Non-Linear Vibration
Prof. S.K. Dwivedy
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 2
Derivation of Non Linear Equation of Motion
Lecture - 6
Ordering Techniques

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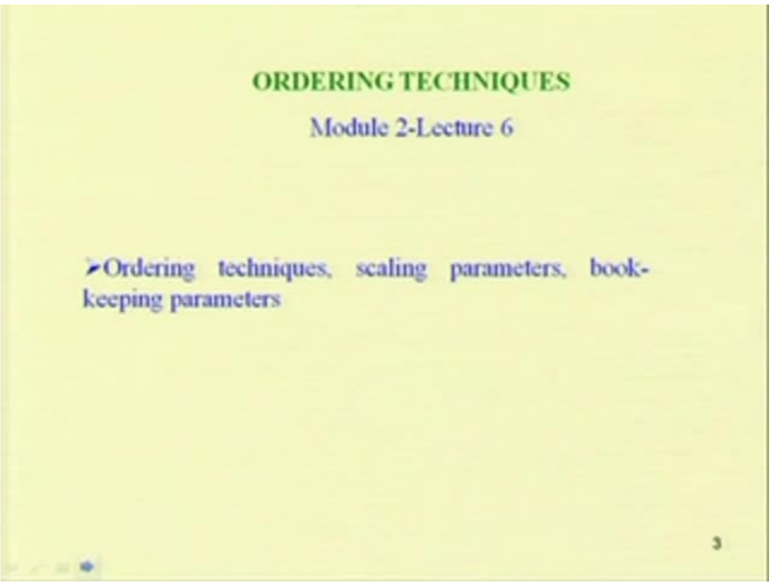

Welcome to today class of non-linear vibration. So, today class we are going to discuss about different ordering techniques in the non-linear equations, what we have derived before using different methods. So here, we will discuss about the scaling parameters, book keeping parameters, and also will discuss about different non-linear equations used in this course.

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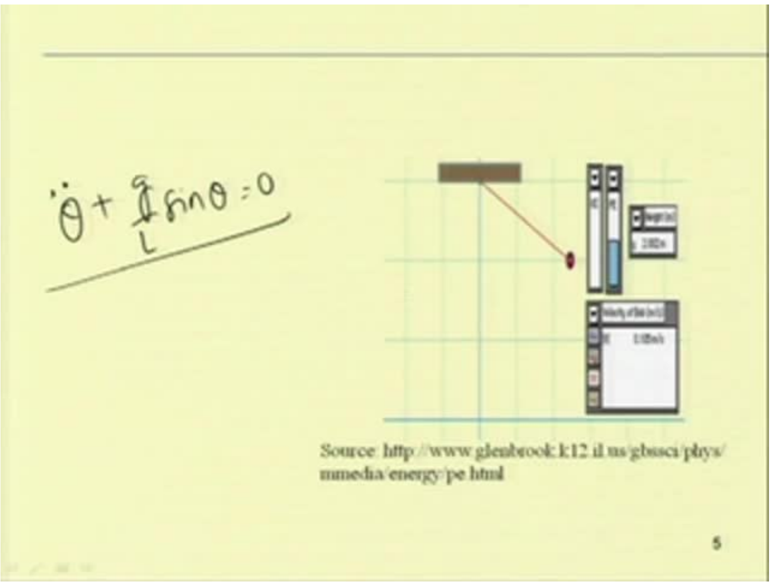

So, before going for this ordering technique, so let us see, what are the systems we have already discussed. So, for examples we have discussed about the simple pendulum. So in case of simple pendulum, already you have seen how we have written the equation motion in a non-linear form. The linear equation motion already you have this, already you know, so it can be written in this form that is theta double dot plus g by l sin theta equal to 0. So here for small value of theta, so sin theta equal to theta and this equation reduces to that of a harmonically excited system that is theta double dot plus g by l sin theta equal to 0. So, the motion is simple harmonic. So but if theta is not small, in that case one has to expand this term sin theta up to its third order or fifth order or higher order and write the governing equation motion.

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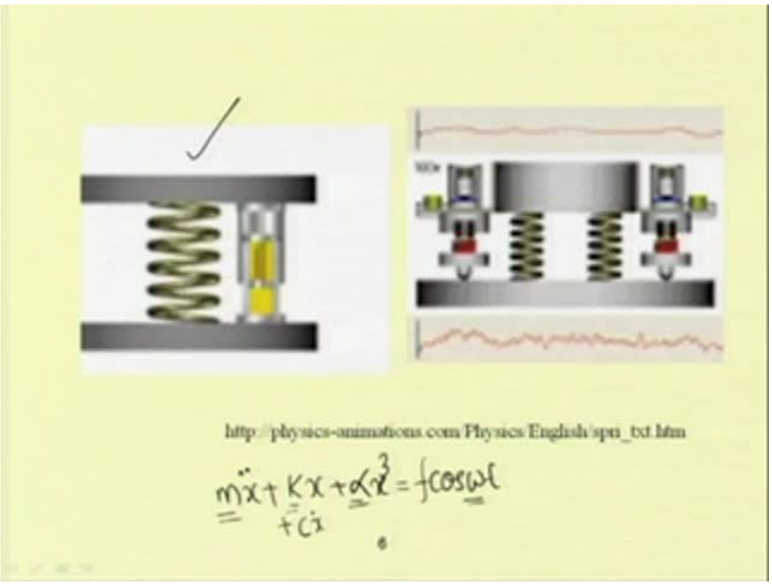

Similarly, one can see this simple spring mass damper system. So in this case, the spring may be a linear spring or it may be non-linear spring; so depending on the hardening or softening type of springs, so one can write different equation of motion. So in this case the equation of motion one may write the in the simplest form that is in the form of Duffing equation, which can be written in this form that is m x double dot plus k x plus alpha x cube, so if I will write a cubic non-linear term so for free vibration, I can write this is equal to 0 or for force vibration, I can write this equation in this form that is f cos or sin omega t. So, here f is the forcing parameter or amplitude of forcing, omega is the frequency and m k, m is the mass of the system, k is the stiffness of the system, and alpha is the coefficient of the non-linear term.

So, we can add the damping also to the system, so by damp adding damping this left hand side I can add c x dot term. So, in this way we can write the equation of motion, so for this simple spring mass damper system similarly, when we have number of springs added or number of dampers are added to the system. So we can write the equivalent stiffness and equivalent damping of the system, and taking the non-linearity effects into account we can write the equation of motion.

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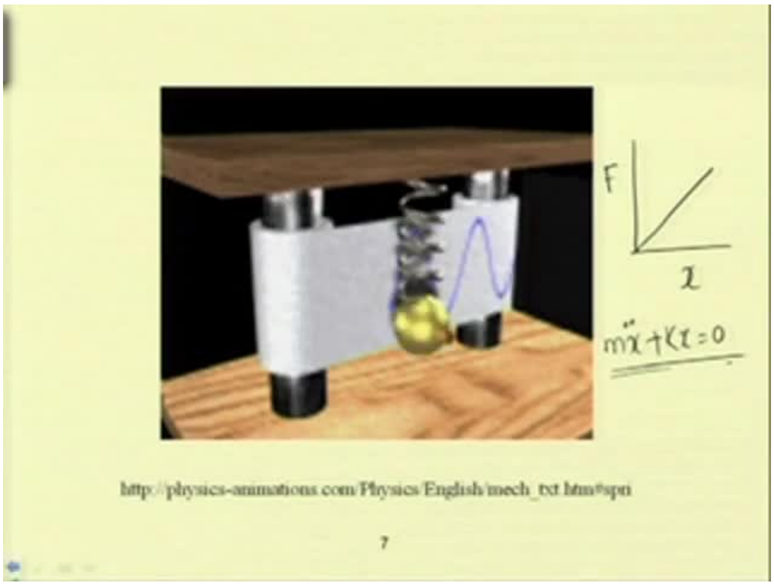

So one can observe, so incase of incase of this spring mass system, so when it is executing the simple harmonic motion that means when the spring we have considered to be linear. So, in this case if of the spring force this spring force let f is the spring force versus this displacement, so if this is linear, so then we have the equation motion in this form that is m x double dot plus k x equal to 0; and in this case, the system will exhibit simple harmonic motion as one can observe from this plot.

So in this plot, so it shows a sinusoidal curve, so when we have the linear spring, then we can have a simple harmonic motion. But if the spring is not linear, then we can we can have a non-linear type of response, which we have to, we have to determine how the response of the system is obtained with time, how the system response or how the system behaves with time. The dynamics of the system can be obtained from either one can plot this dynamics of the system by doing this experiment or analytically one can determine the response of the system.

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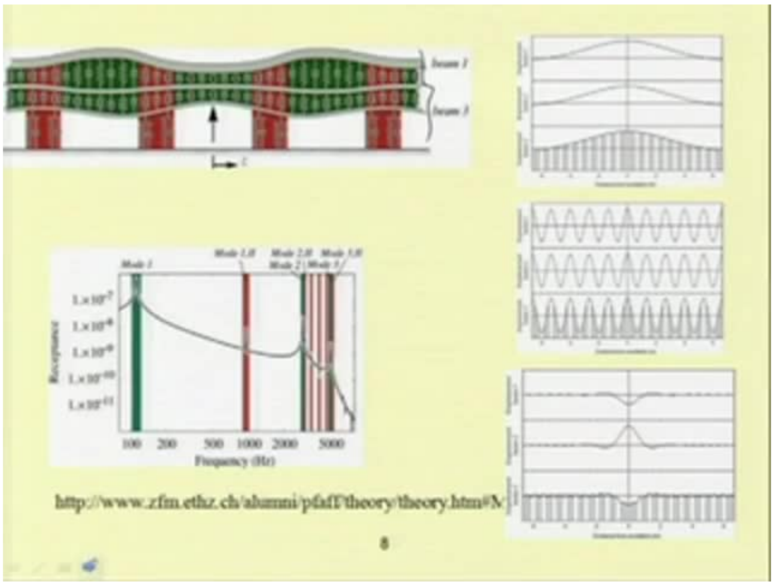

So similarly, we have studied other different types of systems also so, here this is a free system which can be modeled as a beam with many spring and damper system in between, so one can have different layers of the beam, so here one has put three layers of beam beam one, beam two, beam three or the soil can be modeled, sometimes the soil or breeze can be modeled in this way. So in that case also, one can find the response of the system, so the response may or may not be simple harmonic. So, if one consider the spring and damper to be non-linear, then one can obtain different type of response.

So, what type of response one will get that thing we will study in next module, but in this module we are interested to know about the equation of motion. So, already we have seen we can use different type of equation, different type of methods. For example: Newtons second law, D'Alemberts principle, or Lagrange principle, and Hamilton principle to derive the equation of motion.

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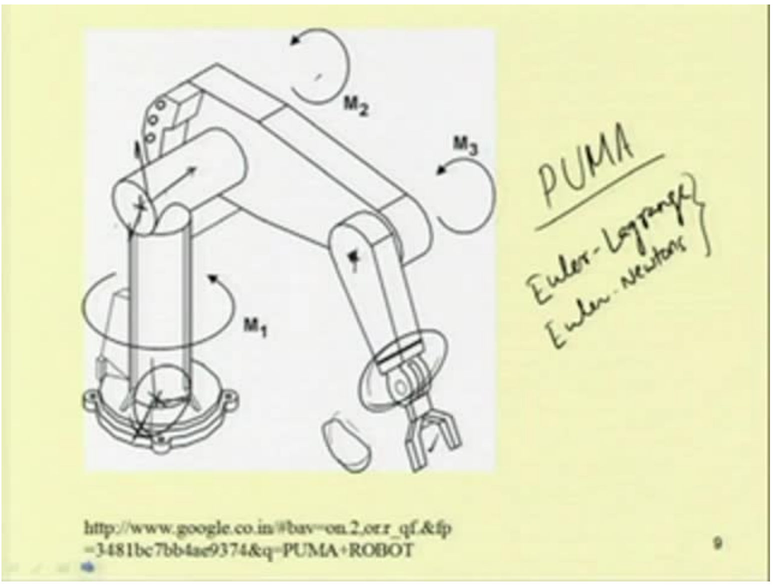

So, after deriving this equation of motion so, we can find the coefficient of this equation to see the, or to perform the ordering of the equation, so before that let us see some of other type of systems. So, in previous systems what we have seen so here, we can use fix coordinate systems to find the equation motion but, sometimes it may required to define the moving coordinate system.

For example, in this case of this puma robot puma is programmable universal PUMA programmable universal manipulator machine for assembly, programmable universal machine for assemble, so this puma robot one can use. So here another type of robot that is a humanoid robot, also can see the image of humanoid robot, or in this case of puma robot so, we can fix different coordinate system. So this coordinate systems can be fixed at the joints, so in this case the, so this is the fix one can fix the fix coordinate system here, so also one can fix the coordinate system here also in the joint, so we have six different joints; so, six joints are there in this puma robot.

So, here we have three joints, so yaw pitch and role motion in the wrist and in addition to that we have three degree of freedom here. So, this is this rotates about the space, also it rotates about the space and it has a revalued joint here also. So, we have three major motions about this three axis and we have yaw pitch and role motion above the wrist axis so, we have six degrees of motion and it can take any object from the so, it can take so using these end effecter it can take any object from its work space.

So while, deriving the equation motion for this type of system, so here one has to use this Euler Lagrange type of equation or Euler Newton type of Euler Newton formulation to derive the equation motion. So, we have not studied this equation Euler Lagrange or Euler Newton formulation, the interested reader can read those things from the books robotics related books. So, in this case so we have one moving coordinate system and fix coordinate system fix, this is the fix coordinate system in addition to we have this moving coordinate system.

So to find what is the torque required at this joint that means we are putting a motor at this joint, we have to put a motor at this joint, also at this joints we are putting different motors; so, at this joints we required to find what is the torque required at those joints to manipulate this robot. So, to find this torque so we have to write the dynamic equation of motion so, by finding this dynamic equation motion so one can one can obtain the response of the system.

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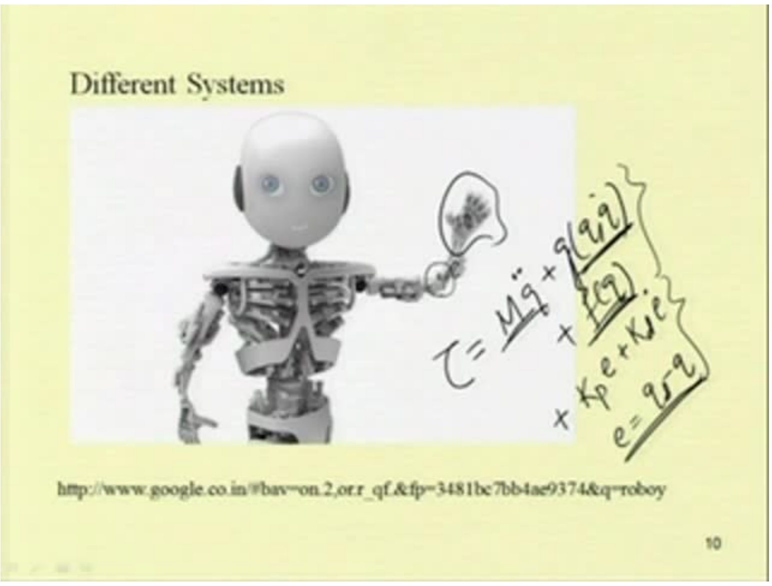

So similarly, in case of this humanoid robot to move this joint so for example, these fingers to manipulate these fingers so in that case one has to know, so what is the torque or force has to be applied by the tendant. So, there are several tendents in this humanoid robot, so in this humanoid robot also one can put the motor also at these joints, so one has to find what is the force or torque required at the joints in this robot.

So, in that case one has to write the equation motion for example, one can write the equation motion by using this equation that is tau equal to M q double dot plus g q q dot plus g or f q equal to 0 f q so, here this is the inertia force so, this will contain the potential potential function like k x spring and for example, in it is equivalent to this stiffness of the system so, this contain the gyroscopic force gyroscopic force due to gyroscope couple or coriolis component, and by taking this dynamics into account one can find the torque required sometimes it is not possible to model this equation directly.

So, for that purpose one has to use this control parameter or one can use this control k p let, this proportional gain so k p into e, so if there is some error plus then k d into e dot so, where e is the q desired minus q actual, so error is q desired and q actual and q is the joint variable, so M is the mass matrix, so if taking all these into account so, one can write the dynamic equation and after getting the dynamic equation, so one can check whether the system vibrates or not by solving this equation motion.

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So, in this way one can or another example is also there this is a satellite system so, in the satellite system these are the solar panels, and also we have different mechanism to activate these things, so one has to write the equation motion for the system to study the dynamic of the system.

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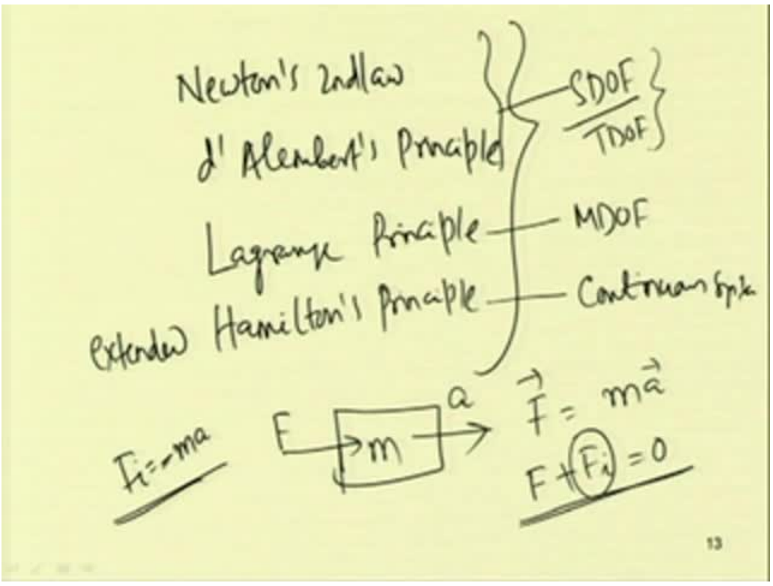

So, to find the equation motion for different systems already we know we can use this Newton’s second law, for Newton second law, then d’ Alembert’s principle, then so Lagrange principle, and Hamilton’s principle extended Hamilton principle. So, already we have discussed about these methods and we know that this Newton second law or d’Alembert principle we can use for systems with few degrees of freedom. For example: single or two degrees of freedom system, and this Lagrange principle can be conveniently used for multi degrees of freedom and this Hamilton principle one can use conveniently for continuous system so, continuous system so in so, here this is multi degree of freedom system and these two for single or two degrees of freedom system.

So, in case of Newton second law so we are drawing the free body diagram of the system, after drawing the free body diagram of the system, we write we separate the whole assembly by drawing the free body diagram and then we write the force equilibrium equation. For example, we write the external force will be equal to mass into acceleration, if this external force F is acting on the system and it is accelerated with acceleration a so, you can writing the equation motion in this form.

And in d’ Alembert principle so we can write this external force plus this internal, this is the inertia force equal to 0. So, this is d’ Alembert principle where this F i that is the inertia force equal to mass into acceleration and it takes place in a direction opposite to that of acceleration, so we can put a negative sign. So, by using this d’ Alembert principle or Newton second law we can derive the equation motion for single or two degree of freedom system conveniently.

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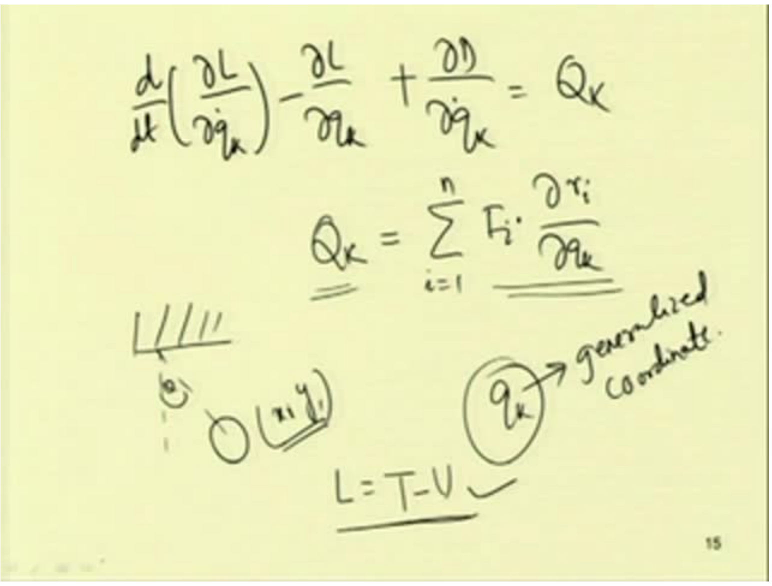

Similarly, in case of Lagrange principle so we can write the equation motion in this form that is d by d t of del L by del q k dot minus del L by del q k plus if damping is present in the system, we can write del D by del q k dot equal to q k so, where this q k that is the generalized force is written as summation i equal to 1 to n so F i dot del r i by del q k. So, here this let n number of forces are acting on the system then writing the position vector as r i.

So, one can do this dot product to find this generalized force q k where q small q k is the generalized displacement or generalized vector so, this is the generalized coordinate; q k is generalized coordinate for example: we have already discussed that incase of the simple pendulum so generalized coordinate is theta, theta is the generalized coordinate instead of putting this x and y x i y i or x 1 y 1 one can put this theta as the generalized coordinate, so here x and y are the physical coordinate. So, here L capital L is the Lagrangean of the system that is equal to kinetic energy minus potential energy of the system.

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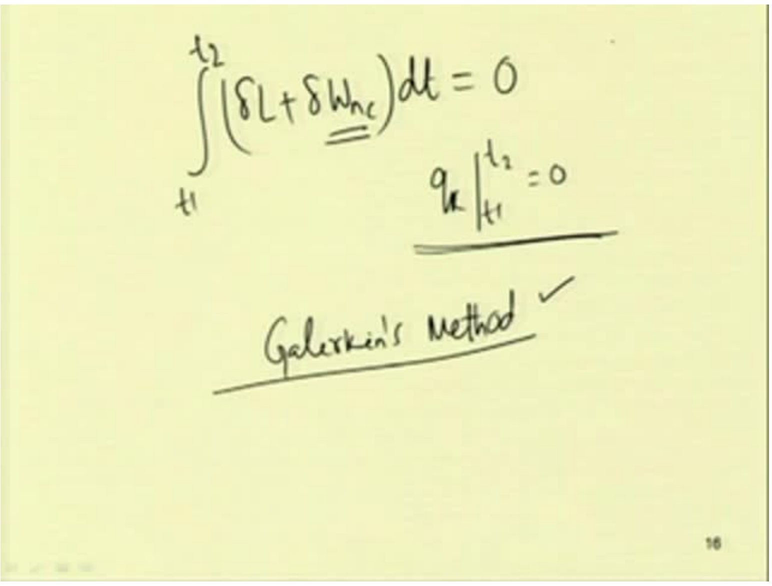

Similarly, one can derive the equation motion by using Hamilton principle so, in case of Hamilton principle so the equation can be derived by using this formula, integration t 1 to 2 delta L plus delta W n c d t equal to 0 so, where this q k vanishes at t 1 or q k t 1 t 2 equal to 0 so q 1 vanishes at the t 1 and t 2 so, L is the Lagrangean of the system, delta w n c is the non conservative, non conservative work done by the system so, by using this extended Hamilton principle also we can derive the equation motion.

But, in case of continuous system unlike discrete system we have the equation motion is a function of both space and time so, in case of continuous system it is a function this displacement parameter is a function of both space and time, so in that case to reduce it to its temporal form already, we have discussed that we can use this, general we can use the Galerkin's method, generalized Galerkin method so we can use the Galerkin's method to reduce the equation from its space and time to its temporal form. So, in this way we can derive the equation motion, this is just review of what we have studied in this module, so in this way we know how to derive the equation motion of the system.

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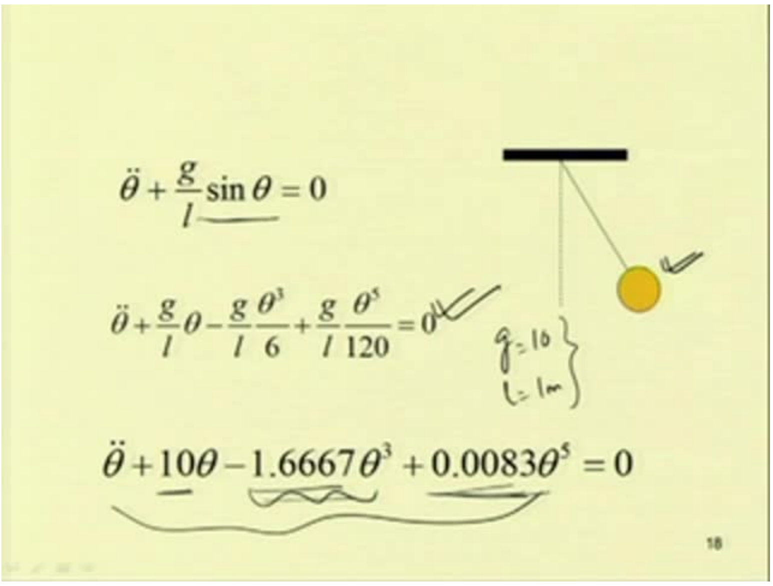

For example, in this case of simple harmonic, in this case of simple pendulum the equation motion can be written in this form, theta double dot plus g by l sin theta equal to 0. Now, expanding this sin theta we can write theta double dot plus g by l theta minus g by l theta q by 6 plus g by l theta q by 120 equal to 0. So, now we are going to study how you can order this equation so, that our analysis will be easier or we can do the analysis in a simplified way.

So, here one can by taking this g equal to let us take this g equal to 10 meter per second square and l equal to 1 meter. So, taking this example g equal to 10 meter per second square and l equal to 1 meter so, this equation can be written in this form, that is theta double dot plus 10 theta minus 1.6667 theta q plus theta 0.0083 theta to the power of 5 equal to 0. So, looking at this example looking at this coefficient so, coefficient of theta equal to 10, coefficient of theta q equal to 1.667, so which is almost 10 times less than that of the linear coefficient, that is coefficient of theta. And this term that is the coefficient of the theta to the power of 5 which is 0.0083 so, it is almost 1000 time less than this linear term.

So, one can conveniently neglect this term that is theta to the power theta to the power 5 and one can write this equation in this form that is theta double dot plus 10 theta minus 1.6667 theta q equal to 0. But, still one can neglect this term also this coefficient of as the coefficient of theta q is very small, that is almost 10 times or 7 8 times less than that of linear term.

So, in that case one can write the linear equation that is equal to theta double dot plus 10 theta equal to 0, so that equation already we have discussed that in the linear case, that is theta double dot plus 10 theta equal to 0. But if we are not neglecting this term, that is we are assuming that theta to be small, then we have to take into account these terms. So, in that case we can keep up to order of this coefficient of theta q, that is we may have to keep the equation in this form or write the equation in the form, that is theta double dot minus 10 plus 10 theta minus 1.6667 theta cube equal to 0.

But still if you require more precision we have to keep all the term that is we may keep up to this quintic order, so this equation what equation we have written before, that is theta double dot plus theta minus 1.6667 theta cube plus 0.0080 theta to the power of 5 equal to 0, so in this way we have to keep up to quintic order.

So, the in analysis of non-linear system it is very important to check the coefficient of the terms; coefficient of the so here the coefficient of theta is 10 theta q equal to 1.6667 and theta 5th equal to this. So, we have to write this coefficient in such a way that we should not feel to neglect these terms, so to write this equation in a modified form. So, we can use some scaling factor or book keeping parameter. So, let us assume that or let us take this theta equal to p y.

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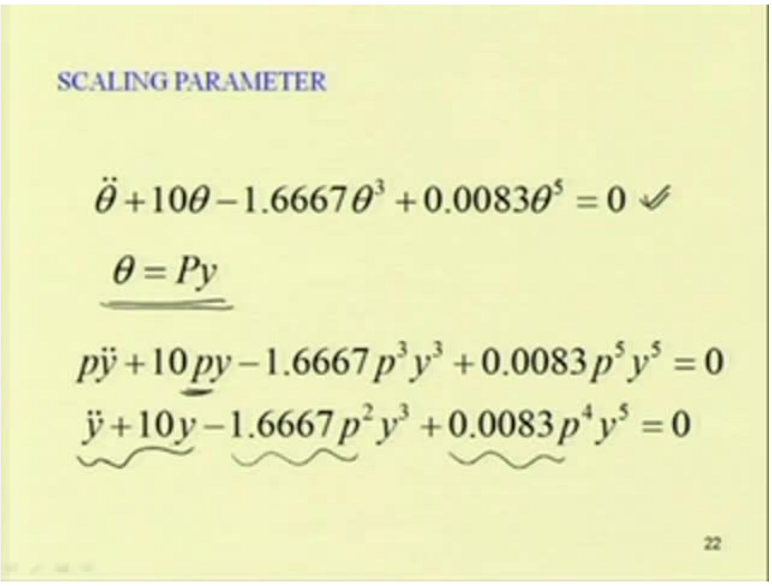

So by taking this theta equal to p y so let us use taking theta equal to p y so, you can substitute it in this equation. So, now the equation become p y double dot plus 10 p y minus 1.6667 p cube y cube plus 0.0083 p to the power 5 y to the power 5 equal to 0; now, by suitable choosing the parameter p so we can manipulate these coefficients. Now, this equation is written now dividing first dividing this p throughout this equation, so we can write this equation in this form, that is y to the power y double dot that is d square y by d t square plus 10 y, so this is the linear part, so you just see in the linear part we do not have this p term but, in the non-linear part we have the p term remaining.

So, in this cubic order we have this p square term that is minus 1.6667 p square and this quintic term we have this 0.0083 p to the power 4; now, by suitable change choosing this p parameter, so we can order this equation or we can write this coefficient in such a way that we so, this coefficient will be of the same order as that of the liner part. That means so, we have to choose p in such way that this 1.6667 p square may be nearly equal to 10. So, let us take some value of different value of p and check how the equations look like.

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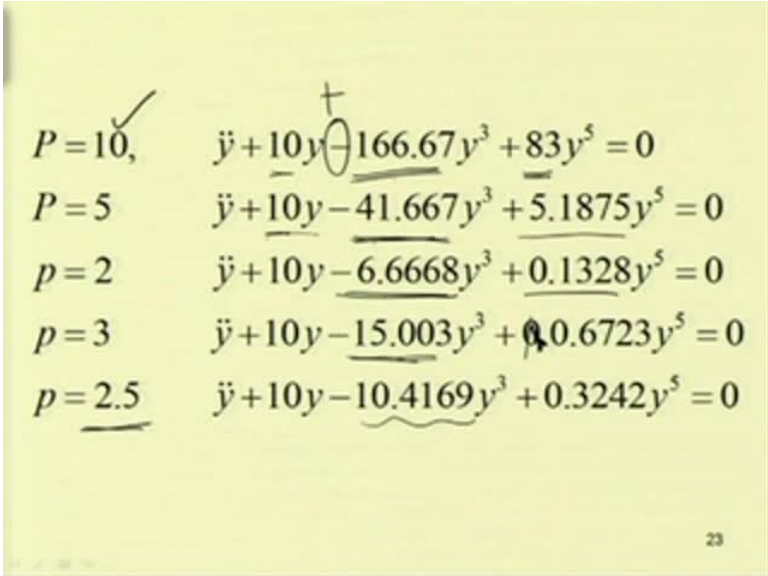

So, for example: taking p equal to 10, so if you take p equal to 10, so this equation now becomes, y double dot plus 10 y minus 166.67 y cube plus 83 y to the power 5 equal to 0, so in this case, one may note that these two coefficients are very large in comparison to linear part so, that means this will behave as a strongly non-linear system, so if you take this p equal to 10.

So, now taking p equal to 5 so, you can write the same equation in this form that is y double dot plus 10 y minus 41 667 y cube and 5.1875 to the power 5 equal to 0. So, in this case one may note that, while this quintic coefficient of the quintic order, that is 5.1875 is almost near to or of the same order as that of the linear term, that is 10, so but this cubic term is 4 times, so this is 41 and this is 10.

So, it is 4 times higher than that of the linear part. So, if we are interested to keep this coefficient to that of the linear order so, we can now play with this number p, and we can choose this parameter in such way that they will of the same order. For example, taking this p equal to 2, now we have the coefficient of the cubic order less than that of linear part that is 10. So, this becomes 6.6668 and the quintic part becomes 1. 0.1328. So, now almost it is this quintic part is hundred times less than that of the linear part, but this cubic term is of the same order as that of the linear part.

And by putting p equal to 3, so we can get this cubic order to be 15.003 which is slightly higher than this linear part, but this quintic part equal to 0.6723 so, by choosing p equal to 2.5 so, we got almost the cubic order term to be same as that of the linear part, this is 10.412969; so, this equation becomes y double dot plus 10 y minus 10.4169 y cube plus 0.3242 y to the power of 5. So this way by taking different value of p, so we can write the same equation in different form.

So, where one can have a system similar to that of a hardening spring, or one can have or one can have system with that of a softening spring, or one can write this or one can have this coefficient very high or low order. So, this hardening or softening spring term will come let us take this p equal to minus, so if you take p equal to minus 10 so, instead of this minus sign we can get this plus sign so, in case of minus we are getting the softening type of spring but, if we have this plus sign we can get this hardening type of spring. So, if we take p equal to minus 10 so, in that case okay p square will be equal to 100 so, it will not be negative but, we but, we can have this fifth order term so, fifth order so this becomes forth order, so in this case, in this particular example, as we have this p square and p forth.

So, by taking different value of p whether positive or negative, we can get only this equation or the coefficient negative, that is y so always we can have the so, we will not get a positive term or positive coefficient, and so the spring type will be that of, if you compare this thing with that of a Duffing equation, this will be this will show the response as that of a softening type of spring.

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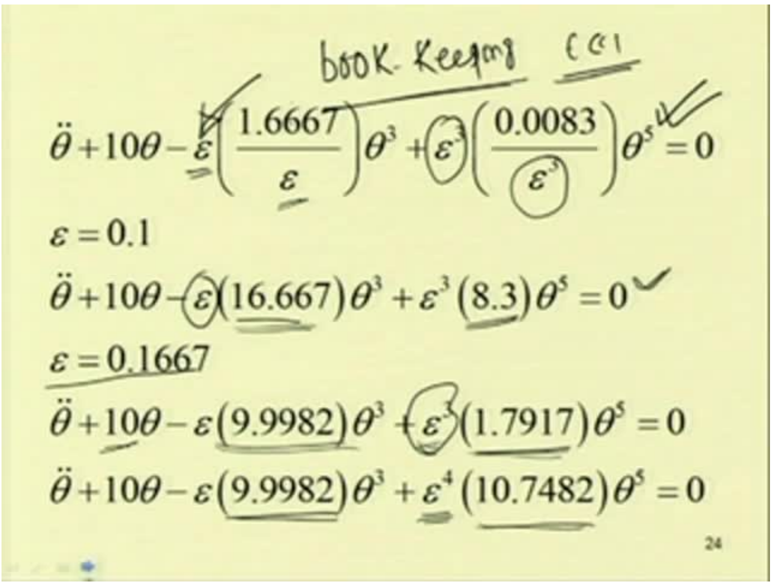

And okay so now, we can have different type of equation, we have seen how by taking different type of different value of p, we can control the coefficient, and we can get different equations. So, let us use another form, so the same equation theta double dot plus 10 theta minus 1.6667 theta cube plus 0.0083 theta fifth can be written in this form also, by dividing and multiplying this term by epsilon, where epsilon is a book keeping parameter which is very very less than 1; so, we can write the equation in this form, that is we are we can write this equation, theta double dot plus 10 theta minus epsilon; so, we have multiplied epsilon and divided epsilon here.

So, now by dividing epsilon in this equation so what we can write so, we can get this term equal to 16.667 which is almost of the same order as that of 10. And here, also in this case by dividing this term by epsilon cube, you just see we have multiplied this term by epsilon cube, and divided this term by epsilon cube, so our coefficient so, here the coefficient becomes 8.3 and here the coefficient becomes 16.667.

So, one can write this same equation in this form, that is theta double dot plus 10 theta minus epsilon into the 16.667 theta cube plus epsilon cube into 8.3 theta to the power 5 equal to 0, so here this term 8.3 and 16.667 cannot be neglected with respect to the term 10. But, when you are writing the equation in this previous form that means, theta double dot plus 10 theta minus 1.6667 theta cube plus 0.0083 theta to the power 5 equal to 0, so there is a tendency always to neglect the cubic and fifth order term.

But, when we are using this book keeping parameter epsilon, so as the coefficient are of the same order as that of the linear part, we will so there will be no tendency to neglect these non-linear terms. So, now let us take another parameter that is epsilon equal to 0.1667. So, in that case we have this equation or the coefficient becomes 9.9982, so which is almost near equal to 10 and this coefficient becomes 1.7917, so in this way by using this epsilon parameter; so this epsilon is known as a book keeping parameter.

So, by using this book keeping parameter which is a small number, that is less than so, this epsilon always less than, very very less than 1. So, by using this book keeping parameter, so we can order the equation motion. So, here we know now by using the scaling parameter, or by using this book keeping parameter we can order the equation of the motion equation motion.

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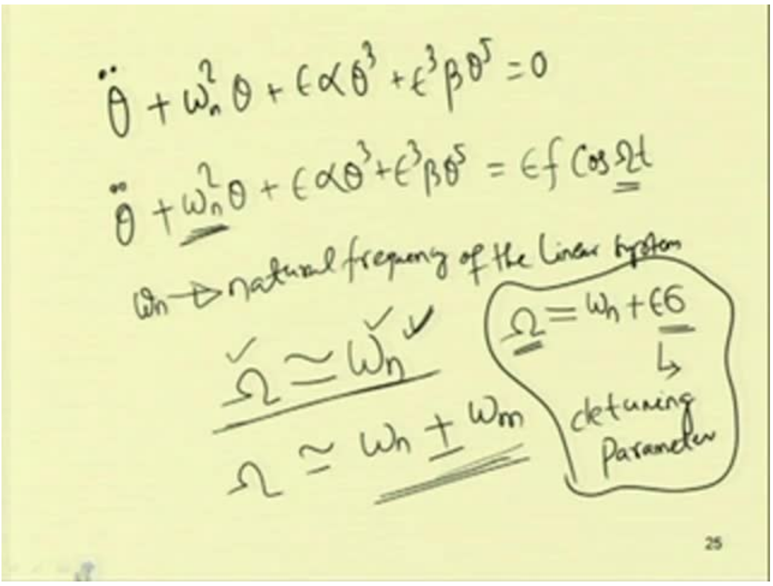

So, it is the same equation what we have written here, so we have used, the coefficient of theta to the power 5 as that of the order of epsilon cube also, one can write so, when I have written it is of the order of epsilon cube so, then the coefficient becomes 1.79, but this is not the same order as that the linear part that is 10.

So, I have to increase this value or so, I have to write this is of the order of epsilon to the power 4, so the same equation can be written in this form, that is theta double dot plus 10 theta minus epsilon into 9.9982 theta cube plus epsilon to the power 4 10.7482 theta to the power 5 equal to 0.

So, we can write our equation so, that is let me write a general equation in this form, that is theta double dot plus omega n square theta plus epsilon alpha theta cube plus epsilon cube beta theta to the power 5 equal to 0. So, if we use the forcing in this system so we can write this equation in this form also, omega n square theta plus epsilon alpha theta cube plus epsilon cube beta theta to the power 5 equal to so, I can take this forcing to be of the order of epsilon so, then I can write this is equal to epsilon f cos omega t; so here this omega that is the external frequency, and this omega n square or from this this omega n so, this is the natural frequency of the linear system so, natural frequency of the linear system.

So, most of the times in case of one linear system we may be interested to study the external frequency to be nearly equal to that of the natural frequency, or we may be interested to study the summation of the natural frequency sum of different natural frequency. For example, in continuous system we will have a number of natural frequency of the system, so in that case, one may be interested to study different frequency. So, in first case so that is when, omega nearly equal to omega n to study the nearness of this external frequency to that of the natural frequency, one can use a parameter so that is known as detuning parameter. So, one can write this omega equal to omega n plus epsilon sigma so, here sigma is known as the detuning parameter. So, one can use detuning parameter to write the nearness of the natural frequency to that of the external frequency.

Similarly, in this case that is when omega nearly equal to omega n plus omega n, so one can have a combination type of resonance condition, so in this case omega nearly equal to omega n one can have primary resonance condition. And in this case one can have this combination resonance condition, so in case of combination resonance condition.

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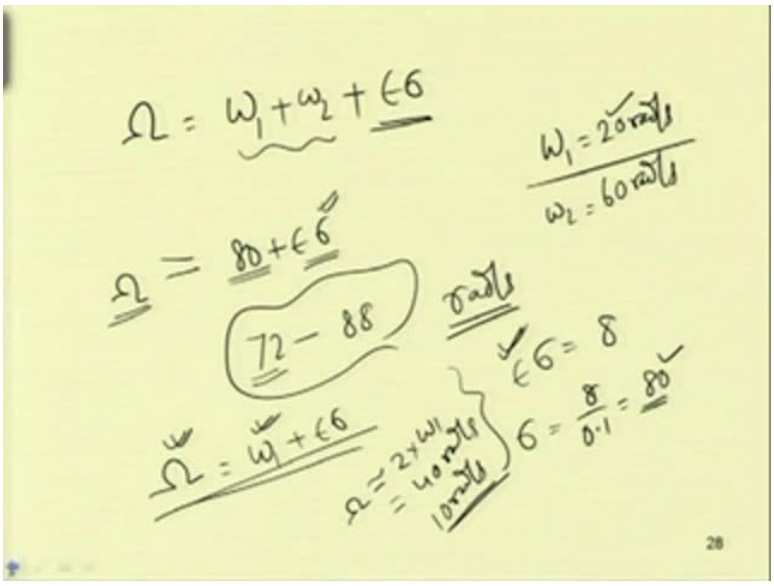

Also, one can study this omega nearly equal to so, one can study omega nearly equal to…Let, me take only the first two modes, it will be omega 1 plus omega 2 plus epsilon sigma so, this will so, in this case so we have taken only the first two mode. So, this external frequency nearly equal to summation of the first two modes frequency and we are studying near the so, we are studying near the summation of this first two mode. So, here epsilon is the book keeping parameter and this sigma is the detuning parameter.

For example: so let omega 1 equal to, so let us take omega 1 equal to 20 radian per second and this omega 2 equal to 60 radian per second. So, in that case our omega so, we can study this omega nearly 620 plus 60 that is 80 plus epsilon sigma. So, to study the nearness of this so generally one can take this near to 10 percent of this thing so, we can so if you are taking 10 percent of 80, so that means we can vary this thing from 72 to 88. So, we have to study the frequency near to 72 to 88 radian per second so, in that case we have this epsilon sigma equal to so, when it is 88 so our epsilon sigma equal to 8 by taking this epsilon equal to 0.1; so you have this sigma equal to 8 by 0.1; so this is equal to 80.

So, we can have a value of sigma nearly equal to 80 to half this omega that is the external frequency near to 88 or near to 72. In case of 72 also this epsilon sigma so, you will have a negative sign here, so this epsilon sigma equal to 8, and sigma will be equal to 80. Also, we can vary this detuning parameter to study the resonance different resonance condition, so previously I told about the primary resonance condition that is omega equal to omega 1 plus epsilon sigma so, when this natural frequency nearly equal to the external frequency.

So, in case of a forced vibration of the system so, we can have the primary resonance so, this is primarily observed, in case of the linear system but, in case of non-linear system in addition to this primary resonance, so one may have secondary resonance. For example, one can have this sub harmonic oscillation, or the super harmonic oscillation corresponding to so corresponding so, for example, this for example, this omega 1, so if omega 1 nearly equal to 10 or 10 radian per second, so we can have the secondary oscillation or secondary resonance condition.

Also, we may have so omega 1 so, this capital omega nearly equal to twice the omega 1 that is nearly equal to 40 radian per second so, we may have sub harmonic resonance and when it is equal to 10 radian per second, we may have the other type of resonance condition, that means when it is away from the natural frequency that is away from the 60 also, we can resonance condition. So in case of non-linear system, so we can have this super harmonic and sub harmonic resonance condition, so now let us see some commonly used non-linear equations, and some system which behave or which give rise to those type of equations.

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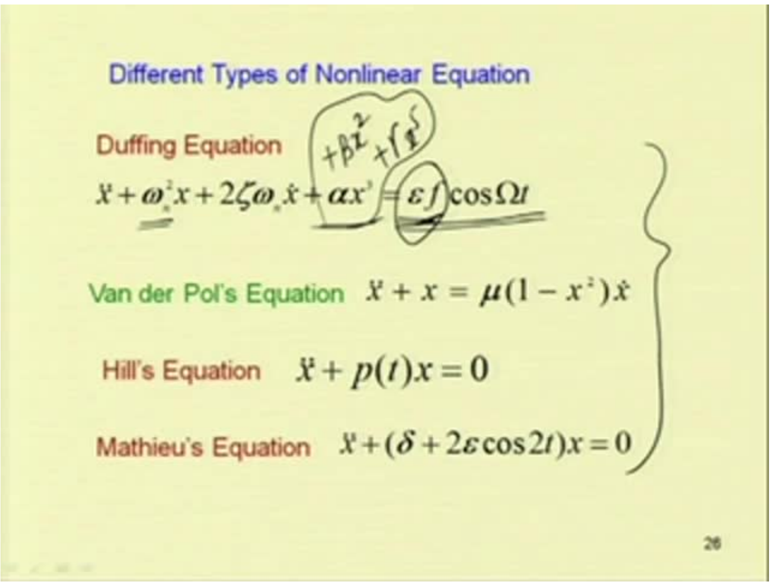

So, our familiar equation motion what we have studied till now is the Duffing equation. So, the commonly used different type of non-linear equations are this Duffing equation, Van der Pol’s equation, Hill’s equation, and Mathieu’s equation, and a combination of all these equation. So, for example: in spring mass damper system we have this, so in case of these spring mass damper system the equation is that of the Duffing equation type, so in this Duffing equation type we can have this quadratic non-linearity, or we can have cubic non-linearity.

So, this is cubic non-linearity one can add, one can add quadratic non-linearity, or quintic type of non-linearity, what already we have seen gamma x to the power 5 so, by adding different type of non-linearity to the system, so we can write this Duffing equation; the simplest type of Duffing equation is x double dot plus omega n square x plus 2 zeta omega n x dot plus alpha x cube equal to 0; so that is for the free vibration case.

And incase of force vibration, we can have the forcing term in the right hand side, so epsilon f cos omega t in the equation, so a number of variation of this equation can be written. So, this Duffing equation for free vibration by putting equal to 0, Duffing equation for force vibration can be written in this for force vibration, also we can have two variation, so in this case we have the strong non-linearity or weak non-linearity, so incase of weak non-linearity, we can remove this term epsilon and we can write the equation, so that is f cos omega t.

And in case of strong non-linearity, we have f cos omega t where, this f is of the same order as that of the omega n square, and in case of weak non-linearity we have epsilon f that means, this epsilon f will be atleast one order less than that of omega n square similarly, we can have multi frequency excitation.

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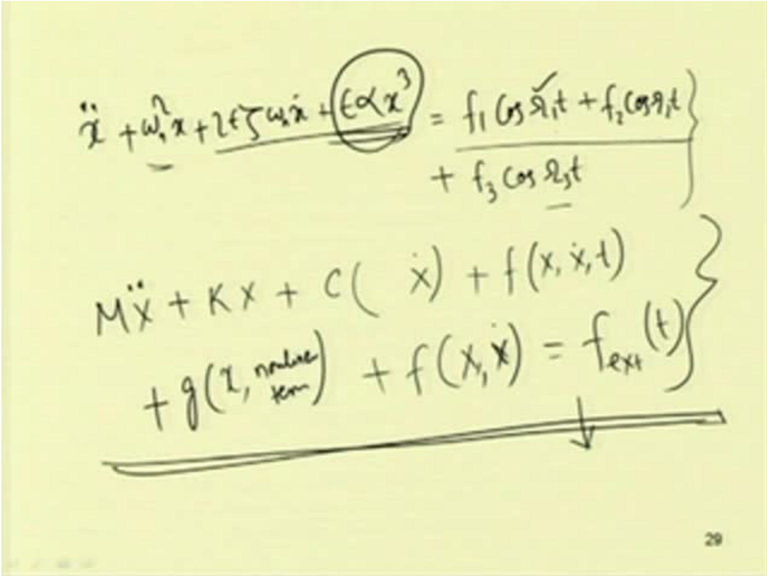

So here, we have a single frequency excitation. So, we can write the equation in this form, that is x double dot plus omega n square x plus two epsilon zeta omega n x dot plus epsilon alpha x cube; here we have taken one order less in cubic non-linearity, so it may be written equal to f 1 cos omega 1 t plus f 2 cos omega 2 t plus, so you can put two or three terms f 3 cos omega 3 t.

So, in this case for example, in case of a linear system already we know if this equation is this in this form, that is x x double dot plus omega n square x equal to f 1 cos omega 1 t plus f 2 cos f 2 cos omega 2 t. We can use super position theory to write the solution of the system, but when we have this non-linear term in this equation we cannot use the super position theory, and one has to find the response for each case.

That means one has to find the response of the system, first for f 1 f 1 cos omega 1 t, so one has to find the equation motion by considering all the effects, so in this case one may note that the resonance condition may occur, when omega 1 nearly equal to omega n square. Similarly, omega 2 nearly equal to omega n square omega 3 nearly equal to omega n square also, it may occur when the combination of this omega 1 plus omega 2 will be nearly equal to that of omega n square.

So, one can have combination type of resonance also in this multi frequency excitation, so one can have simultaneous resonance condition, simultaneous resonance condition means this omega 1, omega 2 and omega 3 capital omega 1, omega 2, omega 3; simultaneously will be as they are independent; they simultaneously equal to that of the natural frequency, and so in that case one can have the simultaneous resonance condition.

Similarly, one can have this, one can have this is this equation is written for a single degree of freedom system, so one can write the same equation that for a multi degree of freedom system by writing this x as in the vector form. So, that means x double dot plus omega n square; so in this case, this omega or I can write this equation in this form, that is M x double dot plus k x plus so, I can have some other matrix, that is so which will contain this damping c is a function of x x dot. And we can have this f another matrix or damping matrix will be function of x dot, we can have this forcing term is a function of x x dot t, and another forcing term which may be a function of x and x dot.

Similarly, we can have this external force f external so, is the function of time, so this may contain this x x dot, and we can some term also which is function of g, so it is a function of x and x higher order like so, it may contain the non-linear term like x cube x square non-linear term. So, x square x cube some non-linear term so, this will be function of some non-linear term. So, in this way one can write the equation motion for multi degree of freedom system so, writing the equation of multi degree of freedom system, then one can reduce this equation by using model reduction method. There are several model reduction method available, and then one can use this ordering technique to order this equation. So, after ordering these equation one can then go for the solution technology which will be, which we will discuss in the next module.

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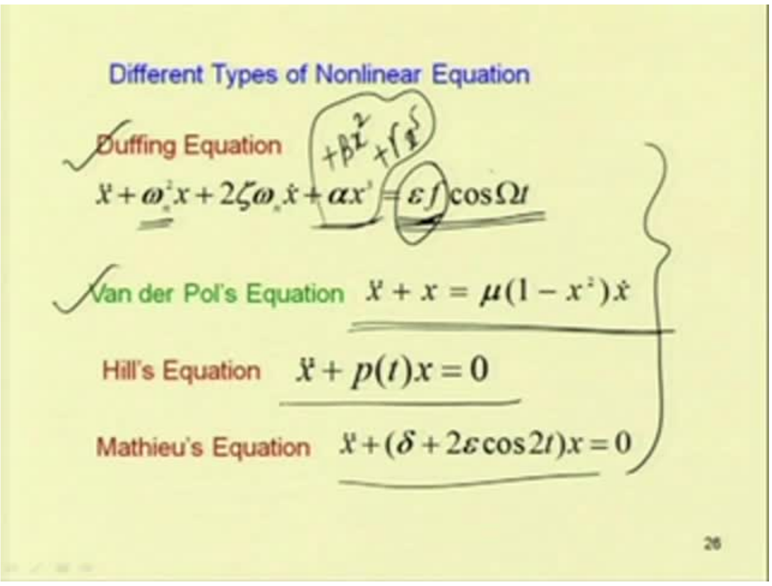

So, instead of using this Duffing equation, there are some more types of equation which we have also considered. This is Van der Pol type of equation x double dot plus x equal to mu into one minus x square into x dot, one can use a combination of this Duffing equation, and Van der Pol equation similarly, one can have this Hill's equation in this form, that is x double dot plus p t x equal to 0 and Mathieu equation in this form, that is x double dot plus delta plus 2 epsilon cos 2 t x equal to 0.

But, by adding this Mathieu equation, or Hill's equation, or Van der Pol equation with Duffing equation, and taking different type of forcing parameter, and ordering this equation one can have different sets of equation equation of motion which will correspond to different type of non-linear systems, but one can study in this course. So, in this way one can order the governing differential equation motion by either using the scaling parameter, or by using or by using this book keeping parameter. Sometimes both scaling parameter and book keeping parameter has to be used to order this system.

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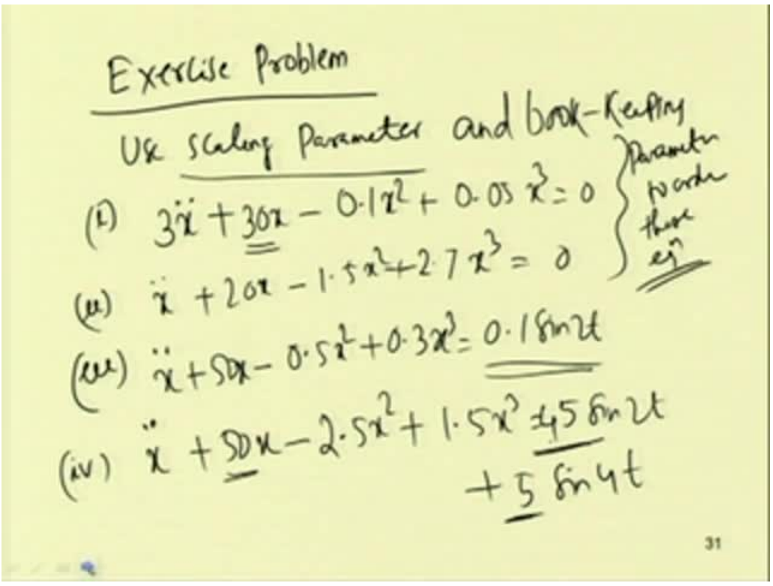

So, let us take some exercise problems to so let us, take exercise problem so, use scaling parameter so, one can use scaling parameter to write this equation. So, the equation 3 x double dot plus 30 x minus 0.1 x square plus 0.05 x cube equal to 0; x double dot plus 20 x minus 1.5 x square plus 2 . plus 2.7 x to the power cube equal to 0. So, x double dot plus 50 x minus 0.5 x square plus 0.3 x cube equal to 0.1 sign 2 t. So, these two are for free vibration; this is for force vibration. So, one can write one more equation that is for multi frequency. So, x double dot plus so this is 50 x, 50 x minus 2.5 x square plus 1.5 x cube equal to so, let us put some strongly non-linear system that is 5 or 45 sin 2 t plus let me put another forcing term, which is of 5 sin 4 t.

So, in this case this forcing term that is 45 sin 2 t, so the coefficient is of the same order as that of the linear part, that is why this of the same order as that of the linear part, but this forcing the coefficient of this forcing term is 10 times less than that of the linear part, that is why one can use a book keeping parameter epsilon in this case. So, in this case use the scaling parameter and book keeping parameter to order… Keeping parameter to order these equations to make the non-linear terms as same as that of the linear part. So, in this way we have studied in today class, we have studied the scaling parameter and book keeping parameter and we have seen different type of non-linear equation such as, Duffing equation, Van der Pol equation, Mathieu equation, then Hills equation, and a combination of all these equations; and one can know different type forcing para forcing also, single frequency multi frequency different resonance conditions, that is primary resonance condition, secondary resonance condition, sub harmonic resonance, super harmonic resonance condition; and in addition to that we have this combination type of resonance also, we have simultaneous resonance conditions observed in this case of non-linear systems.

So, in the next module we are going study about how to solve these equations these non-linear equations by using both qualitative and quantitative methods. So, in quantitative methods will study different perturbation techniques, and harmonic balance method to find the solution of the equations. So, in the forth module we are going to or in the fifth module we are going to study different non-linear techniques to find the solution of this non-linear equations.

Thank you.