Non-Linear Vibration

Prof. S. K. Dwivedy
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 3
Solution of Nonlinear Equation of Motion
Lecture - 8
KBM Method of Averaging

Welcome to today class of Non-linear Vibration. So, today we are going to study KBM method that is Krylov Bogoliubov and Mitropolski method of averaging. So, this is one of the solution methods for this non-linear equation of motion.

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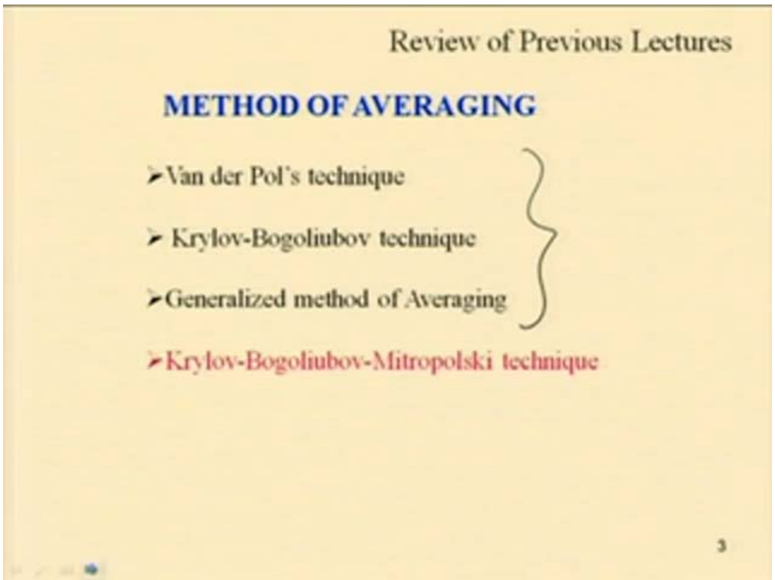

So, previously we have studied Van der Pol's equation, Krylov Bogoliubov technique and generalized method of averaging. So, these are different methods of averaging, and today class this Krylov Bogoliubov and Mitropolski technique will be dealt with. So, this is an extension of this Krylov Bogoliubov technique, which was developed in 1947 by this Krylov Bogoliubov, and later it was modified by this Bogoliubov one Mitropolski in 1961 and Mitropolski further used this is for non stationary system in 1965. So, in this method, so this is the extension of this Krylov Bogoliubov method.

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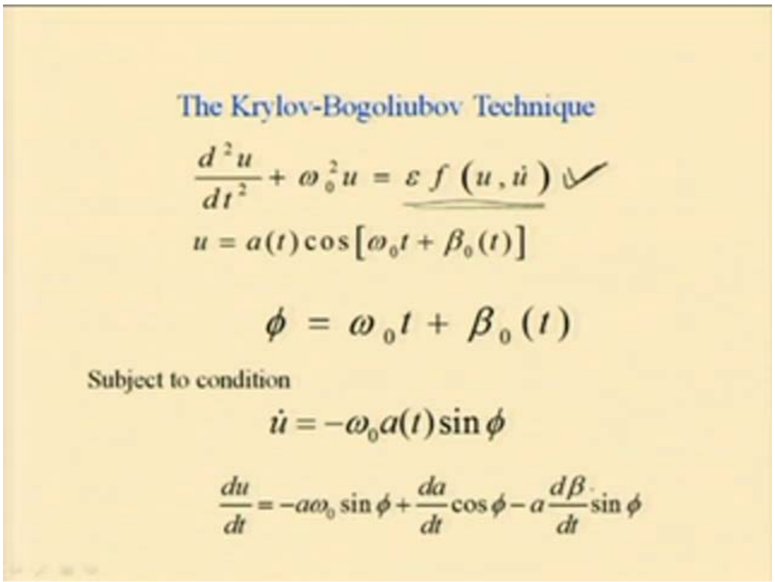

And already we know in case of the Krylov Bogoliubov method, if we are considering the differential equation in this form that is d square u by d t square plus omega 0 square u equal to epsilon f, which is a function of u and u dot. So, then we are assuming the solution to be a t cos omega 0 t plus beta 0 t. So, this is, in this case initially by assuming this part equal to 0, the solution can be written the as a cos omega 0 t plus beta, where, a and beta are constant. But when we are taking this right hand side term; that is epsilon f which is a function of u and u dot, then instead of assuming this a and beta to be constants, we are assuming this a and beta are function of time and for small non-linear terms, so we are assuming this a and beta are slow slowly varying function of time.

So, in this case by taking this phi equal to omega 0 t plus beta 0 t so this equation written in the form u equal to a t cos phi. So, in this case, they assumed that is Krylov and Bogoliubov; assumed that this u dot can be written in this form; that is u dot equal to minus omega 0 a t sin phi and this d u by d t equal to minus a omega 0 sin phi plus d a by d t cos phi minus a d beta by d t sin phi.

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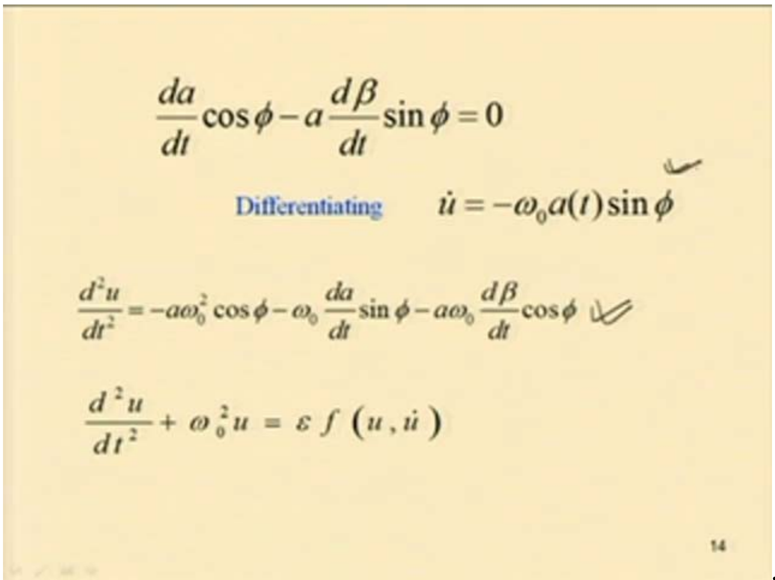

So, following in this way so, one can write this d square u by d t square equal to minus a omega 0 square cos phi minus omega 0 d a by d t sin phi and minus a omega 0 d beta by d t cos phi. So, substituting these equations that is, d square u by d t square and this u dot equal to minus omega 0 a t sin phi.

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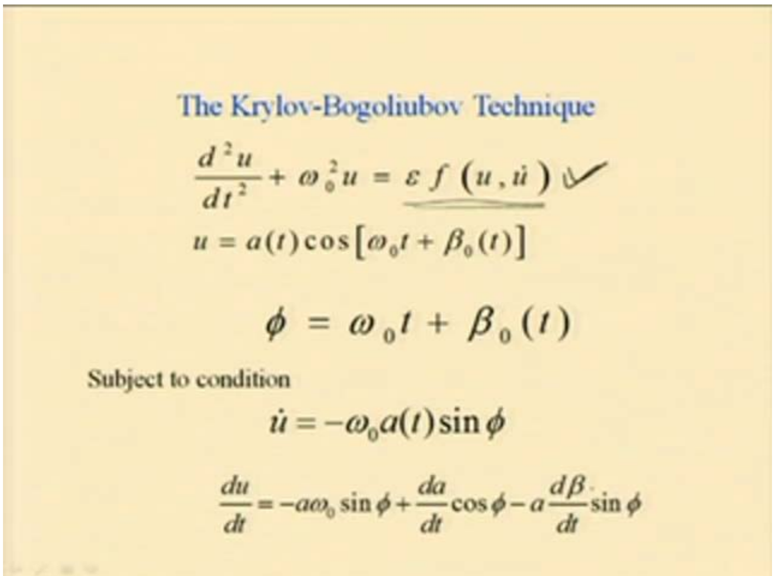

And also this d u by d t equal to minus a omega 0 sin 5 plus d a by d t cos phi minus a d beta by d t sin phi in the governing equation.

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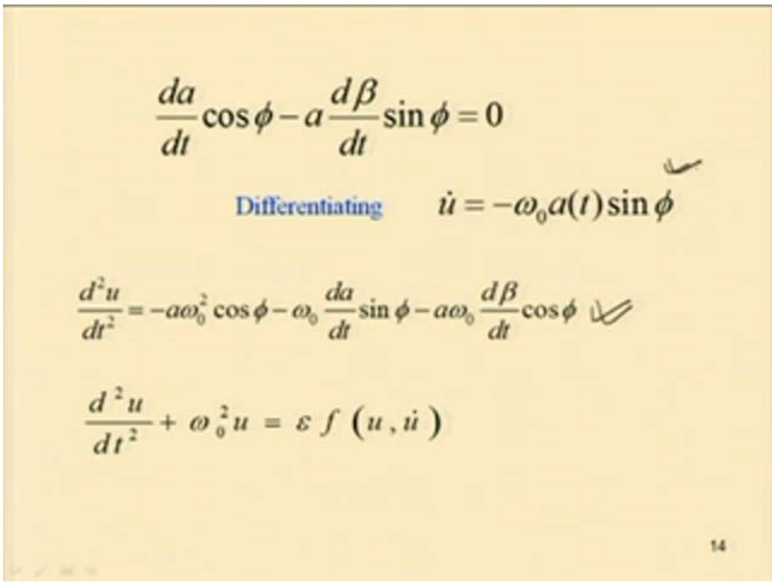

Then, one can write this equation.

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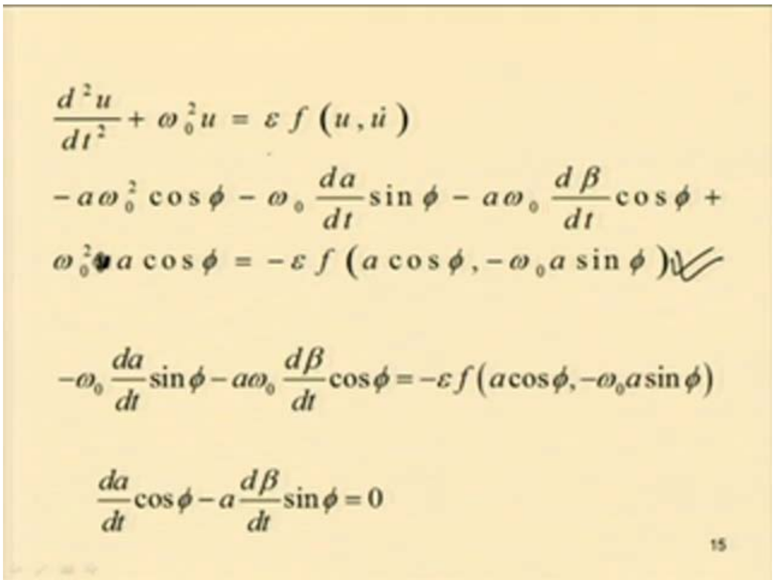

So, one can write the equation or one obtain this equation. So, were one can write this minus a omega 0 square cos phi minus omega 0 d a by d t sin phi minus a omega 0 d beta by d t cos phi plus omega 0 square u cos phi equal to minus epsilon a a cos phi and minus omega 0 s sin phi. So, for u one substitute this a cos phi. Now, from separating this equation one can write this minus omega 0 d a by d t sin phi.

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$$\frac{da}{dt} = -\frac{\varepsilon}{\omega_0}\sin\phi f\left(a\cos\phi, -\omega_0 a\sin\phi\right)$$

$$\frac{d\beta}{dt} = -\frac{\varepsilon}{a\omega_0}\cos\phi f\left(a\cos\phi, -\omega_0 a\sin\phi\right)$$

$$\frac{da}{dt} = -\frac{\varepsilon}{2\omega_0}\left[\frac{2}{T}\int_0^T \sin\phi f\left(a\cos\phi, -\omega_0 a\sin\phi\right)dt\right]$$

$$\frac{d\beta}{dt} = -\frac{\varepsilon}{2a\omega_0}\left[\frac{2}{T}\int_0^T \cos\phi f\left(a\cos\phi, -\omega_0 a\sin\phi\right)dt\right]$$

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So, one can equate the order of epsilon and write the equation and from those equations one can find this d a by d t equal to minus epsilon by omega 0 sin phi f a cos phi minus omega 0 sin phi. Similarly, d beta by d t equal to minus epsilon by a omega 0 cos phi f a cos phi minus omega 0 a sin phi.

Here, it is assumed that as a and beta are slowly varying function of time so, this d a by d t will be of the order of epsilon and so, the variation very small so, that one can average this with a period 0 to t and one can write this d a by d t in this form. So, in this Krylov Bogoliubov method so one can take this integration that is d a by d t equal to minus epsilon by 2 omega 0 2 by t 0 to t sin phi f a cos phi so, this is for u and this part is for u dot. So, this one can integrate it from or one can average it out in a cycle from 0 to t and one can write the expression for d a by d t and d beta by d t. So, for different function f if substituting this for u a equal to u equal to a cos omega phi and u dot equal to minus a omega 0 sin phi one can find this d a by d t and d beta by d t and from this by integrating one can find a and beta. So, after finding a beta one can find the solution of a system. So, this is the method of Krylov Bogoliubov but, in case of Krylov Bogoliubov and mitropolski so, there is a modification of this equation.

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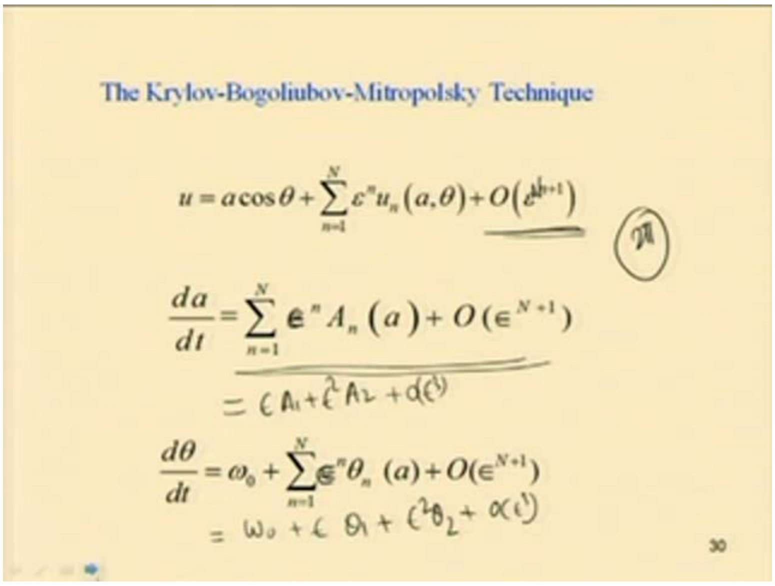

So, instead of using that integration so here, this is the solution can be assumed in this form. So the solution can be assumed as, a cos theta where theta equal to here omega 0 t. So, one can assume this u equal to a cos theta plus summation n equal to 1 to n epsilon n u n. Where, u n is a function of a and theta and one can neglect the order of epsilon to the power n plus 1. So, considering up to nth mode so, one can neglect up to neglect after n plus 1 so, one can write this is capital n plus 1.

So, this in this case this a and theta are function of time so, this d a by d t can be written so, this d a by d t so your u n is a function of and theta and this a. So, Krylov Bogoliubov according to this technique one can take this d a by d t in this form so, here u n is a periodic function of period 2 phi. Now, by taking this d a by d t in this form that is n equal to 1 to n epsilon n so, epsilon is the book keeping parameter so epsilon to the power n a n which is a function of a only. Similarly, d theta by d t equal to omega 0 plus n equal to 1 to n epsilon n theta n so, this is this epsilon is the book keeping parameter. So, here it is assumed this n capital n and theta n are function of a only. So, for example, if one take n equal to 2 so, this thing can be written as d a by d t can be written as epsilon a 1 plus epsilon square a 2.

Similarly, this theta d theta by d t can be written as omega 0 plus epsilon 1 theta 1 plus epsilon square so, epsilon so, this is epsilon theta 1 plus epsilon square theta 2. So, here we are neglecting the higher order that is order of epsilon cube is neglected here. So, by neglecting the order of epsilon cube one can write this d a by d t equal to epsilon a 1 plus epsilon square a 2. Similarly, this d theta by d t is written as omega 0 plus epsilon theta 1 plus epsilon square theta 2. So, instead of integrating as in case of this Krylov Bogoliubov method so, here it is written in this expansion form this d a by d t and d theta by d t are written in this expansion form.

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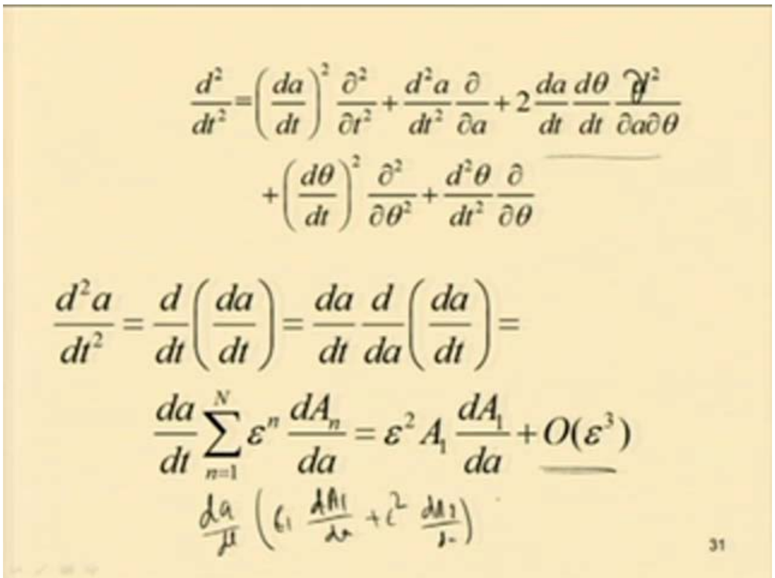

And now, one can write this d square by d t square equal to so, this d square d t square can be written d by d t of d by d t and by expanding that thing one can write this will be equal to d a by d t square into del square by d t square plus d square a by d t square into del by del a plus 2 into d a by d t. So, 2 into d a by d t into d theta by d t into d del square by so del square by del a del theta plus d theta by del t whole square into del square by del theta square plus square theta by d t square into del by del theta. Now, d square a by d t square can be written as d by d t of d a by d t.

So, again we know this d a by d t we are writing in the form of epsilon a 1 plus epsilon square a 2 plus epsilon cube a 3 if we are taking 3 terms or if we are taking n term it can be written in the summation form. So, one can write so, this d square a by d t square equal to d a by d t into d by d t of d a by d t. So, it will be equal to d by d t d a by d t into so, this n equal to 1 to n epsilon n d a n by d a. So, as we are substituting this d a by d t in terms of a n that is a 1 a 2 a 3 a 4 or a n. So, this d square a by d square can be written d a by d t n equal to 1 to n epsilon n d a n by d a. So, one can put for n equal to 1 so, this will become so, one can write this d a n by d a so d a by d t so, already d a by d t is written in this form that is epsilon a 1 epsilon square a 2. Now, differentiating that thing so, one can write so this will be epsilon square a 1 into d a 1 by d a plus order of epsilon cube. So, one can write expand this part that is d a by d t so, this thing can be written d a by d t so, for this part it will be equal to epsilon d a 1 so, one can write expanding this thing so, it can be written epsilon 1 d a 1 by d a plus epsilon square d a 2 by d a.

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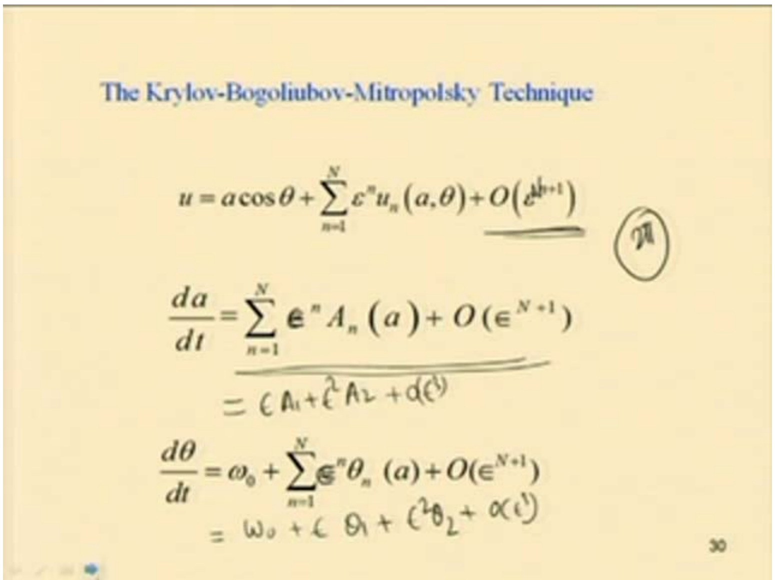

So, in this form by substituting this thing in this equation it can be written it is equal to epsilon square a 1 d a 1 by d a.

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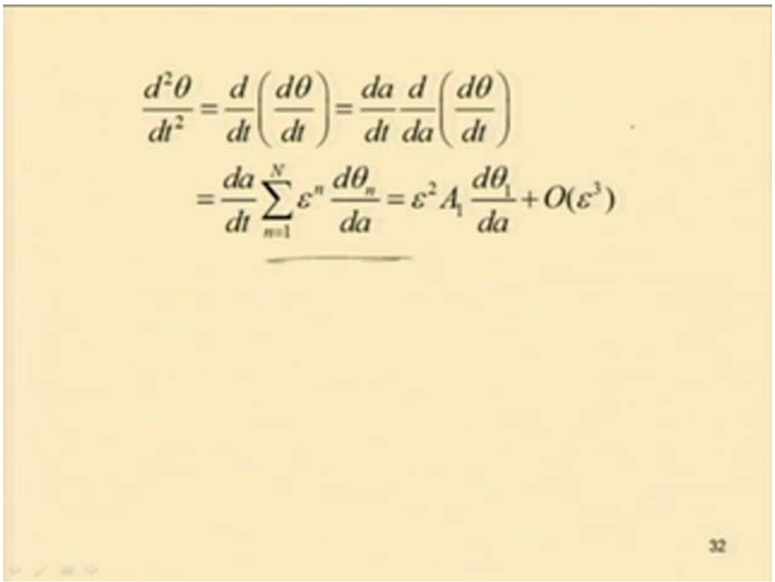

Similarly, d square theta by d t square can be written as d by d t of d theta by d t so, this is equal to d a by d t into d by d a of d theta by d t so, this will be equal to so, already we know this d theta by d t in this form so, it can be written as d by d t of this so, it will be by expanding this thing one can write so this is equal to epsilon square a 1 d theta 1 by d a. Now, substituting these equations in the governing equation and separating the order of epsilon one can get a set of equations and from those set of equation if one can kill the secular terms so, one can find the required equations for the governing solution of the governing equation.

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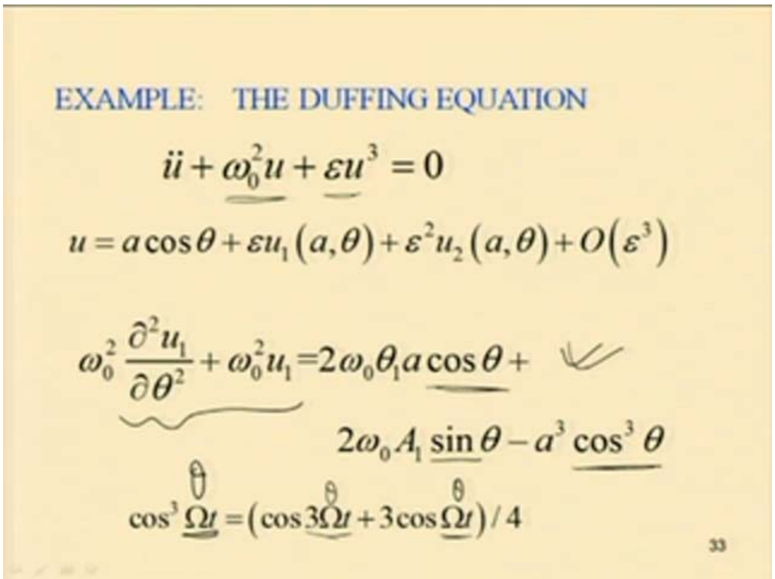

So, we can see one example that is the duffing equation let us take the cubic order polynomial. So, in this case one can write this u double dot this equation can be written in this form that is u double dot plus omega 0 square u plus epsilon u cube equal to 0. So, in this case we can assume let us assume up to second order so, we can write this u equal to a cos theta plus epsilon u 1 a theta plus epsilon square u 2 a theta.

Now, for this u double dot by substituting this u double dot means d square u by d t square so, one can write thing d square u by d t square. So, this thing can be written as d a by d t whole square plus del square u by d t square plus d square a by d t square del u by del a plus 2 d a by d t del theta by d t del square u by del a d del theta plus del theta by d theta by d t whole square into del square u by del theta square plus d square theta by d t square into del u by del theta.

But, in this case now again substituting this d square a by d t square equal to epsilon square a 1 d a by d a 1 d a and d square theta by d t square equal to epsilon square a 1 d theta 1 by d a and substituting this u equal to a cos theta plus epsilon u 1 plus epsilon square u 2 and separating the order of epsilon 1 can obtain a set of equations. Now, one can see so, for d square u we have i have shown you that equation what you have to use for omega 0 square u simply multiplying this omega 0 square term one can get this omega 0 square a cos theta plus omega 0 square u 1 a theta plus omega 0 square epsilon square u 2. Now, for this term epsilon u cube now, one can cube this term so, it will be a plus b so, it can follow the form a plus b plus c whole cube.

So, one can find the cube of this term and by expanding this and then, separating the order of epsilon; so, one can get the first equation, one can get this is the first equation So, it will be omega 0 square del square u 1 by del theta square plus omega 0 square u 1 so, this will be equal to 2 omega 0 theta 1 a cos theta plus 2 omega 0 a 1 sin theta minus a cube cos cube theta.

Now, writing this cos cube theta equal to cos 3 omega t so, for this omega t 1 can put theta so cos cube theta can be written as cos 3 theta plus 3 cos theta. So, substituting this cos cube theta equal to 3 cos theta plus cos 3 theta plus 3 cos theta by 4. So, one can write this equation that is omega 0 square del square u 1 by del theta square plus omega

0 square u 1 equal to 2 into omega 0 into theta 1 a cos theta plus 2 omega 0 a 1 sin theta minus a cube into a cube by 4 into cos 3 theta plus 3 cos theta.

Now, due to the presence of this term that is sin theta and this cos theta so, the solution will leads to infinity because the left hand side we have this term is omega 0 del square u 1 by del theta square plus omega 0 square u 1.

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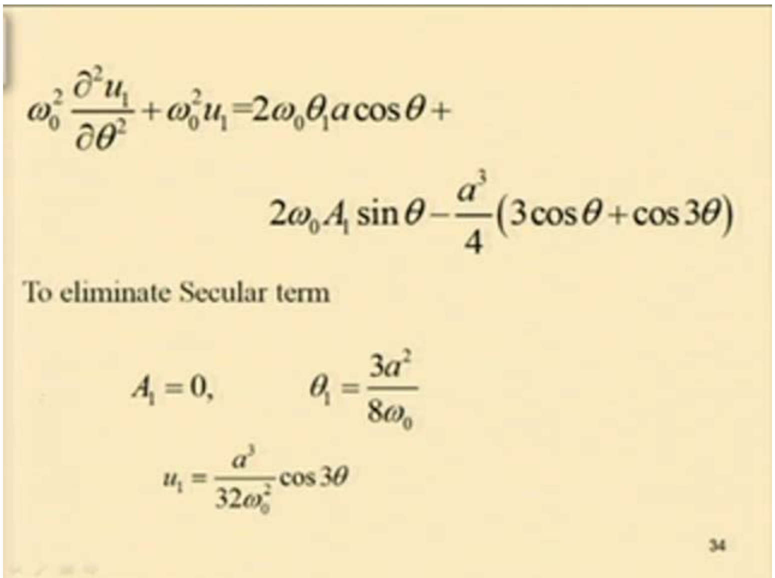

So, if we will take this cos term then in this case so, we will have for example, so in this case as the left hand side we have omega 0 square. So, in the left side we have omega 0 square del square u 1.

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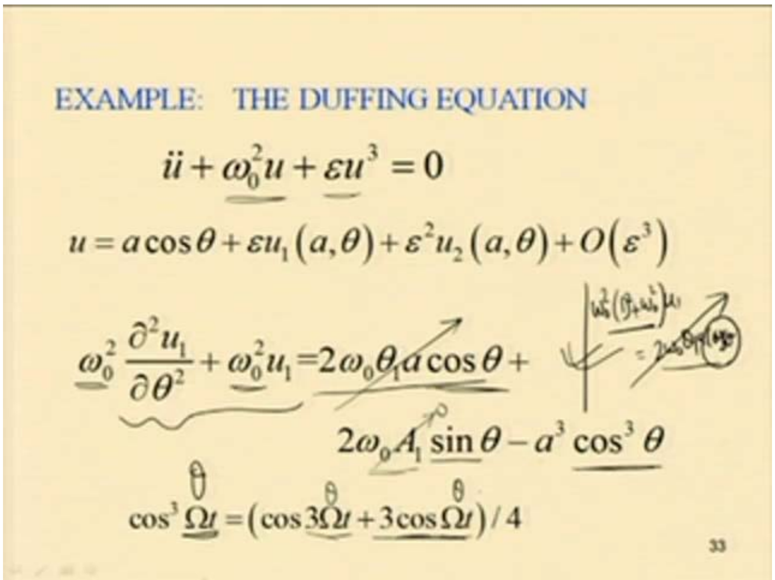

So, we can see this term so, this is omega 0 square, omega 0 square del square u 1 by del theta square plus this omega 0 square we can take common. So, in this case it will become omega 0 square into del square u 1 by del theta square plus u 1. So, the auxiliary equation becomes d square plus u 1 so, d square plus u 1 will be equal to so, d square plus so, the auxiliary equation I can write in this so, this is omega 0 into d square plus omega 0 square so, u 1 so, this will be equal to 2 omega 0 theta 1 a cos theta. If you take only this term so this due to the presence of this theta so, theta equal to our omega t plus omega 0 t so, as it contain this theta contain omega 0 t.

So, this will be a secular term as we can by substituting this omega 0 here so, d square will be equal to minus omega square so, this term will leads to so this term will infinity. So, this will be a secular term. Similarly, this will be a secular term and for cos cube theta we have substituted this term that is 3 cos theta so, that will also be a secular term. So, to eliminate the secular term we have to equate the coefficient of sin theta equal to 0 and coefficient of cos theta equal to 0. So, by equating the coefficient of sin theta equal to 0 so, that mean 2 omega 0 a 1 equal to 0 so, we have this a 1 equal to 0. So, we got this condition that a 1 equal to 0. Otherwise, if i not putting a 1 equal to 0 so, one can find the solution to be infinite this is not possible in particle system so, this term has to be eliminated that means a 1 equal 0.

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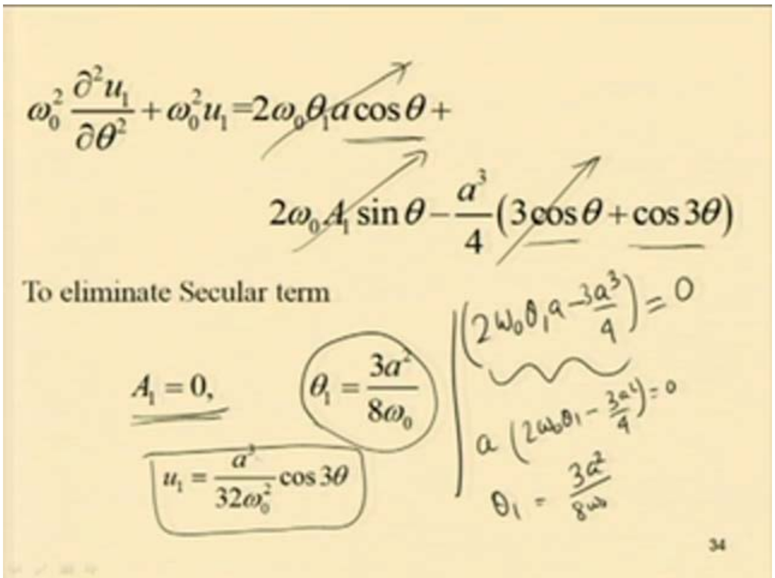

Similarly, taking this term and the part of this a cube 3 by 4 3 by 4 cos theta so, one can write the second equation so, after substituting this cos cube theta one can written this equation in this form. Now, equating the coefficient of cos theta so this is cos theta and this term is also cos theta so, one can write this 2 omega 0 theta 1 a minus this a cube by 4 3 a cube i so this is 3 a cube i 4 so this is cos theta.

Now, to eliminate the secular term so, we have to substitute this part equal to 0. So, by writing this is equal to 0.So, we can write by taking this a common so, one can write a into 2 omega 0 theta 1 minus 3 a square by 4 equal to 0 or either a equal to 0 or so a equal to 0 corresponding to the trivial solution. Otherwise, one can write this theta 1 equation to 3 a square by 4 into 2 that is, 8 omega 0. So, we got this condition that is theta 1 equal to 3 a square by 8 omega 0 and a 1 equal to 0. So, by eliminating this secular term we got this expression.

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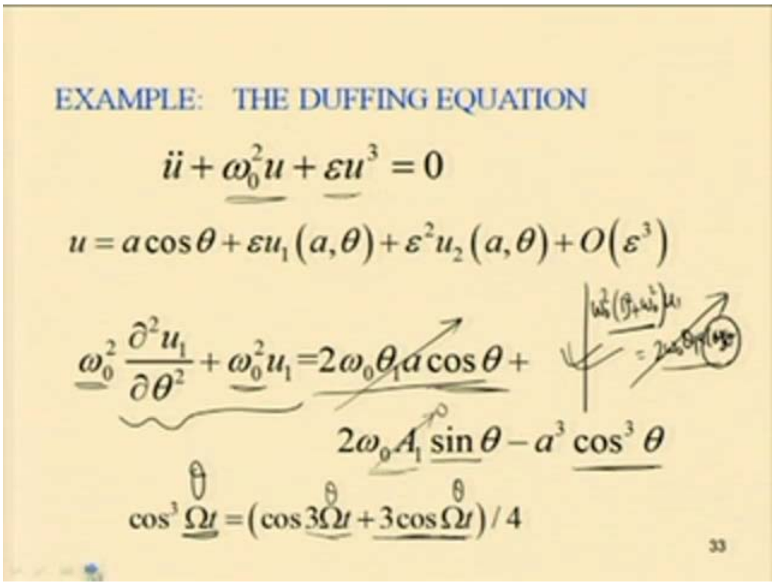

So, in this case instead of taking 2 terms if one take one term only that is u equal to a cos theta plus epsilon u 1 a theta then, one can find one can go up to this term and one can find the solution so, the solution could have been u equal to a cos theta plus epsilon u 1.

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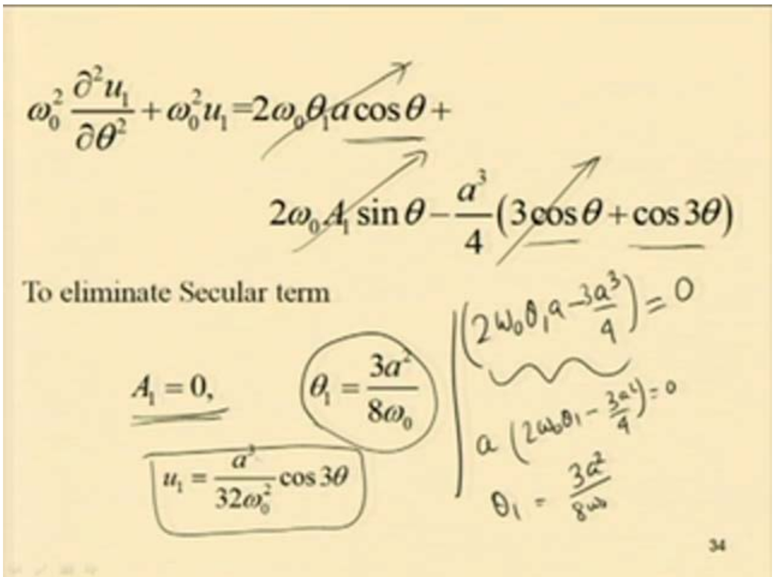

So, in this case after eliminating this secular term the remaining terms will be so, this term is a secular term this is also a secular term and this is also a secular term. Now, the remaining term is this that is cos 3 theta minus a cube by 4 cos 3 theta. So, the solution will be the equation remaining equation becomes del square u 1 by del theta square plus omega 0 square u 1 equal to minus a cube by 4 cos 3 theta. So, the solution of u 1 will be equal to a cube; so, this is a cube by so, this is 4; so and we have to divide this omega 0 square so this becomes so in this case this becomes a cube by 4 cos 3 theta by so, if I will write the auxiliary equation so it will be by d 0 square plus omega 0 square.

Now, in place of d 0 square I have to substitute this 3 square that is 3 square equal to 9 so, minus 9 omega 0 square so, minus 9 omega 0 square plus omega 0 square this becomes minus 8 omega 0 square so, 8 into 4 8 into minus 4 and this has a minus sin so, this becomes u 1 equal to u 1 become a cube by 32 omega 0 square cos 3 theta. So, we obtain this u one equal to a cube by 32 omega 0 square cos 3 theta and this a 1 equal to 0.

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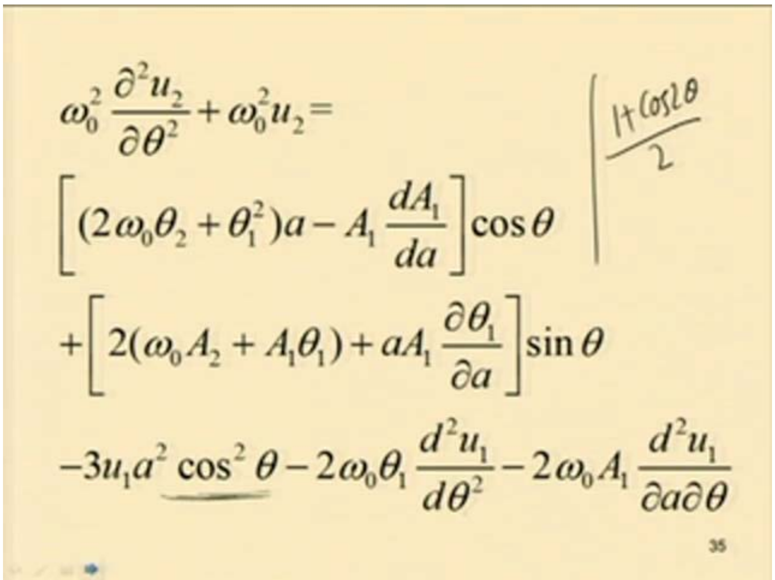

Now, by substituting this u 1 in the second equation so, in this case this is the first equation the second equation we obtain to be this that is your omega 0 square del square u 2 by del theta square plus omega 0 square u 2. So, this is will be equal to 2 omega 0 theta 2 plus theta 1 square a minus a 1 into d a 1 by d a into cos theta plus 2 omega 0 a 2 plus a 1 theta 1 plus a 1 del theta 1 by del a sin theta minus 3 u 1 a square cos square theta minus 2 omega 0 theta 1 d square u 1 by d theta square minus 2 omega 0 a 1 d square u 1 by d a d theta. So, in this case we can substitute this cos square theta so, in terms of cos theta we can write so, 1 plus cos 2 theta so, this cos square theta can be written as 1 plus cos 2 theta by 2. So, we can substitute that thing in this equation.

And now, similar to the previous case we can see that the coefficient of this cos theta or the terms containing this cos theta and sin theta should be eliminated to eliminate the secular terms. Otherwise, these terms will lead to the infinite response which is not practically possible. So, one has to eliminate the secular term so, to eliminate the secular term so, we can have we have to substitute this part equal to 0. Similarly, we have to substitute this part equal to 0 so, by substituting this is equal to 0 so, what you can get so, you obtain.

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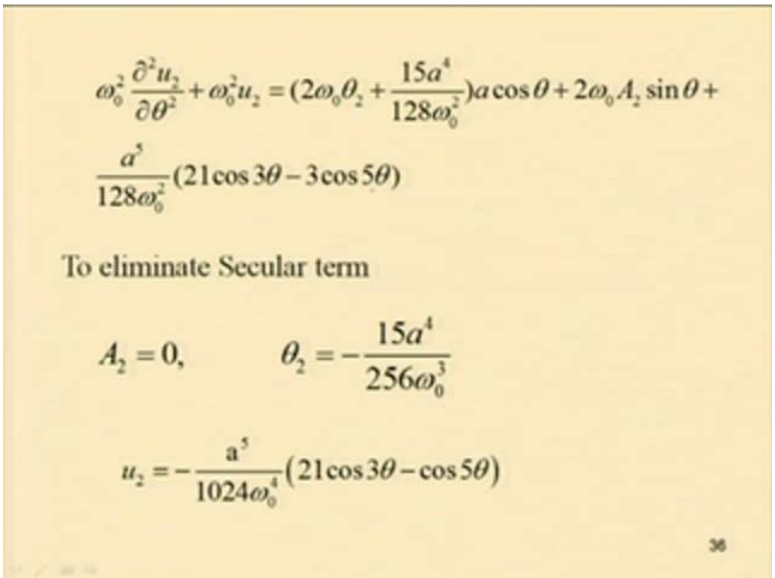

So, in this case we can obtain this so, this is the equation after simplification. So, after simplification the equation reduces to this omega 0 square del square u 2 by del theta square plus omega 0 square u 2 equal to 2 omega 0 theta 2 plus 15 a forth by 128 a 0 omega 0 square into a cos theta plus 2 omega 0 a 2 sin theta plus a to the power 5 128 omega 0 square into 21 cos 3 theta minus 3 cos 5 theta.

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$$\omega_0^2 \frac{\partial^2 u_2}{\partial \theta^2} + \omega_0^2 u_2 =$$

$$\left[(2\omega_0\theta_2 + \theta_1^2)a - A_1 \frac{dA_1}{da}\right]\cos\theta \qquad \left| \frac{1+\cos 2\theta}{2} \right.$$

$$+\left[2(\omega_0 A_2 + A_1\theta_1) + aA_1 \frac{\partial \theta_1}{\partial a}\right]\sin\theta$$

$$-3u_1 a^2 \cos^2\theta - 2\omega_0\theta_1 \frac{d^2 u_1}{d\theta^2} - 2\omega_0 A_1 \frac{d^2 u_1}{\partial a \partial \theta}$$

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So, we obtain this thing by substituting the value of u 1 what we obtained before so, this u 1 equal to a cube by 32 omega 0 square cos theta. So, we have substituted in this equation in this part.

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$$\omega_0^2 \frac{\partial^2 u_2}{\partial \theta^2} + \omega_0^2 u_2 = (2\omega_0\theta_2 + \frac{15a^4}{128\omega_0^2})a\cos\theta + 2\omega_0 A_2 \sin\theta +$$

$$\frac{a^5}{128\omega_0^2}(21\cos 3\theta - 3\cos 5\theta)$$

To eliminate Secular term

$$A_2 = 0, \qquad \theta_2 = -\frac{15a^4}{256\omega_0^3}$$

$$u_2 = -\frac{a^5}{1024\omega_0^4}(21\cos 3\theta - \cos 5\theta)$$

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And now by clubbing the coefficient of cos theta and sin theta so, we have written this equation in this form. So, the resulting equation becomes this omega 0 square del square u 2 del theta square plus omega 0 square u 2 equal to 2 omega 0 theta 2 plus 15 a fourth by 128 omega 0 square a cos theta plus 2 omega 0 a 2 sin theta plus a to the power 5 by 128 omega 0 square into 21 cos 3 theta minus 3 cos 5 theta.

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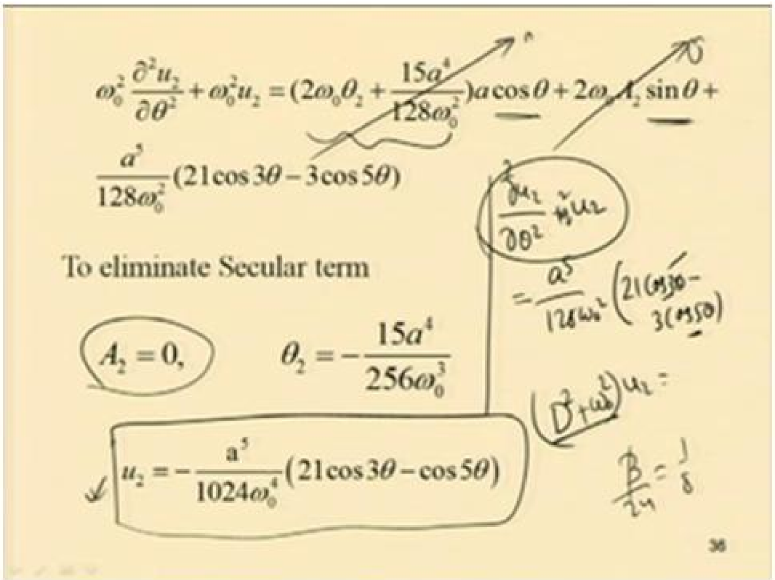

So, similar to previous case here also we have to eliminate the coefficient of cos theta and sin theta separately. So, eliminating the coefficient of sin theta so, we can write this a 2 equal to 0 because the coefficient of sin theta equal to 2 omega 0 a 2 so, this a 2 should be equal to 0 so, a 2 so, we have to we have put this a 2 equal to 0. Now, similarly, killing this coefficient of this cos theta term that is, 2 omega 0 theta 2 plus 15 a fourth by 128 omega 0 square so, we can put this term equal to 0 so, by substituting that thing we can write this theta 2 so, this theta 2. Now, it becomes minus 15 a fourth by 256 so, this 2 can be multiplied here. So, this theta 2 becomes 15 minus 15 a fourth by 256 omega 0 square is here and omega 0 multiplied again so, this becomes minus 256 omega cube.

So, we can write now these terms have been eliminated so, as this term and this terms we have already eliminated the resulting equation becomes omega 0 square del square u by del square u 2 by del theta square plus omega 0 square u 2 equal to a to the power 5 128 omega 0 square into 21 cos 3 theta minus 3 cos 5 theta. Now, the solution of this u 2 so, we can so, the auxiliary equation so, we have this by dividing this omega 0 square so, we have this equation del square u 2 by del theta square plus. Now, we have only this u 2 so, this will becomes so, we can write this term so, that means we can write this is equal to a to the power 5 by 128 omega 0 square into 21 cos 3 theta minus 3 cos 5 theta so, in this case cos 3 that cos 3 so, this is omega 0 square we will have. So, in this case so, by substituting for this for this term that is your or auxiliary equation will be D square plus omega 0 square so, as the auxiliary equation will be this into u 2 will be in the right hand side.

So, in place of this d square we have to substitute this square of this 3 or square of this 5. So, we have to substitute minus 9 omega 0 square or we have to substitute in this case we have to substitute omega. So, we have to substitute in second case we have to substitute minus 25 in the first case we have to substitute minus 9 so, minus 9 plus 1 that is 8. So, if you multiplied that thing it becomes 1024 omega 0 fourth. And similarly, this 3 by so, here 24 3 by 24 so, this 3 by 24 so, this becomes 1 by 8 so, this 1 by 8 when it is multiple with this 256 this becomes also 1024. So, one can obtain this u 2 so, this u 2 becomes minus a to the power 5 1024 omega 0 omega 0 to the power 4 into 21 cos 3 theta minus cos 5 theta. So, we now obtain u 2 also previously we obtain u 1 and this theta 1 and theta 2. Similarly, we obtain this a 1 and a 2.

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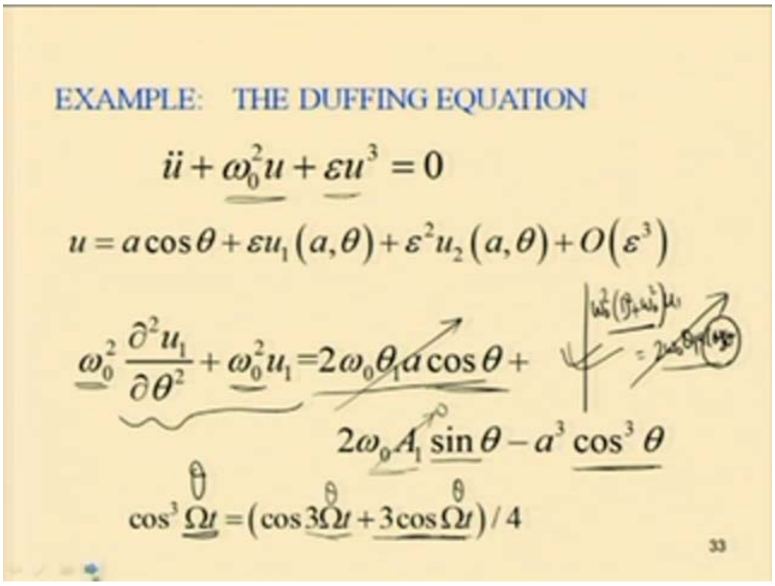

So, after obtaining all these things so, one can now refer back to the original equation that is this u equal to a cos theta plus epsilon u 1 plus epsilon square u 2. So, we already know u 1, expression for u 1 and expression for u 2.

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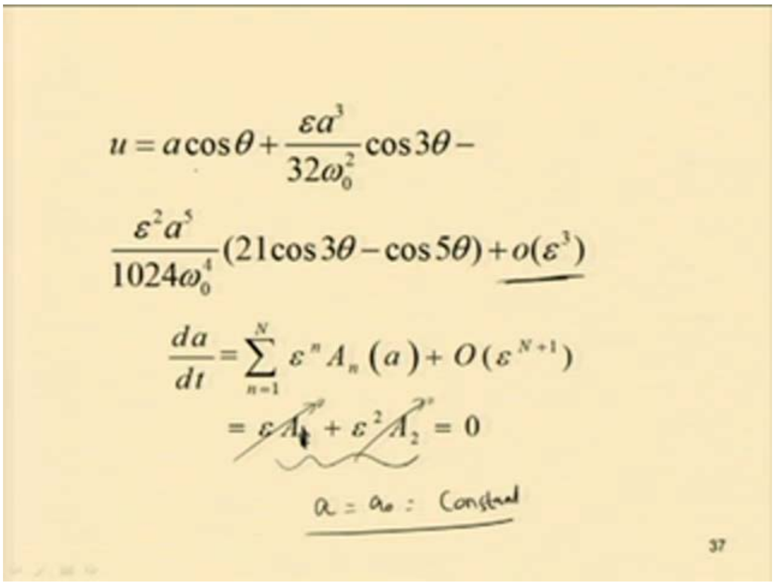

So, by substituting these expressions we can write. Now, let us substitute this expression and see. So, we can write this u equal to a cos theta plus epsilon so, for u 1 we are substituting a cube by 32 omega 0 square cos 3 theta and for this u 2 we have substituted this equation. So, it becomes epsilon square a to the power 5 by 1024 omega 0 fourth into 21 cos 3 theta minus cos 5 theta. So, this is now so, we are we have neglected the order of epsilon cube. So, in this case still now, we do not know what is a and theta.

Now, we can substitute so or we know this d a by d t equal to epsilon a 2 plus epsilon a 1 plus epsilon square a 2 and already we obtain this a 1 equal to 0 and a 2 equal to 0 previously by eliminating these secular terms we already got a 1 equal to 0 and here in the second case we got this a 2 equal to 0. So, by substituting these 2 so, we can get so this part equal to 0 and this a 2 also equal to 0 so, this d a by d t equal to 0. As d a by d t equal to 0 so, by integrating one can get so, a will be a constant so on one can write this a equal to a 0 so, this is a constant. So, in this equation u equal to a cos theta so, here we obtain this a to be a constant.

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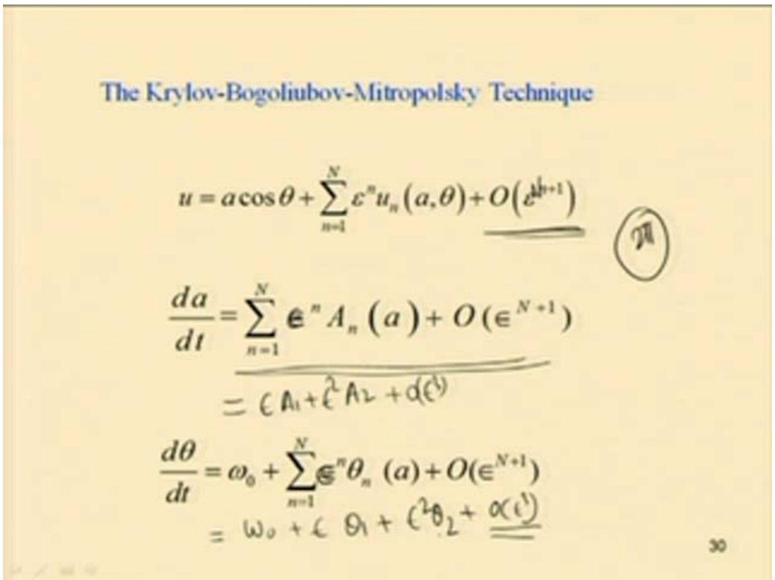

Similarly, now for theta so, this d theta by d t we can write so, following this equation so, we can write this d theta by so, we know this d theta d t expression so, we can write d theta by d t equal to omega 0 plus epsilon theta 1 plus epsilon square theta 2 plus higher order terms.

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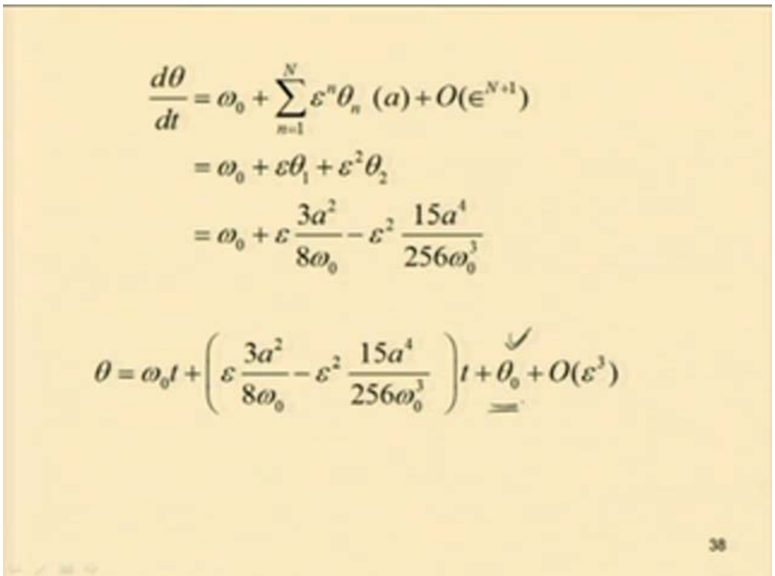

So, already we obtain this theta 1 and theta 2 so we can write so, this in this case we can write this d theta by d t equal to omega 0 plus epsilon 1 theta 1. So, for theta 1 we have written this is equal to 3 a 3 a square by 8 omega 0. So, for theta 2 we have written this epsilon square into 15 a to the power 4 by 256 omega 0 cube. So, already we know a is constant so, we can write this theta will be equal to by integrating this thing so, we can write so this will be multiplied by t plus theta 0 so theta. So, this expression of theta becomes omega 0 t plus epsilon into 3 a square by 8 omega 0 minus epsilon square into 15 a fourth by 256 omega 0 cube into t so plus this theta 0.

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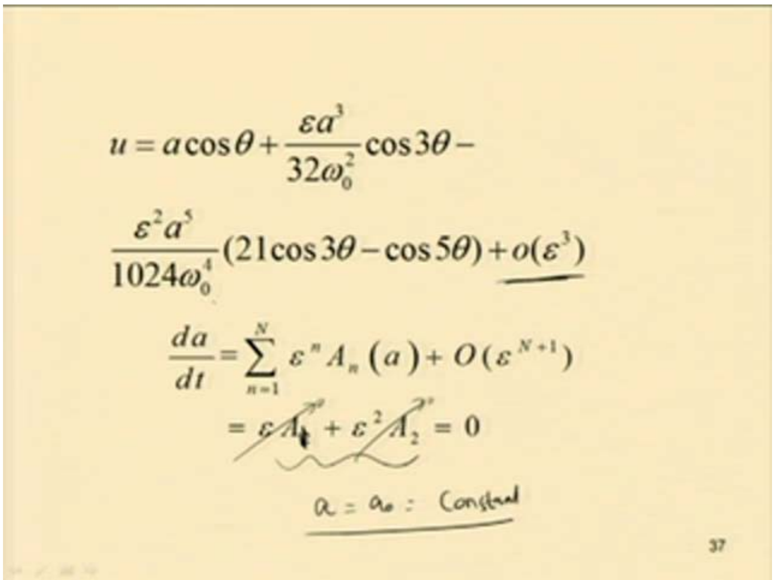

So, from initial condition one can get this a 0 and theta 0 and one can write the final expression of the system in this form that is, u equal to a cos theta plus epsilon a cube by 32 omega 0 square cos 3 theta minus epsilon square a to the power 5 by 1024 omega 0 fourth into 21 cos 3 theta minus cos 5 theta.

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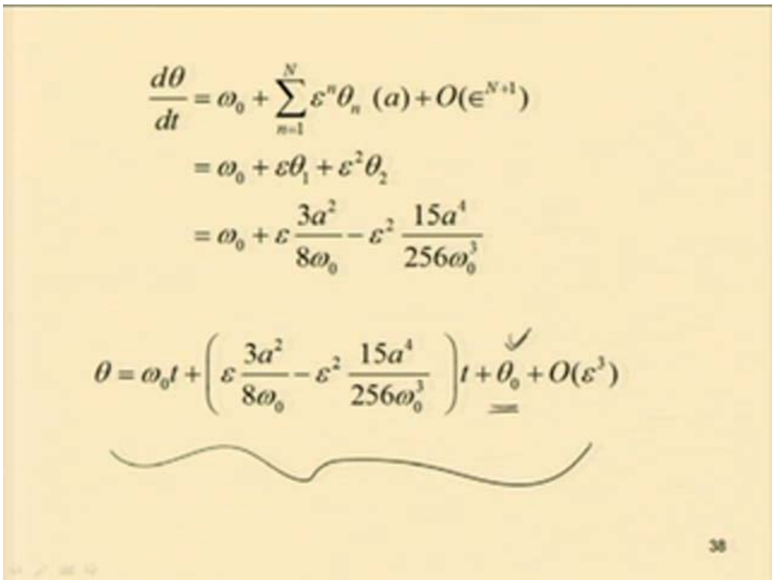

Here, theta equal to this. So, in this way by using this k b m method that is Krylov Bogoliubov and mitropolski method. So, one can find the solution of a non-linear equation.

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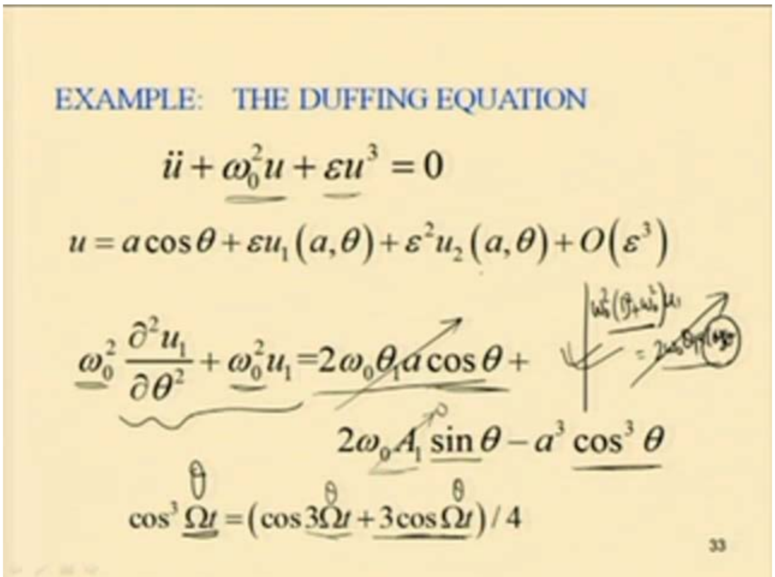

So, one can go up to any accuracy by increasing this order of epsilon. So, instead of taking a single term or 2 term one can take higher order terms to find the solution of the governing equation. But, adding higher order terms will make the calculation or make the computation very cumbersome so, in that case one can use the computer to find the solution in case of the higher order terms. So, for manual calculation one may go up to the second order term and one can find the solution.

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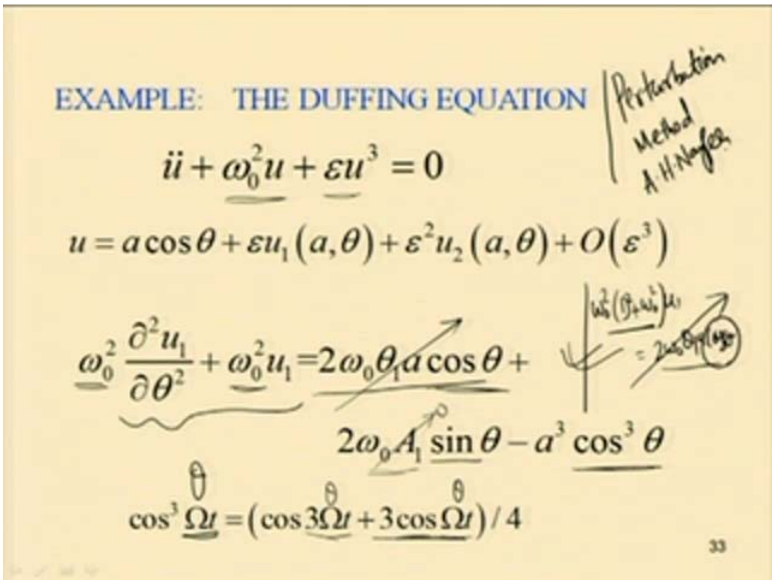

So, this method has been followed from the book by Nayfeh perturbation method so, from the book of perturbation method so, it is followed from the example by the book perturbation method by A. H. Nayfeh.

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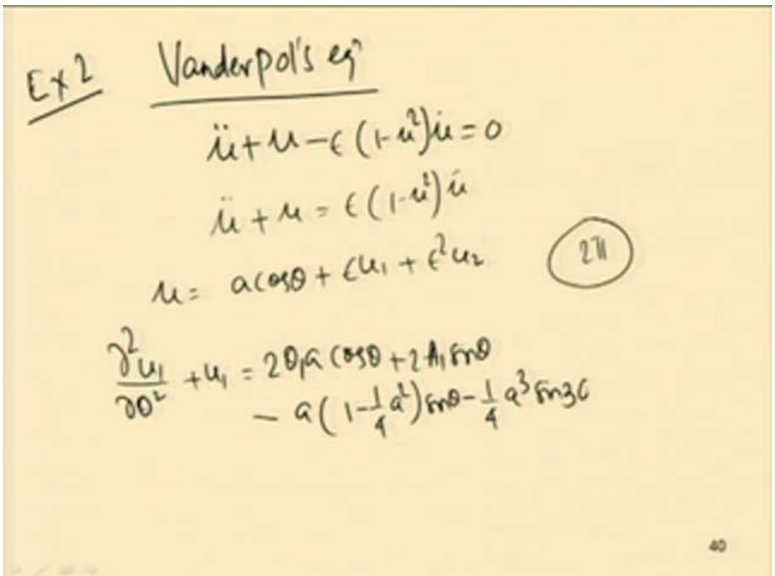

So, we can take another example so, that is van der pol equation also, let us take the second example that is van der pol equation and in this case also we can write the equation in this form though van der pol equation can be written in this form that is so, let us take the second example of van der pol's equation, example 2.

So, in this case the equation can be written in this form; u double dot plus u so, minus epsilon into 1 minus u square into u dot equal to 0. So, this is the equation or this equation one can write in this form also u double dot plus u equal to epsilon into 1 minus u square into u dot. Now, by substituting u equal to a cos theta plus epsilon u 1 plus epsilon square u 2 where this u n are function of periodic function of theta so, with a u 1 u 2 are periodic function of theta with a period of 2 phi. So, following the previous case so we can write this equation by substituting this equation in the first equation we can write this del square u by so we can write this equation in this form so, del square u 1 by del theta square plus u 1 will be equal to 2 theta 1 a cos theta plus 2 a 1 sin theta minus a into 1 minus 1 by 4 a square sin theta minus 1 by 4 a cube sin 3 theta.

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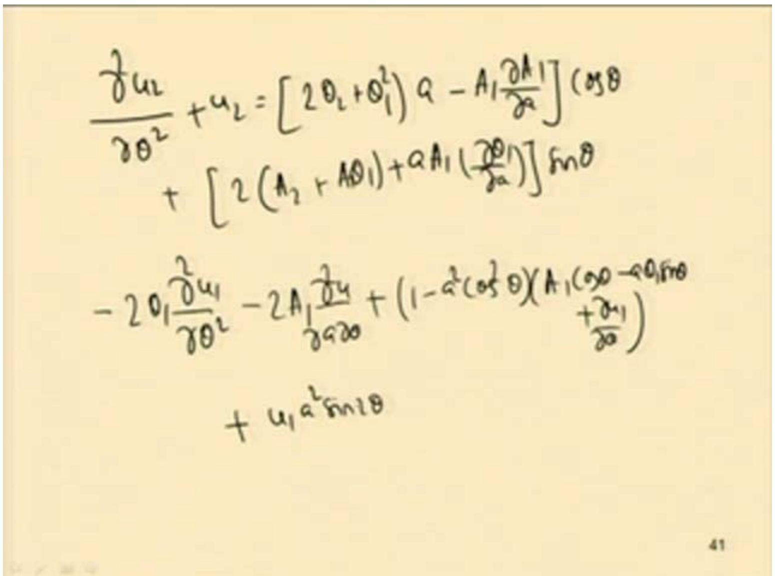

Similarly, the second equation can be written in this form that is del square u 2 by del theta square plus u 2 so, this will be equal to 2 theta 2 plus theta 1 square into a minus a 1 into del a 1 by del a small a. So, this multiplied by cos theta. Similarly, plus 2 into a 2 plus a 1 theta 1 plus a 1 del theta 1 by del a so, this into sin theta. So, this equation again can be written in this form that is minus 2 theta 1 del square u 1 by del. So, the first equation so, here we can write it again to in this form that is 2 theta 1 del square u 1 by del theta square minus 2 a 1 del square u 1 by del a del theta plus 1 minus a square cos square theta into a 1 cos theta minus a theta 1 sin theta plus del u 1 by del theta so, plus u 1 a square sin 2 theta.

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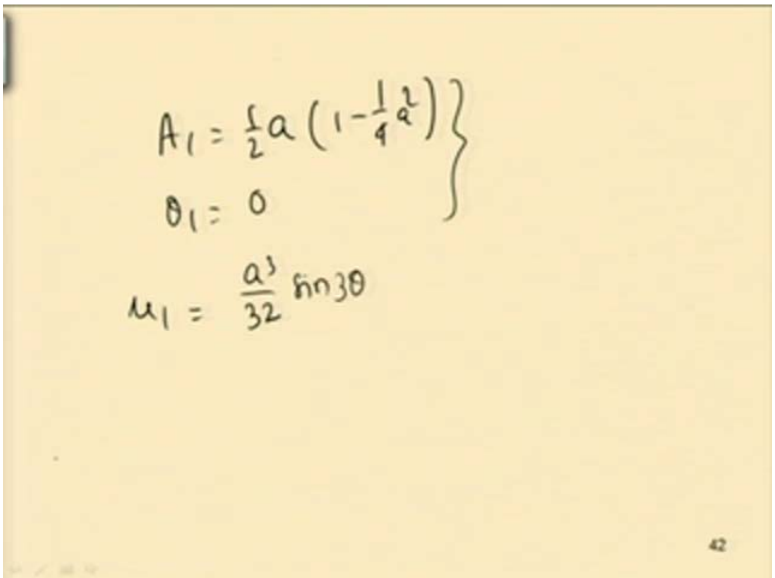

So, from this we can obtain by eliminating the secular term so, we can write this a 1 equal to half a 1 minus 1 by 4 a square and this theta 1 equal to 0. Now, we can write this u 1 equal a cube by 32 by eliminating the secular term we can write this u 1 equal to a cube by 32 sin 3 theta.

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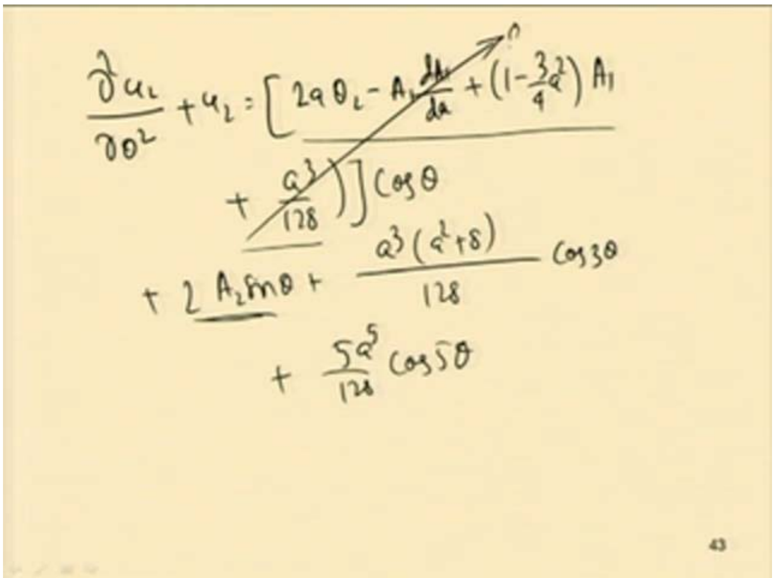

So, eliminating secular term so, we can write this a 2 equal to 0; and also we can write this theta 2 so from this we can write so, from this expression we can find theta 2; that is 2 A theta 2 minus a 1 into d A 1 by d a plus 3 plus 1 minus 3 by 4 a square a 1 plus a cube by 128 this thing should be equal to 0.

And substituting this equation in the second equation so, we can write this del square u 2 by del theta square plus u 2. So, this is equal to 2 a theta 2 minus a 1 into d a 1 by so, this is d a 1 by d a plus 1 minus 3 by 4 a square into a 1 plus a cube by 128 into cos theta plus 2 A 2 sin theta plus a cube into a square plus 8 by 128 cos 3 theta plus 5 a to the power 5 by 128 cos 5 theta. So, following the previous method here also we can eliminate the secular term. So, the secular term can be eliminated by eliminating the coefficient of cos theta and sin theta so, here coefficient of sin theta equal to a 2 so, we have to eliminate this term a 2. So, we have to put this, a 2 equal to 0. Similarly, by eliminating the coefficient of cos theta so, we have to eliminate this part. So, from this already we know the expression for a 1.

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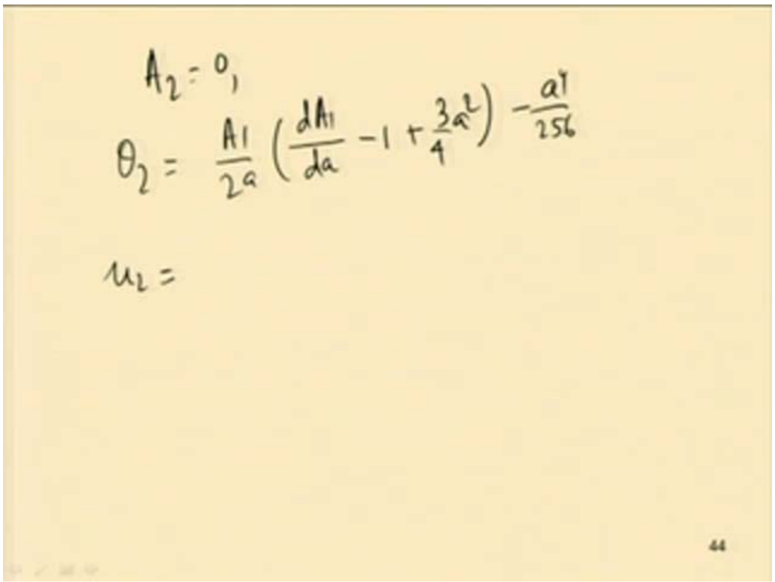

So, by writing the expression for a 1 in this case and eliminating the secular term so, we can write this, a 2 equal to 0.

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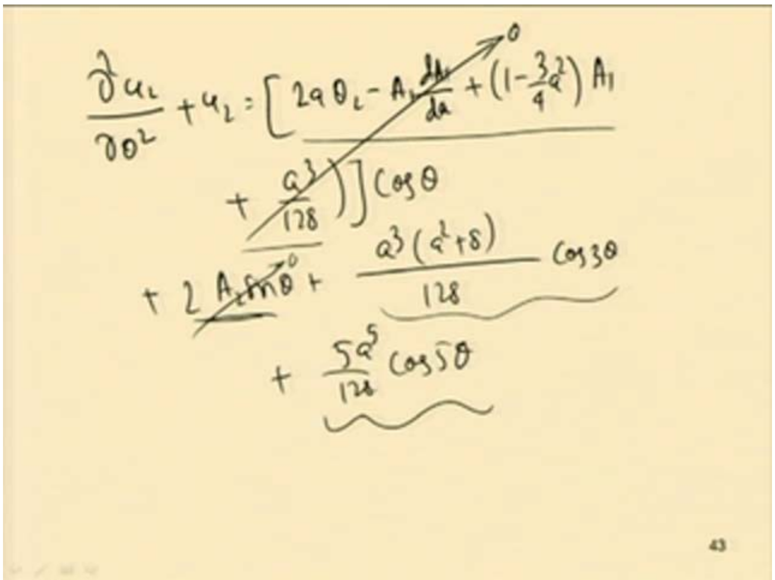

And also you can write this theta 2. So, from this expression we can find theta 2 that is, 2 a theta 2 minus A1 into d A1 by d a plus 1 minus 3 a square A1 plus a 3 by 128, this thing should be equal to 0. So, by equating this part equal to 0 that means, coefficient of cos theta.

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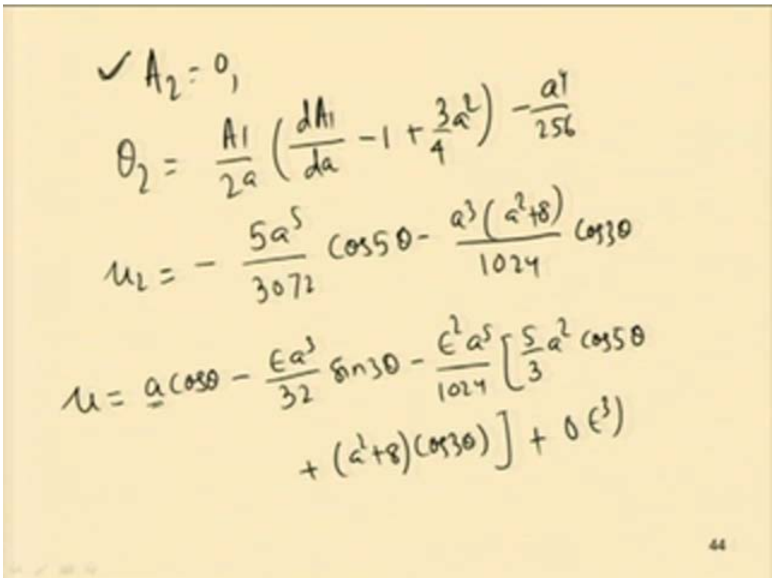

So, by equating this equal to 0, we can write theta 2 equal to so, this will be of theta 2 expression. So, theta 2 equal to A 1 by 2 a into d A 1 so, this becomes by d A 1 by d a minus 1 plus 3 by 4 a square minus a to the power 4 by 256. Now, as we have obtained this or eliminated this term and this these 2 terms we have eliminated. So, the remaining terms we have 2 remaining terms that is this term and this term. So, from this we can write the expression for u 2. So, we can write this u 2 will be equal to so, in this case the auxiliary equation comes d square plus 1. So, by substituting for d square for this term minus 9 so, this becomes by 8 so 128 into 8. Similarly, in this case this is minus 25 minus 25 plus 1 this becomes minus 24 so, minus 24 so 5 by 128 and in the bottom or denominator part we have this minus 24.

So, we can write this expression for u 2 in this form so, u 2 will be equal to minus 5 a to the power 5 by 3072 cos 5 theta minus a cube into a square plus 8 by 1024 cos 3 theta. Now, already we have this expression for A 1 A 2 theta 1 theta 2 and u 1 u 2. So, we can write this u so, this is equal to a cos theta u equal to a cos theta epsilon u 1 plus epsilon square u 2 so, or this we can write this is equal to minus a cos theta minus epsilon a cube by 32 sin 3 theta minus epsilon square a 2 the power 5 by 1024 into 5 by 3 a square cos 5 theta plus a square plus 8 into cos 3 theta so, this into so this plus order of epsilon cube. So, in this case till now we do not know what is a and theta.

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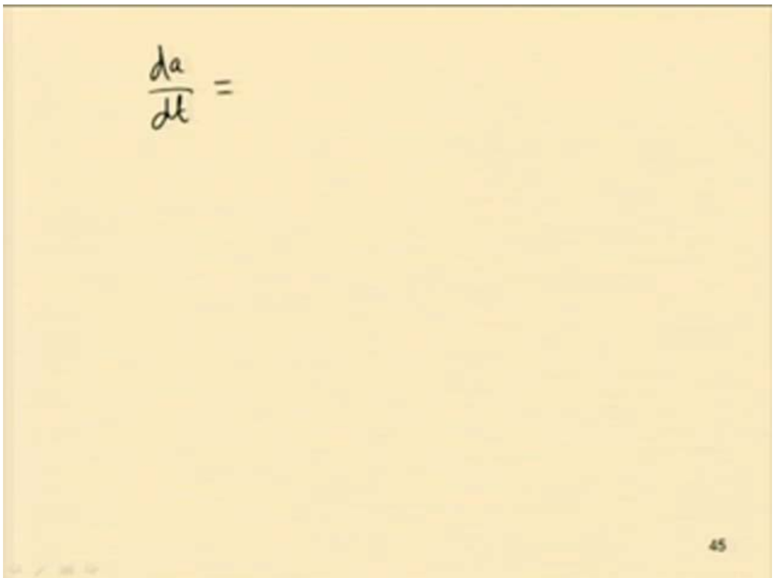

And we can write this a and theta in this form because we know d a by d t. So, we know this expression so d a by d t we know so d a by d t equal to epsilon.

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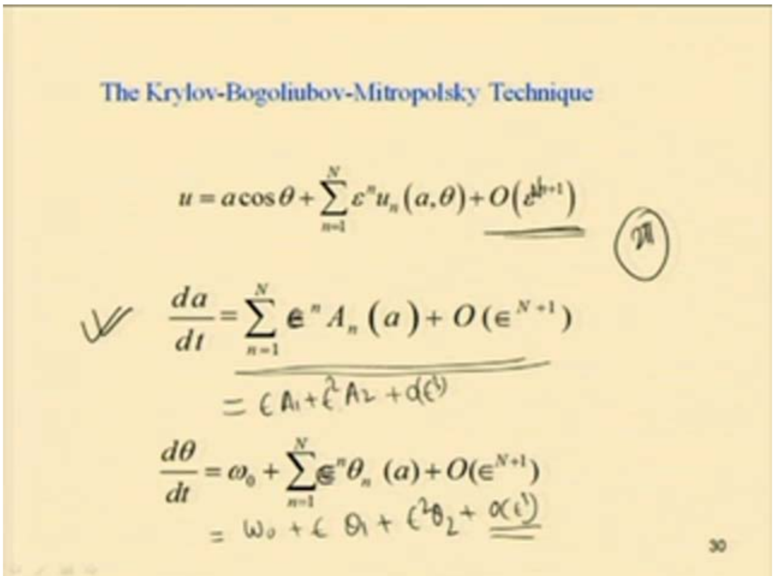

So, from the previous expression for d a by d t so, this is the equation for d a by d t alright we can you know the expression for d a by d t what we have assumed. So, d a by d t equal to epsilon a 1 plus epsilon square a 1 plus epsilon square a 2 and already we know the expression for a 1 a 2. Similarly, d theta by d d theta by d t equal to omega 0 plus epsilon theta 1 plus epsilon square theta 2.

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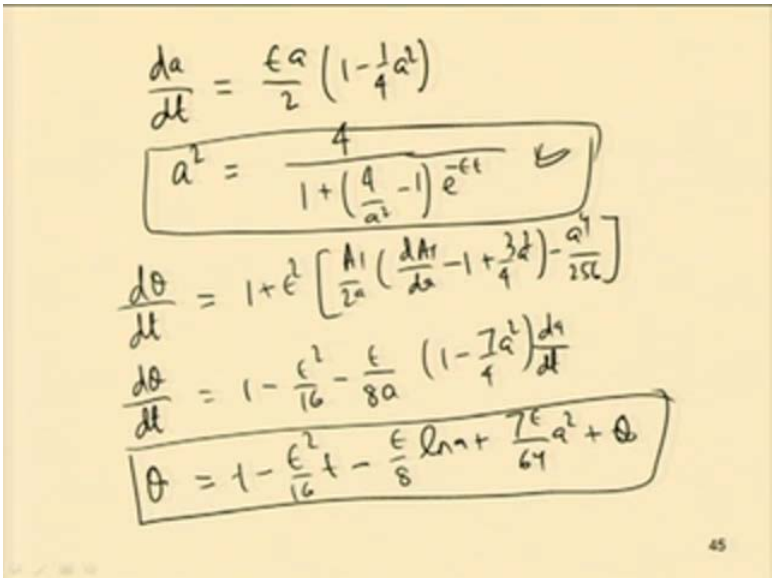

So, from this expression we can write this d a by d t will be equal to so, this d a by d t equal to we can write this is equal to epsilon a by 2 into 1 minus 1 by 4 a square and this a square can be so from this d a by d t equal to epsilon a by 2 1 minus 1 4 a square. Now, we can integrating this so we can write a square equal to by taking this a square so, we can find this a square equal to 4 by 1 plus 4 by a square minus 1 e to the power minus epsilon t. So, this is the expression for a square we obtained. Similarly, writing by d theta by d t so, we can write this is equal to 1 plus epsilon square into a 1 by 2 a into d a 1 by d a minus 1 plus 3 by 4 a square minus a to the power of 4 by 256.

So, from this we can write this d theta by d t so, this will be equal to 1 minus epsilon square by 16 minus epsilon by 8 a into 1 minus 7 by 4 a square into d a by d t, already we know d a by d t. So, we can find this theta equal to t minus epsilon square by 16 t minus epsilon by 8 l n a plus 7 epsilon by 64 a square plus epsilon 0 or plus theta 0. So, this is the expression for theta and already we got the expression for a square.

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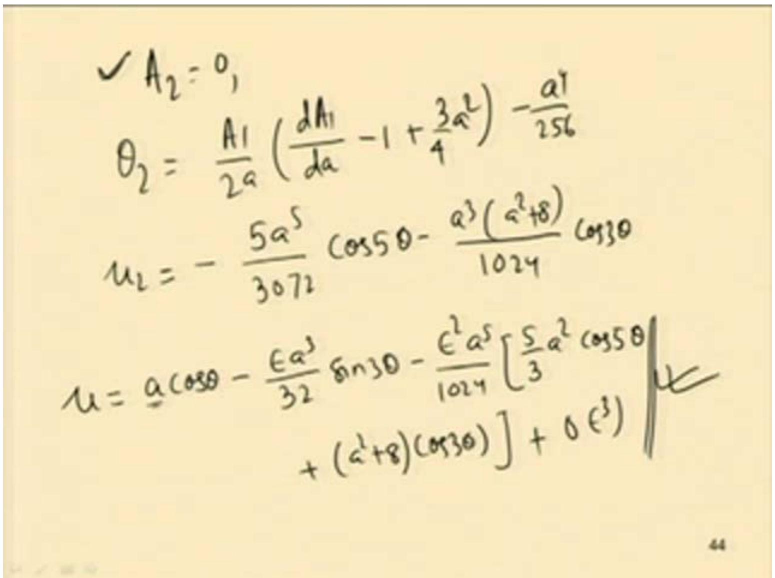

And we obtain the expression for u by this way. So, this gives the solution for the van der pol equation.

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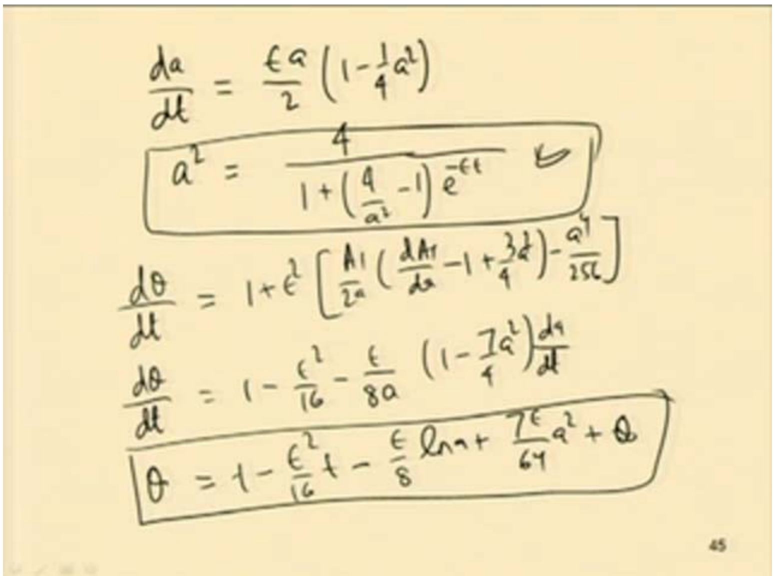

So, in this way by using this K B M method this is Krylov Bogoliubov mitropolika method so, one can find the solution of a non-linear equation up to any higher order precession. So, in the next class we will study some other methods or recently used methods like this incremental harmonic, balance method and intrinsic harmonic balance method and some other newly developed method using this non-linear solution of non-linear equation motion.

Thank you.