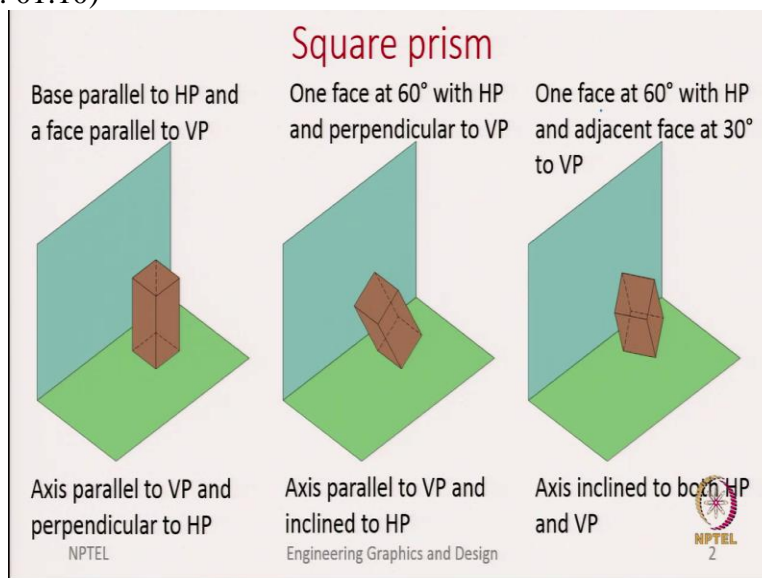


**Engineering Graphics and Design**  
**Professor Naresh V Datla**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology, Delhi**  
**Lecture 20**  
**Projection of Solids**

Welcome back, we are on week 4, where we are discussing about orthographic projections. In the last two lectures we have discussed, why we need to focus on solids. Because then we can use a combination of these solids to develop few complex objects. And we also looked in the previous class, how to construct as well as project polygons. In today's class, we will be discussing about projection of solids.

With this background of already knowing how to construct and project polygons, projection of solids become a little easier. We will be essentially reviewing those concepts and then say how do we extend it to draw as well as project solids. So, we will be essentially looking at three examples. The first example is a square prism.

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So, what I will be showing is the square prism something like this, let us assume that this cross section is square and this is the height of this prism. So, we can show this prism in different orientations. So, the first one says that the axis is parallel to the vertical plane. So, let us say this is the height. And let us say this is the vertical plane. So, the axis is now parallel to the vertical plane. And this axis is perpendicular to the horizontal plane.

So, now we are placing it in a neat orientation, which is simple to draw the projections. But now, let us say if it is inclined to one of the planes. So, here it says the axis is parallel to the vertical plane. Now, let us say I am tilting this, so this axis will be at an angle to the horizontal plane. So, it is not as simple as the first orientation. But it is reasonably simple, because it is only inclined to one of the planes.

But now, let us say if it is inclined to both horizontal and vertical plane. So, now what I will do is, I will rotate this object. So, I will maintain this angle with the horizontal plane and then rotate it. So, if I wrote it in this fashion, now it is, this angle makes an angle with the horizontal plane, and it will also make an angle with the vertical plane. So, how do we define this object in a particular orientation?

In these three examples, I have defined it using the axis. But is that the only way to define these square prisms? We can use the base of the object, we can also use the sides of the object. We can use the face of the object. So, let us look at the same things defined using different features of

this prism. So, in this simple orientation, we can define it saying that the base is parallel to the horizontal plane.

And a face, so one of the face is perpendicular to the, parallel to the vertical plane. So, let us say I am talking about this face. So, this face is parallel to the vertical plane. So, that is how I can define instead of using the axis. Similarly, in the second orientation, how do I define this? So, one way of defining it, is to say one face is at an angle of 60 with the horizontal plane. So, let us say we are talking about this face.

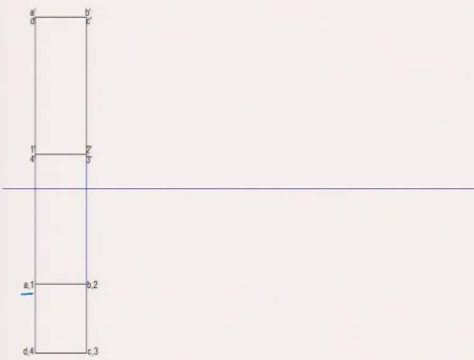
So, we are saying that this face makes an angle or 60 degrees. So, the angle between this phase and the horizontal plane is 60 degrees. And it also talks about the same face saying that this face is perpendicular to the vertical plane. So, now we are able to define this orientation just using this face. And in the last orientation, it says, again, we can use this saying that now we rotated it. So, this face is at an angle of 60 with the horizontal plane, and adjacent face.

So, this is one face adjacent to it is either this or this. So, let us take this, this face is at an angle of 30 with the vertical plane. So, let us say this face is making an angle of 30 with the vertical plane. So, you use one face to define the angle with a horizontal plane, and the other face to define the angle with the vertical plane. These are just one way of defining the angles. But as you can see, there are many possibilities. You can define it using the edges or the face and the base. What we will be now solving in the next slide is how to do projections of these final orientation.


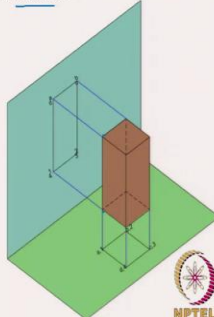
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### Square prism

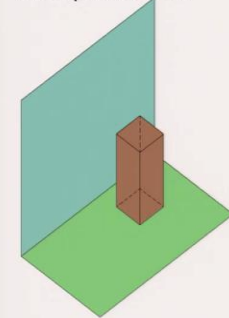
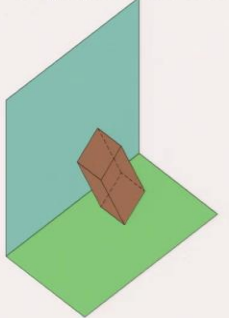
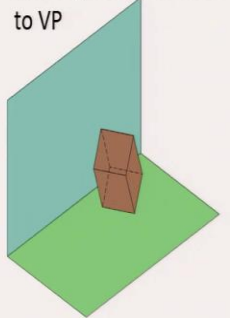
Draw projections of square prism with one face  $60^\circ$  to HP and adjacent face at  $30^\circ$  to VP. Base side is 40 mm and height is 60 mm.




1. Top view  
2. Front view



### Square prism

Base parallel to HP and a face parallel to VP	One face at $60^\circ$ with HP and perpendicular to VP	One face at $60^\circ$ with HP and adjacent face at $30^\circ$ to VP
		
Axis parallel to VP and perpendicular to HP	Axis parallel to VP and inclined to HP	Axis inclined to both HP and VP

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### Square prism

Draw projections of square prism with one face  $60^\circ$  to HP and adjacent face at  $30^\circ$  to VP. Base side is 40 mm and height is 60 mm.

1. Top view  
2. Front view  
3. Turn front view about edge 14

So, I have the question like this saying that draw the projections of a square prism, with one face  $60^\circ$  to horizontal plane. And adjacent face at  $30^\circ$  to vertical plane. So, if you think about the question, you will guess that we are talking about the last orientation in the previous slide. So, if I go back, this is exactly what we are being asked in the next slide. So, instead of directly solving the problem in this complicated orientation, what I do is, I break it down into three steps.

The way we also broke it down for polygons. Saying that step one, let us make sure so axis or the base is properly oriented with both the planes, horizontal and vertical planes, either parallel or perpendicular. And in the next step, let us make only one of the, making an angle with one of the

planes, either horizontal or vertical. And lastly, we will come to the problem where it is inclined to both planes.

So, the first step we are saying is, we start with a simple orientation. In this case, we are saying that, I forgot to mention that the base side is 40 mm and the height is 60 mm. This helps us to draw this prism. So, first is the simple orientation, we can either start with the front view or the top view. Randomly, I am starting with the top view. So, the top view will be whatever this cross section is. This, we said is a square.

So, we will have the square projections. So, first, let us draw the square projections. Here the namings are the top face has the points as a b c d. And the bottom face has the points 1, 2, 3, 4. So, the top base and the bottom base, the top base is a b c d, bottom base is 1, 2, 3, 4. So, this is the way we denote it saying that since the project line first touches the top face, point A. And then the same line touches point 1. We write it as, a,1. Same with all the other corners.

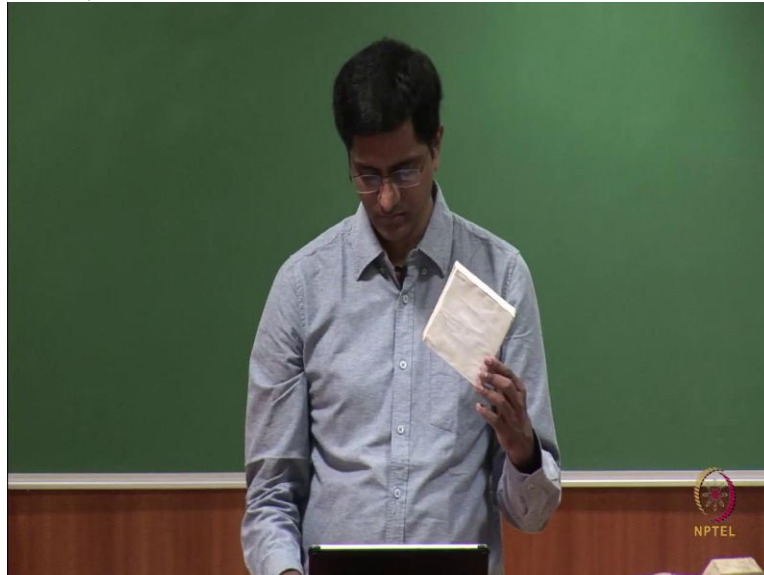
So, once we covered the top view, the next is to capture the front view. To get the front view, I can use the projection lines from the top view. With that, I will be able to draw the top view. So, the top view is the height and the width. So, step 1 is done, where we captured both the top view and the front view. What do we do next? We said we will orient it such that it only makes 1 single angle with a plane.

So, this is that angle. So, in this angle, which one do you want to start with, the front view or the top view? So, before rotation and after rotation. So, before and after, do you see the same shape? If you see the same shape, is it in the front view or the top view? The answer is you see the same shape of this rectangle in the front view. The only difference is that shape is now rotated. So, it is like a rigid body rotation, which means that shape will remain the same, only thing is that orientation of the shape will be different.

So, let us capture that. So, we say that we turn the front view, because the shape is the same, way simply turn it. We turn it about this line 1, 4. So, I capture the front view. So, once I captured the front view, then it will be a little easier for me to capture the top view. Because, now I know the points in the front view I can use those projection lines to find where are these points in the top view. So, now I need to draw the top view by using projection lines from the previous top view

and projection lines from the current front view. So, once I intersect these, now I go even now I am able to capture the top view with one rotation.

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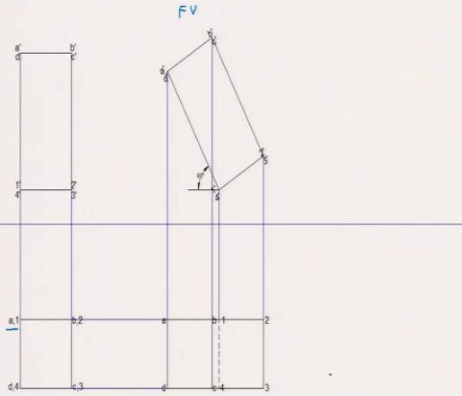
**Square prism**

Draw projections of square prism with one face  $60^\circ$  to HP and adjacent face at  $30^\circ$  to VP. Base side is 40 mm and height is 60 mm.

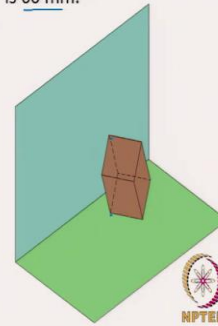
1. Top view  
2. Front view  
3. Turn front view about edge 14

## Square prism

Draw projections of square prism with one face  $60^\circ$  to HP and adjacent face at  $30^\circ$  to VP. Base side is 40 mm and height is 60 mm.

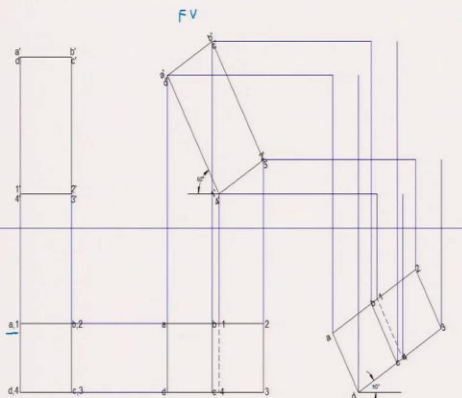


1. Top view
2. Front view
3. Turn front view about edge 14
4. Draw top view
5. Turn top view by  $30^\circ$

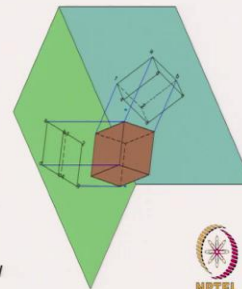


## Square prism

Draw projections of square prism with one face  $60^\circ$  to HP and adjacent face at  $30^\circ$  to VP. Base side is 40 mm and height is 60 mm.

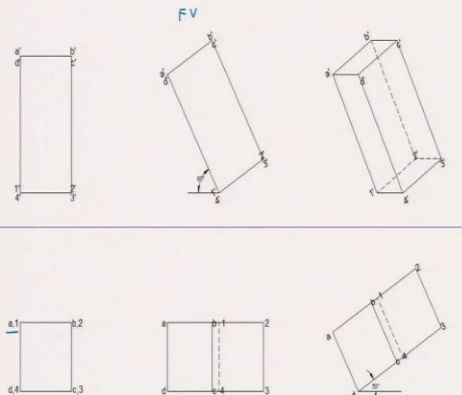


1. Top view
2. Front view
3. Turn front view about edge 14
4. Draw top view
5. Turn top view by  $30^\circ$
6. Draw front view

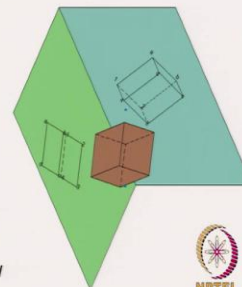


## Square prism

Draw projections of square prism with one face  $60^\circ$  to HP and adjacent face at  $30^\circ$  to VP. Base side is 40 mm and height is 60 mm.



1. Top view
2. Front view
3. Turn front view about edge 14
4. Draw top view
5. Turn top view by  $30^\circ$
6. Draw front view





Now, let us come to the final step, which is the actual question to be solved. So, now we say, after the single rotation, we do the second rotation. So, once you do the second rotation, will the front view shape remain the same, or the top view shape remain the same? So, that is a question to be thought, I can again show you.

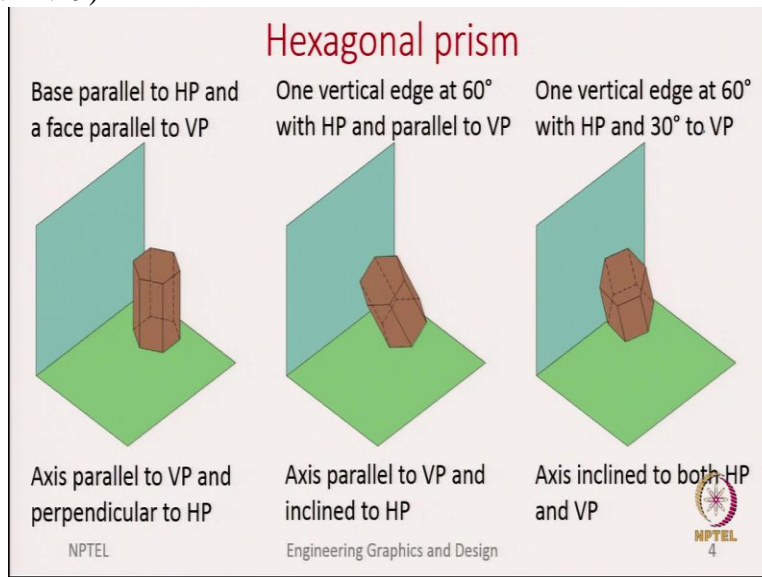
So, we are going from here before rotation to after rotation. Obviously, the front view does not be the same. So, what is remaining the same is the top view. So, I can go front and back here to make you convince that the top view shape remains the same. So, once you identify that the top view shape remains the same, that is where we start with, in the last.

So, we say that turn top view by 30 degrees. Because that is the angle which is given in the question. So, this is the angle 30 degrees, which is given. And what I am simply doing is trying to copy the top view from step 2 as a top view in step 3. The only difference is, I just need to make sure that this is rotated by an angle of 30 degrees. And last step is of course, you need to draw the front view but now it is manageable.

Because we can now project these points from the current top view and the front view of the previous turn. These are the projection lines from the previous front view and the current top view. So, once I identify the points, it is easier for me because I just need to connect those points to get the view. So, this is the view in the final step, the front view in the final step.

I hope now I am able to convince you all, that there is a systematic way, which you, if you follow, you will be able to capture the views of simple objects as well as complex objects. When it is not properly aligned with the projection planes. Let us say, if it is at an angle to the projection plane with one plane, or it is at an angle with both the projection planes. Next, we will follow the same procedure for the second example, which is hexagonal prism.

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Again, before going to projections, let us make sure that the visualization is correct. So, we will start with the properly aligned hexagon. So, here is the hexagon we have. So, I am saying the first step is we will make sure that it is properly align. So, let us say this is how you see the front view. And this is what I see in the top view. And, of course the axis is parallel to the vertical plane, and this axis is perpendicular to the horizontal plane.

So, in the next step, I am saying that I will tilt this hexagonal prism by  $90^\circ$ . So, as you can see, this rotation is becoming parallel to the vertical plane, before and after. So, now what happened to the axis now we have an axis parallel to the vertical plane, but at an angle with the horizontal

plane. And lastly, we are making an angle not just with the horizontal, but also with the vertical plane. So, what I will do is, I will rotate this.

So, when I am rotating this, if you are looking from the top, it is symmetric, I mean, which means the rotation is parallel with the horizontal plane. So, here is the middle picture, as you see on the slide, and this is after rotation. So, we will come back to the prism in the next slide. But we said instead of the axis, we can also define this prism using other features, like the base face or the edges. Now, let us look at it.

So, the first simple orientation we say can be defined with the base, saying that the base is parallel to the horizontal plane, and the face is parallel to the vertical plane. What he meant by is, one of the face let us say this face, is parallel with the vertical plane. And for the second orientation, we can say one vertical edge at 60 degrees. So, in this case, we are looking at this edge, this is the vertical edge.

We are saying that this makes an angle of 60 degrees with the horizontal plane. And this vertical edge is also parallel to the vertical plane. So, this edge makes an angle of so and so with the horizontal plane and parallel with the vertical plane. Finally, saying that one vertical edge makes an angle of 60 with horizontal plane and 30 with the vertical plane. So, we are defining the whole object using one single edge.

Initially this edge is vertical, afterwards it is inclined with horizontal plane. In the last orientation we are saying that, it is inclined at so and so, with the horizontal plane as well as with the vertical plane. I hope now, the visualization is clear about this hexagonal prism in the three different orientations. Because, in the next slide we will be looking at the projection of this prism in these three orientations.

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### Hexagonal prism

Draw projections of hexagonal prism with one vertical edge at 60° to HP and at 30° to VP. Base side is 40 mm and height is 60 mm.

1. Top view
2. Front view
3. Turn front view
4. Draw top view

### Hexagonal prism

<p>Base parallel to HP and a face parallel to VP</p> <p>Axis parallel to VP and perpendicular to HP</p> <p><small>NPTTEL</small></p>	<p>One vertical edge at 60° with HP and parallel to VP</p> <p>Axis parallel to VP and inclined to HP</p> <p><small>Engineering Graphics and Design</small></p>	<p>One vertical edge at 60° with HP and 30° to VP</p> <p>Axis inclined to both HP and VP</p> <p><small>NPTTEL</small></p>
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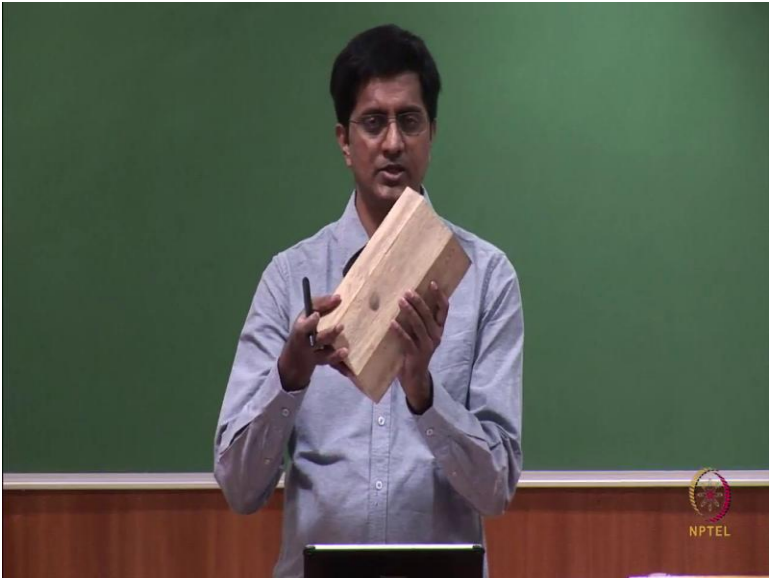
So, this is how we get to the, describe the problem. Saying that, we need to draw the projection of an hexagonal prism with one vertical edge at 60 degrees to horizontal plane and 30 degrees to vertical plane, side is 40 and height is 60. So, this is exactly what we have seen previously. So, we are talking, describing the last orientation. But we said instead of solving it directly, where it is too complicated to solve it, we will simplify it by making the, by starting from a very simple orientation and gradually coming to the final orientation.

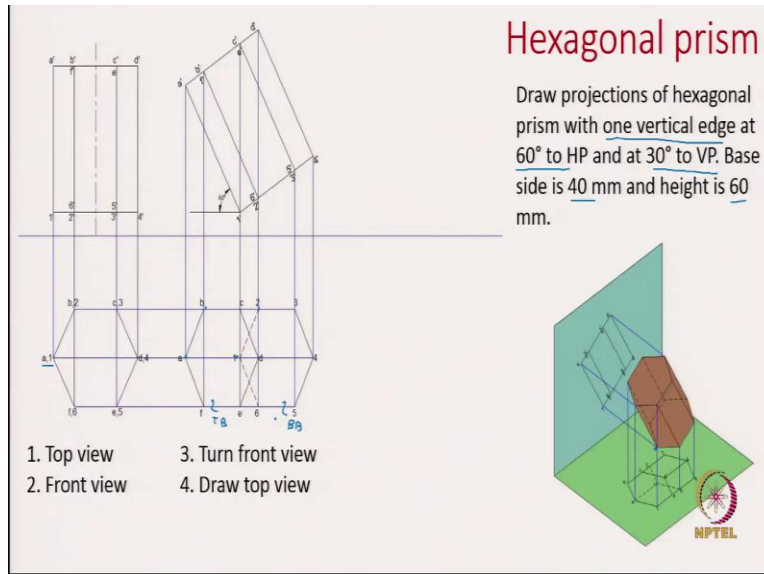
So, the first step is, of course, the simple orientation. And here, do we want to start with the front view or the top view? The answer is, we want to start with the top view. The reason being in the top view, what you see is a hexagon. Because the top base and the bottom base, both of them are parallel to the horizontal plane. So, first we captured the top view, and this is the top view. Again, the top base is described by these points a to f.

And similarly, the bottom base is described by the points 1 to 6. And then the sequence follows, a ,1. Because we first see the top base, and then the bottom base, if you follow the projector line. So, after capturing the top view, we can now use this information to draw the front view, because then we can use these projectors. So, we capture the front view. And to do that we use these projectors.

Now, we have captured both the top view and front view in the simple orientation. I think this is a more straightforward procedure. But now, comes the second orientation where we tilt it. So, we are now talking about this vertical edge, between these two points. And we say that, that makes an angle of 60 degrees with the horizontal plane. So, what do we capture now, the front view or the top view first? Which of them will be the same as in the previous step?

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So, again, I am bringing the object. So, this is the step 1, and this is step 2. So, from step 1 before rotation to after rotation, will the front view remain the same or the top view remain the same? We see that, this hexagon in the top view now becomes a squeezed hexagon. It is not the same, it does not retain the true shape. But, the front view before and after rotation retains the same shape. The only difference is, the shape is rotate.

So, now let us capture the front view, turn front view. So, this is how we captured the front view. And next, we follow the points both on the top and on both the bases, the top base and the bottom base. How do we do that? By projectors. So, first I will draw projectors from the current front view and then projectors from the previous top view. The intersection gives me the new points in the top view.

So, now what I am doing is, I get two squeezed hexagons. So, this corresponds to the top base, and this corresponds to the bottom base. So, first I capture the two hexagons. Of course, these are not the real hexagons, these are the squeezed hexagons, let us call it. So, once you captured these two hexagons, how do you make it a solid, we just need to join the corresponding points. So, for example, you just need to draw a line between a and 1, b and 2, similarly c and 3 and so on so forth. So, once you connect those lines, you get the complete solid.

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### Hexagonal prism

Draw projections of hexagonal prism with one vertical edge at  $60^\circ$  to HP and at  $30^\circ$  to VP. Base side is 40 mm and height is 60 mm.

1. Top view
2. Front view
3. Turn front view
4. Draw top view
5. Turn top view by  $30^\circ$
6. Draw front view

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### Hexagonal prism

Draw projections of hexagonal prism with one vertical edge at  $60^\circ$  to HP and at  $30^\circ$  to VP. Base side is 40 mm and height is 60 mm.

1. Top view
2. Front view
3. Turn front view
4. Draw top view
5. Turn top view by  $30^\circ$
6. Draw front view

NPTTEL

So, now we are started with a simple orientation, then we tilted it, I one angle with one plane. And then comes the final orientation, where it is inclined to both planes. How do we do that? We say, this is the step 1, step 2. And in the step 3, we rotate it. So, before and after rotation, will the top view or the front view remain the same?

So, I am not talking about the complete view, but the shape. So, if you notice, clearly, the top view retains the shape. Whereas the front view, there is a change in shape. So, we will start with the top view, for step 3. So, turn the top view by 30 degrees. So, if you compare the top view

from step 2, and step 3, the shape remains the same. The only difference is, now we are at an angle of 30 degrees.

So, once we capture the top view, last thing is to capture the front view. Now, it is becoming more easier, because now we know the procedure. We just need to follow the procedure by saying that, we will follow all these 12 points of the prism, by projectors. First, we will have projectors from the previous front view, and then projectors from the current top view. The intersections will give you the points in the final orientation.

So, once you join them, you first get the top base and then the bottom base, one needs to connect it with the corresponding points on both bases to get the solid. So finally, what after removing these projector lines, this is how it looks like in the final orientation, the front view and the top view. So, what we have achieved in this slide is we are able to solve a problem, which initially appeared more complex or more difficult. But then by following a systematic procedure, we are able to capture the views, systematically starting with a simple orientation and then inclined only with one plane and then finally inclined with both the planes. So, we are done with the second example.

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Draw the projections of a cone, base 40 mm diameter and axis 50 mm long, when it is resting on the ground on a point on its base circle with the axis making an angle of 30° with the H.P. and 45° with the V.P.

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Engg. Graphics and Design	Week 4	DATE: 17/06/2021





We are going to the last example, which is the cone. So, here we have a cone. The question says, we need to draw the projections of a cone. The base diameter is 40 mm. And the length of the, or the height of the cone is 50 mm. So, the axis is 50 millimeters long, in other words, the height is 50 mm. So, we need to do this when it is resting on the ground, on a point. So, it is not that the whole thing is resting on the ground.

It is resting on the ground, only at a point. But that can have many combinations, it can rest anyway. So, how is it resting? So, the base circle makes an angle  $\theta$  with the horizontal plane. So, the base circle makes an angle of 30 degrees with the horizontal plane. So, if this is the horizontal plane, it is making an angle of 30 degrees. And it is also making an angle of 45 degrees with the vertical plane.

So, which means currently, if I simply turn it in this fashion, it is still perpendicular to the vertical plane. But once I rotate it, then it makes an angle. So, it is still resting on the ground with one angle and the second angle. So, now let us see how do we represent it. We start with the reference line. And of course, again we will break it down into three steps, saying that the first step is simply oriented or oriented in a very simple manner.

Which is the base is parallel or same with the horizontal plane. And the axis is perpendicular to the horizontal plane and parallel to the vertical plane. So, for this it is easy to capture the front view and the top view. The top view will be circle, with diameter as the base diameter. And the

front view will be a triangle. The height of the triangle will be the height of the cone, and the width of this triangle will be the diameter of the base circle.

So, we start with the front view, then use projection lines, identify the center of the circle and then draw the top view. So, the next thing we said is we will rotate it. So, that the base makes an angle of 30 degrees with the horizontal, before rotation and after rotation. Again, the same question, so in which view is the shape being the same? The front view or the top view? So, in the front view you see a triangle. After rotation, what do you see?

You still see a triangle. However, in the top view, before rotation, you see a circle. But after rotation, what do you see? Let us only focus on the base circle. So, before rotation is a circle and after rotation, if you look from the top, since it is an inclined circle, what you actually see is an ellipse. So, we will start with the front view, because it is easy to capture the triangle, the shape being remaining the same.

First, we will draw a inclined line of 30 degrees. And once we draw this inclined line, it will be easy to draw this inclined triangle. So, this is the point about which it is being rested on. So, once we are done with the front view, then how do we do the top view? To do the top view, we know that first we will start with the base. And the base here is what? An ellipse, how do we draw an ellipse? We first, in the previous view where we had a circle, we divided it into 12 equal parts.

So, we had these points one going from 1. Of course 2, 3, 4, 7, 10, 11 and 12. Similarly, these points you project it onto the triangle, so that you identify these points. Now, by using these projection lines, both from the front view as well as in the top view of the previous step, we identify the intersection of those points. So, once we know the new points, we using freehand we join those 12 points on the circle to draw the ellipse first.

And then, identify where is the apex. So, for the apex you follow this vertical line. And similarly, from this horizontal line starting from o, and you identify the apex. So, once we know the apex, we just need to join it with two lines. Such that, it gives the appearance of a solid. First the ellipse, then the apex once you get the ellipse and the apex then you need these two incline lines to capture the complete cone.

So, now we are done with the step two with both the front and top views. Now, coming to the last view, saying that it is now inclined to the vertical plane. So, before rotation and after rotation, which shape will retain the same? Where will you have the true shape? Will it be in the front view or the top? So, if you follow the clearly, you will notice that the top view will retain the, same shape.

So, first let us draw the top view. Now we identified it is at an angle of 45 degrees. So, we first draw the ellipse. And then capture the, by the length of the height of the cone, we capture the apex. And once you capture the ellipse and the apex, you can draw these inclined lines to make, complete the cone. So, in the last step, we are done with the top view. Once the top view is done, the front view you can capture by again following the same procedure.

Where, all the twelve points on the ellipse are first traced, using projectors. And the intersections gives you the new shape. So, you first follow the ellipse and then the apex. And once you get that, connect the apex with the ellipse using the inclined lines. So, finally, you get the cone, both in the front view and the top view, in the final orientation. So, as you can see the procedure remains the same, no matter what your object is. The first two examples are examples of a prism, where the bottom base and the top base are the same. In this example, we have an example of a pyramid, where the base cross section decreases as you go along the height of the pyramid. So, it is a circular pyramid or in other words a cone.

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### Summary: Projection of solids

**Orientation of object**

- Axis parallel to one projection plane and perpendicular to other
- Axis parallel to one projection plane and inclined to other
- Axis inclined to both projection planes

**Examples**

- Projection of square prism
- Projection of hexagonal prism
- Projection of cone

NPTELEngineering Graphics and Design

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So, finally, what do we summarize? So, we say that the object orientation can be either parallel or perpendicular to the projection planes. In which case, drawing the projections become easy. The next difficult case, is the axis is parallel to one of the plane, but it is inclined to the other. And finally, when the axis is inclined to both horizontal and vertical planes. So, what we are saying is, this is gradually we are showing what is a simple orientation and a more complex or a more difficult orientation.

Where to get the projections becomes a little tedious, but it is achievable by following the procedures we discussed in the class. So, we also looked at three different examples, the projection of a square prism, hexagonal prism and a cone. So, we noticed that in all these examples, we followed the same step where we divided this problem into three simple steps saying the simple orientation, orientation with only one angle and the final, final orientation with two angles with the projection planes.

So, what we achieved in the last two lectures and today's lecture is, we are able to show how to define the position and orientation of a solid. We are also able to capture the projections of these solids, not only in the simple orientations but also in a more complicated or difficult orientations. So, the take home point is, to say that if you are able to deal with regular solids, the same principles can be extended to any real objects.

Because most of the real objects, as we notice, are composed of these simple or regular solids. So, in the next lectures, we will be looking at some practice problems, where we will be solving these examples on a paper with a pencil. So, that you too can follow and practice at least one or two examples. And then there are some other examples which you, you should be following it or practicing it by yourself.

Here again, I am repeating the same thing in this course, saying that this is a practical course. It is not just about the theory, but the more you practice, that is when you feel more comfortable with this material. So, I am encouraging you one more time to keep practicing it on a paper. The first couple of things will be a learning exercise, but afterwards you will be more comfortable with these procedures. Thank you for your attention.