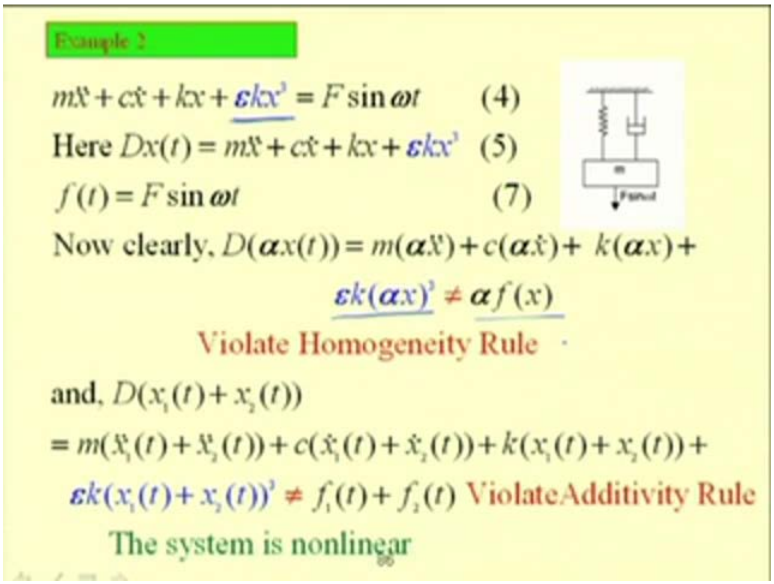

So, in that case one can write a non-linear equation but, now one can compare the order of non-linearity. Here, I have used k one equal to point one but, if I will take a value if I will take a higher value then one can see the linear portion is getting less and less. Now, let us see you take this example let us take this example, m x double dot plus c x dot plus k x we have taken a non-linear spring. So, due to that non-linear spring this term is added to the system. Now, the system equation becomes m x double dot plus c x dot plus k x plus epsilon k x cube equal to f sin omega t. So, d x t can be written like this. So, f t equal to this so let us see whether this homogeneity and additive rules are violated or not.

So, to check the homogeneity rule let us replace this x t by alpha x t. When you replace this x t by alpha x t we got this expression. So, this is equal to m alpha x double dot plus c alpha x plus k alpha x plus epsilon k alpha x cube but, this epsilon k alpha x cube is not equal to alpha x cube so to obey this homogeneity rule. It should have been alpha epsilon k x cube. So, this is not equal to alpha f x so for that purpose this does not obey this homogeneity rule. Similarly, one can see that it violates the additive rules. Also, today's class we have briefly reviewed the two degree of freedom system, multi degree of freedom system and continuous systems so by using this model analysis principle we have seen that one can reduce a coupled equation to a set of un coupled equation and solved those equations.

As that of single degree of freedom systems and in case of continuous system one can use or one can solve those continuous systems by using different principles. So, one can use different approximate principle to find the system response and this non-linear system and linear system can be differentiated by using the superposition rule. While in case of linear system obey the superposition rule. So, we have seen with these example that the non-linear the superposition is violated. So, in case of non-linear system or one can distinguish a non-linear system by applying this superposition rule to its differential equation of motion. So, next class we will see that of the different phenomena associated with the non-linear system.

Thank you.