Non-Linear Vibration
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Module - 3
Solution of Nonlinear Equation of Motion
Lecture - 2
Solution of Nonlinear Equation
of Motion Using Numerical Technique and
Straight Forward Expansion Method

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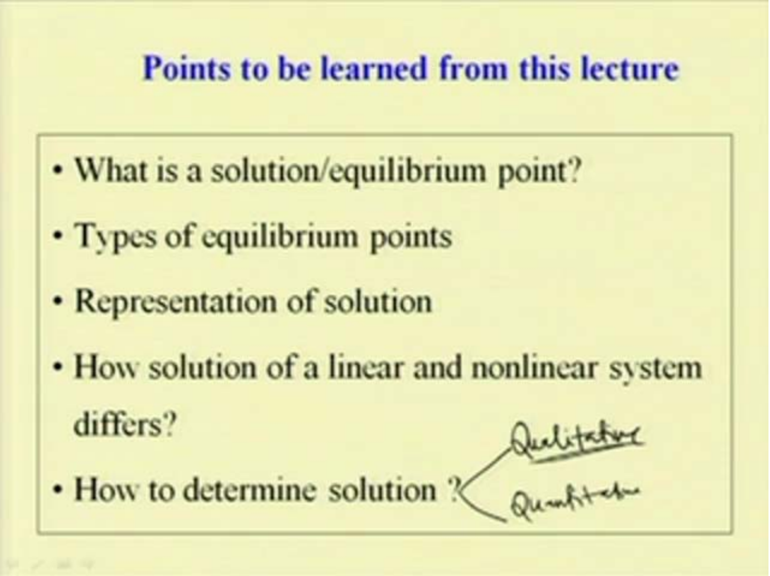

Welcome to the second class of this non-linear vibration of module 3. So, in this class we will continue, the solution procedure of the non-linear equations; and in the previous class, we have reviewed or we know about, what is a solution or equilibrium points, types of equilibrium points, representation of the solution, and how solution of a linear and non-linear systems differ? And how to determine the solution? So, in the last class I told you, to determine this solution, so we can apply or we can go for a qualitative analysis or we can have this quantitative analysis.

So, in case of the qualitative analysis, we have seen how to plot the phase portrait to find the response of the system? And also to find the equilibrium points, let us take few examples: few more examples and let us see, how to how these equilibrium points are determined?

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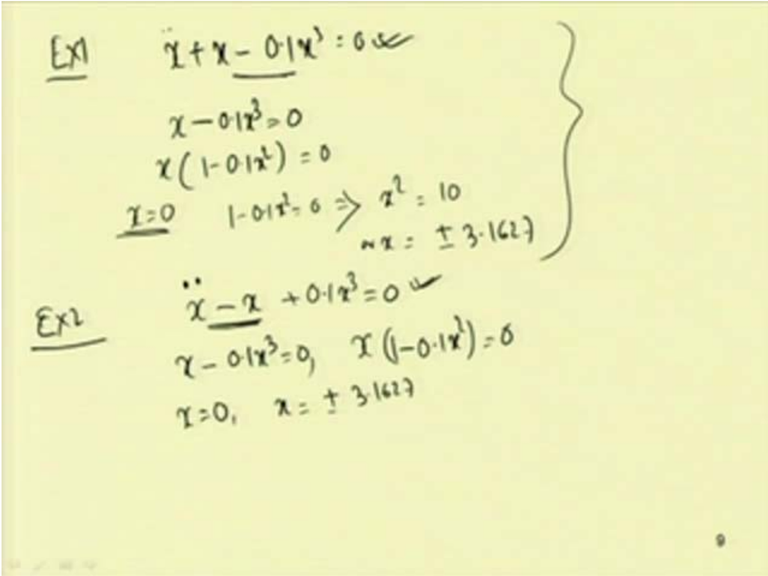

For example, so let us take this example: Let, we have a equation x double dot plus x minus 0.1 x cube equal to 0. So, as told before in case of the equilibrium position, or in case of the equilibrium solution, so if it is steady state solution, then it will not be a function of time. So, this x double dot or any function of any variation of x, with time should be neglected, so in this case, we can neglect this x double dot equal to put x double dot equal to 0.

And the remaining thing, we can write as x minus 0.1 x cube equal to 0; and this will give rise to x into 1 minus 0.1 x square equal to 0 or so, in this case we can have this x either x equal to 0 or, this 1 minus 0.1 x square equal to 0, which will give rise to x square equal to 10 or x equal to plus minus 3.1627. So, in this case, we have 3 values of this x so, these 3 are the equilibrium position for this equation.

So in this equation, x equal to 0 refers to the trivial solution, and x equal to plus minus this 3.1627 are the non trivial solution of the system. So, one can use this qualitative analysis procedure, to find the phase portrait of this equation, and one can check, whether these points correspond to this center saddle point? and one can find the separate ricks also in this case? So, let us take another example: So in this case, let us take this equation. In this form, x double dot minus x plus 0.1 x cube equal to 0. So, in the previous case, the this is similar to, this doffing equation with a subtending type of spring with cubic nonlinearity. This is we have this minus 0.1 x cube.

And in this case, we have taken a equation, we have this linear stiffness part is written as negative. So, in this case for this steady state solution, this x double dot will be equal to 0, and this equation can be written as so x minus 0.1 x cube equal to 0. So, we can have the solution x or x into 1 minus, so x into 1 minus 0.1 x square equal to 0 or, we can have x equal to 0 and x equal to plus minus 3.1627. So in this case also, we have same solution; that is x equal to 0 is a solution and x equal to plus minus 3.1627 another two solutions; so these are the equilibrium points of this equation.

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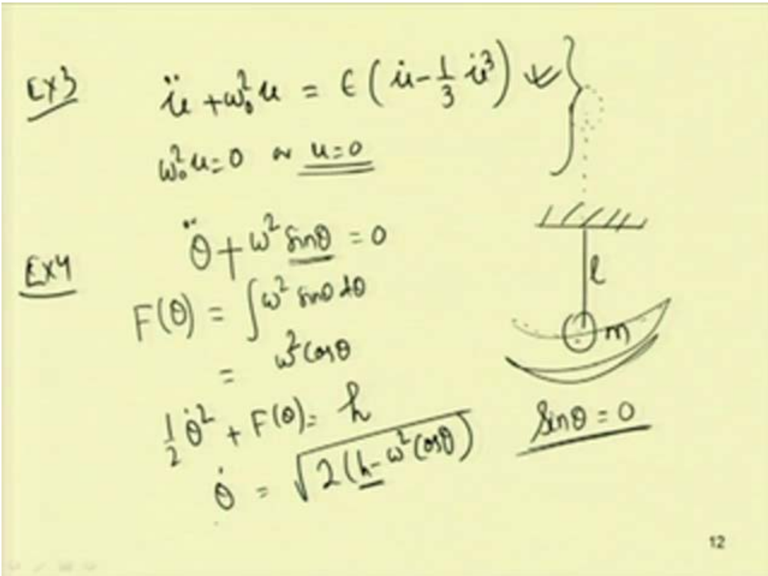

So, let us take another example; so in this case; so let us take the example; example 3: So, in this case u double dot plus omega 0 square u equal to epsilon into u dot minus, let me take this u dot u cube. So, in this case for steady state, one can put this u dot and u double equal to 0. So by putting, u dot and u double dot equal to 0. So, one can obtain the equilibrium solution as omega 0 square u equal to 0 or u equal to 0. So, this u equal to 0 represents the equilibrium solution in this case.

So, one can obtain; so this is the 100 pole equation, and the previous two equations are doffing equations. And, one can obtain the response of the system by using qualitative analysis or quantitative analysis. And today class, we will see, how you can use a straight forward expansion method for find the solution of the system? Also, one can go for this numerical analysis method, or numerically, one can solve these equations to obtain the solution of the system. And, let us take another example: In which, one has to plot this potential or one has to do the qualitative analysis for a simple pendulum.

So, in case of the simple pendulum, the equation can be given. So, in case of a simple pendulum, the equation motion of this, so this has a mass m, and so the system has a mass m and length is l. So, the equation of motion can be written as theta. So if it is moving with theta, this equation is theta double dot plus omega square sin theta equal to 0. So, where omega square equal to g by l, so g is acceleration due to gravity, and l is the length of the simple pendulum.

So, in this case, so if one do this qualitative analysis, so here one can find this f theta equal to, that is the potential function equal to integration of this f u d u. So that means, integration of this omega square sin theta d theta, so that thing can be written in this form. So, omega square cos theta, so the equation can be written by integrating this equation. Already you we know, that it can be written in this form. So, ½ theta dot square plus f theta will be equal to h or one can write this equation, or one can write this theta dot equal to root over 2 into h minus omega square cos phi. Here, one may note that, unlike in case of simple degree of freedom system, when we are taking sin theta equal to theta, here one can takes this instead of taking sin theta equal to theta, one can expand or one can take it directly.

And, one can find this expression for theta dot equal to 2 into h minus omega square cos theta. So, for different value of h, so one can plot this flow, that is theta dot. And, one can find the equilibrium position. For example: In this case, the equilibrium position, one can have two equilibrium position one, so by putting this theta double dot equal to 0. The equilibrium position corresponds to sin theta equal to 0.So, sin theta, so these correspond to sin theta equal to 0. So, in this case sin theta equal to 0, at theta equal to 0 and 180 degree. And so, one can have two positions; so this is one equilibrium position and the other equilibrium position will be this one. So, one can have two equilibrium position, one correspond to theta equal to 0 and other correspond to theta equal to phi.

So, it may be so when, we will study the stability analysis or when we will carry out the stability analysis of the system, we can see that this equilibrium position is a stable equilibrium position, while the other equilibrium position is an unstable equilibrium position. So, this equilibrium position represent in the absence of this damping. So, this will represent a center and the motion will be periodic. So, one can have a periodic motion but, this equilibrium position will be an unstable saddle point. So, one can plot this theta dot versus this theta to qualitative analyze, the throw or trajectory of this system.

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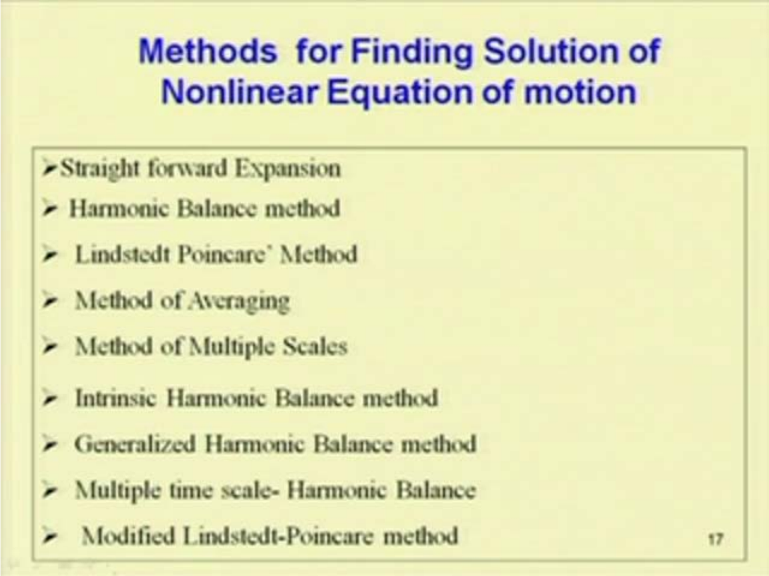

And, one can do this numeric. Last class several methods has been pointed out, which can used for solving this non-linear equation motion. So, these methods include the straight forward expansion method, harmonic balance method, lindstedt Poincare method, method of averaging, method of multiple scales, intrinsic harmonic balance method, generalizing harmonic balance method, multiple time scale harmonic method and modified lindstedt-Poincare method. And before, studying this straight forward expansion, let us take or let us study about the numerical methods are by using the numerical method. We can find the solution of these equations.

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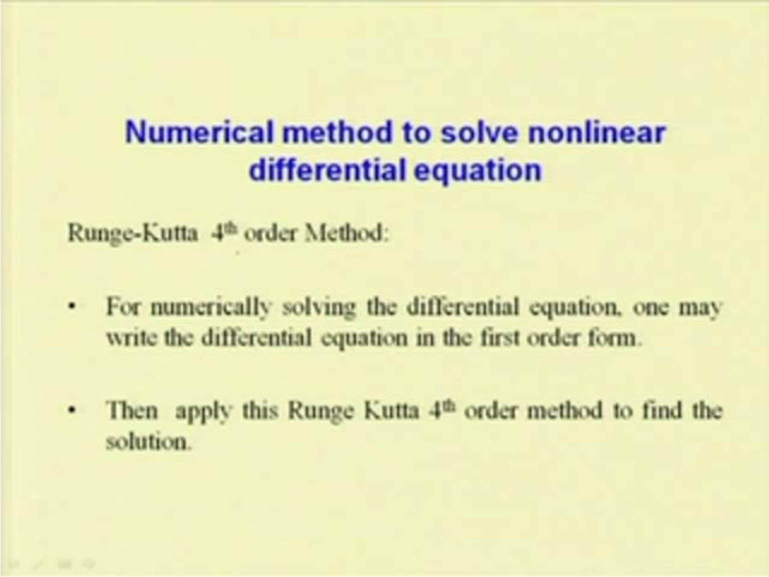

So, in case of the numerical method, let us take this Runge-Kutta method, for Runge-Kutta 4th order method for finding the solution.

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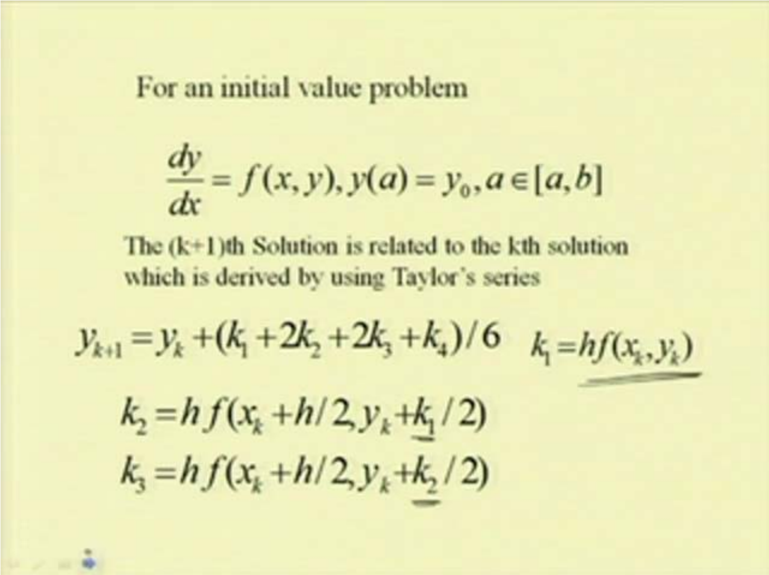

So in this case, in case of the Runge-Kutta method, so one can solve this initial value problem d y by d x. In this form, d y by d x equal to f x y, where y a equal to y 0, and this a will be within a and b. So, this d y by d x while solving this equation, d y by d x equal to f x y. So, it has four steps; so one can find the k plus 1 eth solution as a function of k eth solution as follows; so y k plus 1 will be equal to y k plus k 1 plus 2 k 2 plus 2 k 3 plus k 4 by 6, where k equal to h into function of x k and y k. So, in this case this k 2 can be written as h into f x k plus h by 2 y k plus k 1 by 2.

So, this function evaluated at, x k plus h by 2 and y k plus k 1 by 2, will be used to find this k 2 constant k 2, similarly constant k 3 can be found by putting this or evaluating this function at x k plus h 1 by 2 and y k plus k by 2 k 2 by 2; so this is k 2 and this is k 1 and this k 4 also can be obtained. So, by using this k 1, k 2, k 3, k 4 in this equation, so one can evaluate this y k plus 1 as a function of y k. And, one can find the solution of any governing equation of motion, any differential equation motion.

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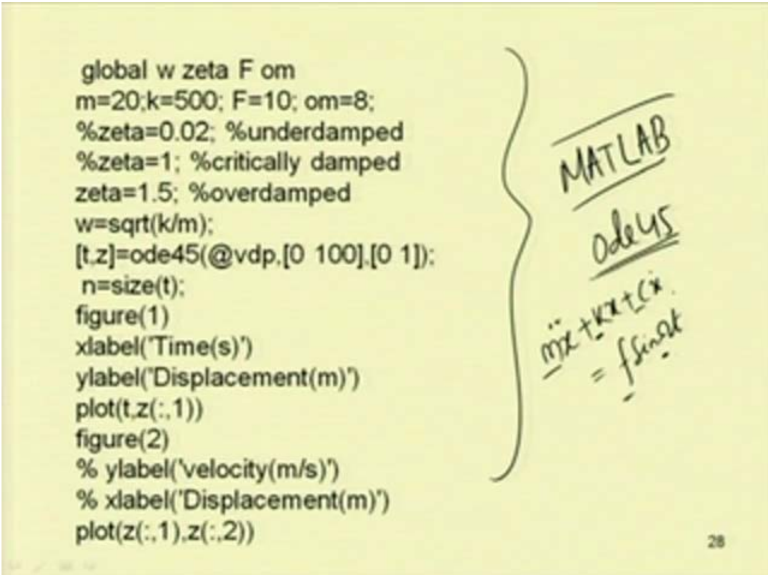

Or one can develop a code, using this MATLAB to find the response. So, in MATLAB a function ode 4 5, one may use to solve these equations. So, this equation may be one can solve a linear equation or non-linear equations; so one can write a simple code in MATLAB to find this. So, here let us use this equation or use this MATLAB function to solve this linear equation first. So, let us solve this equation m x double dot plus k x plus c x dot equal to f sin or cos omega t f sin omega t. So, in this case for a given value of m k c and f and omega, one can find x. So, first one has to reduce this equation, this second order equation to a set of first order equation; so after reducing this equation to a set of first order equation, one can find it is solution. So, to reduce this thing into a set of first order equation, so one can write this equation in this form.

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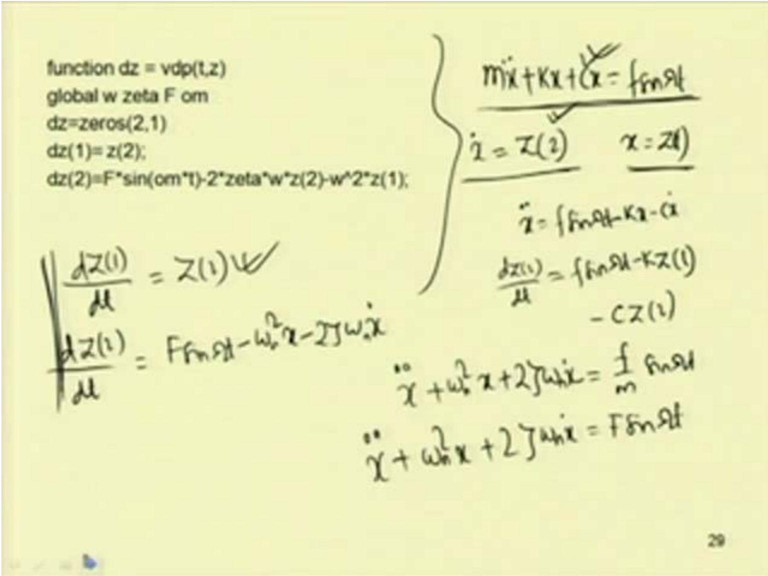

So, one can write this equation ok. So, let me write this equation, so the equation is m x double dot plus k x plus c x dot equal to f sin omega t. So, one can write this x, let one put this x dot equal to z 2 and x equal to z 1. So, if one substitute x equal to z 1 and x dot equal to z 2, then this equation can be written in this form. So, x dot equal to z 2 or one can write this x double dot equal to f sin omega t minus k x minus e x dot, so this thing can be written this x double dot as x dot is written z 2, so this x double dot equal to d z 2 by d t. So, this is d z 2 by d t, so this will be equal to so d z 2 by d t will be equal to f sin omega t minus k. So, for x 1 put this equal to z 1 minus c, for x dot 1, put it z 2 or this equation this equation also can be written in the form: x double dot plus omega n square x plus 2 zeta omega n x equal to f by m sin omega t, so this f by m can be written in this form.

So, x double dot plus omega n square x plus 2 zeta omega n x equal to f sin omega t, so substituting this x equal to z 1, and x dot equal to z 2.So, one can get two differential; first order differential equation: the first differential equation is this one, that means d z 1 by so, first differential equation equal to d z 1 by d t equal to z 2, and second equation becomes, d z 2 by d t, so this becomes f sin omega t minus omega n square x minus 2 zeta omega n 2 zeta omega n x dot. So, by putting these two equation in a function, so here in MATLAB; this function written is written as, v d p. So, function d z equal to v d p t z and using this global value this or this w represent this omega n, and this zeta f, f is the amplitude of the expectation, and omega o m is the frequency of this excitation.

So, by putting this as global variable, so one can write d z equal to 0, so initially it as, we have two equations. So, we can put it 0’s to 1, so d z 1 equal to z 2, this is the first equation. And, the second equation is written d z 2 by d t; as d z 2 equal to f sin omega t minus 2 zeta omega z 2 minus omega square z 1. So, using this function one can find, one can use this function in this ode 45; this is the command for finding this thing. So, d z equal to omega o d e 45 v d p, so by taking this initial condition, so one can take the time step and initial condition and find the response of the system, so taking it 0 to 100 time step.

And, 0 to 1 as the initial condition, so one can find the response, so n equal to by putting this n equal to size t. So, here so 3 cases have been solved; one first case by taking m equal to 20, k equal to 500, f equal to 10, and omega equal to 8. So, the response have been found for the under dumped case by taking zeta equal to 0.02 ,critically damped case zeta equal to 1, and zeta equal to 1.5 for the over damped case. So, the response curves have been plotted both in phase portrait and time response.

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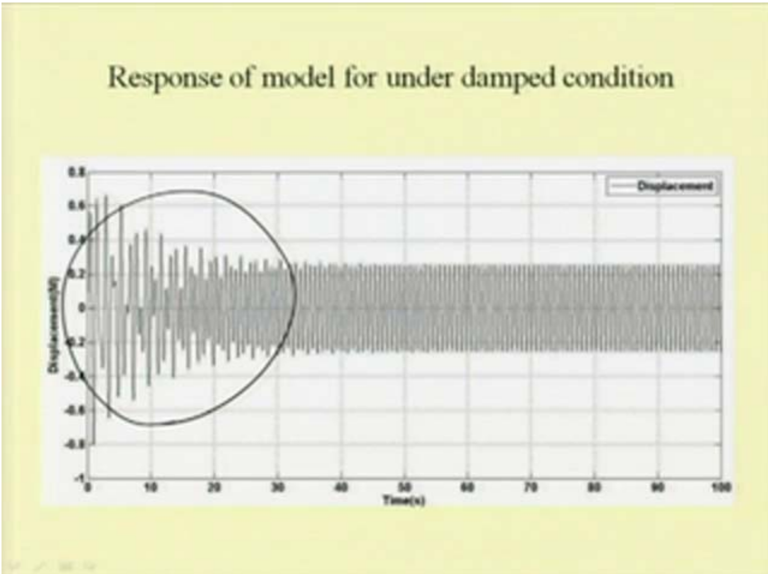

So, let us see this thing. So, for a under damped case so for a under damped case: one can see the initial part, which represent the transient part of the response, so it is varying with the initial condition, and it decreases, and finally it approaches a steady state response. So, this is the state response raised after several time steps. So, one can find the steady state response by using this method also, this is the transient part, this is the steady state part and this shows the time response of the system.

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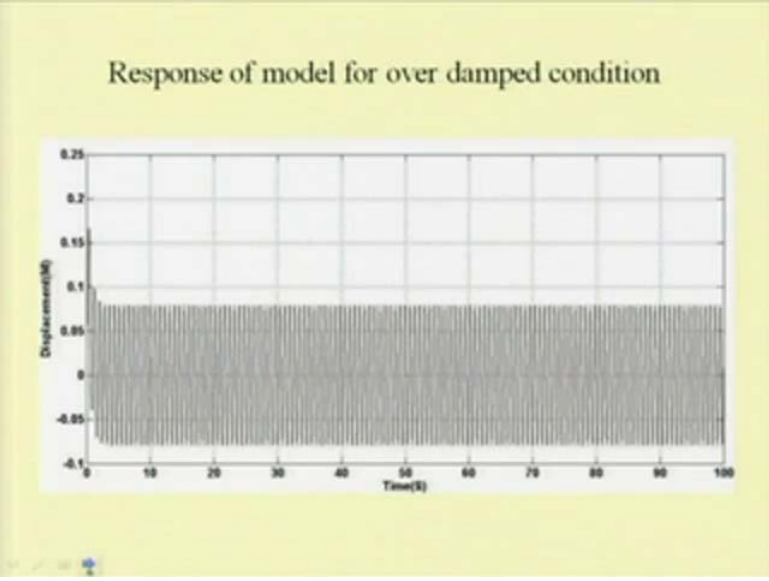

So, one can find the time response for the critical critically damped case. So, this is represent the critically damped case; so here the displacement in meter, and time in second. Similarly, one can find the response for the over damped case.

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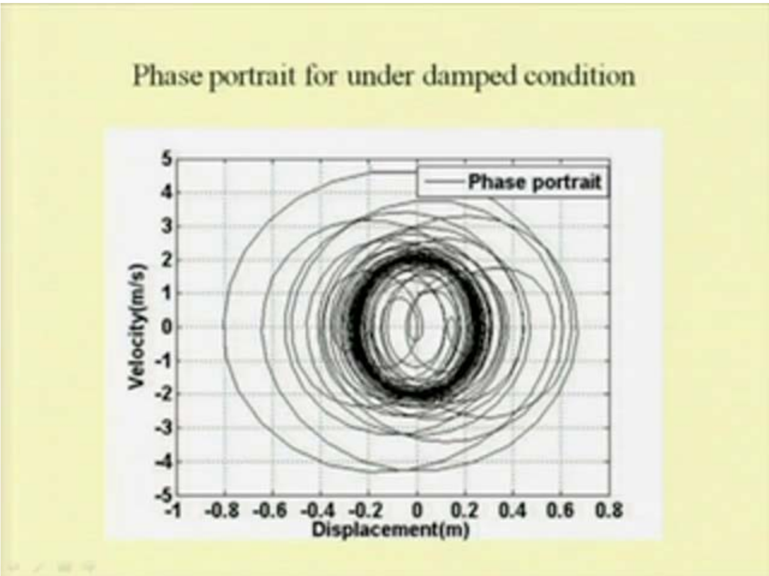

And, if one plot the phase portrait; so this phase portrait is plotted along with the transient part. That is why, it shows both transient and the steady state response part. This curve shows, the steady state part and starting for a initial condition. So, this shows the both transient and steady state part for this damped condition.

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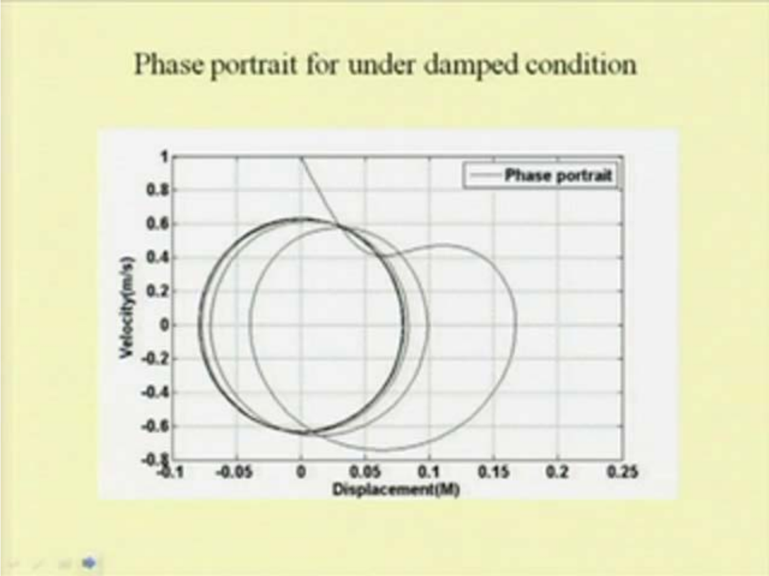

And, for the critically damped condition, so this response, this phase portrait represent the phase portrait for the critically damped case. And this represent for the under damped case.

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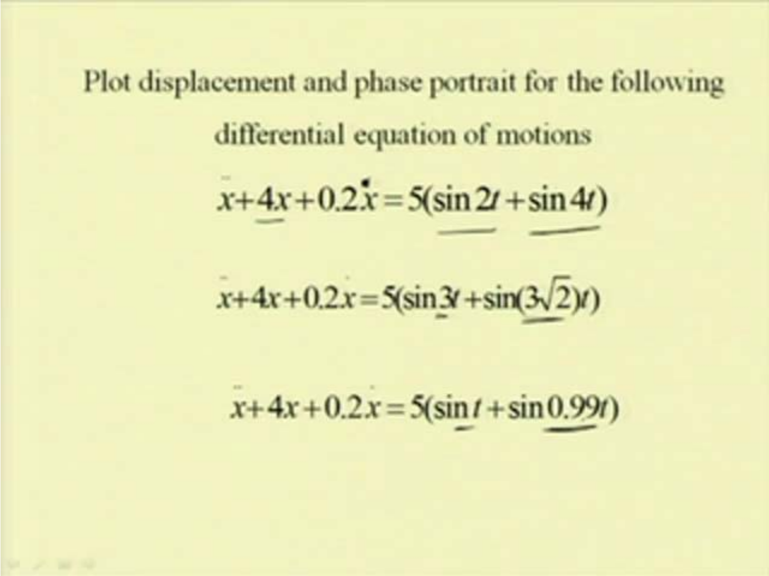

So let us see, three more examples. So, in this case instead of taking a single forcing function, if one take two forcing function, so this is a 2 frequency excitation case. So, in this case one can note that, this x double dot plus 4 x plus 0. 2 x 0. 2 x dot; here this 4 is the omega n square, so omega n equal to 2. The nature frequency of the system equal to 2. And in this case, it is subjected to a excitation with a frequency 2 and 4 and with amplitude 5. In the second example: So, it is subjected to a frequency of 3 and 3 root 2, and in the third example: it is subjected to a frequency of 1 and 0.99. So, in the first case one except to have two periodic response; one corresponding to this 2 t, and other one corresponding to the 4 t, corresponding to omega equal to 2, and other correspond to omega equal to 4. But, in the second case as the ratios are irrational number; so that means 3 root 2 by 3 equal to root 2.

So, one can get a quasi periodic response in the solution. In the third case, the response will be a periodic or one can get a beating type of response, as the frequencies are very near to each other, so this is 1 and this is 0.99. So one has a beating frequency of 0.01.So, by using this Runge-Kutta method, the response have been plotted, so in the three cases it has been plotted.

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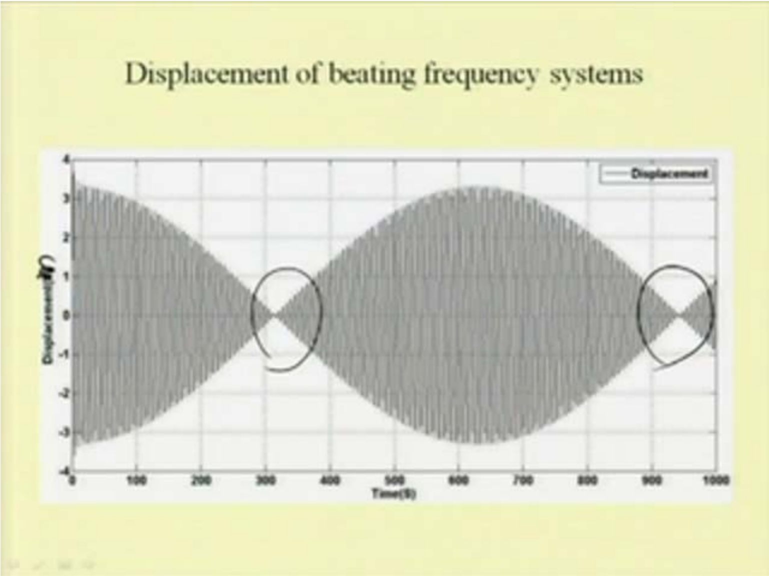

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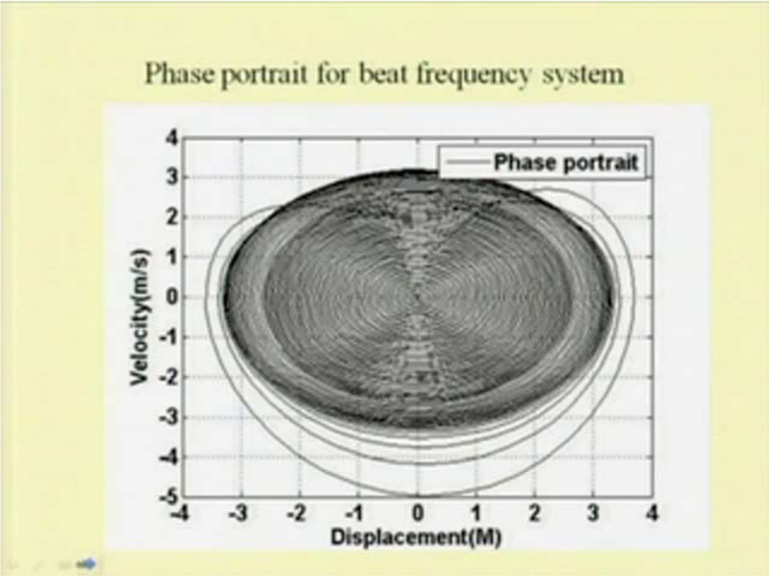

So, this is the phase portrait and time response. So, quasi periodic response and so, this is the phase portrait for the quasi periodic response; and this is the response for the beating type of beating type beating phenomena. This shows the constructive and destructive inference type of things, and when one has frequency nearly equal to so, the as the beating frequency is 0.1; this shows the response, and one find the amplitude; maximum amplitude and minimum amplitude. In this case, so by using this method this Runge-Kutta method, one can find the time response, phase portrait of the system. So, in addition to finding this equation of the linear system, one can use the same equation for the non-linear case also.

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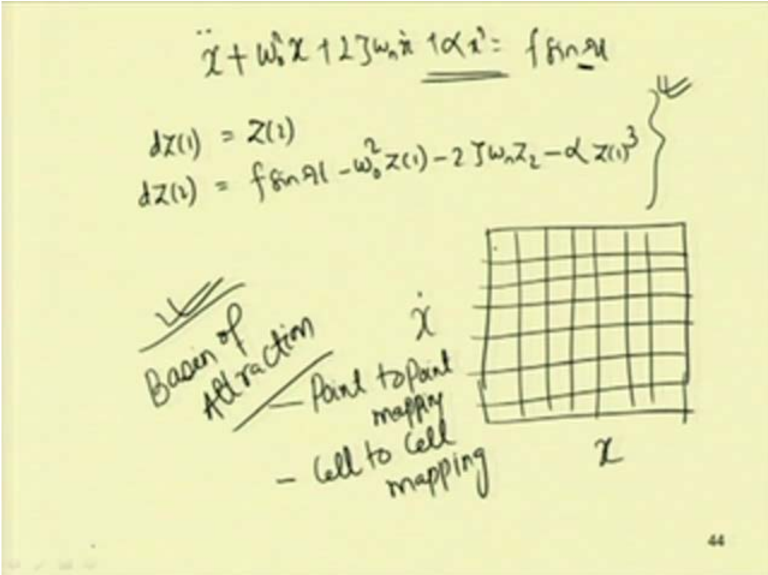

Let us take, the doffing equation. So, in case of the doffing equation; so x double dot plus, let us take a equation in this form: x double dot plus omega 0 square x plus 2 zeta omega n x dot plus alpha x cube, let it is equal to f sin omega t, so in this case, so one can write those two function; that is d z 1 equal to z 2, and the second equation d z 2.

One can write this in this form, so it will be f sin omega t minus omega 0 square z 1 minus 2 zeta omega n z 2 minus alpha z 1 cubed, but it may be noted, that in this case as one can have multiple solutions. Unlike in case of the linear system, what we have seen before, so it depends on the initial condition. What we are supplying to the system? So, while solving so depending on the different initial conditions, so one can have different phase portraits and different time response of the system.

So later, when we study about different resonance conditions, and then we will know that. This equation will have many different types of solution, depending on the resonance conditions, and for that matter, for a particular value of forcing frequency omega, for a particular value of forcing frequency omega as it have several solutions. So, one can plot or one can plot the phase portrait, in which taking different initial conditions. One can have the equilibrium positions.

So, one can find the equilibrium position, so by using this Runge-Kutta method. So, one can find the steady state solution, and as it has several solutions; so depending on the initial condition; it will go to a particular solution. So, this plot, which shows the variation of the trajectories depending on the initial conditions are known as basin of attraction basin of attraction.

So, by plotting this basin of attraction, so one can find so one can find the trajectories starting form a particular point and ending at the equilibrium position. So, this basin of attraction there are several methods to plot, this basin of attraction. So, which we will study in later module but, one may note that, the methods may be a point to point mapping method, point to point mapping or cell to cell mapping method. So, one can use either a point to point mapping method or cell to cell mapping method, to plot this basin of attraction for a non-linear system. To find all the equilibrium positions. So later, we will see how to solve this equation by using this Runge-Kutta method and qualitatively qualitatively.

Already, we have seen one example of the Duffing equation, to plot the phase portrait and to study the phase portrait also graphically. There are several methods available to study this phase portrait. So, one such method is the method of isoclineso. In case of, method of isoclines, so one can let us see the method of isoclines.

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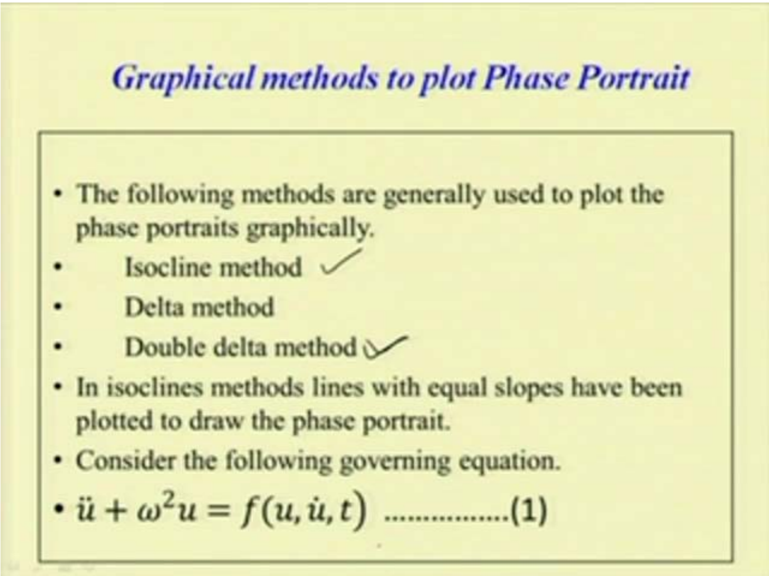

And so, there are several methods. So, one method is method of isoclines, second method is delta method. Then, we can have the modified delta method and we can have the double delta method. So, today class we will see about this isocline method. And, then we will study the straight forward expansion method. So, in case of method of isoclines; iso means similar or equal, so isocline method one has to plot lines with equal slopes, and draw the phase portrait. So let us consider, this equation u double dot plus omega square u equal to f u u dot and t.

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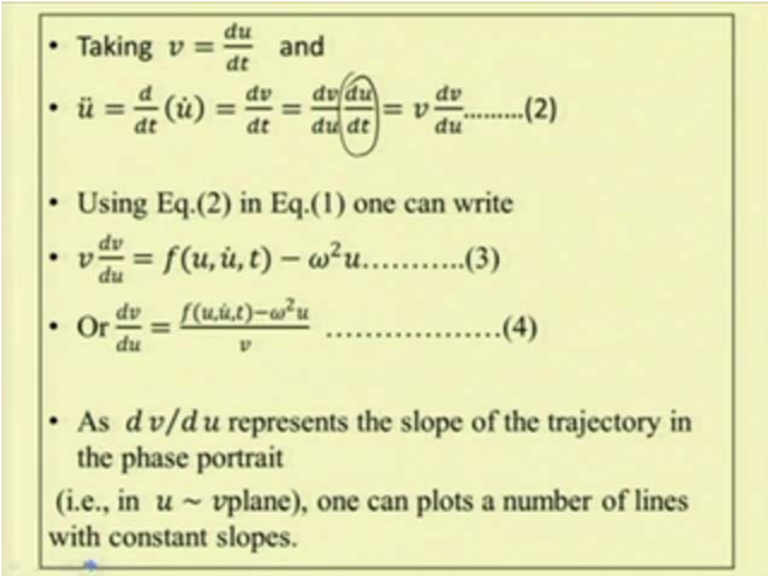

So, in this case by taking v equal to d u by d t. So, one can write this u double dot equal to d by d t of u dot, so this is equal to d v by d t equal to, one can write this is equal to d v by d u into d u by d t, and as we are taking this d u by d t equal to v. So, one can write this u double dot equal to v into d v by d u, so by taking this equation or substituting this equation in the first equation, one can write v d u by d t equal to f u u dot t minus omega square u or this d v by d u equal to f u u dot t minus omega square u by v as d v by d u, represent the slope of the trajectory in the phase portrait. So, one can plot this phase portrait in u v plane, and one can find this phase portrait.

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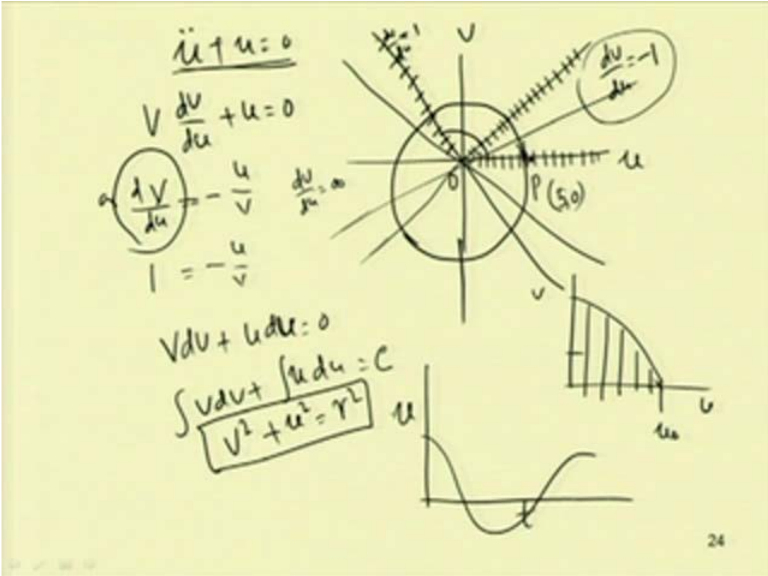

So, for example let us take the equation, u simple linear equation: u double dot plus u equal to 0. So in this case, one can write this equation in this form for u double dot. One can write this is equal to v into d u by v into d v by d u, so v into d v by d u plus u equal to 0 or one can write this v d v by d u equal to minus u by v, now by taking different value of d v by d.

So, if one take this d v by for example, if one take this d v by d u equal to 1, so then this will be equal to u by v or if one plot this curve u by v, so let this is v and this is u, so this represent a curve, this represent a curve u v; so this is a this represent a straight line with 135 degree slope; so it makes an angle 135 degree with the u axis. Similarly, one can take different value of this d v by d u and plot several lines, so let one take this d v by d u equal to minus 1. So in that case, this will be a line with 45 degree. Similarly, by taking 3 by 2; that means d v by d u equal to 1.5.So, one can get another line, so this way one has to in this method, one has to draw several lines, and this so represent so when this represent v equal to, so this is v equal to 0 ,v equal to 0 or by putting this d v by d u equal to infinity; so this line represent, so the first line what we have plotted with 135 degree, so then this this is d v by d u; so d v by d u equal to 1.

So, this is d v by d u equal to minus 1, and this represent this d v by d u equal to infinity, so after getting these curves. Now, so for example, taking this d v by d u equal to 1, so this represent u equal to minus v, now one can find, now one can draw several lines. So, on this curve draw several equi-space lines with a slope d v by d u equal to, so this d v by d u equal to 1, so by drawing several lines with slope equal to 1. So, one can draw several lines 1, means so it will make the slope will make 45 degree with this u axis. So, one can draw this way.

Similarly, for this line one can draw, so as d v by d u equal to infinity. So, it will come in a straight way; so this way one can fill this curve with equating equal distance spaced lines, with the slope given by or the slope taken here. So, by putting these lines, now by starting at a point. So, let us start at a point this, so let us start at a point p, which represent some initial condition, for which this u and v are given; let u and v, so let this is 5 0. So, then one can find the next point, so this curve will start here.

And, if one has several lines. Then one can have a curve, one can obtain the phase portrait, so by joining these lines, so in this case actually this should be this should represent a circle with, so this should represent a circle, with radius (()). Because, in this equation v d u by d v by d u plus u equal to equal to 0.So, one can write this v d v, so v this represent v d v plus u d u equal to 0, u d u equal to 0 or by integrating this thing, one can get, so by integrating one can by integrating this equation v d v plus integration v d u ,so one get a constant. So, this or one get a constant.

So, let us write the constant as c. So, this is v square or one can write this equation v square plus, so this is u v square plus u square equal to r square, so in this case, where r is a constant. So in this case r equal to 5, so by using this method, one can by using this method of isoclines; so one can find the phase portrait of this system. So, after finding this phase portrait, one can find the time and then plot the response of the system.

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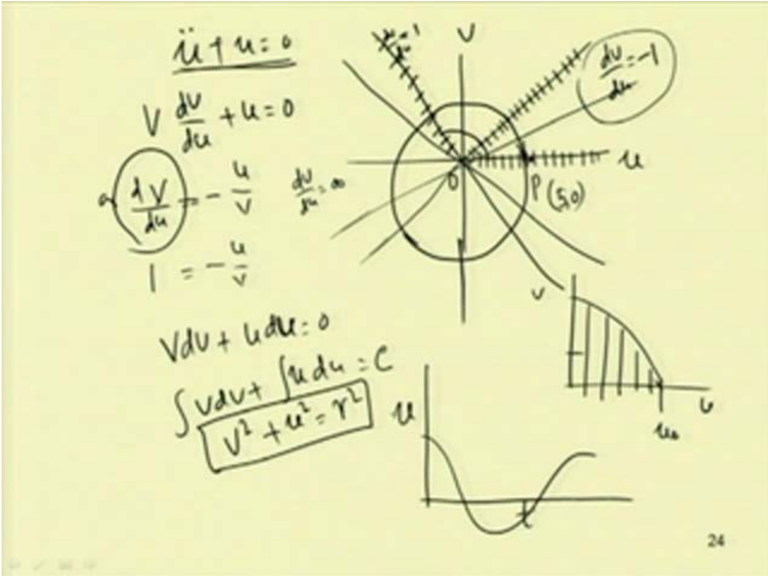

So, to find the time one can take, let us take this one quadrant of this, and divide this phase portrait; that is v and u into several time, several steps. So, by taking several steps, let at so this is at u so u, so this represent u 0; so this is u 0; that is initial condition so u 0, so at so one has to find, what is the time period or what is the time or with time how this response varies? So, to plot this y versus t, to get the time response, so one has to plot this u versus time, so for that purpose one should know, what is this time? So, this time can be obtained from this equation.

So, one can find this time from this equation d v by d u equal minus u by v or one can get this v equal to d u by d t, so as v equal to d u by d t, and the slope is known, so one can find this time. So, this time will be equal to this h by v, so by taking this average velocity as v or for this from this to this taking this velocity as v 1, so it will be h by v 1. So, for this so the time will be equal to h by v 1 plus h by v 2, so in this way one can find this time, so after finding this time; so as one knows, what is the position, that is u, so one can plot this curve so in this particular case.

So, it will be it will follow the sinusoidal curve; that is y equal to or u equal to a sin omega t plus v cos omega t, so this is the method of isoclines. So later, we will study the method, del delta method and double delta method or modified delta method. And, today class now, we will study about the straight forward expansion method. To find the response of the system. So, in case of the straight forward expansion as pointed out in the last class.

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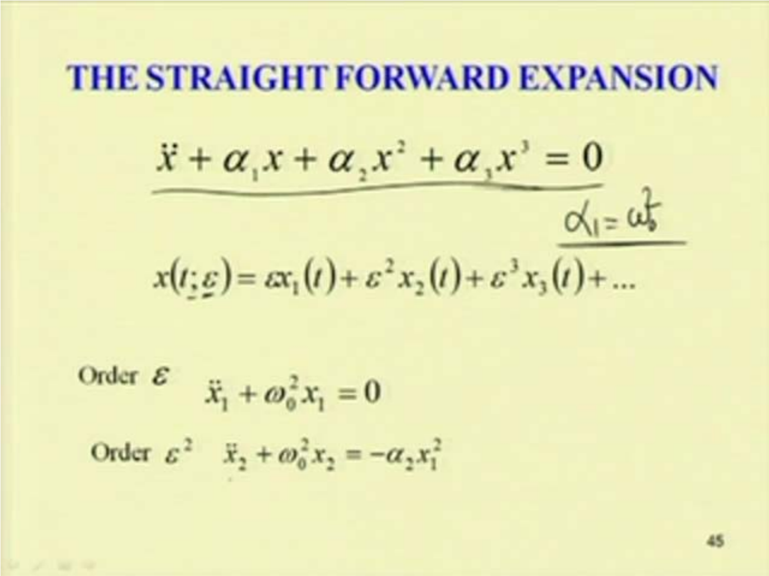

So, in case of the straight forward expansion method, let us take this equation x double dot plus alpha 1 x plus alpha 2 x square plus alpha 3 x cube equal to 0.So, one can put this book keeping parameter or one can use this book keeping parameter, to write this equation also. In that case, one can use some book keeping parameter to represent the nonlinearity of the system. So, in this case let us take this alpha 1 equal to omega 0 square.

So, by taking this alpha 1 equal to omega 0 square and one can expand this equation or one can write this response x, which is a function of time t, t is the variable and epsilon is the parameter, that is why one use this semicolon instead of a comma. So, this x t semicolon epsilon, so can be written in this form epsilon x 1 t plus epsilon square x 2 t plus epsilon cube x 3 t, and if one expand this thing or one substitute this thing in this equation. So, then one can write this x double dot equal to epsilon x 1 double dot plus epsilon square x 2 double dot and plus epsilon cube x 3 double dot plus alpha 1 into x plus alpha 2 into the square of these three terms. So, one can take higher so more terms also by taking only these three terms, so and expanding this equation one can write this is this will be epsilon square x 1 square epsilon 4th x 2 4th and epsilon to the power 6 x 3 square.

So, plus 2 into epsilon cube x 1 x 2 plus 2 into x epsilon 4th x 1 x 3 and plus 2 into epsilon to the power 5 into x 3 x 2, so in addition to that, if one take this term, then it will be alpha 3 into the cube term of this equation. So, by substituting this equation in this and ordering of the order of epsilon, so one can find the first equation, one can get that is equal to x 1 double dot plus omega 0 square x 1 equal to 0, so in the order of epsilon square one get the equation x 2 double dot plus omega square x 2 equal to minus alpha 2 x 1 square.

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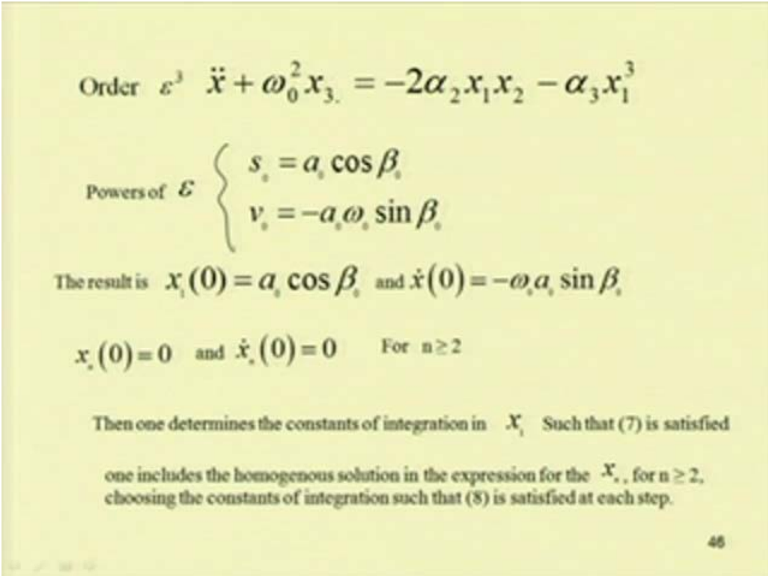

And, order of epsilon cube, one can get x double dot plus omega 0 square x 3 equal to minus 2 alpha 2 into x 1 x 2 minus alpha 3 into x 1 cube, so for a given initial conditions, that is at late at time t equal to 0. So if it is given that, the steady state response, so this is equal to s 0 equal to a 0 cos beta 0, because one can find for this order of epsilon.

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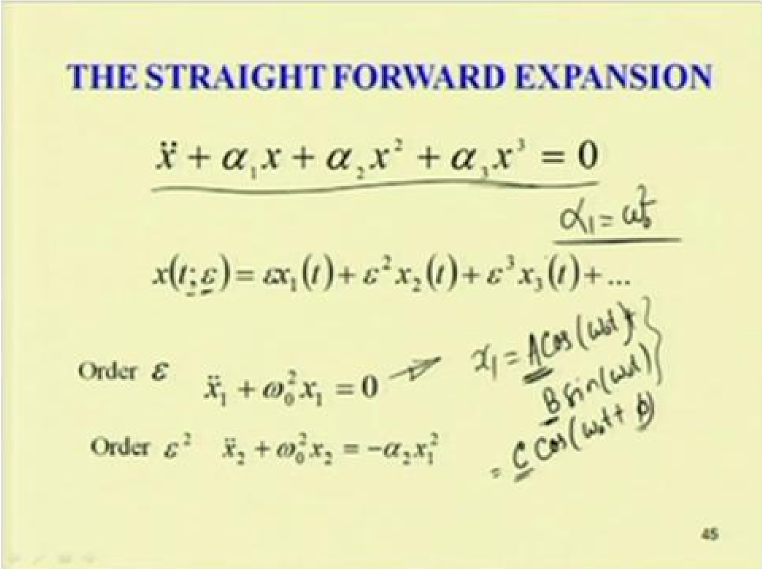

So, the solution of this equation, one can write x 1 equal to, so one can write the solution equal to A cos omega 0 t plus phi or a cos omega 0 t plus B sin omega 0 t, where A and B, so A cos omega 0 t plus B sin omega 0 t or one can write this equation in this form using another constant; that is C cos omega 0 t phi, so as it is a second order differential equation. So, one can has two constant; so either one can use this constant A, B or one can use the constant C and phi; so both these constant can be obtained from this initial condition.

So, for the given initial condition, one can substitute it in this equation and find this A and B or C and phi. So, the solution so there are two procedure; one can adopt in the first case, one can use so there are two procedure in the first case; one can substitute the initial condition first, and find the coefficient A and B, and then proceed for the order of epsilon square or in the second case; one can find or substitute this initial condition, after finding all the solutions by taking first taking this equation, and instead of finding this initial condition, substitute this equation in the order of epsilon square and order of epsilon cube, and at last find these coefficients or find these constants.

So, one can use either the first procedure, in which in the first stage itself, one can find this constant A and B or C and phi, or in the second case; one can find this constant at the end condition. So, one can it can be shown that, in both the cases, one can obtain one can obtain same solutions for this equation.

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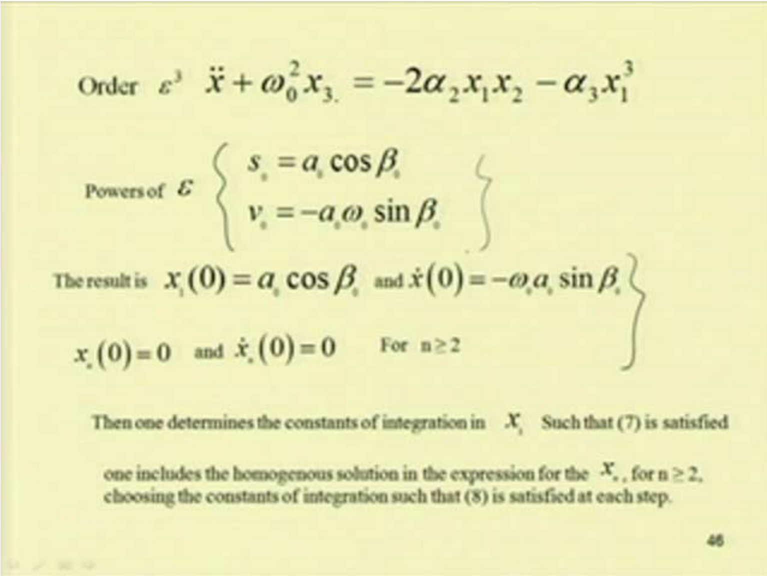

So, one can have this initial condition. This way and one can write this x 1 0 equal to a 0 cos beta 0 and x dot 0 equal to minus omega 0 a 0 sin beta 0, and the higher order term; that is x 2 0 equal to 0 and x dot x n dot 0 equal to 0, by putting this n equal to n greater than 2 for n greater than 2; one can take this and one can substitute these two equation in this order of epsilon. so after taking this equation, one can one can substitute this equation in the order of epsilon square and order of epsilon cube to find the solution.

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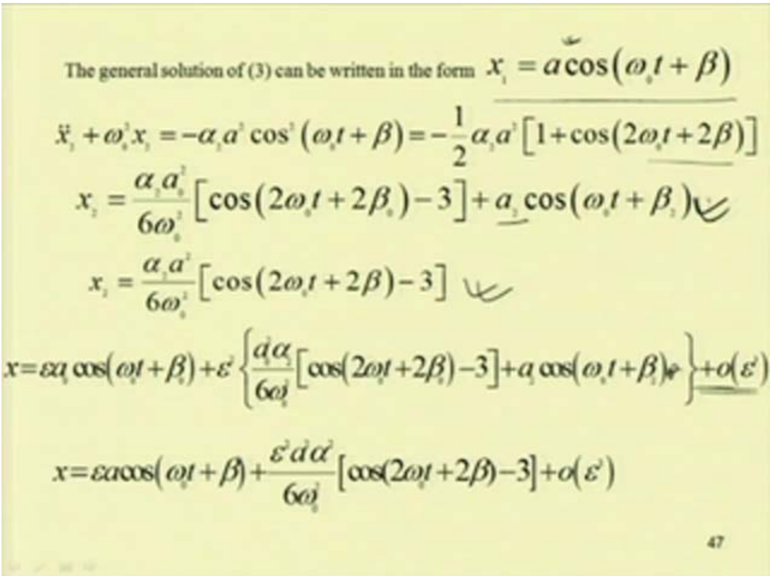

So, the general solution in this case, one can find the General solution can be written in this form: x 1 equal to a cos omega 0 t plus beta, and this x 2 double dot plus omega square x 2 can be written as, minus alpha 2 a square cos square omega 0 t plus beta, so which will be equal to minus half alpha 2 a square into 1 plus this cos square can be written in this form: 1 plus cos 2 omega 0 t plus 2 beta, so x 2 can be so by taking this thing, which depends on the initial condition a and beta, so one can find the particular integral of the other part.

So, one can find the particular solution of this equation; so the particular solution of this equation can be written either in this form or in this form. So, one can write this particular integral x 2 of this equation by putting this thing cos, so this is constant part. So, this is minus 1 by 2 alpha 2 a square, so plus minus 1 by 2 alpha 2 a square into cos 2 omega 0 t plus 2 beta, so for this part, one can write the solution to be alpha 2 a 0 square by 6 omega 0 square into cos 2 omega 0 t plus 2 beta 0 minus 3 plus for the other part.

It can written plus a 2 into cos omega 0 t plus beta 2, so instead of writing this constant a 2 beta 2, one can follow the other procedure to write this x 2 in this form, instead of writing a 0 a 2. So, one can directly write this is equal to alpha 2 a square by 6 omega 0 square into cos 2 omega 0 t plus 2 beta minus 3, so in the final stage one can find this a and beta or in the first stage one can after finding this, one can substitute it in this equation and one can obtain.

So, the finial expression for x 1 can find in this form. So, epsilon a 0 cos omega 0 t beta 0 plus epsilon square alpha 0 square alpha 2 by 6 omega 0 square cos 2 omega 0 t plus 2 beta 0 minus 3 plus a 2 cos omega 0 t plus beta 2 minus, so this is order of epsilon cube; and neglecting this order of epsilon cube, one can have this equation or one can write this same equation in this form; that is equal to epsilon a cos omega 0 t plus beta plus epsilon square a square alpha square by 6 omega 0 square into cos 2 omega 0 t plus 2 beta minus 3. So, either one using this first one can write this equation, using this second one can write this equation, so writing these equation.

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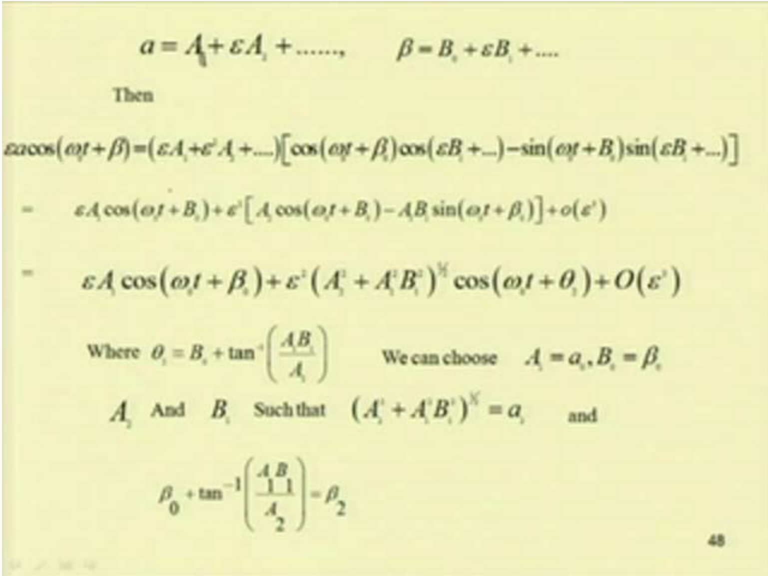

Now, taking this a equal to A plus epsilon A 2 and beta equal to so, a equal to A 1 plus epsilon A 2 and beta equal to B 0 plus epsilon B 1, this so this way, then one can write this epsilon a cos omega 0 t plus beta equal to, so one can expand this equation in this form; so one can write this epsilon, so this will be epsilon A 1 plus epsilon square A 2 into, so this term so by substituting this equation one can write this equation, so where one can further modify this equation and find these things.

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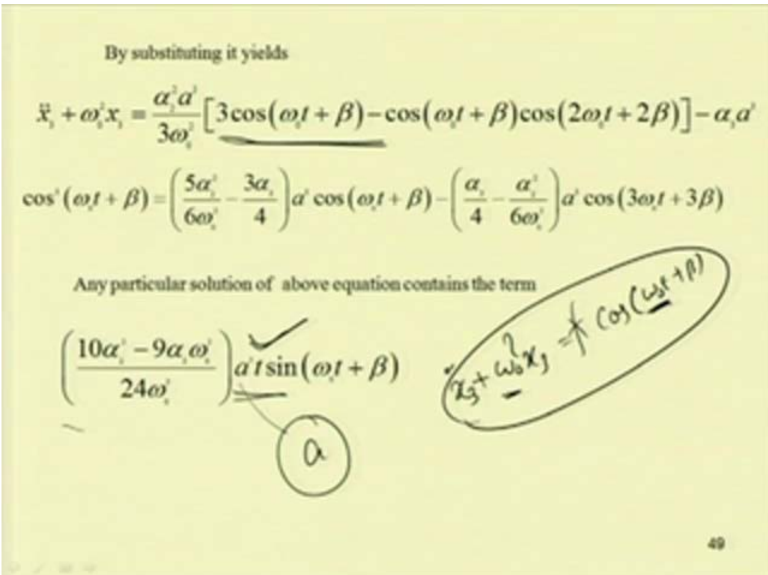

So by substituting, those equations one can find this and substituting in the third equation. One can write this equation in this form x 3 double dot plus omega 0 square, x 3 equal to alpha 2 square alpha 2 square a cube by 3 omega 0 square into cos 3 cos 3 cos omega t plus beta minus cos omega 0 t plus beta into cos 2 omega 0 t plus 2 beta minus alpha 3 a cube cos cube omega 0 t plus beta equal to so, this will be equal to so these terms. But it may be noted that in this equation, so any particular solution of this equation, so because it contains a term 3 into cos omega 0 t plus beta, so the equation, x 3 double dot plus omega 0 square x 3, which contain a function, which contain a term, so which is let me put some f, so this is cos omega 0 t plus beta as this is omega 0 and this is omega 0 square.

So, it will lead to resonance condition and the particular solution of this part will contain a term, which will increase with time. But in actual case, we have observed that the solution is by is bounded, but by doing this straight forward expansion method, so we have see the solution in unbounded, as it varies time. But in actual straight so but, in actual qualitative analysis, one can find a center, in this case while the straight forward expansion method is showing the response to be increasing. So the straight forward method is not giving this correct result in this case.

So this is due to the fact that the constant or the amplitude response, what we have considered that is a we have not considered it be a function of frequency, a is not varying with frequency that for that purpose, so the expansion method, what we did is not correct. So lindstedt so, he has modified this method and by taking a frequency dependent amplitude term, so he has modified this method; and next class, we will study that method that is the lindstedt Poincare method for finding the response of the non-linear equation.

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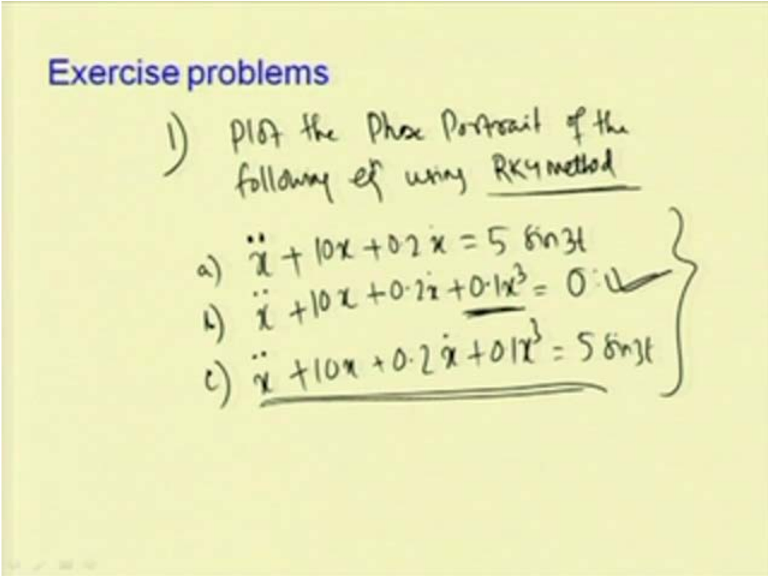

So, you can take few exercise problems to solve. So, plot the phase portrait of the equation, portrait of the following equation following equation; using RK4 method. So, in case of RK4 method, directly you can use the equations, what is shown, you can develop your own code or you can use mat lab to plot this thing. So, let you plot for this 1 x double dot plus 10 x plus 0.2 x dot equal to 5 sin 3 t, then x double dot plus 5 x plus 0.2 x dot plus 0.1 x cube, so this is duffing type.

So equal to 5 sin 3 t, so in this case, so this is the non-linear term added to the previous equation. So, previous linear equation we have plotted, and in this equation, we have a non-linear term 0.1 x cube added to the system. So, by plotting, so you just check with different initial condition. So, what is the response you are getting. So taking different initial condition, plot this phase portrait; first you take this term this forcing term equal to 0, and in third you just take a forcing term 10 x plus 0.2 x dot plus 0.1 x cube equal to 5 sin 3 t. So, solve these three problem using this RK4 method, then solve the last problem using the straight forward, solve the second problem using the straight forward expansion method.

Find, first find the equilibrium point in this case; to find this equilibrium point put x double equal to 0 x dot equal to 0, find x, and then plot the equilibrium points, and check with the straight forward expansion method. Whether you are getting the same equilibrium point or not. So next class, we will study about the lindstedt Poincare method.

Thank you.