Non-Linear Vibration

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Module - 3
Solution of Non-Linear Equation of Motion
Lecture - 4
Method of Multiple Scales

Welcome to today class of non-linear vibration. So, we are continuing with the solution of non-linear equation of motion and today class I will tell you about the method of multiple scales.

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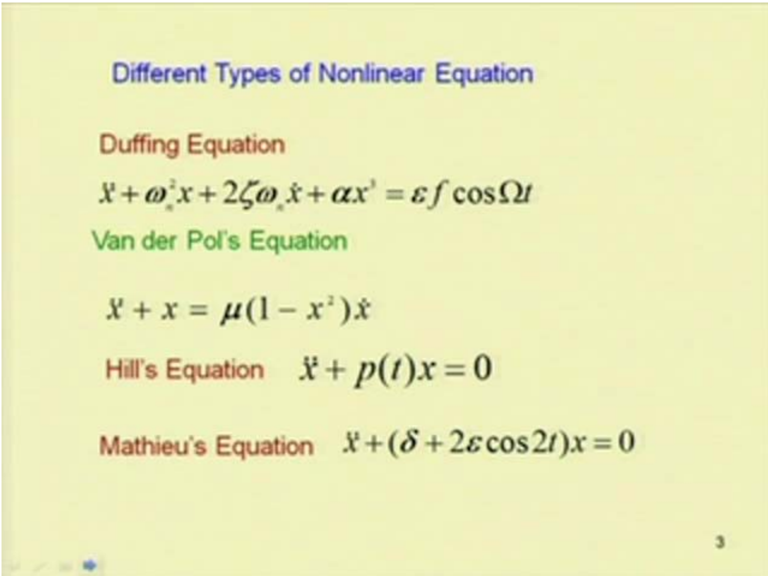

So, before we have studied different type of non-linear equations such as duffing equations van der pol equation hills equation and Mathieu equations and we have developed these qualitative analysis and quantitative analysis for solution of these equations.

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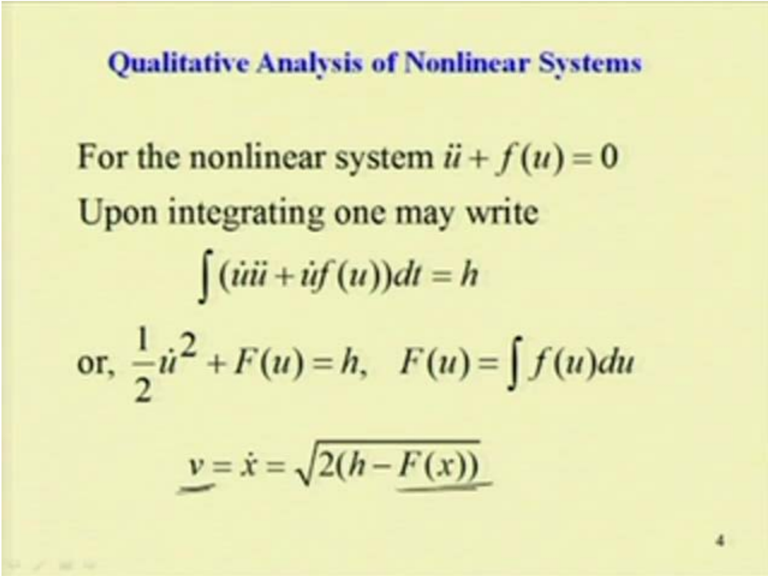

So, in case of the qualitative analysis, we have found the potential function and after finding the potential function then we have found this velocity that is x dot in terms of the total energy and the potential function or potential energy and for a conservative systems we know how to draw the phase portrait of the system or qualitatively how to analyze the system.

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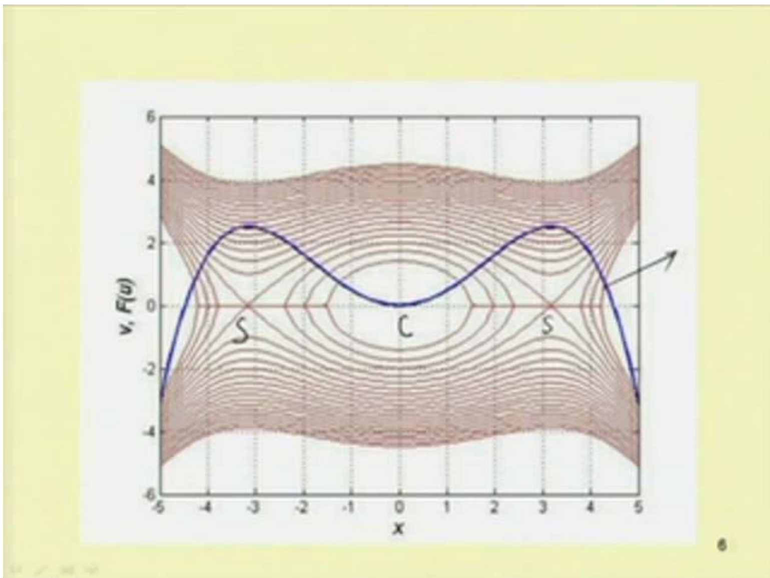

So, with the help of this example. So, we have studied the flow or the response of the of a conservative system. So, these cause. So, the potential well or potential function of the system and these shows how it will behave in case of qualitative analysis. So, this is the saddle point this is centre and this is also another saddle point. So, saddle Point correspond to maximum potential energy and correspond to the minimum potential energy we have center. So, this is in case of the qualitative analysis in quantitative analysis. So, we have studied different type of. So, we have studied different type of perturbation techniques.

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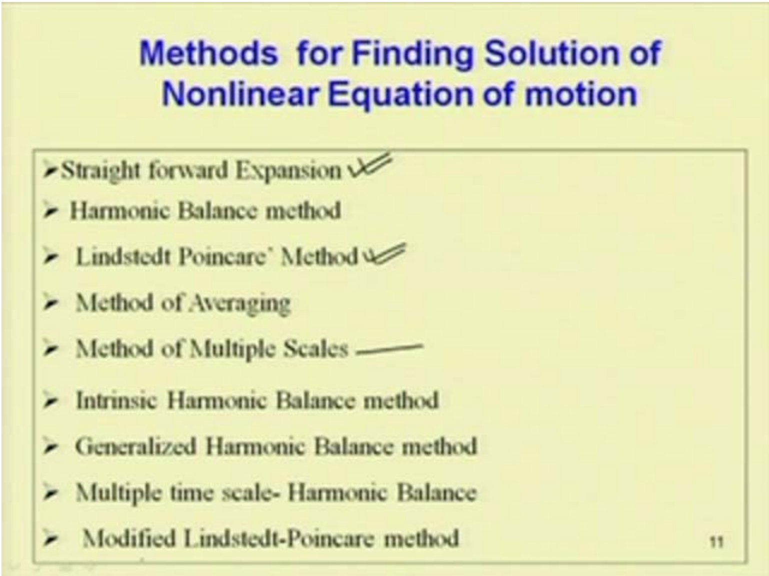

So, we have studied the straight forward expansion and Lindstedt Poincare method and today class we are going to study this method of multiple scale. So, in case of the straight forward expansion method.

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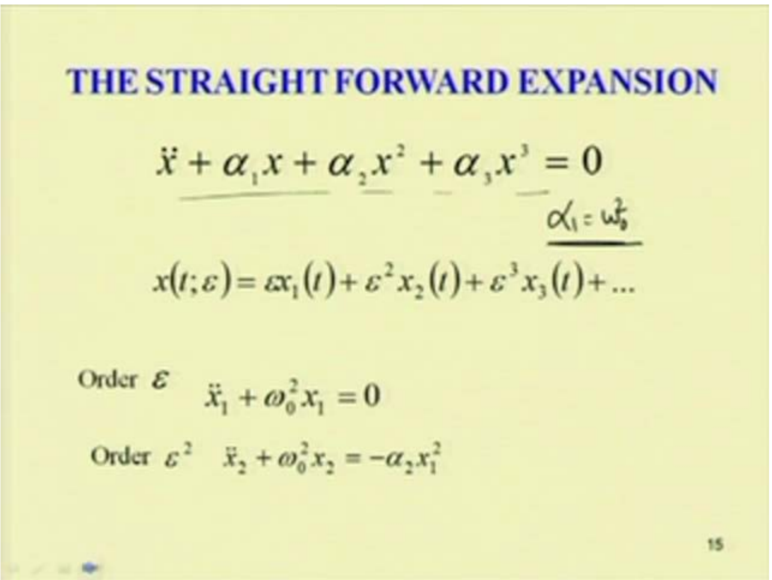

So, while solving this equation by substituting so, by substituting the equation in case of the straight forward expansion let us consider this equation where x double dot plus alpha 1 x plus alpha 2 x square plus alpha 3 x cube equal to 0 where alpha 1 we have taken it equal to omega 0 square. So, in this case by substituting this equation x t epsilon where epsilon is a book keeping parameter. So, one can write this x t epsilon equal to epsilon x 1 t plus epsilon square x 2 t plus epsilon cube x 3 t or one can write the higher order terms also so by substituting this equation in the equation differential equation and ordering the of the order of epsilon and epsilon square and epsilon cube.

So, one get a set of equations and now by substituting the set of equation first equation that is x 1 double dot plus omega square x 1 and substituting the solution in second equation and similarly substituting the solution of the second equation in the third equation that is the order of x epsilon cube.

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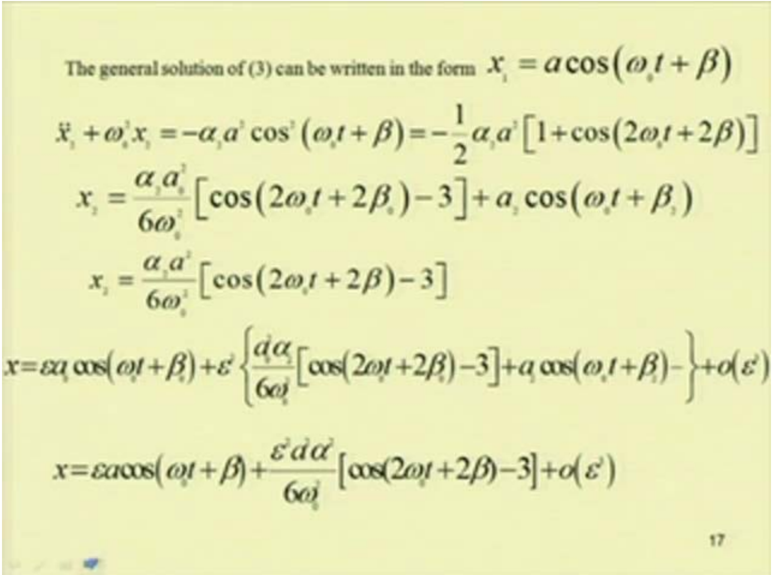

So, one can find the required response of the system, but in this case we have we have observed that by substituting this way. So, we have some term which grows with time. So, so those are known as the secular term and. So, due to this presence of this secular term. So, this straight forward expansion method is not suitable and here we have observed that in case of this straight forward expansion method. So, the secular terms. So, the response amplitude what you obtained or the frequency of the response is not a function of amplitude of the response.

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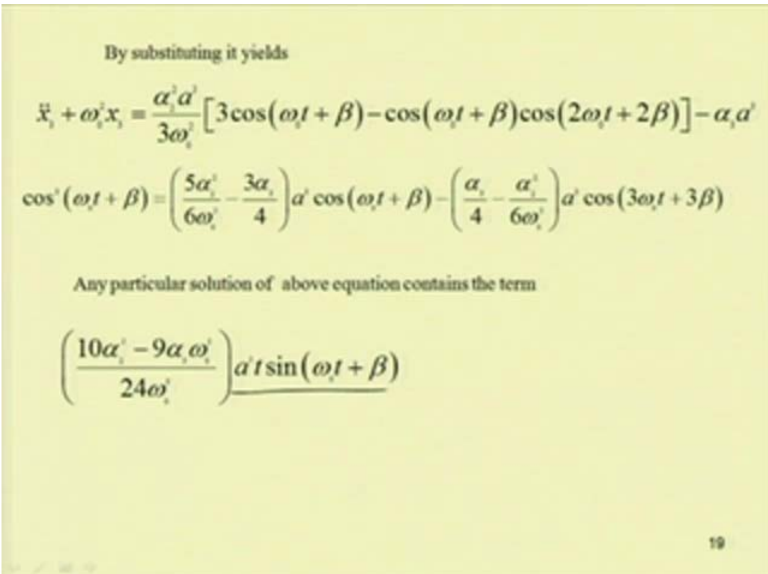

So, in case of the non-linear system generally we observe that phenomena that is the response of the system is a function of response of the system is a function of the amplitude. But as this is not showing this wave here. So, this method has to be modified and in the last class we have seen by modifying this method we have use this Lindstedt Poincare technique. So, Lindstedt Poincare they have modified this straight forward expansion method by substituting or by taking another term.

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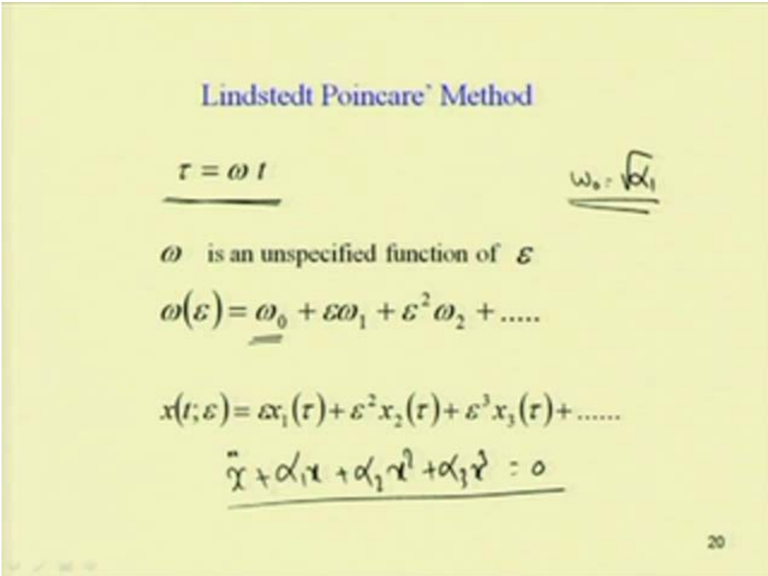

So, first in Lindstedt Poincare techniques. So, we have taken a non-dimensional time that is equal to omega into t where t is the time and omega is the frequency and this omega is written in terms of omega 0 plus epsilon omega 1 plus epsilon square omega 2 where this omega 0 is known to us. So, omega 0 equal to root over alpha 1. So, that is the coefficient of this alpha 1 the coefficient of the linear term in the governing equation. So, taking this omega equal to omega 0 plus epsilon omega 1 plus epsilon square omega 2 and also taking this x t epsilon equal to epsilon x 1 tau plus epsilon square x 2 tau plus epsilon cube x 3 tau and substituting it in the equation x double dot plus alpha 1 x plus alpha 2 x square plus alpha three x cube equal to 0. So, we get a set of equations by ordering the order of epsilon. So, in this case by substituting this. So, we have seen and by ordering, this is order of epsilon and order of epsilon square.

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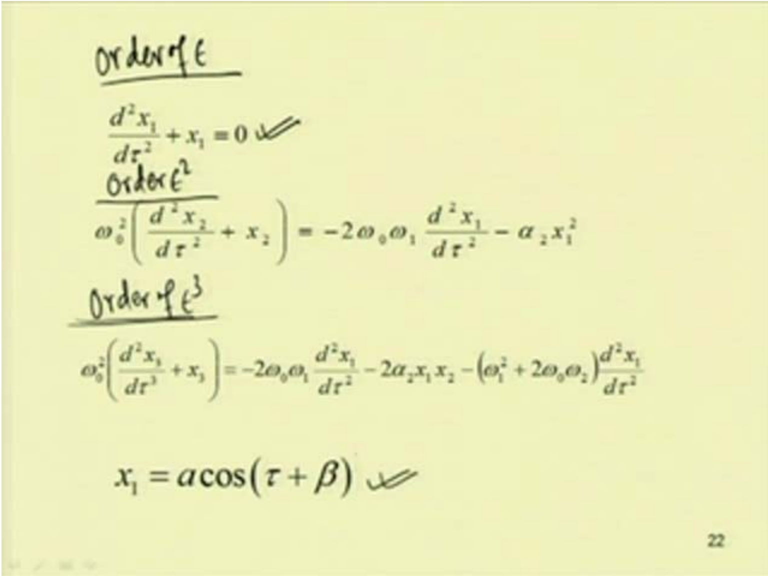

So, we can write order of epsilon order of epsilon square and order of epsilon cube. So, one can use some symbolic software package to find or to expand this thing for the higher order terms. So, for order of epsilon 1 can find the solution easily as this is d square x 1 by d tau square plus x 1 equal to 0. So, one get the solution x 1 equal to a cos tau plus beta and now substituting this equation in the second equation one get a secular term.

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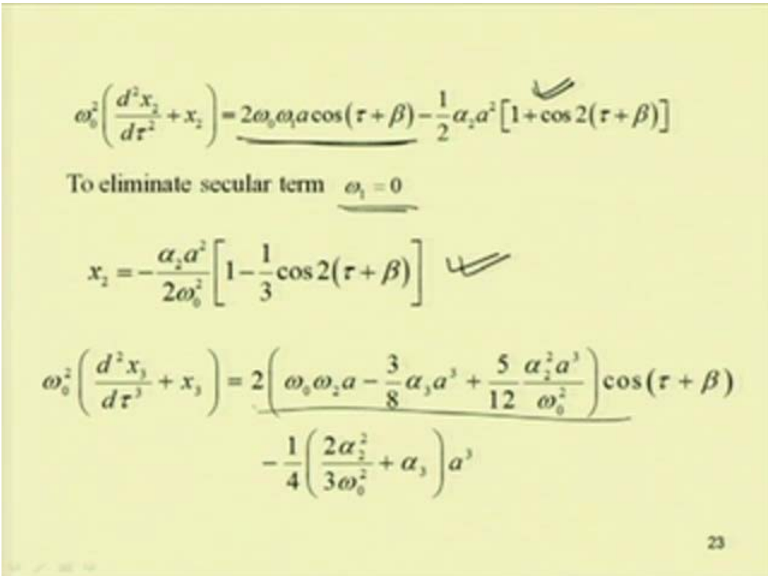

So, now, eliminating this secular term. So, one get the expression for. So, to eliminate the secular term. So, one get this omega 1 equal to 0 now the this party the secular term and eliminating that thing the remaining part is this and solution of the remaining part one can write x 2. So, substituting this x 1 and x 2 expression in the previous equation that is the order of epsilon cube and to eliminate this secular term. So, this term is the secular term. So, we have last class that this is the secular term now to eliminate this secular term. So, we can by eliminating the secular term we get the frequency relation.

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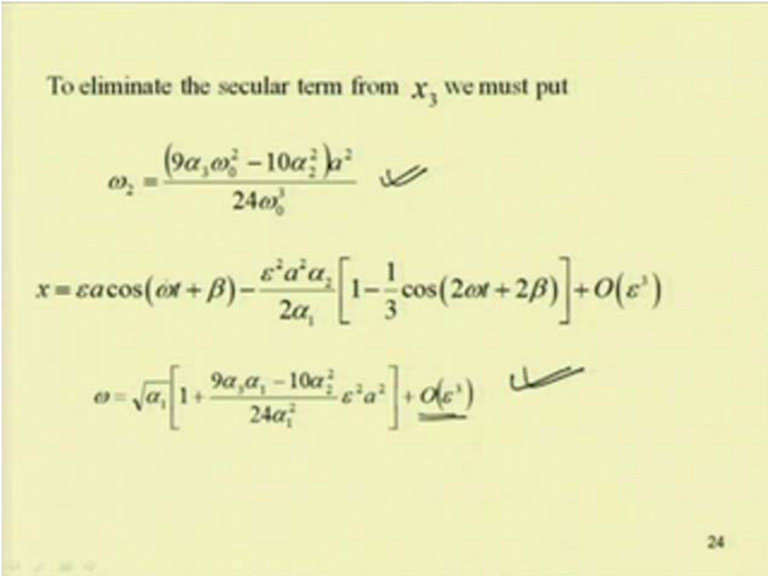

So, from this we obtain this omega 2 equal to this now the expression for x can be written as. So, as already we know the expression for x 1 and x 2. So, we can write x equal to epsilon x 1 plus epsilon x 2 plus order of epsilon cube. So, one have this expression and where this omega can be given by. So, omega can be given by omega 0 plus epsilon omega 1 plus epsilon square omega 2 already omega 1 equal to 0.

So, by substituting this omega 1 omega 0 omega 1 and omega 2 in the expression of omega 1 can get this frequency relation. So, this frequency relation shows the dependence of the amplitude response on the frequency; that means, the amplitude response of the amplitude response of the system influence the frequency of the system in case of the non-linear system, but in case of the linear system the frequency is independent of the amplitude of the excitation.

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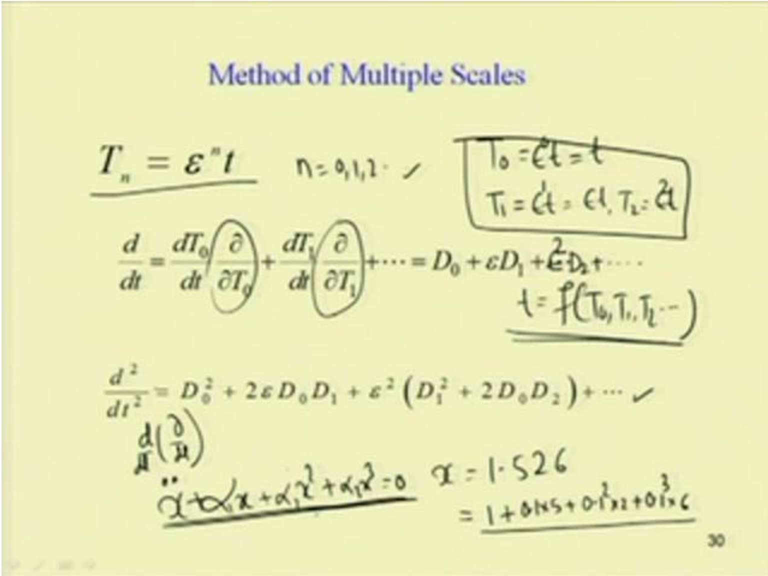

So, today class we are going to study about the method of multiple scales. So, in case of the method of multiple scale. So, we are taking different time scales for the solution of the governing equation motion. So, here we are taking a term time time scale different time scale like this that is T n equal to epsilon n to the power t. So, where n equal to 0 1 2 like this. So, T 0 equal to epsilon T. So, epsilon to the power 0 that is. So, epsilon to the power 0 that is equal to 1 can write. So, T 0 equal to epsilon to the power 0 into t. So, epsilon to the power 0 equal to 1.

So, this is equal to T. So, T 0 that is the lower time scale equal to t similarly T 1 will be equal to epsilon to the power 1 into t. So, that is epsilon t and T 2 will be equal to epsilon square t. So, this is similar to the hour hand minute hand and second hand of the watch. So, for example, we want to write a term we want to write a solution let solution x equal to 1.5 to 6. So, in this case if I will take this epsilon let epsilon equal to 0.1. So, if epsilon equal to 0.1 the solution can be written equal to. So, x will be equal to 1.

So, this is the linear part plus I can write this 5 equal to let. So, I can write this is equal to 0.1 into 5 plus 0.1 square into 2 plus 0.1 to the power cubed into 6. So, this 1.526 I can write in this form that is 1 plus 0.1 into 5 that is 1 plus epsilon into 5 plus epsilon square into 2 plus epsilon cube into 6. So, to get more accurate term. So, or up to higher order decimal I can take epsilon to the power 4 epsilon to the power 5 and in that order. So, if I want to keep this solution up to this third order. So, I can take. So, this is the linear part plus the other parts I can write 0.1 epsilon into this 5 plus epsilon square into 2 and plus epsilon cube into 6.

So, by... So, in this way I can scale. So, I can use different scale for writing these terms. So, by writing a higher order. So, the otherwise this term one can safely one can neglect this 6 with respect to this 1.5. So, to take into account the higher order terms. So, one can take different time scales and one can write the equation motion or one can write first the time scale and then this differential can be written in this form. So, d by d t now one can write equal to d by d T 0 by d t into. So, this will be. So, as t is a function of T 0 T 1 now one can write this t is a function of T 0 T 1 T 2 and higher order terms.

So, d by d t can be written as d T 0 by d t into del by del T 0 plus d by d T 1 d d T 1 by d t into del by de l T 1. So, in this way by substituting this d t d by d T 0 as D 0 and d by del by del T 1 as D 1 also one or del by del T 2 as D 2 one can write this equation in this form D 0 and as d T 0 by d t equal to T 0 equal to t. So, d T 0 by d t equal to 1 similarly d T 1 by d t equal to epsilon.

So, one can write this equation equal to D 0 plus epsilon D 1 plus 1 can add the term epsilon square D 2 and higher order terms. So, in this way one can write or one can use different time scales T 0 T 1 T 2 or higher higher order terms to write the equation of motion to now this d by d t is written in this form similarly D square by d t square can be written as D 0 square plus 2 epsilon D 0 D 1 plus epsilon square into D 1 square plus 2 D 0 D 2. So, this thing can be obtained by. So, d square by d t.

Square one can write this as d by d t of d by d t and now by substituting this previous equation in this equation. So, one can write this D square by d t square equal to D 0 square plus 2 epsilon D 0 D 1 plus epsilon square D 1 square plus 2 D 0 D 2 and the higher order terms. So, now, substituting these equations in our original equation which we are going to solve that is x double dot plus alpha 1 x plus alpha 2 x square plus alpha three x cube equal to 0. So, let us call this equation. So, in this equation now my substituting this d by d t and D square by d t square term in this way.

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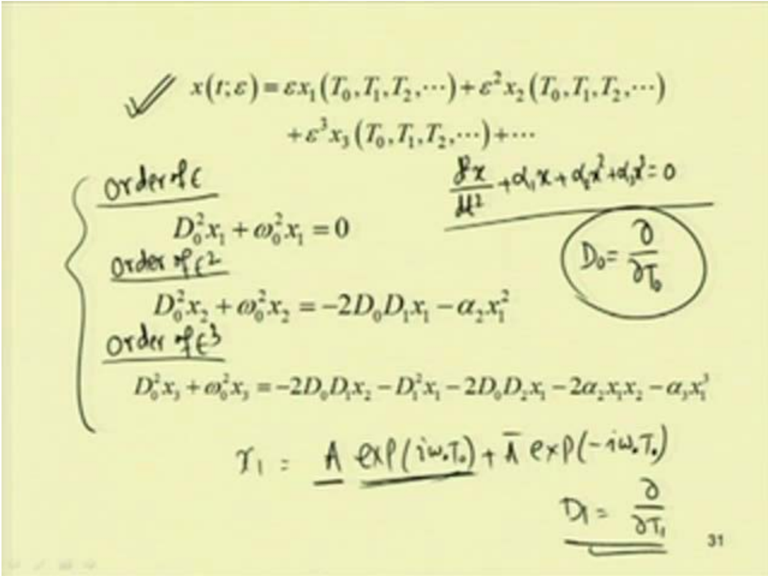

So, one can write the equation and also by substituting like in case of the Lindstedt technique and in case of straight forward expansion we can write this x t epsilon equal to epsilon x 1 T 1. So, in this case this x 1 is also a function of T 0 T 1 T 2 and higher order terms plus epsilon square x 2 into T 0 T 1 T 2 plus epsilon cube x 3 into x epsilon cube x 3 which is a function of T 0 T 1 T 2 and higher order terms. So, in case of Lindstedt Poincare techniques we have not taken this x 1 x 2 x 3 to be function of different time scales.

But in this case we are taking this x 1 x 2 x 3 to be function of different time scale and x 1 equal to epsilon x 1 x equal to epsilon x 1 plus epsilon square x 2 plus epsilon cube x 3 now this substituting these in this equation x double dot that is D square. So, one can write this equation this way. So, D square x by d t square plus alpha 1 x plus alpha 2 x square plus alpha 3 x cube equal to 0. So, in this equation by substituting this D square by d t square equal to D 0 square plus 2 epsilon D 0 D 1 plus epsilon square D 1 square plus 2 D 0 D 2 and x equal to epsilon x 1 plus epsilon square x 2 plus epsilon cube x 3 and separating the order of epsilon. So, one can write. So, this is order of order of epsilon this term is order of the epsilon this term will be order of epsilon order of epsilon square and this is order of epsilon cube . So, now, one can get these set of these equations. So, for order of epsilon 1 can write the equation D square x 1 plus omega 0 square x 1 equal to 0. The solution of which we already know that x 1 will be equal to a cos omega t a cos omega 0 t.

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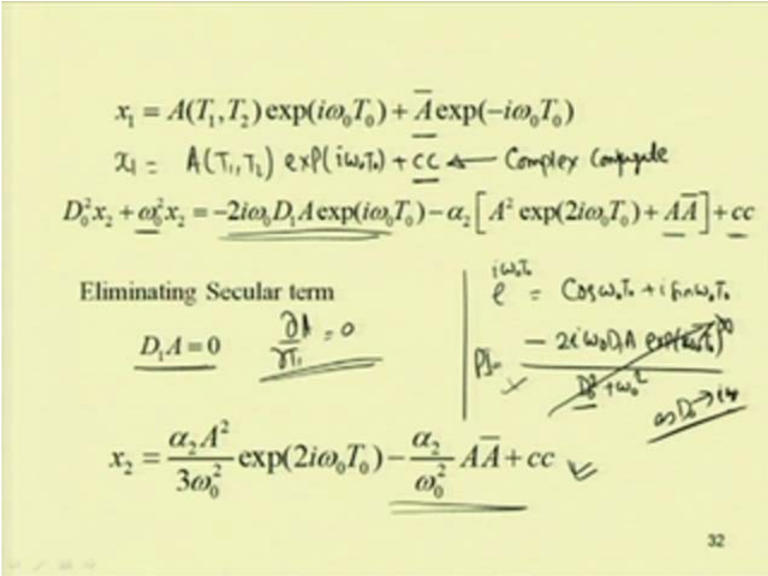

So, instead of writing this in terms of cos and sine we can express that thing in terms of the in terms of the exponential function; that means, we can write this x 1 equal to A as a is a constant; that means, a should not be. So, from this equation A should not be a function of T 1. So, this is x 1. So, when we have written in this form. So, D 0 square x 1 plus omega 0 square x 1 as D 0 equal to d by. So, as d zero. So, already you know. So, d. So, this is del by del T 0. So, D 0 equal to del by del T 0. So, the constants would not be a function of T 0. So, this constant is a function of may be a function of T 1 T 2. So, this x 1 equal to A T 1 T 2 e to the power i omega 0 T 0 plus a bar e to the power minus i omega 0 T 0.

So, as you know e to the power i omega 0 T 0 equal to cos omega 0 T 0 plus i sine omega 0 T 0. So, the harmonic solution of this equation that is a cos omega 0 T 0 or a sin omega 0 T 0 can be written by using this exponential function. So, e to the power i omega 0 T 0 equal to cos omega 0 T 0 plus i sin omega 0 T 0.

So, one can write x 1 equal to A e to the power i omega 0 T 0 plus A bar where A bar is the complex conjugate of a and the complex conjugate of I omega 0 T 0 equal to minus A minus i omega 0 T 0. So, instead of writing x 1 A x 1 equal to A T 1 T 2 e to the power i T 0 plus A bar e to the power minus i omega 0 T 0. So, one can write this is equal to A T 1 T 2 e to the power i omega 0 T 0 plus complex conjugate c c. So, the c c terms is or complex conjugate of the preceding term. So, the c c represent the complex conjugate term of the preceding terms that is a T 1 T 2 e to the power i omega 0 T 0 which is equal to a bar e to the power minus I omega 0 T 0.

So, now, by substituting this equation in the previous equation that is of the order of epsilon square. So, we can write. So, this equation represent D 0 square x 2 plus omega 0 square x 2 equal to minus 2 D 0 D 1 x 1 minus alpha 2 x 1. So, as already we have seen this x 1 equal to A e to the power i omega 0 T 0 plus c c or A bar e to the power minus i omega 0 T 0. So, differentiating this thing with respect to. So, as A is a function of T 1 and T 2. So, only this term can be differentiated with respect to this D 1.

So, where D 1 equal to. So, as we know D 1 equal to del by del T 1. So, only a term can be differentiated with respect to T 1. So, x 1 can be written as D 1. So, D 1 x 1. So, D 1 x 1 will be equal to D 1 A e to the power i omega 0 T 0 plus D 1 A bar e to the power minus i omega 0 T 0. So, this i e to the power i omega 0 T 0 as they are not function of time T 1.

So, they can be treated as constant and one can differentiate this D 1 with respect to a now operating this D 0. So, one can write. So, one can differentiate this part as this is a function of T 0 and one can write this term minus 2 D 0 D 1 x 1 in this form. So, this will be equal to. So, D 0 square x 2 plus omega 0 square x 2 will be equal to minus 2 I omega 0 D 1 A e to the power I omega 0 T 0 minus alpha 2 into A square e to the power 2 i omega 0 T 0 plus A into a bar plus it is complex conjugate.

So, this complex conjugate will be equal to. So, this is minus. So, minus minus. So, plus 2 i omega 0 D 1 A bar e to the power minus i 2 omega 0 T 0 and for this part also you can write the complex conjugate. So, one can write this complex conjugate which is the complex conjugate of the preceding terms. So, in this case one can observe that the coefficient of x 2 equal to omega 0 square. So, any term containing e to the power any term containing this i omega 0 will be. So, this term this term will be a secular term because e to the power. So, if you will find the particular integral of this term. So, the particular integral term containing this e to the power i omega 0 T 0.

So, one can write. So, this particular integral will be equal to. So, 2 i omega 0. So, this is D 1 A e to the power i omega 0 T 0. So, y. So, particular integral of this part will be equal to y D 0 square plus omega 0 square and in this case for D 0 we have to substituting this i omega 0. So, square of. So, D 0 square will be equal to minus omega 0 square. So, minus omega 0 square plus omega 0 square this is equal to 0.

So, the particular integral tends to… So, this tends to infinity as D 0 you are substituting equal to i omega 0. So, this is a secular term. So, one must eliminate this term to have a bounded solution as we have already seen from the qualitative analysis our solution should be bounded and for that reason. So, these terms would be equal to 0. So, to set this term equal to 0 as this exponential function cannot be 0 and this omega 0 is not equal to 0. So, one should set this D 1 a should be equal to 0, but D 1 a equal to 0; that means, D 1 equal to del by del T 1 a equal to 0. So, the. So, that this a is a constant where a should not be a function of t one. So, already we have seen that a is not a function of T 0 and now from this equation also we have seen that a is not a function of T 1 also. So, now, the solution of this equation now eliminating this part. So, the remaining part we have this D 0 square x 2 plus omega 0 square x 2 equal to minus alpha 2 into a square e to the power 2 i omega 0 T 0 plus A A bar and we can find the particular integral corresponding to this term and which is which can be given by this. So, here x 2 equal to alpha 2 a square by three omega 0 square e to the power 2 i omega 0 T 0 minus alpha 2 by omega 0 square A A bar plus it is complex conjugate.

So, one can find. So, in this case as this is e to the power 2 i omega 0 T 0. So, similar to the derivation we made here. So, here in the denominator part this D 0 square will be replaced by minus it will be D 0 should be replaced by 2 i omega 0. So, this D 0 square becomes minus 4 omega 0 square. So, plus omega 0 square. So, this becomes minus three omega 0 square. So, if you are you have a minus term. So, minus minus plus. So, that is why this x 2 becomes alpha 2 a square by three omega 0 square into e to the power 2 i omega 0 T 0 similarly for this part a A bar. So, as this will be a constant. So, one can take this D 0 square omega 0 square by taking this omega 0 square common. So, one can write this thing equal to 1 plus D 0 square by omega 0 square and taking that thing to numerator and expanding or using this binomial theorem.

So, one can find this particular integral to be this. So, the particular integral for x 2 become alpha 2 a square by 3 omega 0 square e to the power 2 i omega 0 T 0 minus alpha 2 by omega 0 square A A bar plus its complex conjugate. So, now, substituting this expression for x 1 and x 2 in the order of epsilon cube term that is D 0 square x 3 plus omega 0 square x 3 equal to minus 2 D 0 D 1 x 2 minus D 1 square x 1 minus 2 D 0 D 2 x 1 minus 2 alpha 2 x 1 x 2 minus alpha 3 x 1 cube and taking the note that D 1 a equal to 0. So, we have seen this D 1 A equal to 0 from previous secular term eliminating the secular term. So, by substituting those equations in this equation.

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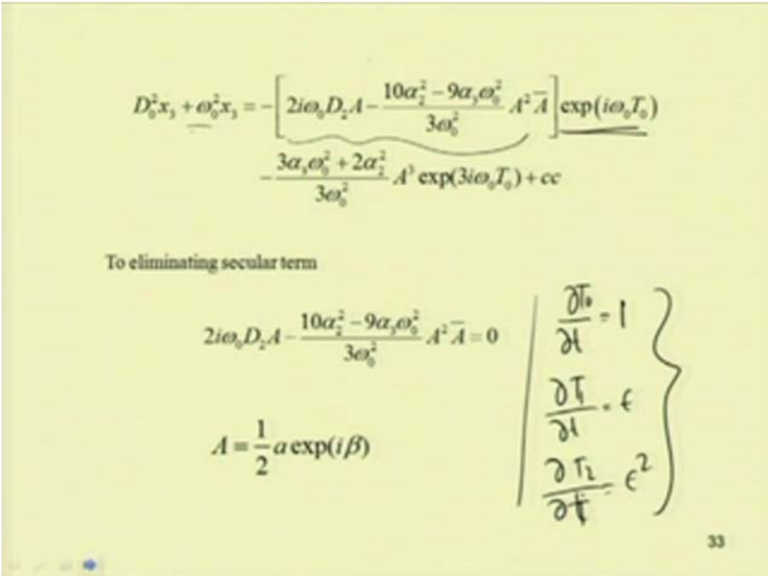

So, one can write this D 0 square x 3 plus omega 0 square x 3 equal to minus 2 minus 2 I omega 0 d 2 a minus ten alpha 2 square minus 9 alpha three into omega 0 square by three omega 0 square plus a square a bar into e to the power I omega 0 T 0 minus three alpha three omega 0 square plus two alpha 2 square by three omega 0 square a cube into e to the power three I omega 0 T 0 similar to previous case here.

Also one can observe that the coefficient of this should be eliminated to avoid the secular term. So, this is a secular term as the coefficient this omega 0 square. So, here the coefficient of x three equal to omega 0 square and in this e to the power we have the term i omega 0. So, this will lead to a secular term because we have to substitute this I omega 0 for this D 0 which will lead to the response to be infinity or unbounded. So, to have a bounded solution. So, we should eliminate this term. So, this is this. So, to eliminate the secular term. So, we should set this part equal to 0.

So, by setting this part equal to 0 we can write 2 i omega 0 D 2 a minus ten alpha 2 square minus 9 alpha three omega 0 square by three omega 0 square a square a bar equal to 0 . So, now, we can assume the solution a equal to a equal to half a e to the power I beta because when we have assumed the solution of x 1 x 1 equal to A T 1 T 2 e to the power I omega 0 T 0 plus a bar e to the power minus I omega 0 T 0 which was this solution of this equation D 0 square x 1 plus omega 0 x 1 equal to 0 also.

Here, we should have two constants for this solution of this equation. So, this a can be written as a real part and imaginary part. So, which are the two unknowns we have to find. So, for that reason. So, one can write this a equal to a in polar form that is a equal to half A e to the power i beta. So, now, substituting this a equal to half A e to the power I beta in this equation and recalling that this recalling that this del by del or del T 0 by d t. So, del T 0 by d t equal to 1 and del T 1 by del t equal to epsilon and del T 2 by del del 2 by del t equal to epsilon square. So, by recalling this thing and substituting this in this equation. So, one can separate the real and imaginary parts by separating the real and imaginary parts. So, one can write omega a dash equal to 0. So, already we have seen this a is a is a function a is not a function of T 0 and T 1. So, it should be a function of T 2 and the higher order higher time scales that is 3 T 4 T 2 T 3 T 4 maybe function of the higher time scales, but it is not a function of T 0 and T 1. So, as it is not a function of T 0 and T 1. So, this a and beta are function of T 2.

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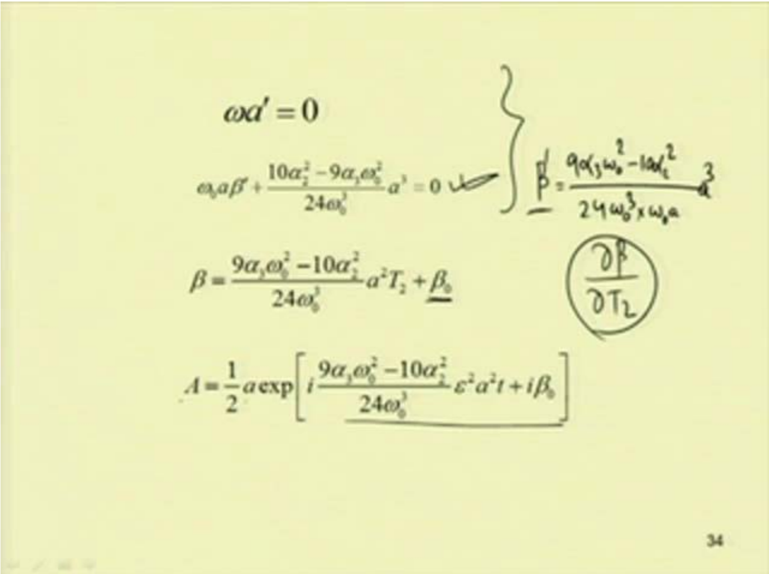

So, a and beta are function of t two. So, we can write by separating this real and imaginary parts. So, we can write omega a dash equal to 0 and omega 0 a beta dash plus ten alpha 2 square minus 9 alpha three omega 0 cube by twenty four omega 0 cube a cube equal to zero. So, these are by setting the real and imaginary part equal to zero. So, from this equation omega a dash equal to zero. So, this shows that a should be a constant and from this equation one can find beta equal to. So, beta . So, one can write this beta dash equal to. So, beta equal to. So, this part that is minus or one can write taking this minus inside. So, one can write this equal to 9 alpha 3 omega 0 square minus ten alpha 2 square by twenty four omega 0 cube a cube. So, beta dash equal to. So, by by omega 0 a. So, this becomes. So, beta dash becomes 9 alpha 3 omega 0 square minus 10 alpha 2 square by this 24 omega 0 fourth into this a and a cube. So, one can delete one. So, this will be into a square. So, now, one can write this equation.

So, now, integrating this thing one can write this beta equal to 9 alpha three omega 0 square minus ten alpha 2 square by twenty four omega 0 cube a square into T 2 as beta dash beta dash equal to del beta by del t two. So, one can get. So, this is this remain as a constant. So, into T 2 plus this beta zero. So, one can write beta equal to this and a is a constant. So, one can write this capital a which is equal to half a e to the power I beta in this form. So, a equal to half a e to the power i. So, for beta one can substitute this. So, here we have two unknowns that is a and beta zero. So, this a and beta 0 can be found from the initial conditions. So, now, substituting this a in the previous equation.

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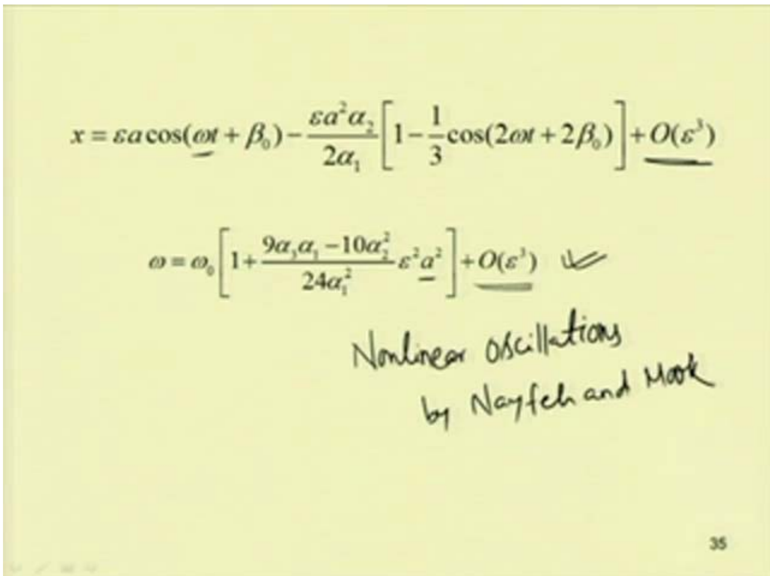

So, we can write the expression for x in this form. So, x will be equal to. So, we know that x equal to epsilon x 1 plus epsilon square x 2 and the higher order terms. So, by substituting this a in this expression x 1 that is a e to the power I omega 0 T 0 plus a bar e to the power minus I omega 0 T 0 and x 2 equal to and x 2 equal to alpha 2 a square by three omega 0 square e to the power two I omega 0 T 0 minus alpha 2 by omega 0 square into a a bar plus it is complex conjugate.

So, we can write the expression for x equal to epsilon a cos omega t plus beta 0 minus epsilon a square alpha 2 by 2 alpha 1 into 1 minus 1 by 3 cos two omega t plus 2 beta 0 plus order of epsilon cube. So, here this frequency omega equal to omega 0 into 1 plus 9 alpha 3 into alpha 1 minus 10 alpha 2 square by 24 alpha 1 square into epsilon square a square plus order of epsilon cube. So, in this case we have seen this frequency is a function of this amplitude. So, already this equation is similar to that we have obtained in case of the Lindstead poincare technique. So, in this way one can use method of multiple scale to solve the differential equation non-linear differential equation. So, this derivation is followed from the book of non-linear oscillations by Nayfeh non-linear oscillations by Nayfeh and Mook a h Nayfeh and d t Mook and one can see some other references also.

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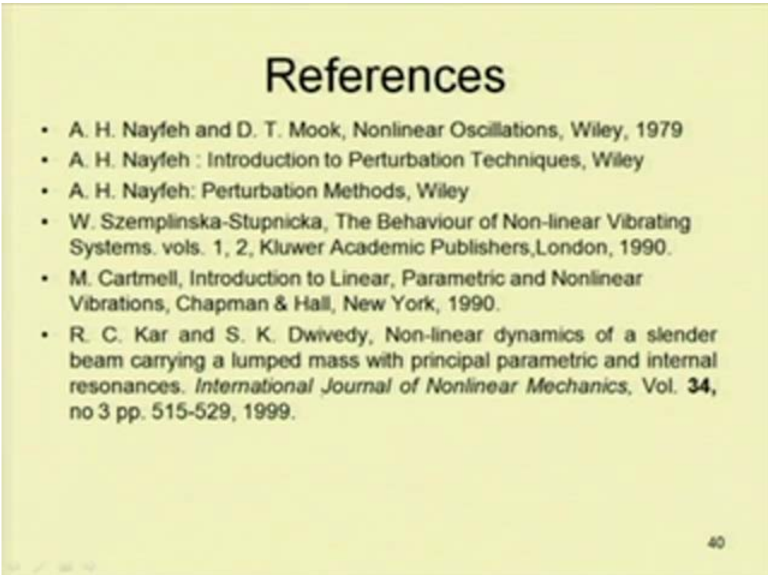

So, a h Nayfeh and d t Mook non-linear oscillations and then also one can follow the book by introduction to perturbation techniques by a h Nayfeh also one can see this perturbation method by a h Nayfeh also this w Szemplinska-Stupnicka the behavior of non-linear vibrating systems volume one and two also the book by M cartmell introduction to linear parametric and non-linear vibrations and some of other publications like this by r c kar and S K Dwivedy non-linear dynamics of a slender beam carrying lumped mass with principle parametric and internal resonances.

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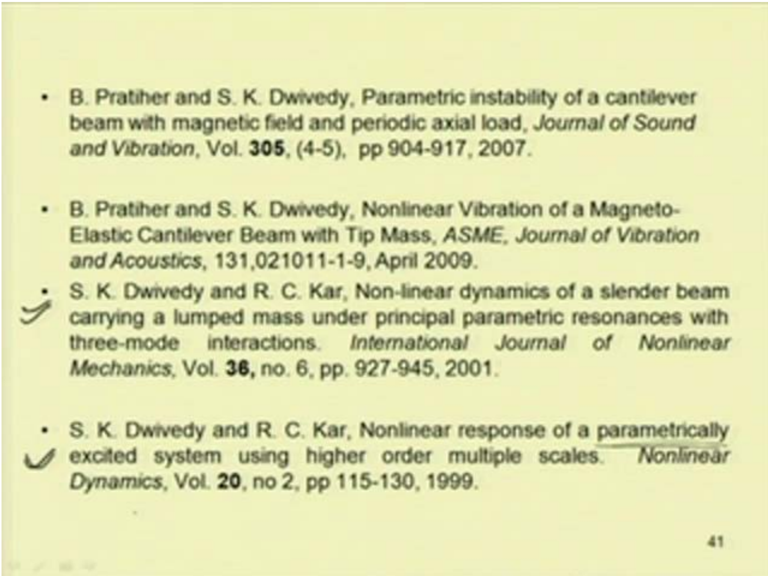
- B. Pratiher and S. K. Dwivedy, Parametric instability of a cantilever beam with magnetic field and periodic axial load, *Journal of Sound and Vibration*, Vol. **305**, (4-5), pp 904-917, 2007.
- B. Pratiher and S. K. Dwivedy, Nonlinear Vibration of a Magneto-Elastic Cantilever Beam with Tip Mass, *ASME, Journal of Vibration and Acoustics*, 131,021011-1-9, April 2009.
- S. K. Dwivedy and R. C. Kar, Non-linear dynamics of a slender beam carrying a lumped mass under principal parametric resonances with three-mode interactions. *International Journal of Nonlinear Mechanics*, Vol. **36,** no. 6, pp. 927-945, 2001.
- S. K. Dwivedy and R. C. Kar, Nonlinear response of a parametrically excited system using higher order multiple scales. *Nonlinear Dynamics*, Vol. **20**, no 2, pp 115-130, 1999.

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So, which was published in international journal on non-linear mechanics also this by b Pratiher and S K Dwivedy parametric instability of a cantilever beam with magnetic field and periodic axial load which was published in journal of sound and vibration also this paper by Pratiher and Dwivedy non-linear vibration of a magneto-elastic cantilever beam with a tip mass. So, this is published in a s m e journal of vibration and acoustics and one can refer this paper Dwivedy and kar non-linear dynamics of a slender beam carrying a lumped mass under principle parametric resonances with three-mode interaction and other paper also Dwivedy and kar non-linear response of a parametrically excited system using higher order multiple scales. So, in this case different type of multiple scale higher order multiple scales have been stated. So, two different types of multiple scales has been shown in this paper for the first time in this paper the higher method of multiple scales using these two methods were applied for a parametrically excited systems. So, now I will give you one example. So, that is the paper by this r c kar and S K Dwivedy non-linear dynamics of a slender beam carrying a lumped mass with principle parametric and internal resonances and in this system we will apply the method of multiple scale to find the response of the system.

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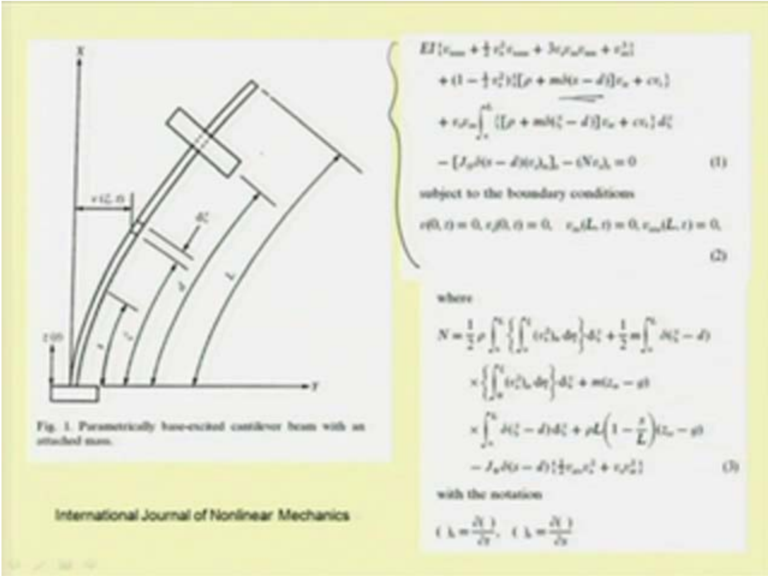

So, this is the system. So, in this case this is a slender beam carrying a lumped mass in which the base is excited. So, base excite is on z t can be written as. So, this z t equal to. So, z 0 sine omega t. So, this is this is harmonically excited base. So, this is a cantilever beam where the beam is excited harmonically and as the beam is excited harmonically. So, the governing equation of motion which is is one can find the detail in that paper. So, can be written in this form. So, e I del fourth b by del x fourth plus half b x a square into del cube b by del x cube plus three into del b by del x plus del square b by del x square plus del cube b by Del x cube plus del square b by del x square whole cube plus 1 minus half del b by del x whole square plus rho into rho plus m delta s minus d into b del square b by del t square plus e into del b by del t plus del b by del x plus del square b by del s square integral s to l rho plus m delta zeta minus d into b del square b by del t square plus c plus del b by del t.

So, in this equation… So, this is this is a non-linear equation. So, in this non-linear equation. So, a may the team mass rho is the mass per unit length of the beam and here this derived delta function is used to show the position of this team mass at an arbitrary position.

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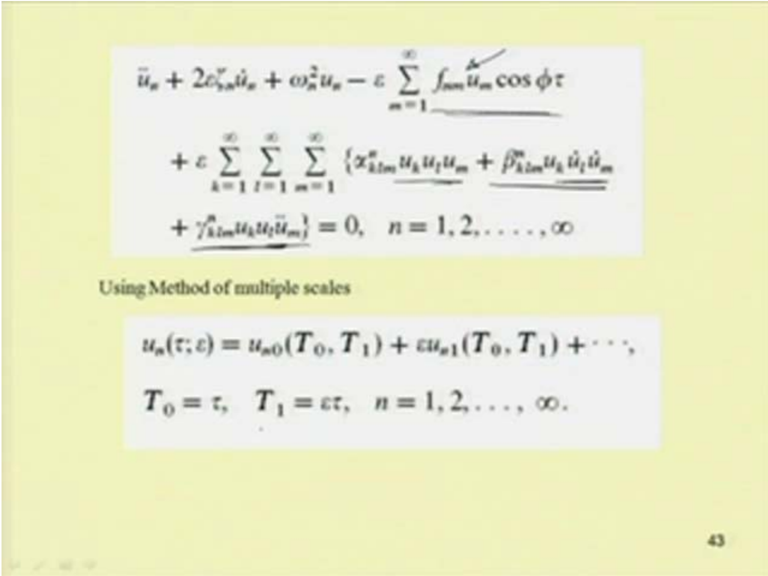

So, by using this governing equation and in this governing equation one can use this (()) procedure to reduce this spacio temporal equation to its temporal form. So, this is the temporal equation of motion. So, in this case the temporal governing equation of motion is del square u by del del square u by del t square plus two epsilon zeta n del u.

By del t plus omega n square del u n by del u 1 minus epsilon into m equal to one to infinity f n m u m cos phi t plus epsilon summation k equal to one to infinity l equal to one to infinity and m equal to one to infinity alpha k l m n into u k into u l into u m plus beta n k l m u k u l dot u m dot plus gamma k l m u k u l u m double dot. So, due to the presence of this term. So, this is u k u l u m. So, this is cubic geometric type of non-linearity and this is the product of two velocity term. So, this is inertia type of non-linearity and this is also product of this inertia into this u k and u m. So, these are the. So, this is the inertia non-linearity this is also inertia non-linearity and this is geometric non-linearity. So, in this equation we have inertia non-linearity and geometric non-linearity along with a number of forcing function. So, the previous equation what we have studied. So, we have not considered the frequency. So, we have not considered the forcing term there also in this case this forcing term is a function of u. So, in this case this forcing term is a coefficient of u. So, that is why as the coefficient of the response as this parameter this periodic parameter is the coefficient of the response.

So, this equation is similar to the Mathew Hill equation what you have seen before and in addition to that we have this non-linear term. So, this is a this equation in addition to the Mathew Hill equation we have additional non-linear term . So, now, by using this method of multiple scale by writing this u l tau epsilon equal to u n 0 plus epsilon u n one where t T 0 equal to tau and tau one equal to epsilon t and substituting this and proceeding like the previous case.

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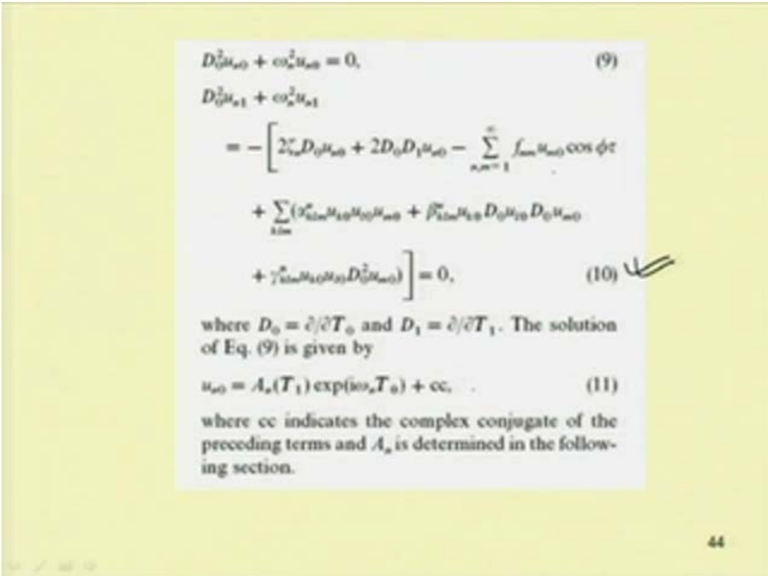

$$D_0^2 u_{n0} + \omega_n^2 u_{n0} = 0, \qquad (9)$$

$$D_0^2 u_{n1} + \omega_n^2 u_{n1} = -\Big[2\zeta_n D_0 u_{n0} + 2D_0 D_1 u_{n0} - \sum_{n,m=1}^{\infty} f_{nm} u_{m0} \cos\phi\tau + \sum_{k,l,m} (\alpha_{klm}^n u_{k0} u_{l0} u_{m0} + \beta_{klm}^n u_{k0} D_0 u_{l0} D_0 u_{m0} + \gamma_{klm}^n u_{k0} u_{l0} D_0^2 u_{m0})\Big] = 0, \qquad (10)$$

where $D_0 = \partial/\partial T_0$ and $D_1 = \partial/\partial T_1$. The solution of Eq. (9) is given by

$$u_{n0} = A_n(T_1)\exp(i\omega_n T_0) + cc, \qquad (11)$$

where cc indicates the complex conjugate of the preceding terms and A_n is determined in the following section.

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So, we can write by separating the order of epsilon. So, we can write this equation in this form. So, D 0 square u n 0 plus omega n square u n 0 equal to 0 D 0 square u n 1 plus omega n square u n one equal to this. So, the solution of this thing can be written as n T 1 e to the power I omega n T 0 plus it is complex conjugate now substituting this equation in this equation and by substituting this equation in this equation and one can see it will contain several secular term and in addition to that there will be some mixed secular term.

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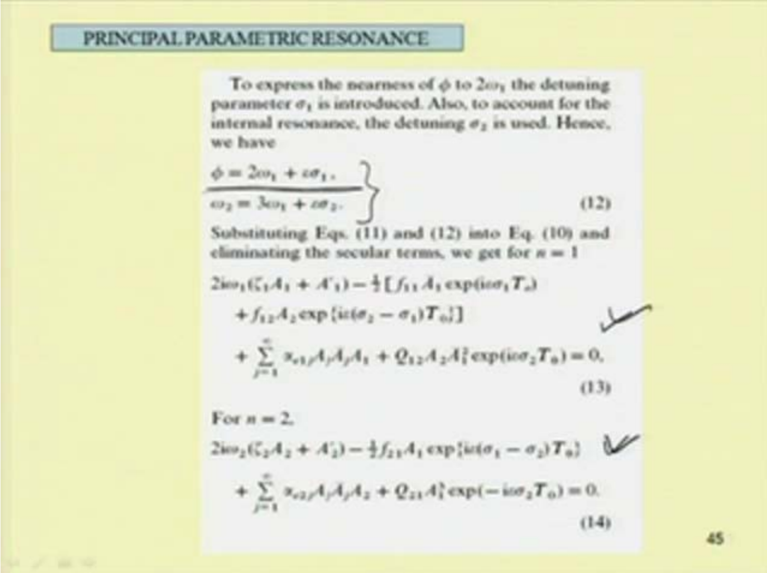

So, this mixed terms will come when we consider this phi equal to two omega 1 plus epsilon one and in this case in this particular case. So, we have considered the internal resonance also. So, in case of the internal resonance. So, we have considered omega 2 nearly equal to 3 omega 1 plus epsilon sigma two were the sigma 1 and sigma 2 are the detuning parameter to express the nearness of this phi two to omega 1 and omega 2 near to this 2 3 omega 1.

So, in this particular example we have placed this attest mass in such way that the natural frequency of the system are in the ratio that is the second mode natural frequency and the first mode natural frequency are approximately in the ratio of three is to one. So, to take into account this approximate. So, we have taken this phi equal to two omega 1 plus epsilon sigma one and omega 2 equal to three omega 1 plus epsilon sigma two. So, by taking this nearness of this phi that is the external excitation frequency and the second mode natural frequency in this way and substituting this in the previous equation this equation. So, to eliminate the secular term we have a set of equations. So, for n equal to one we have this equation. So, n equal to one correspond to the first mode n equal to two correspond to the second mode and n equal to n greater than three.

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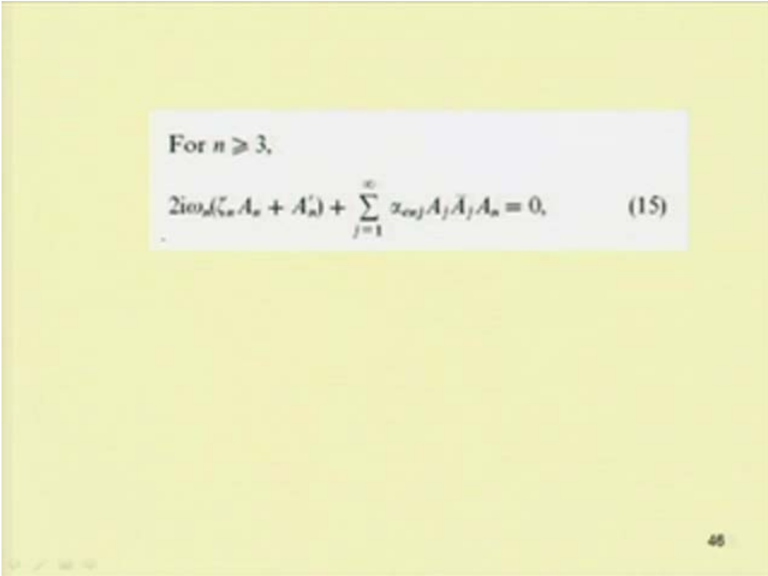

So, those are for third fourth and higher order modes. So, from this one can see that these higher order terms can be the response of the higher order terms will be very small. So, one can take only these two resonance condition as we are considering phi nearly equal to two omega one. So, this case is referred as the principle parametric resonance case. So, this is principle parametric resonance of the first mode. So, later we will study in detail about the parametrically excites systems. So, we will see how the parametrically excited system behave. So, in this particular example we are intended to show how this method of multiple scales can be applied to a particular type of differential equations.

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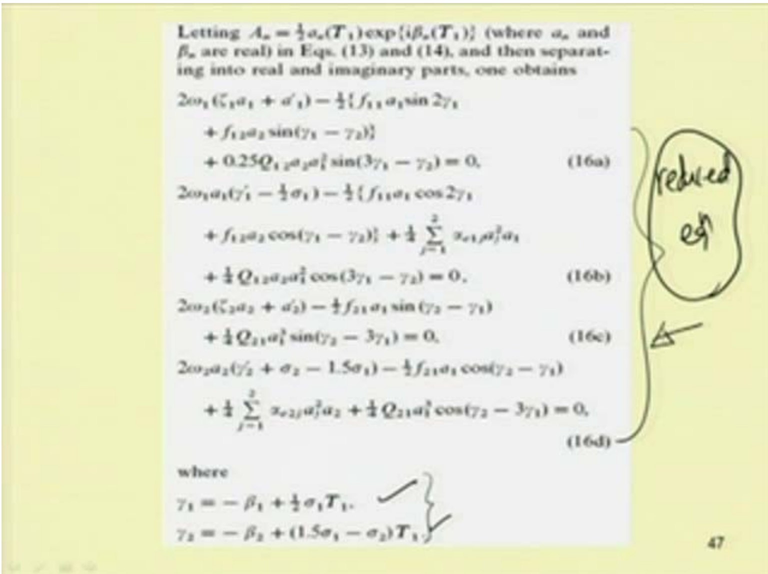

So, by substituting this a n equal to half a n e to the power a n e to the power I beta n we can write these equations. So, these two equations can be written and now by separating the real and imaginary parts by separating the real and imaginary parts. So, we can write these four equations. So, you can get a set of four equation. So, from the set of four equations we can we have to find the response of the system. So, to find the response of the system. So, from these equations we can observe that we have the terms a. So, this equation can be written a dash equal to a function of a one a two a one a two and beta one beta two or this beta one beta two also can be written in terms of the in terms of gamma one and gamma two where gamma one equal to minus beta 1 plus half sigma one T 1 and gamma two equal to minus beta two plus this. So, writing this beta in terms of this gamma one and gamma two to remove this time dependent term.

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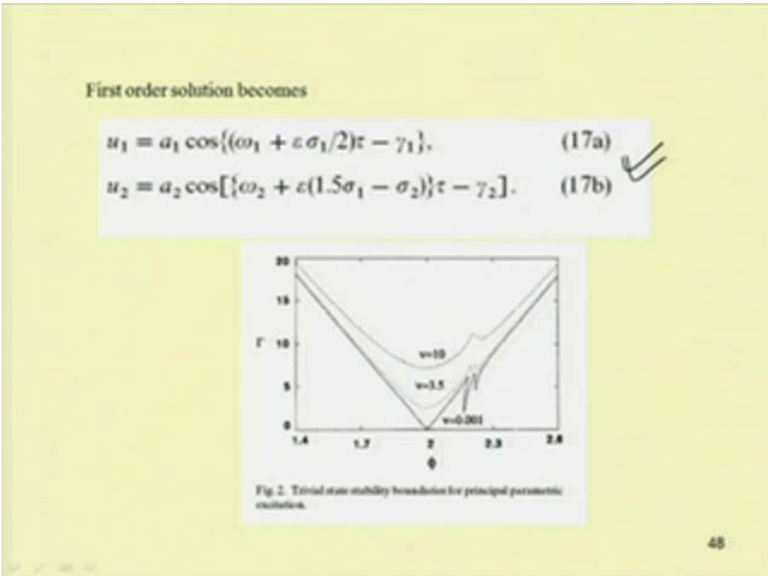

So, one can have several four equations now from these four equation for steady state one can find the solution in this from. So, in this way one can use method of multiple scales to find the response of the system and later we will see how these equations these reduced equations. So, these four equation what we obtained are known are the reduced equation. So, these equations these reduced equations that are one can use for studying the steady state solution. So, for steady state as a dash gamma dash a two.

Dash and gamma two dash will be equal to 0 this will leads to a set of non-linear algebraic equation. So, these non-linear algebraic equation can be solved to find the response of the system some typical response of this particular system is shown in this in this curve.

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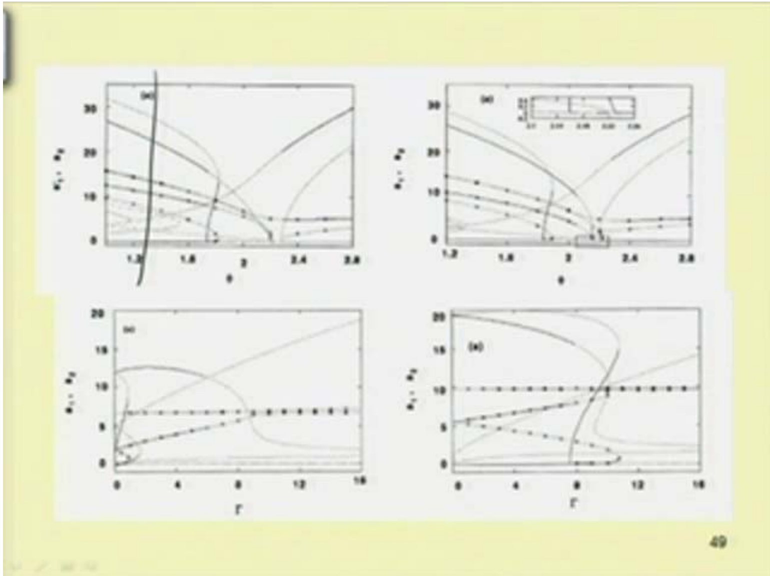

So, these are some typical response curve. So, one can see multiple solutions for a particular value of phi. So, as in case of non-linear we know. So, there will be multiple solution these this example shows how one can obtain multiple solution in this case.

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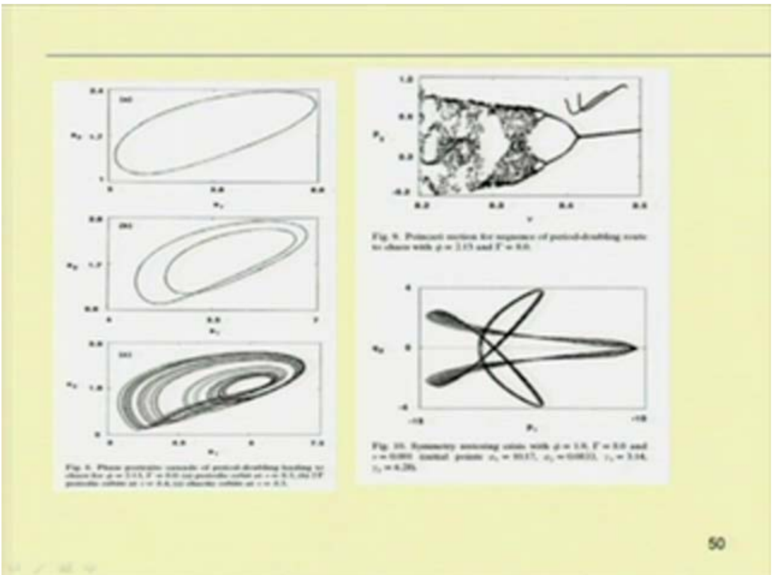

So, in addition to this fixed point response. So, in this case also it has been shown. So, the system will have some periodic response periodic to periodic and also finally, it have some chaotic response. So, this is the poincare section. So, here the poincare section is shown which clearly defeat initially a periodic solution then we have two periodic solution which bifurcates to have multiple solutions and finally, chaos. So, this is a period doubling route to chaos which we will study at a later stage.

So, these are some of the other chaotic type of attractor obtained in this type of system. So, these are some of the chaotic attractors and chaotic responses obtained in this system.

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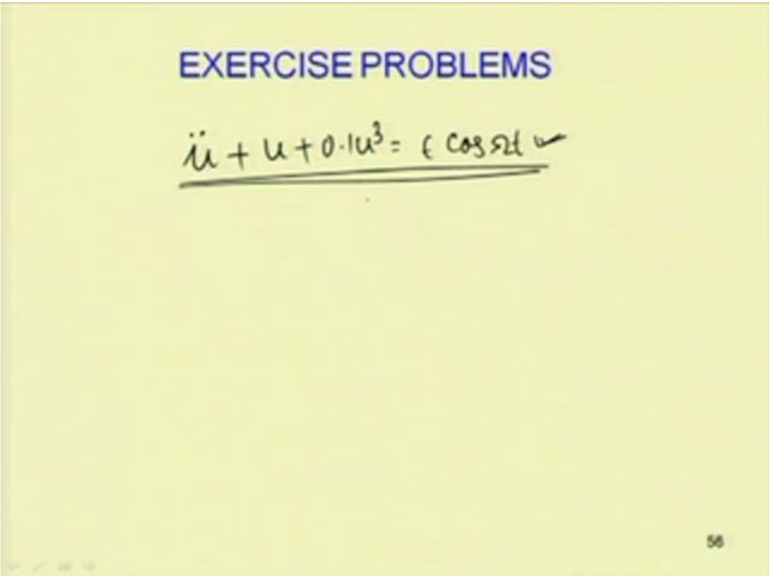

So, one can take one exercise problem. So, in this case one can solve this equation duffing equation with the forcing term epsilon double dot let this equation is written as u double dot plus u plus 0.1. So, let it is 0.1 u cube equal to epsilon epsilon cos omega t. So, this is a force vibration equation.

So, one can solve this equation to find the response of the system by using method of multiple scale. So, next class we will study about the harmonic balance method and how this harmonic balance method can be abrogated by using different type of other methods also we study in this model.

Thank you.