Non-Linear Vibration
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Module - 3
Solution of Non-Linear Equation of Motion
Lecture - 1
Straight Forward Expansions

Welcome to this non-linear vibration course. And today we will study about this module 3 and lecture 1 of this course. So, in this non-linear vibration previously, we have studied 2 modules in which in the first module I have given the introduction about his non-linear vibration; and in the second module I told you about how to derive the equation motion for these non-linear systems.

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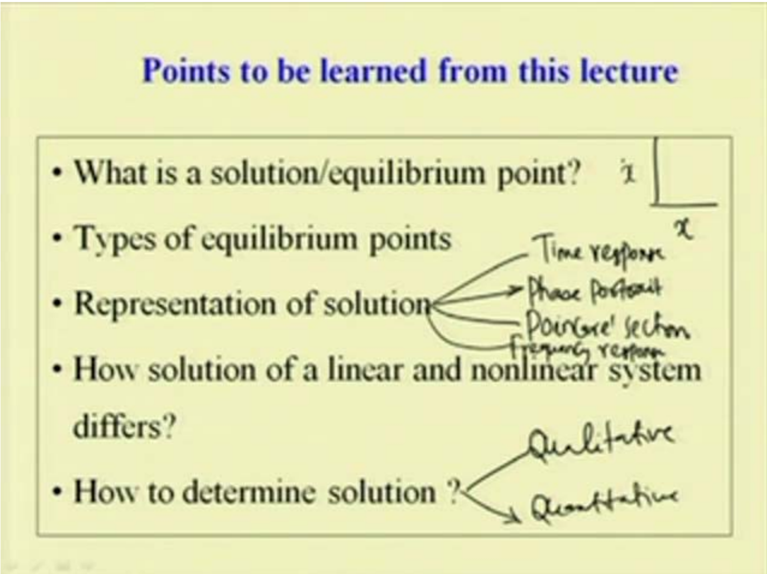

So, in those cases, we have studied many different types of equations. For example, we have studied this Duffing equation.

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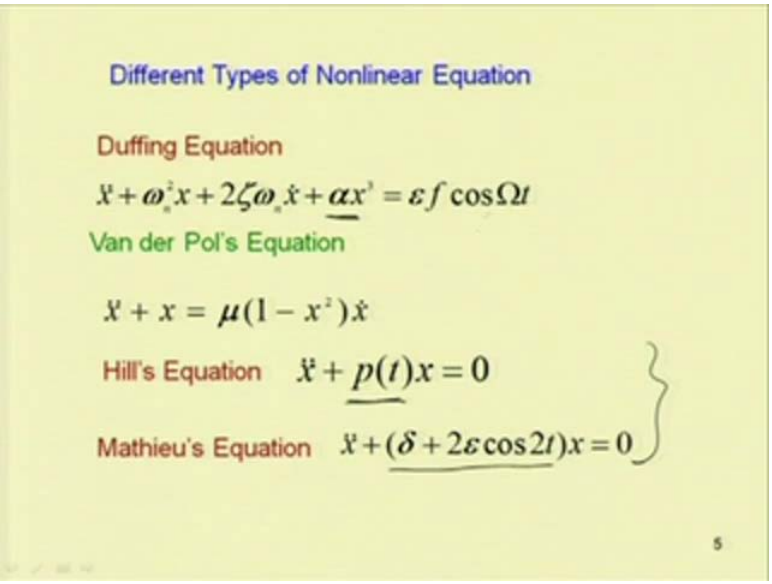

So, where the equation can be written in this form that is, x double dot plus omega n square x plus 2 zeta omega n x dot plus alpha x cube equal to epsilon f cos omega t, where this x is the response of the system, f the forcing function amplitude of the forcing, and omega the frequency of the forcing function. Here, alpha x cube the non-linear term added to the linear equation x double dot plus omega n square x plus 2 theta omega n x dot equal to epsilon f cos omega t. So, due to this presence of this non-linear term alpha x cube so, the system behavior will be non-linear. Also, we have briefly studied what will be the difference between these linear system and non-linear systems.

Similarly, in case of the van der pol's equation, the equation can be written in this form that is x double plus x equal to mu into 1 minus x square into x dot; and hill's equation x double dot plus p t x equal to 0; and Mathieu equation x double dot plus delta plus 2 epsilon cos 2 t equal to t into x equal to 0. So particularly in this case of hill's equation and Mathieu equation, one can find a term or parameter which is coefficient of x. So, in this case p t x is the coefficient of x and in the second case, in case of the Mathieu hill Mathieu equation, this delta plus 2 epsilon cos omega t is coefficient of x. But particularly, this 2 epsilon into cos 2 t term which is a time varying term is coefficient of x.

So, unlike in the first equation that is the Mathieu equation or the van der pol equation here a time varying term is a parameter which is coefficient of x that is why these 2 equations are parametrically exited system. And the other systems are either it can be a force type equation or it can be a free vibration equation, depending on the forcing term used in this equation.

So, in this particular module today class we will study, what is a solution or equilibrium point, types of equilibrium points, representation of the solution, how solution of a linear system and non-linear system differs, how to determine the solutions. So, already we know for the different types of equation the response of equations we can find. So, to find equation solution of those equations so, we can have two different types of solution or 2 different parts of the solution. So, one is the transient part of the solution and other one is the steady state part of the solution. So, at tends to infinity that is time tends to infinity the solution part which we obtained from the governing equation is the steady state part of the solution and before the steady state the response what we got are the transient part of the solution. So, to know what is the equilibrium point. So, in case of the equilibrium point or steady state as this system is not or the governing equation of the system one can find. So, in these equations for example, in case of the Duffing equation, at steady state, no longer it will be a function of time.

So, the velocity term and the acceleration term if you neglect and the remaining things if you can find so, that will give you the equilibrium points of the system. So, for equilibrium points of the system we can put the terms the time burying terms to be 0 and we can find the steady state solution or the equilibrium solution of the system. So, after getting the equilibrium solution of the system, we can find what type of equilibrium points we obtained. So, there are mainly four different types of equilibrium points what we are going to study. So, they are fixed point response, periodic response.

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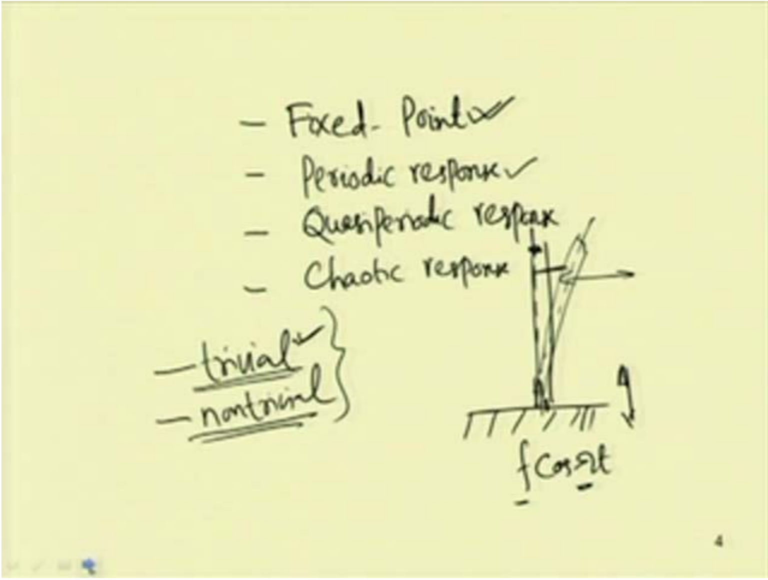

So, this response can be. So, in types one can have this fixed point response then, one can have this periodic response and one can have quasi periodic response or one can have chaotic response of the system. For example, let us take a base excited cantilever beam. So, in this cantilever beam, in this base exited cantilever beam so, let it is base is moving in this direction. So, if the base is moving in this direction in this case the beam may vibrate in these directions or it may vibrate in a transverse direction. So, if the vibration takes place, the vibration what we are going to see may be either it may be a trivial vibration, trivial state or nontrivial state vibration. So, in case of trivial state or let us take the system, in which the beam is vibrating in this direction up and down. So, in this case the as the base is when the base is moving up and down for some frequency and amplitude of this base motion, the beam will move in transverse direction and it may come to this position.

So, this transverse displacement can be found out and the body will be or some frequency and some frequency and amplitude it will oscillate about this point. So, that will be nontrivial state of vibration and if it is only vibration about its own axis so, it will be a trivial vibration. That means the response is 0. So, for the response to be 0 the vibration will be trivial state vibration and when it is moving to a particular distance and vibrate at that point so, that will be nontrivial state of response. So, in case of the trivial or nontrivial state of response, the magnitude if it is fixed, then it will be a fixed point response and if it varies periodically, then it will be a periodic response and if it vary in a quasi periodic manner the response will be quasi periodic and the response may be chaotic also, to know about this periodic or quasi periodic response.

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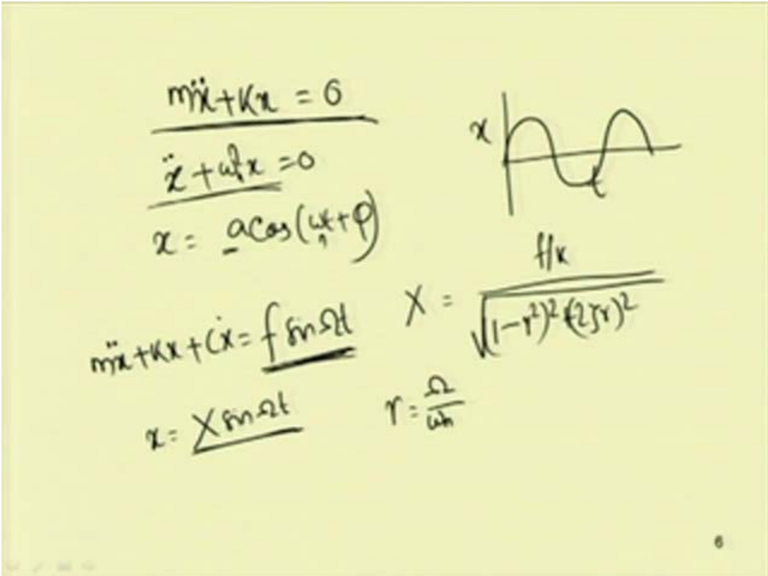

For example, in case of the periodic response so, let us take the simple linear equation m x double dot plus k x equal to 0. So, in this case the equation can be written in this form also x double dot plus omega n square x equal to 0. So, this is for a linear case and it is response and it is response x can be written as a cos omega t plus phi. So, where ‘a’ is the amplitude and this omega so, this will be omega n, omega n t plus phi this is ‘a’ and phi can be obtained from the initial conditions. So, this response is a periodic function. So, the response will be. So, if one plot this x versus t in this case. So, it will burry sinusoidally and the response is periodic. Similarly, in case of this equation m x double dot plus k x plus c x dot equal to f sin omega t so, in this case we have 2 different type of 2 different parts of the response, one is the steady state part and other one is the transient part.

So, the transient part one can obtain by substituting this equal to 0 and for the steady state part so, we know the response can be obtained from this equation. So, x will be equal to for the steady state this amplitude x the solution x can be written as x sin so, it will be x sin omega t and in this case this x the amplitude x can be written as this f by k root over 1 minus r square whole square plus 2 zeta r whole square where, r equal to omega by. So, r equal to this omega by omega n. So, here the steady state response is also periodic. So, if instead of taking a single force single component of the force, if one can take multiple component of the force for example, in case of the in case of let us take the case of Duffing equation.

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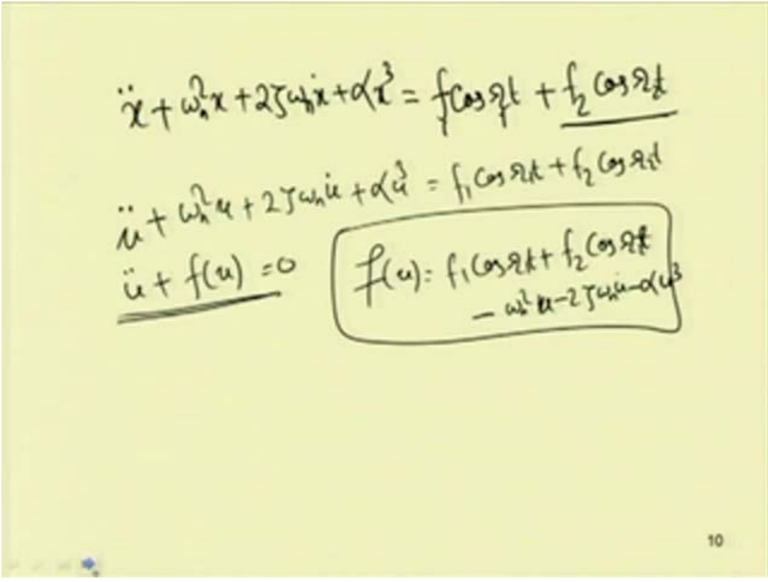

So, in which we can write the equation x double dot plus omega n square x plus 2 zeta omega x dot plus alpha x cube. So, here I can take the equation, if I am taking a first equation I can write this equation in this form. So, this is for a single frequency excitation. And for multi frequency excitation, I can put cos omega 2 t. Or I can add similar terms to this equation to make it 2 frequency, 3 frequency or multi frequency excitation terms.

So, in all these cases the response may be periodic or quasi periodic or chaotic depending on the type of response or type of resonance we are going to consider in this system. So, in case of non-linear system we can get the response either to the steady state response or equilibrium response can either be fixed point periodic, quasi periodic or chaotic depending on the resonance conditions or depending on the control parameters what we are going to study for the system. So, after knowing about the types of equilibrium point so, we should know how to represent the solutions. So, we can either make a qualitative analysis or quantitative analysis for finding the solutions, how to determine the solution. So, we can go for a qualitative method or we can go for a quantitative method for finding the solution. And to represent the solution either we can represent it by the we can have a time response curve or we may go for this phase portrait or we can plot this Poincare section of the system Poincare section of the system to represent the solution of a system. So, one can go for the frequency response also. So, one can represent also using this frequency response curve of the system. So, in case of the time response one can plot the response of the system with time and in case of a single periodic or 2 periodic or quasi periodic or chaotic response, the time response will be different from one equilibrium point to the other equilibrium points. So, in case of the phase portrait we can represent the system in terms of, let x is the response of the system then x dot is the velocity of system.

So, we can represent in a phase portrait by plotting this x versus x dot. So, in case of the Poincare section we sample out the response, time response at particular interval of times and taking those points we can plot the Poincare section of the system. Also, we can go for a frequency response or force response of the system to represent the solution of the system. And already we know how the solution of a linear and non-linear system differs.

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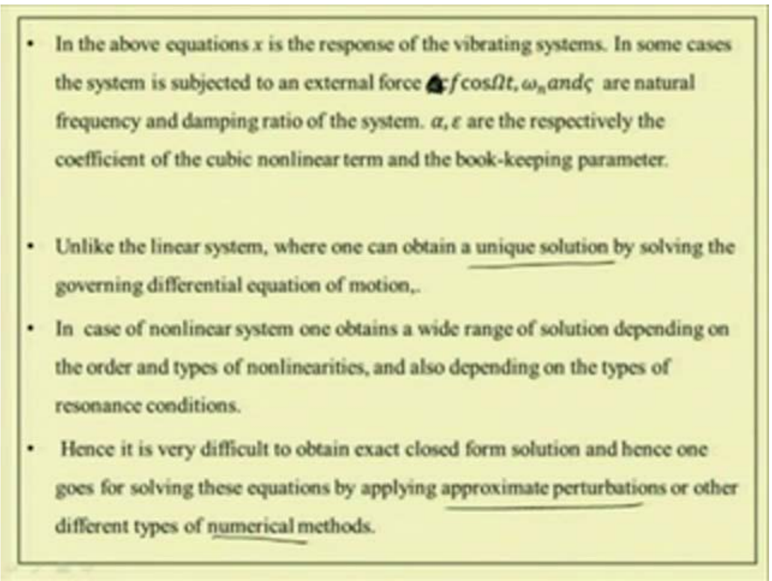

So, there are several point in case of the solution of a linear and non-linear systems. So, in case of a linear system we get we obtain unique solution but in case of a non-linear system the solution is not unique and we can get multiple solutions in case of the non-linear case.

Also in case of the non-linear system one obtain a wide range of solutions depending on the order and type of non-linearity and also depending on the type of resonance conditions. Hence, it is very difficult to obtain exact close form solution and hence, one go for solving these equations by applying approximate perturbations or other different type of numerical methods. So, by using different numerical methods like Runge-Kutta method or different approximates solution perturbation methods one can use to solve these equations.

So, initially we will study qualitative and then, we can go for quantitative methods to study this solution of the non-linear systems. After finding the solution of the non-linear system our work is to check whether the solutions what you obtained are stable or unstable. So, one should go for a stability analysis after finding the response of the system. So, after finding the solutions, we can go for stability analysis or and bifurcation analysis of the system and we can study completely the system response, by studying the equilibrium points and their corresponding stability. So initially, we will now study about the qualitatively the non-linear equations and then, we will go for quantitative analysis of these cases.

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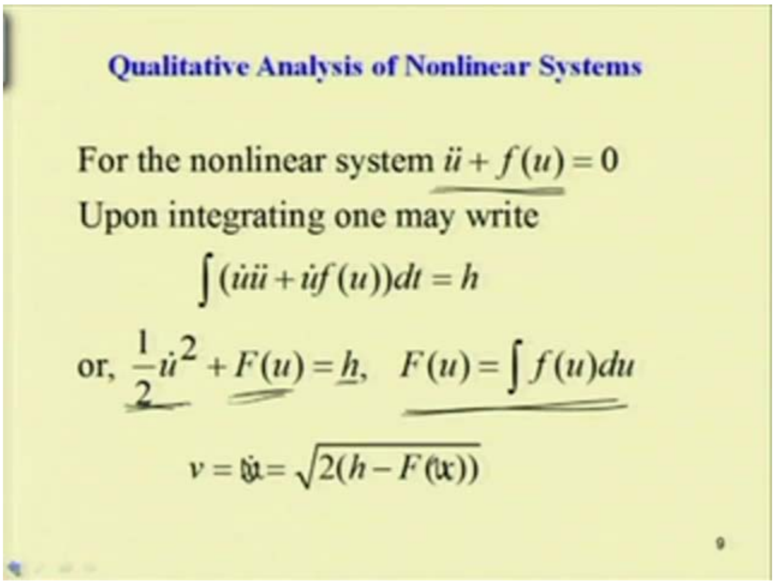

So, let us see this qualitative analysis of this non-linear system. So, let us take a conservative system where, I can write the equation in this form that is, u double dot plus f u equal to 0. So, for example, in case of this equation so, this equation I can write in this form also, that is, u double dot plus omega n square u plus 2 zeta omega n u dot plus alpha u cube equal to f 1 cos omega 1 t plus f 2 cos omega 2 t or u double dot plus f u equal to 0. So, where in this case f u equal to f 1 cos omega 1 t plus f 2 cos omega 2 t minus this omega n square u minus 2 zeta omega n u dot minus alpha u cube. So, one can write all these equations in a form u double dot plus f u equal to 0. Now, multiplying u dot in this equation so, I can write this equation in this form u dot into u double dot plus u dot into f u equal to 0.

And up on integrating this thing one can write this equation equal to integration of u dot u double dot plus u dot f u d t equal to h. So, where, h is a constant and integrating this u u double dot so, one can find this half u dot square plus f u so, half u dot square plus F u equal to h or one can find this f u equal to where this F u equal to f integration of f u d u. So, from this equation one can see that this term correspond to the kinetic energy of the system and this term correspond to the potential energy of the system.

So, kinetic energy plus potential energy of the system is constant that is equal to h. So, for a given value of x that is the potential energy or that is the total energy of the system. So, if one knows this potential function or potential energy of the system so, one can find what is the velocity of the system? So, this velocity term so, one can write this velocity v equal to u dot equal to 2 into h minus f u so, v equal to u dot equal to. So, one can multiply this 2 here. So, it will be u dot square plus 2 f u equal to 2 h or u dot square will be equal to 2 h minus f u 2 h minus 2 into h minus f u and then v equal to u dot will be square root of this.

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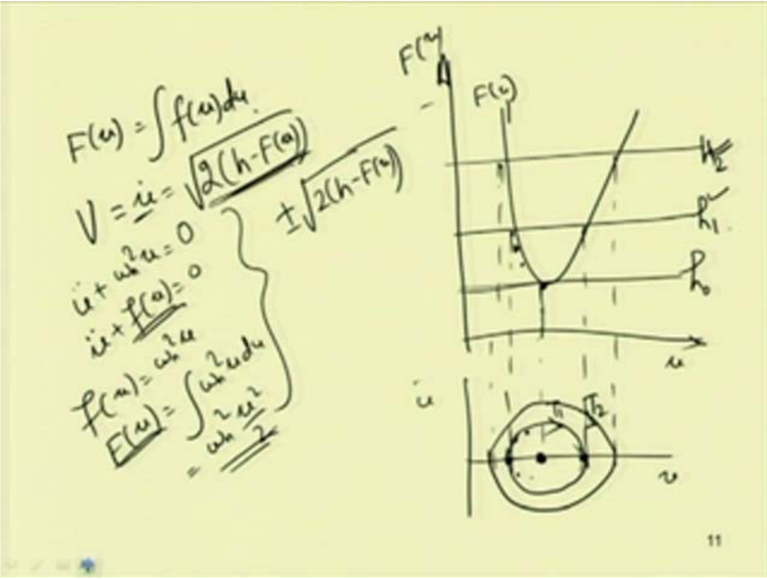

So, after getting this one can. So, v equal to. So, I can write this v that is u dot equal to root over 2 into h minus F u. So, for a given value of h and knowing this potential function of a system, we can find what is u dot of the system. For example, let us take a system let us take a system where, this potential function is represented by this curve. So, let me take so, this is f u, this equation I can write in this form. So, this is f u versus let it is u. Already I told you one can find this f u by integrating.

So, integrating f u d u and we know this velocity v that is equal to u dot equal to root over 2 into h minus f u. For example, in case of the spring mass system I can write this equation in this form u double dot plus u double dot plus omega n square u so, u double dot plus omega n square u equal to 0 or if there is some forcing then, I can put it f cos omega t or f sine omega t then, u double dot plus f u equal to 0. So, in this f u will be equal to. So, f u equal to omega n square u and this capital F u will be integration of this omega n square u d u. So, this can be written as omega n square. So, this will becomes so, u square by 2. So, in this way one can find this f u. So, after getting this f u 1 can substitute it in this equation of velocity that is u dot and find what will be the velocity for a given value of u.

So, let us take this point, the minimum point in this potential curve so, corresponding to this curve. That means, these let us take h equal to so, this is h. So, after getting this h so, let it corresponds to this line correspond to the total energy. So, add this point h equal to f u so, if h equal to f u so, in that case b will be equal to 0.

So, if I will plot this u dot versus u so, corresponding to this I will have 0 responses. That means, the velocity and displacement both are 0 in this case. So, let me take a line here so, this is h 0 so, this point correspond to u 0. And if I will take another point h 1 here so, corresponding to this point so, let us take a point here. So, in this case so, up to this f u equal to 0. So, we have this h 1. So, one can have two different. So, this is h 1 and this is f u so, corresponding to this point so, this will be equal to 0.

Similarly, corresponding to this point this will be the velocity will be equal to 0. So, this velocity equal to 0 and this point velocity equal to 0. See, if I will take any other point away from this. So, let me take a point here. So, if I will take a point here so, corresponding to this as this h is greater than this f u so, this term will be real positive term and the root over can give 2 solutions of this thing, two solutions and one will be. So, it will be plus minus. So, let this value this h minus F u as it is positive so, I can have two value of this corresponding to this and that will be 2 into h minus F u. And we will have 2 real values so, those 2 real values. So, one value I can put it here and other value will be here.

Similarly, corresponding to this point, I will have similarly two more values and if I will go on finding the velocity corresponding to different values of, different values corresponding to this line so, I can have a curve. So, I will two values corresponding to each point and I will have a close curve corresponding to this. So, this close curve represents the trajectory corresponding to h 1. So, corresponding to total energy h 1, one can find the trajectory T 1 which represents the flow or trajectory of u versus u dot.

Similarly, corresponding to another total energy h 2 so, one can find corresponding to this point it will have 0 and corresponding to this point again one have 0. And similarly by proceeding in the similar way so, one can find the trajectory. So, again one can get a close trajectory corresponding to this. So, this trajectory T 2 correspond to this total energy h 2. But, if I take another system in which the potential function system is represented by this, let me take it.

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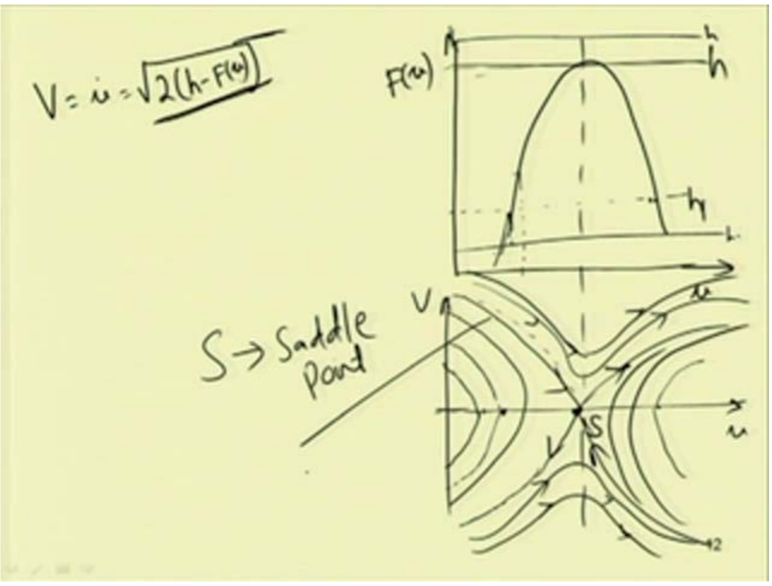

So, in this case so, this is F u and this is u. So, corresponding to this, I can qualitatively analyze the flow or trajectory of the system. So, let me take a point so, this is h 1. So, corresponding to the point where, h 1 and F u have the same value so, our V which is equal to u dot equal to root over 2 into h minus F u h minus F u. So, corresponding to this point we can have we can have so, this is V and this is u. So, let us take this point. So, corresponding to this point we can have 0 velocities. Similarly, corresponding to this point we can have a 0 velocity.

So, here the displacement and slope displacement and the velocity both are 0 corresponding to these 2 points. So, if I will take a point before this where, this F u is less than h that is the total energy. So, this part I can get 2 real roots. But, if I will take a point so, let corresponding to this u if I will take this point. So, in this point this F u is greater than h so, inside term is negative. So, square root of this negative term will leads to imaginary terms. So, I can have. So, corresponding to a term before this I can have the flow, I can have the flow like this or if I will take some other line that is h 0 so, corresponding to that I can another point also. So, this is the trajectory before this and after this also we can have the trajectory like this. And corresponding to this maximum point so, let us take this maximum point. So, corresponding to this maximum point so, h and F u when h and F u value are same so, we can have this point. But before that and after that we can correspond to this, I can put value like this and at this point we will have 0 solution that means, V and u equal to 0 and before that as h is greater than F u.

So, we have 2 values of this. But, if you look at the flow so, one can see that the flow is separated at this point. So, this flow so, it will come in this direction and it will come away along this curve similarly, the flow will be like this. So, at this point the flow moves in this direction or before that it moves in this direction. So, that means, the solution is separated by these two and they are known as separatrix of this solution. And if I will take a value of h which is greater than this so, I can have the flow like this so, I can have this flow corresponding to this. Similarly, before this thing I can if I try to plot all these trajectory, so it may look like this.

So, this point is known as a saddle point or S. So, S represents the saddle point and these lines which separate this are known as the separatrix. In the previous curve one can see so, this point which leads to or which corresponding to the minimum potential energy is known as the centre. So, point corresponding to the minimum potential energy is the center and point corresponding to the maximum potential energy is the saddle point in this case. So, saddle point and centers corresponding to 2 equilibrium points for a given system.

So, if we have given a system first we can find this f u that is, the potential function of the system by integrating this small f u, d u and after getting this capital F u for a particular value of h so, we can find this v. So, after getting this v, by taking different level of energy total energy so, we can find the flow of the system that is, what will be the velocity of the system at a particular instant. For example, in this case correspond to this point we have 0 velocity but, after that we can have a velocity. We have two velocities at as F u is less than h. So, we have two velocities and corresponding to these two we can have these two points. Similarly, up to this I can find 2-2 points and if I will join these points so, I can get this trajectory T 2. So, in case of a system we are we have a maximum potential energy. So, in the previous case we have a system with minimum potential energy that means; potential energy decreases. And in this case we have a maximum potential energy. So, maximum potential energy corresponds to the unstable system. So, in this case corresponding to this maximum potential energy we have a saddle point and corresponding to minimum potential energy we have a center in the system.

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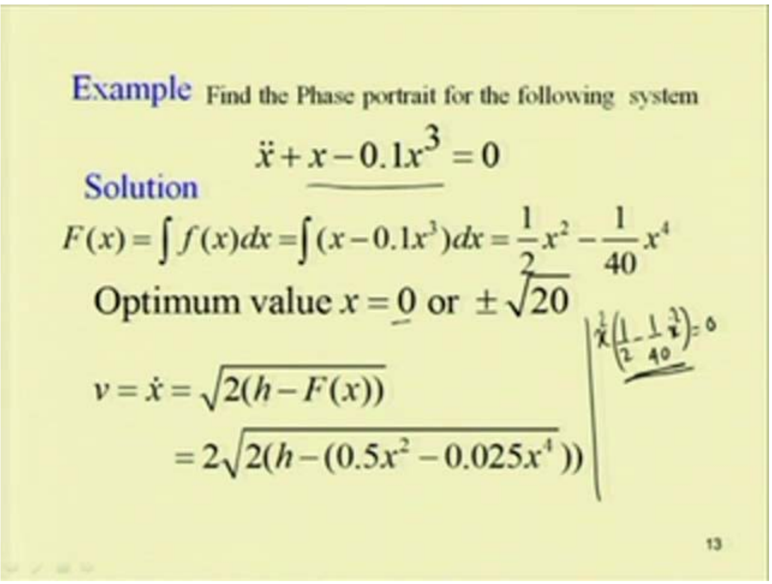

So, let us take one example. So, let we have to find the phase portrait for the system given by x double dot plus x minus 0.1 x cube equal to 0. So, in this case, my f x small f x equal to x minus 0.1 x cube so, this capital f x that is the potential function can be write as integration of f x d x. So, it is integration of x minus 0.1 x cube d x. So, this is, this is equal to half x square minus 1 by 14 x fourth. So, the optimum values can be setting this equal to 0 so, by putting this equal to 0. So, one can take this x square common. So, this is equal to 0.5 minus. So, minus 1 by 40 that is half minus or one can write this way. So, this is equal to x square minus 1 by 40 x square equal to 0 or this gives either x equal to 0 or this inside term equal to 0. So, for the inside term to be 0 so, the x value should be equal to plus minus root 20. So, we can get 3 values of x. So, these are the equilibrium points. So, one is 0 so, 0 is one equilibrium. So, 0 is the one optimum value and other 2 optimum values are plus minus root 2.

So, corresponding to these points we have to find the trajectories of the system. So, the velocity of the system can written as v equal to x dot equal to root over 2 h minus 2 into h minus f x. So, this will be equal to 2 into h minus so, for f x I can put this is equal to 0.5 x square minus 0.025 x fourth. So, corresponding to a value of x so, I can find this x dot and I can plot in this response. So, when this f x is less than this h we can have 2 values of this and when it is greater than that, there will be no flow in the system because this term will be imaginary. So, we can have flow or we can have this velocity only when this h is greater than h is greater than this f x.

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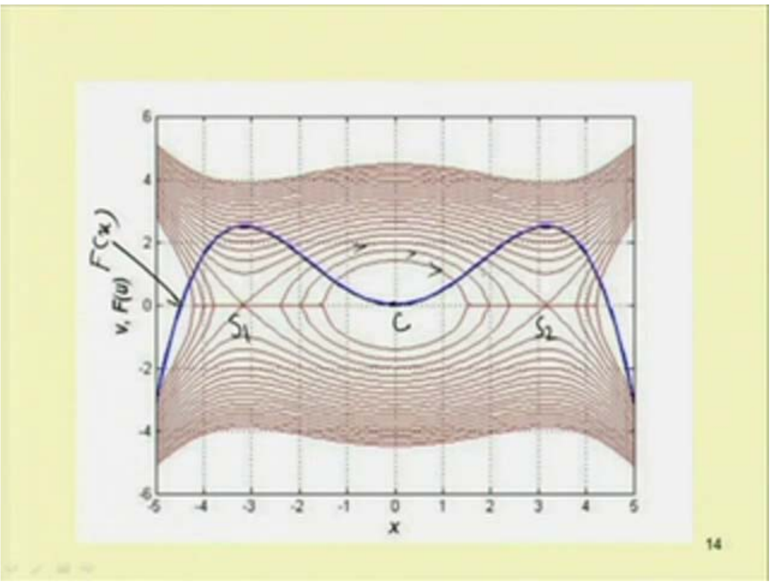

So, if one can plot this F u so, this curve this blue line represent this F u versus u or F x. So, this will be as we are writing u equal to x so, this is f x. So, this curve corresponding to F x and this trajectory is corresponding to v all these trajectory correspond to v. So, corresponding to this minimum point we can have the center at this point and corresponding to these maximum points we can have the saddle points.

So, these two are the saddle point. So, it will be saddle point 1 and this is saddle point 2 and this is center c. So, between these 2 saddle points we have the response periodic response. So, you can observe that the response is periodic here because it repeats. Similarly, here the response is periodic and here the response is periodic and this separatrix which connect this s 1, s 2. So, they separate this periodic response with other type of response. So, these 2 points s 1 and s 2 are unstable points and c is a stable point.

So, for example, if you take a particular value of h here so, when h equal to the same so, we can half this 0 velocity and before and after this point we can have two values. So, one can plot these trajectories corresponding to different values of h and in this case for several values of this potential total energy the trajectories have been plotted. So, in this way one can obtain the trajectories or one can study qualitatively the response of a system. So, in this case qualitatively when we have studied so, we have seen the saddle points and center corresponding to the maximum potential energy and minimum potential energy of the system. So, by plotting this x versus v so, one can obtain this trajectory and one can study the system.

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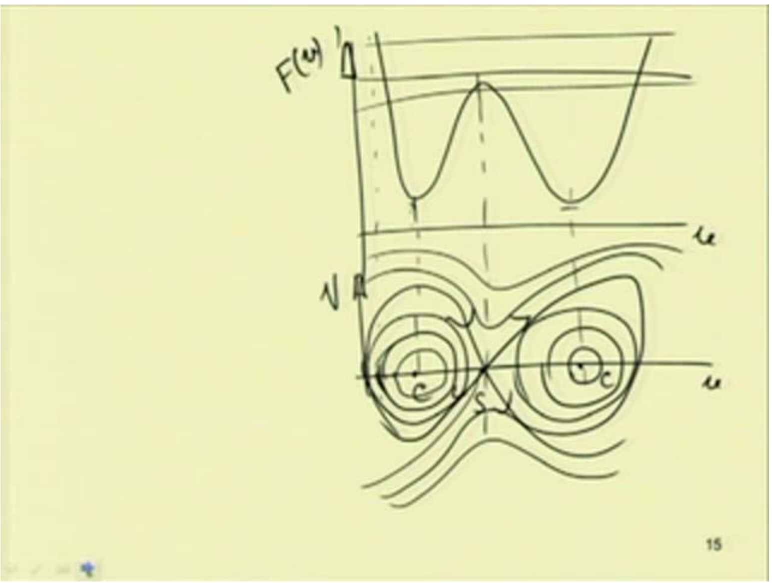

So, let us take some. So, let us take another potential function like this and. So, in this case we have minimum value. So, this corresponds to one minimum value and this corresponds to the second minimum value and this is the optimum value. So, if one tries to plot so, this is F u versus F u versus u. So, in this case if one study qualitatively so, corresponding to this minimum potential energy. So, let us plot this u versus u dot that is v so, corresponding to this one can have a center and corresponding to this one can have, corresponding to this one can have a saddle point. So, there will be separatrix which will separate the flow. So, we will have a center here. So, this is also another center. So, qualitatively if one can study this system one can so, this will be the separatrix and in between the separatrix we have this periodic response. Similarly, here we have periodic response in between this.

So, one side or this side as the flow remain inside this domain so, this is known as homo clinic orbit so, this orbit is known as homo clinic orbit and outside this so, this will be hetero clinic orbits. So, one can have a set of homo clinic and hetero clinic orbits corresponding to different values of h. So, corresponding to this maximum point so, one has a saddle point and corresponding to this and this one has center. So, in this case one has 2 centers and a saddle point. Similarly, if one can take some system where, the intended of taking a conservative system so, if one take a non conservative system.

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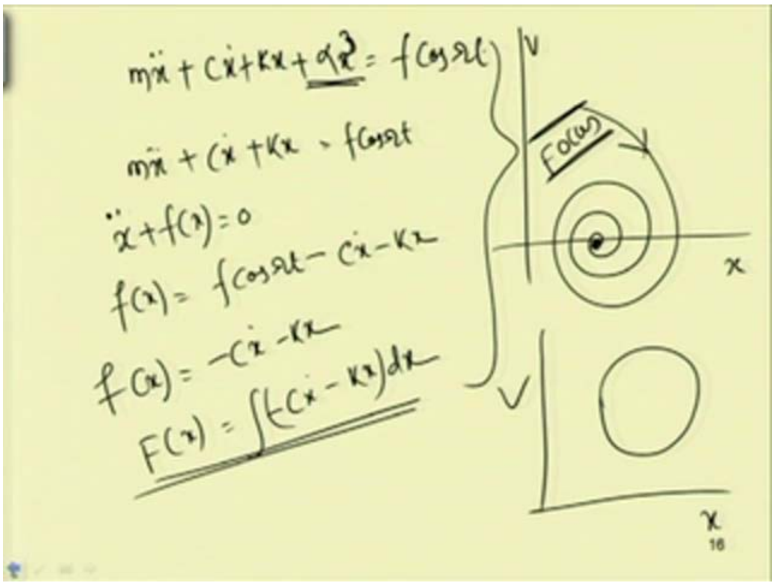

For example, in this case of Duffing equation so, if one take m x double dot plus c x dot plus k x plus this alpha x cube equal to f cos omega t. So, if one take a non conservative system or for the simplicity if one takes only this case of a linear system m x dot plus c x dot plus k x equal to f cos omega t and. So, we can find this x double dot plus this f x equal to 0 where, f x equal to f cos omega t minus m minus c x dot minus k x. So, in this case instead of so, for a free vibration case this equation can be written in this form.

So, f x will be equal to minus c x dot minus k x. And one can find this f x equal to integration of minus c x dot minus k x into d x. And from this one can find the velocity term and in this case instead of getting a center we can see later that one can get a term which is known as focus. So, instead of getting a periodic response so, one can get a response which will die with time. So, if one plot v inverse this x so, with time so, instead of getting a periodic system in case of the. So, in case of a conservative system we obtain one periodic response but, in this case the period instead of getting a periodic response, the response will die out and finally, it will come a fix point. So, this fix point is known as focus. So, later we will study in detail about the different types of response or equilibrium points what you are getting. So, center till now we know about center saddle points, separatrix and focus of the system. So, by doing this qualitative analysis so, one can study the flow of the system with time.

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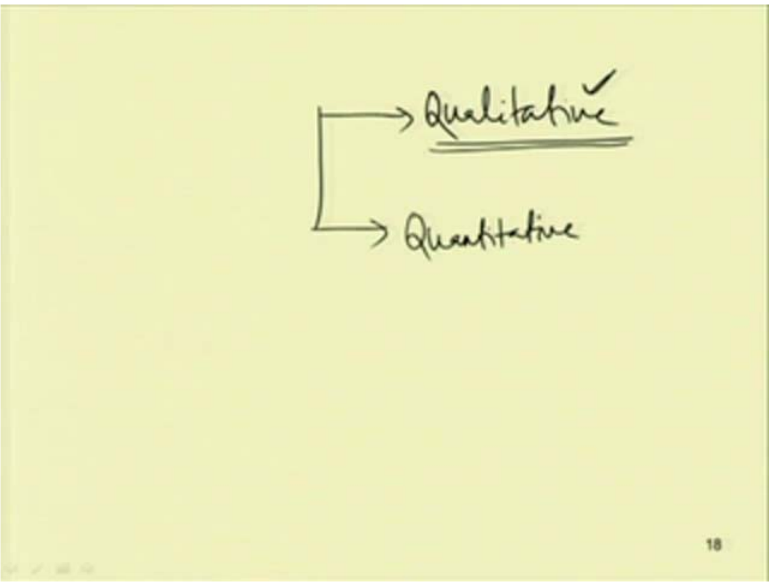

Also, our analysis we know one can do this qualitative analysis and other one this quantitative analysis. So, in qualitative analysis basically we are plotting the phase portrait and knowing how the flow will be there in case of a system. But this qualitative analysis will be difficult very difficult. So, if the non-linear equation order of nonlinearity of the equation increases and one required for that purpose to solve those equation qualitatively by writing some computer code in either using some high level language like C Fortran or one can use this Matlab for finding this response qualitatively.

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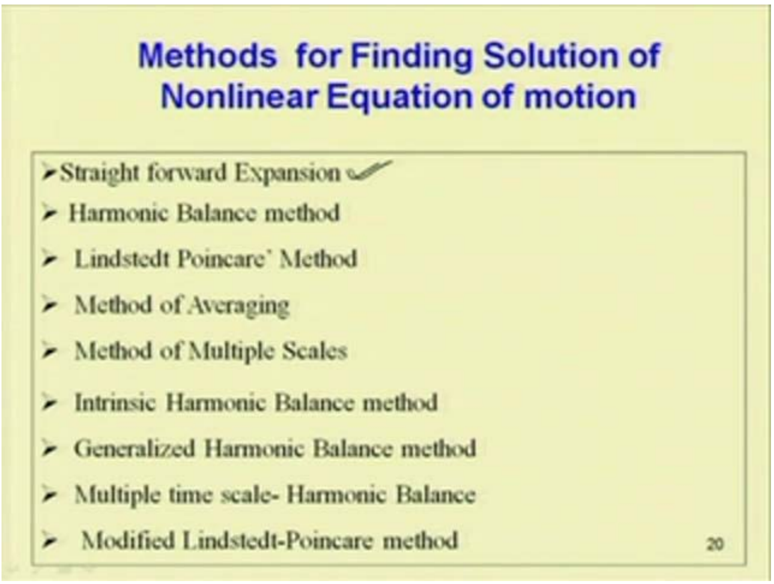

And in case of the quantitative response there are several perturbation methods available or several methods available to study the system. So, for example, one can use this straight forward expansion method. So, in case of the straight forward expansion method, one can write the equation for example.

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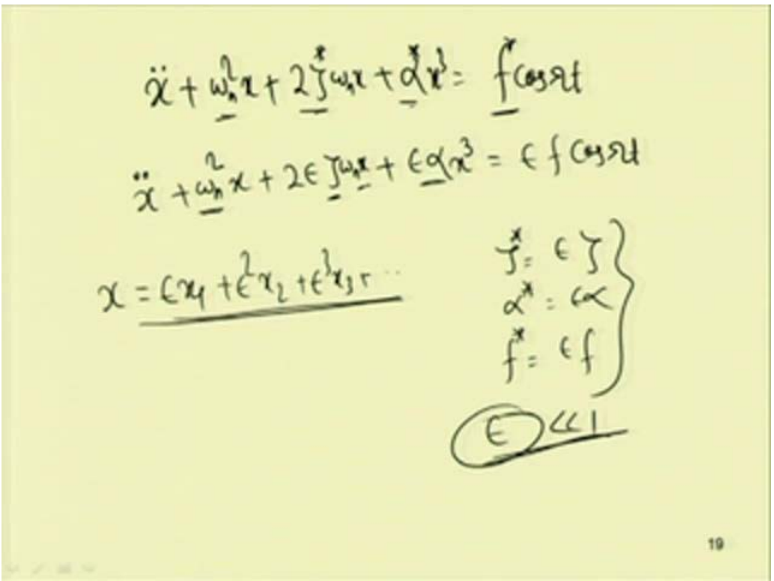

Let us write the equation in this form x double dot plus omega n square x plus 2 zeta omega n x plus alpha x cube equal to f cos omega t. So, in this case first we have to order this equation after knowing the coefficient of each term so, easily one can order this equation. So, let after ordering this equation it is written in this form omega n square x plus 2 epsilon zeta omega n x plus alpha epsilon alpha.

So, let me put this is alpha star this is zeta star and this is f star. So, this equation, I can write this alpha f x cube equal to epsilon f cos omega t square this zeta equal to zeta star equal to epsilon zeta and alpha star equal to epsilon alpha and f equal to f star equal to epsilon f. So, here this epsilon is a book keeping parameter, which is very-very less than 1.

So, this book keeping parameter is used to make this coefficient of this response term equal to or similar to that of the linear term. So, for example, in this case the order of this zeta should be near to that of omega n square and this order of this alpha should be near to or it will be of the same order as that of the omega n square. So, after ordering this equation after ordering this equation so, one can to study it quantitatively one can write this x equal to 1 can expand this x in this form one can expand this x in the form of a Fourier series. So, if one expand this x in terms of a Fourier series then, that method is known as harmonic balance method or if one can expand this equation in this formulae one can write x equal to epsilon x 1 plus epsilon square x 2 plus epsilon cube x 3 like this and do the straight forward expansion and find this x 1 x 2 x 3 and finally, find this x.

So, that method is straight forward expansion. Similarly, one can have this harmonic balance method; in case of harmonic balance method one can use Fourier series to expand this solution and find the coefficient and get the solution. So, next class we will study about this in detail about this straight forward expansion method in which we will see what is the demerits of this method then why we are going for the other methods. So, in this module we will study about this straight forward expansion, harmonic balance method, Lindstedt Poincare method, method of averaging method of multiple scales, intrinsic harmonic balance method, generalized harmonic balance method, multiple time scale harmonic balance method and modified Lindtstedt Poincare methods.

So, today’s class you know about how qualitatively you can analyze a non-linear system. So, after analyzing this non-linear system you can do these exercise problems by finding the trajectory of this simply system. So, these are some of the exercise problem which you can take.

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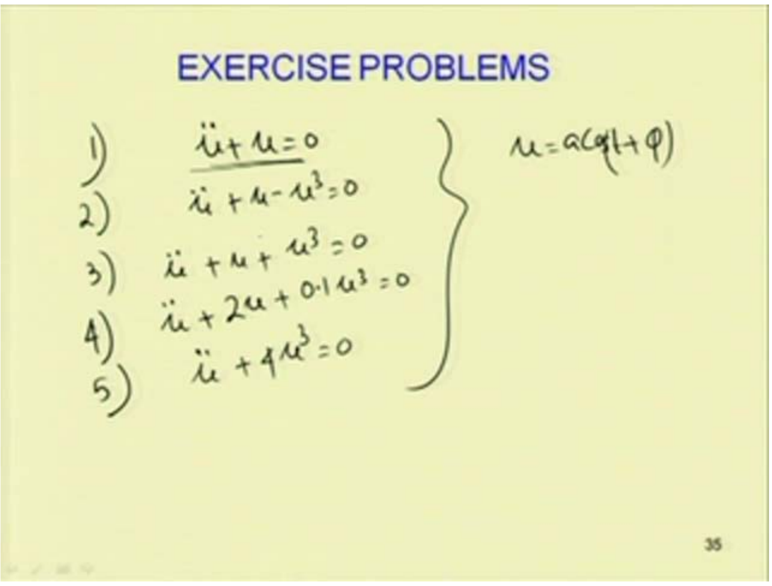

So, in this exercise problem you, so the first exercise problem what you can do you just find the trajectory in this case u double dot plus u equal to 0, second case u double dot plus u minus u cube equal to 0 u double dot plus u plus u cube equal to 0. So, let us take some, so let it is 2 u so, this is omega n square, I have taken 2 u plus let it is 0.1 u cube equal to 0. And the last exercise problem, what you can take so, this is u cube plus u double dot plus 4 u cube equal to 0. So for example, in the first case u double dot plus u equal to 0 so, the solution is already known because this omega n square equal to 1 here, and the solution u equal to a cos or sin omega t a cos t a cos t plus phi.

So, the response is periodic. So, one can get a centre in this case. Similarly, one can solve this non-linear equation first find the optimum points and then, study the type of response so, whether you are getting a center or a saddle point and find this trajectory. So, solve this exercise problem in this case. So, in the next class we are going to study about the straight forward expansion method and what are the demerits we are getting by using this straight forward expansion method. Then, we will study this Lindstedt Poincare method.

Thank you.