Introduction to Uncertainty Analysis and Experimentation
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Module - 02
Error, Uncertainty
Lecture - 06
Sources of Errors. Uncertainty Definitions

Welcome to the next lecture of Introduction to Uncertainty Analysis and Experimentation. We are in the second module looking at Errors and Uncertainty. And, in this lecture we will look at Sources of Errors and Uncertainty Definitions.

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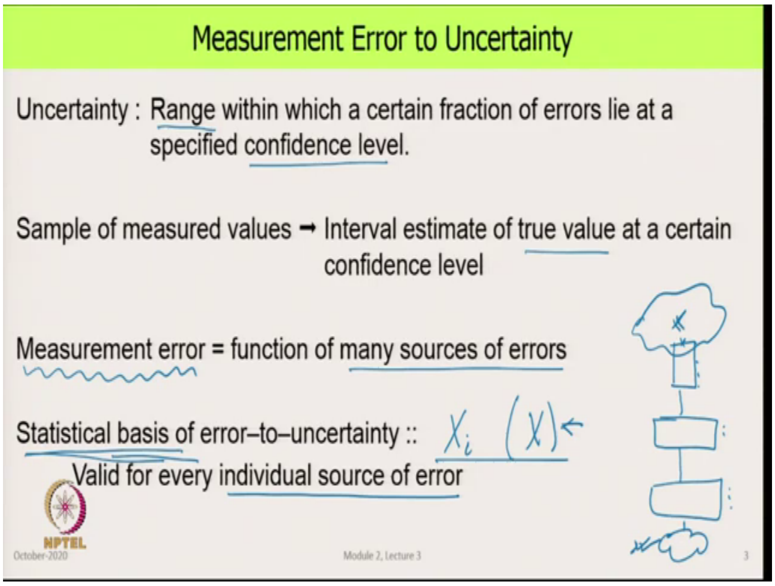

What we have seen so far is that every error has a measurement and we want to develop a process of converting the error into an uncertainty. In the previous lecture, we saw the statistical basis on which this can be done. So, we have a tool now to take data and then report the result or measurement in an uncertainty range. So, our definition of uncertainty is to recap as a range within which a certain fraction of errors lie at a specified confidence level.

We always get a sample of measured values typically one sample that is a single sample measurement and from there we can get the true the interval estimate of true value at a certain confidence level. Now, we ask the question, how do I get the measurement error. We will look at this part in the current lecture.

Begin with we say that the measurement error; that means, error in a measurement is a function of many error sources. And, what we mean here is that, the measurement technique which included say, the physical system that we are looking at, a sensor or an instrument that picks it up. Then there is some electronic signal processing on it and then there is storage or display.

So, all of these could be separate units or all of them could be in 1 unit that could be an instrument or there could be different physically different elements. But in going from here to coming out with a number over here, there are various things happening in this, in this, in this, because of which there is an error that is going through all of this. And, so, what we see here has an error compared to if this were to be measured exactly, which of course, is not possible.

So, how do we treat this? That is what we will look at next. The statistical basis that we have seen in the previous lecture, where we looked at any measure and X i which in statistics terms was just X all the treatment we learnt about it for only one particular measure angle. However, this is equally true for every individual source of error in the measurement or the result.

So, if there was an error introduced over here on some reason, that error would also be treated in the same way, within the same statistical basis that we have learnt in the previous lecture, this is an important thing. Whether that error came from data, from a random source, for a systematic source, we can still model it and look at it as it is statistical basis.

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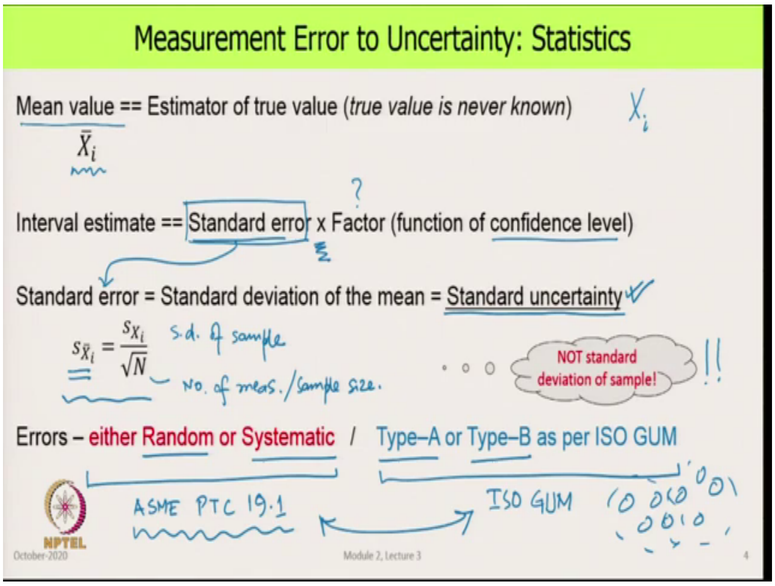

So, what we got so far is that from the sample, we got the mean value. The mean value is an estimator of the true value which of course, is never known and the symbol that we have is right here. X i bar, X i means this is the parameter that we are measuring. The interval estimate the plus minus part of it, this is the standard error multiplied by some factor, which is a function of the confidence level at, which we want the interval.

So, what is the standard error that is what brings us over here? The standard error is the standard deviation of the mean, which henceforth in an uncertainty analysis we call it the standard uncertainty, this word will come all the time.

So, this is important to know and this is the standard uncertainty which is standard deviation of the mean s X i bar, that is the mean which s X i upon square root N with the numerator is standard deviation of the sample, N is sample size or number of measurements in a sample.

This is an important thing to remember, that this part here the standard error is not the standard deviation of the sample, but standard deviation of the mean, this is a very important thing to remember, that this expression tells us that anyway. Now, if we say that there were lots of errors coming into a measurement, then we can classify them in two ways. First either as calling them random or systematic, this classification coming from ASME PTC 19.1.

We could do that or we can classify those very errors the same ones in a slightly different way, which is Type-A or Type-B this is as per ISO GUM. We have two options either to proceed on 19.1 definition basis or GUM. The answer at the end will be the same. We are only taking the same set of errors whatever they may have been say these in one case we classify them as something, in other place we classify them as something else, very small difference between them the treatment is very similar, we get pretty much the same answer.

So, in that sense there is not that much of a difference and we could use either of them. To maintain consistency in this course I have chosen to follow PTC 19.1 terminology, we will look at random and systematic errors.

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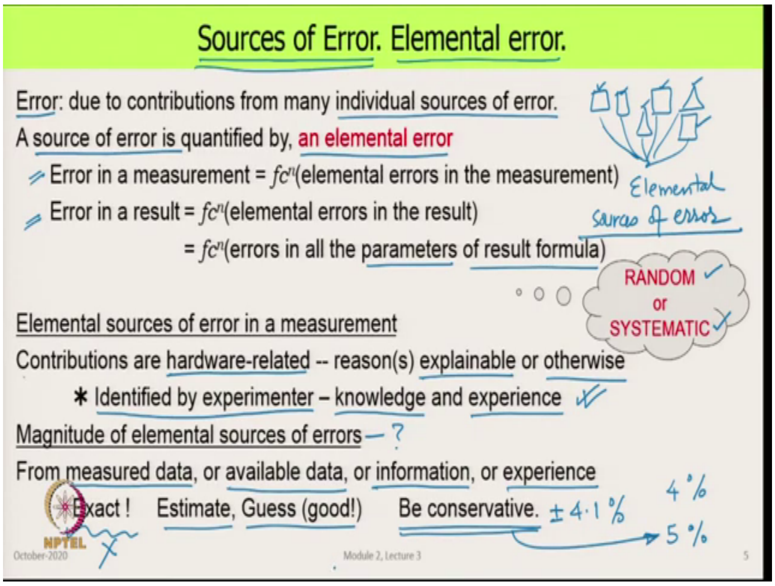

So, every individual reason why there is an error in a measurement that is called an elemental error. And, every factor that causes an error is called a source of error or sources of error. Now, this is something you have learnt in school. In every experiment or even in college many experiments at the end of the experiment you always wrote what are the sources of error and, you left it at that.

In this course we go beyond that, we tried to be more systematic more scientific and try to put a number on each one of them. So, we start with those very things sources of error, each source of error, we will call it as an elemental error source. So, that is what here we are put here.

A source of error is quantified by an elemental error. So, the total error we are now breaking it up and saying this has come from many individuals sources of error. And, all of them in some way contribute to the error in the measurement. And, now we are going to be looking at these which is what we call the elemental sources of error.

So, here is how we will look at this problem? That error in a measurement is a function of elemental errors in the measurement. What that function is we will learn about it. And, that error in the result is a function of the elemental errors in the result just like the measurement. Or alternately we can look at it also as function of errors in all the parameters individual parameters of the result formula.

So, there are two ways of looking at the result. Then as we have already said these errors, they could be either random or systematic, we have to classify them. Then, in every measurement the elemental sources of error where do they come from? It is that it is the hardware and the physical system and to some extent some of the software used, that is what causes the errors to come in.

We may know the reasons qualitatively, we may argue about those reasons from a physical reasoning, but how do we put a number or a quantity on them, that is what we are got to see. For some of them we may know the reasons, but we the theory may be too complicated to say, what is the actual quantum of that error? In some cases we know probably nothing about it, we know that there is an error happening. And, then we also say at best I can estimate that this is about this much, this is perfectly fine.

So, there could be errors which are explainable and we can put a number on it and we can do a cause effect analysis on it, but it is difficult to put a number or it may be that the error research is too complex we know that there is an error, but you do not know how to handle it? So, I say I will find some other way of putting it together.

So, two things are happening now we said we need to identify individual sources of error. So, we call the elemental sources of error. These have to be identified by the experimenter, you are doing the experiment, you are the one who is expected to know, what are the errors that are coming into this.

You do an experiment once, you may not get the feel for it, you do it many times you start getting the hang of it and say look this is what the problem is. These errors you can get a classification of them or a listing of these by knowledge of the process, knowledge of the instrument, knowledge of the physical process of the instrument of the transducer, and from experience.

It is a lot of hit and trial that goes on where things go right things go wrong, that tells you what is it that is happening and where is the error coming from. So, both of these are equally important. How do we estimate the magnitude of elemental sources of errors?

For this we have two or more options. One is we have measured data, that we were able to control all the other errors and see what is the effect of one error. And, say this is the amount of error that is introduced by this particular error in the measurement. This is not always possible, many of the errors are very intractable one cannot even do that.

Some errors can be measured so we will put that as measured errors. And, then we rely on some available data from previous workers, similar workers, elsewhere or from literature and use that as an estimator. It is not the exact answer, but it tells us where the magnitude of error is likely to lie.

And, then information, information about the instrument, information about the electronics, information about the circuitry, all of that will come in and that tells us you know you can get the error estimate from that and of course, lastly from experience.

If, you have been doing an experiment for many years, maybe decades, you have a very good idea how good that setup is. So, there are various ways by which we can put a magnitude on the elemental sources of errors. We will learn about it in one of the later modules in the course.

The point to note is that these are all largely estimates and a very large number of decimal places do not make sense in this, which also tells us that saying that this is the exact error, the exact magnitude of the error just does not make sense. We are quite happy to work with estimates that is what it is.

Estimate is something which tells you approximately this is where it is, but with a reasonable tight range that is what it is. We could be quite happy with having a good guess a lot of engineering is good guesswork actually.

And, finally, if you have to report an error, I have written here that we be conservative. Which means that, if we think that the error was 4.13 4.1 percent plus minus and you have to decide, whether I should report 4 percent 4.5 percent, or 5 percent. And you have two options and somebody says I will report 4 percent, and somebody else says I will report 5 percent.

Now, the implication of this is that for the user who was using this piece of information. If, they think it is 4 percent they will make certain decisions based on this value, not knowing that there is a some chance that it could still be at 5 percent. So, there would be some instances where they could have a problem.

On the other hand, if we reported 5 percent, then this person who worked it and did the next steps of the science or engineering or experiment based on 5 percent, would always be conservative in that the errors will not exceed 5 percent.

So, that is what we mean by being conservative and a good thumb rule in uncertainty analysis is be conservative higher value of uncertainty does no harm, a lower value of uncertainty could lead to a problem. So, that is some of the broad features of putting a magnitude on the elemental sources of error.

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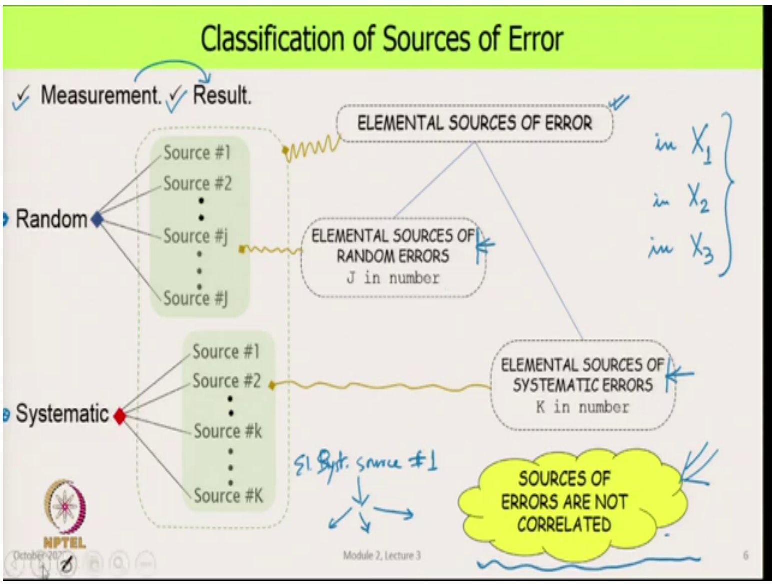

Now, we look at the classification and in this slide I put up various things. And, we say that how can I categorize the errors. This is important because after we categorize it, we will pick one error at a time and find a method to calculate it is magnitude, or estimate it is magnitude. After, we do that for all of them we will go back and compute the error in the measurement and then in the result.

So, this classification here is equally good for a measurement or any measurement in that experiment or in the result formula and also for the result itself, which as we know comes from a mathematical operation on the measurements. So, the two broad categories of errors, we have already seen one is random errors; the other is systematic errors.

Random errors themselves could be many elementary sources of random errors and this we will call as source number 1, source number 2, source number j and so on until source number capital J. So, all these are elemental sources of random error that is what it is here.

Elemental sources of random error and these we say a J in number, which is first source of elemental random error, second source of elemental random error so on and finally, the Jth source of elemental random error. What it tells us is that with if it was a measurement x i, then x i has error, which is coming it is a random error, which is coming from this particular source of error.

So, that is how this whole name comes in. Same thing happens with systematic errors. The source number 1 of systematic error, source number 2 of systematic error and each one of these is an elemental error source and like this we have K numbers of elemental systematic error sources. And, that is what they listed here, that this particular box is elemental sources of systematic errors, which are K in number.

So, K is an integer, J is an integer. And, both these put together, they comprise the superset of elemental sources of error. If, we are looking at a measurement this would be elemental sources of error in a measurement. Typically, that is what we will do. So, if we do it for measurement X 1, which could be some say temperature measurement, then in the experiment you are measuring also flow rate that is parameter X 2, it will have the whole thing done all over again for it.

If, there is a third measurement being done of some dimension, this would be X 3 for that particular measurement we will do all this exercise all over again. Which means as many type measurements or parameters are there in the experiment that many times we have to repeat this whole exercise. And, all of that we are doing only to get a method to identify the errors and then finally, put a magnitude on them.

So, this is what it is how we and what we will put a number on is these things. We will take one source of error at a time, one source of elemental error at a time, starting with say random error and say this is the standard uncertainty with it. The next source of error, which is a random error and put a magnitude on that, after we have exhausted all the random errors we go to the systematic errors we do same thing on that. Then, we have all the elemental errors for that measurement.

So, that is what this picture is showing us, but in writing this in this course, one very important assumption that we are making in all of this is that these sources of errors are not correlated. This is very important. This means, that one error source, say systematic elemental, systematic source number 1, does not affect any of the other error sources systematic or random and vice versa.

No error source affects the error magnitude in any other error source. Put another way every error source is independent. So, this is a simplifying assumption we have made in this course and for good reason this simplifies the mathematics, the treatment of the uncertainty analysis and still gives us a handle on a lot of issues.

However, in many practical situations this may not be justifiable. We would then need the theory of correlated errors, which we do not do in this level one course, you would have to do a higher level course in uncertainty analysis to see how to get correlated errors. For now, we will stick with the fact that all the elemental sources of error that we have looked at they do not influence each other.

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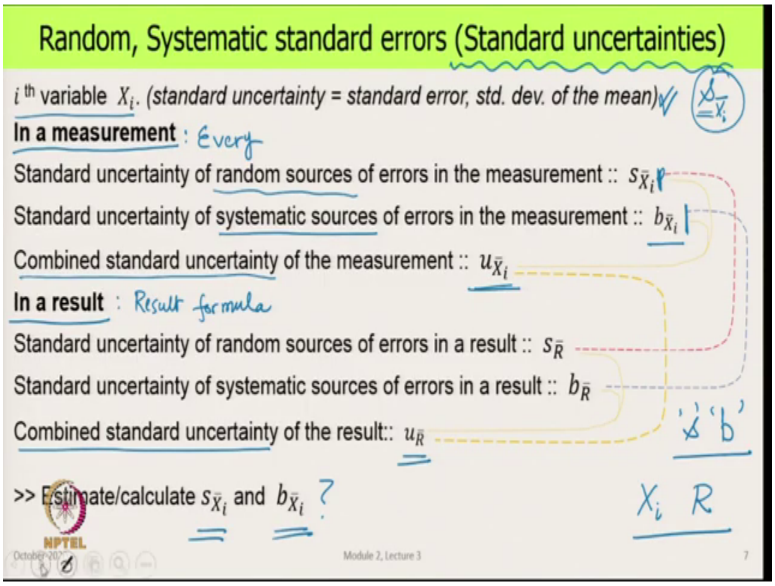

Now, we start looking at how to put symbols on things and what terms do we use? And, we start with random and systematic standard errors, which henceforth now we will call them as standard uncertainties. Now, remember what we got it from statistics standard uncertainties was the standard deviation of the mean; that is what it was.

And, we have used the symbol s in the context of statistics to denote the standard deviation and we said that in X i variable, this was the standard deviation of the mean, which we shall henceforth call the standard uncertainty. So, for each variable say the i th variable X i is the standard uncertainty is the standard error which is standard deviation of the mean, we just repeated that.

In a measurement, we will have the standard uncertainty of random sources of error in the measurement as s X i bar, this is a symbol we use. That means, the standard uncertainty due to all random sources in the measurement all adds up to give us a value, which is over here this one.

Similarly, all sources of systematic errors when put together their standard errors put together, give us the standard uncertainty due to systematic errors of the systematic standard uncertainty that is what we will call it. And, here we use a different symbol. Instead of calling this s, we now call it b just to differentiate that s, henceforth denotes random errors and b denotes systematic errors.

So, now we are deviating from what we saw the symbol systems in statistics, where s was in general used as the standard deviation. Here, we are now saying henceforth we will not follow that norm of statistics. We will say s of X i bar or s in general has to do with the random sources of errors, the standard error. And, b deals with the standard error associated with systematic uncertainties. This is another important thing coming up of what is mean by s and what we mean by b, one is random, other is for systematic.

And, when we combine these two, what we will get is what we call as the combined standard uncertainty of the measurement and this we give with the symbol u.

And, the subscript which is the parameter of interest X i is the parameter of interest as many X i's as many s X i's b xi's u xi's that what it is. So, that is all for a measurement, and for every measurement. So, this is what we are writing once has to be done that many number of times or as many measurements are there in the result formula.

Then, we look at a result, which as we know came from the result formula. And, here we say standard uncertainty of random sources of errors in the result is s R bar v N for denote result by R.

So, we are denoting X i as the parameters R as a result, yet another set symbols we will use and consistently use it in the whole course. Then, standard uncertainty of systematic sources of errors in the result b R following this s b classification, 's' random 'b' systematic. And, combination of both gives the combined standard uncertainty of the result u R bar.

So, all of them have the symbol bar, we are very clear that this is standard error of the mean, that is what we are looking at as the standard error that is where that bar is coming. So, now we go one step back and say how do I calculate these two numbers s X i bar and b X i bar? Then, we will see how to get sr bar br bar. So, that is our next step s X i bar and b X i bar.

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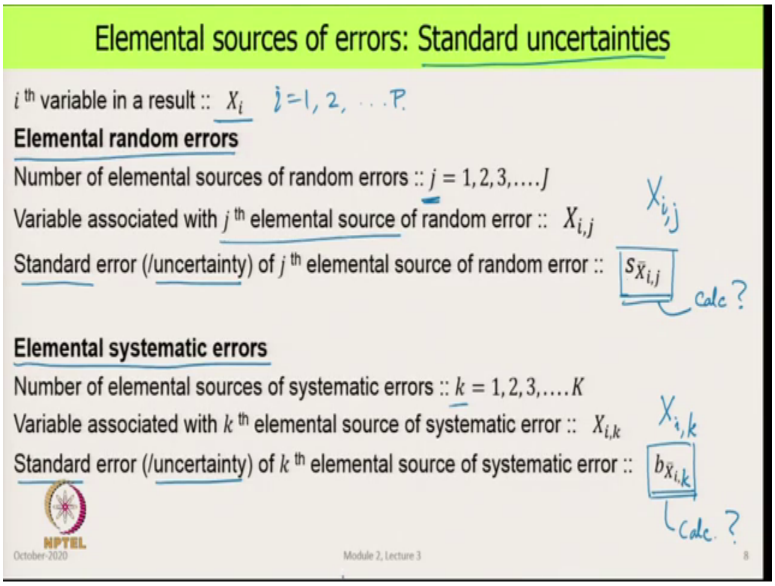

So, to do that, we look at the measurement and start looking at all the elemental sources of errors. And, say, I will calculate the standard uncertainty associated with each elemental source of error. We are looking at the i th variable X i as before i equal to 1, 2 as many parameters are there in the experiment.

Elemental random errors the first thing let us look at it, which means that which are those error sources, which are random in nature. And, we say number of elemental sources of random errors, we will denote it by the symbol small j, which takes values of 1, 2, 3 going up to capital J.

And, then the variable associated with the j th elemental sources of error, we will call it as X ij. X i means, this is X i is the parameter that we are looking at and j tells you j will be 1, 2, 3, 4, 5, it tells you that we are looking at the random error source for X i.

If, there are two sources of random errors we will have X i 1, X i 2 like that and then we will denote the standard error or uncertainties the standard uncertainty, of the jth elemental source of random error as X s of X bar i comma j. This is the number we will want to calculate and say how do I get a value for this? But, the symbol is this is telling you is the standard error of the standard uncertainty in parameter X i and is the jth source of elemental random error.

So, that is one thing we get, then we do the same thing with elemental systematic sources of error. Number of elemental sources of systemic errors we denote by the lowercase k equal to 1, 2, 3 all the way to capital K. The variable associated with the k th element error we call it X i k. K is only to distinguish in the writing of equation analysis, that k is telling us this is a systematic source of error, j will tell us it is a random source.

So, X i j, X i k together of course, they form the total basket of elemental errors in that measurement. And, then the standard uncertainty due to the k th elemental source of systematic error is to have s this now becomes b. So, now that is a job that we have this and this, how do I calculate this and get a number on these, this is what we want to do?

We cannot go any lower than this these are the elemental sources of error. So, we calculate all the s X ijs, all the b X ijs. So, this will be b X ks and then we club all of them together and get that standard error in the standard uncertainty in the measurement. So, that is what all of this effort is for.

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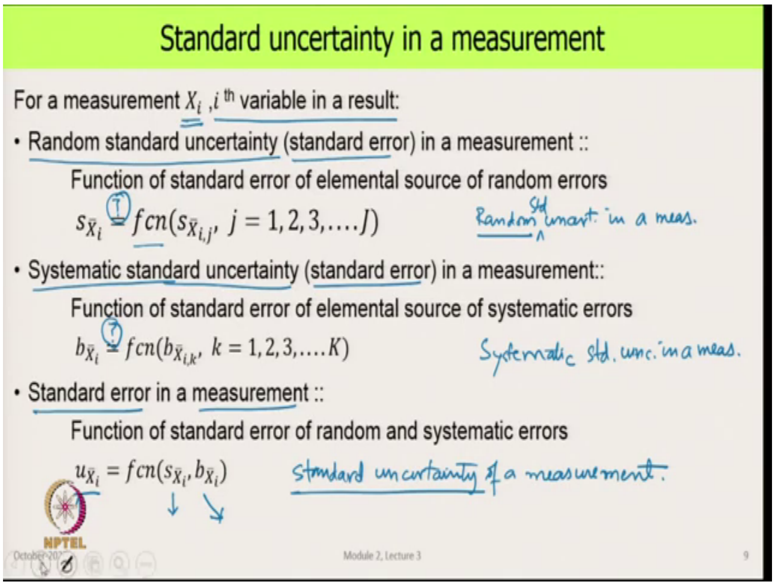

So, here we are standard uncertainty in a measurement, again we are looking at the ith variable in the result, we repeat it for every parameter. The random standard uncertainty in the result or the standard error in a measurement, this is the function of the standard error of the elemental sources of random errors.

That means, this s X i bar, which is the random error in a measurement, random uncertainty in a measurement. And, now when we say these numbers there are number actual numbers on these. This is a function of the elemental sources of random errors, s X i bar ij j 1, 2, 3, 4 say all this way we are denoting it for the random, elemental random sources of error.

What is this function? This is another thing we want to know, we will get this later on. So, that is a question mark. Same thing with the systematic standard uncertainty, we ask what is the standard error or the systematic standard error in a measurement; this was a random uncertainty in the measurement. Now, we ask, what is the systematic standard uncertainty in a measurement. And, like before this is also a function of the elemental sources of systematic errors of systematic uncertainties.

So, the symbol we had b X i bar this is a function of the b X i ks the elemental sources of systematic uncertainty, for every source of systematic uncertainty in the measurement. What is this equation that is also we want to learn. And, when we combine it and set now tell me, what is the standard error in this measurement for this we have the symbol u X i bar. So, u tells you that this is the standard error in a measurement, this is a function of the other two one's, the random standard random uncertainty and the standard systematic uncertainty.

So, combining these two we get the uncertainty in the measurement. So, this is the standard error in the measurement or the standard uncertainty in a measurement, this completes half a story. That we have reached the point where we said, we have so, many parameters that we are measuring, we have done this whole exercise, and now I know the uncertainty or rather the standard uncertainty in every measurement.

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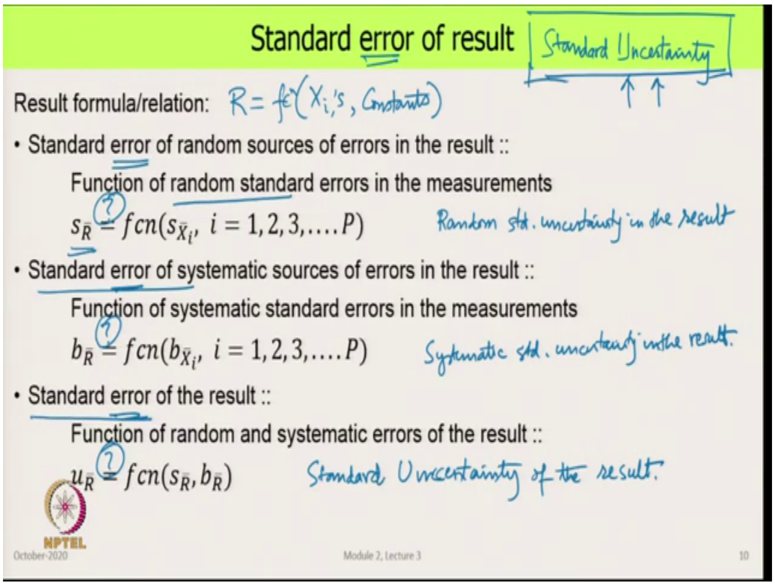

This makes us ready to go to the next step, which is getting the standard error of the result or standard uncertainty in the result. So, the result formula relation that we use this we have said that, R is a function of all the X i s and various constants.

Using that formula, we calculate the random standard uncertainty in the measurements and denote this by s R bar. So, this is the random standard uncertainty in the result. Similarly, we get the systematic sources of error and b R bar is like earlier systematic, standard uncertainty this time it is in the result.

So, these two get calculated and then when we combine them, we get the standard error of the standard uncertainty in the result, which is a function of random and systematic uncertainties of the result. And, this is the standard uncertainty of the result.

So, we want to know all these functions. And, we move on from talking of error to now we talk of all the time the standard uncertainty. Error we will use for defining the elemental sources of error, but all calculations thereafter everywhere, we will be calculating and working with the standard uncertainty which as we have defined the standard deviation of the mean, or the standard error from statistics.

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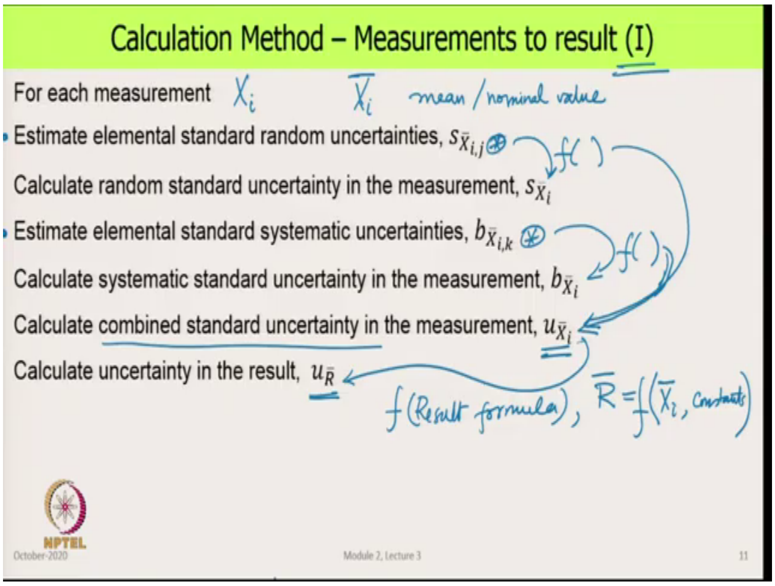

So, this is another major step we have taken we know, how we are going to proceed. And, we can now summarize and say, what is the calculation procedure for all of this. So, everything that we have looked at in the last few minutes, I have summarized it here. So, what do we do? For each measurement, we first calculate the mean, mean value or the nominal value.

Now, we want to calculate the standard error in it. And, for that we go identify all sources of error classify them as random and systematic, for each random error we estimate the elemental standard random uncertainty, which is this one. We calculate the random standard uncertainty using all of these put together, we calculate this some sort of a function between all of them; we estimate.

So, these are our fundamental things. We do the same thing with the systematic uncertainties get the standard error the elemental standard systematic uncertainty and using this we then take a function, and calculate the systematic standard uncertainty in the measurement. And, then we combine these two this comes from here, this joins it here, and these two together give us the combined standard uncertainty in the measurement u X i bar.

And, using this we can go to the result formula, and from there again some function involving the result formula, we calculate this and before that we calculate the mean value of the result. The mean value of the result is nothing, but the result formula calculated at every mean value of the parameters.

So, this is one way it completes the entire story of how to get uncertainty in one result, how to get uncertainty in any measurement that we are interested in. And, using this we then move on to the next step of doing various hypothesis testing, checking of goodness, regressions, whatever else. So, this is one method.

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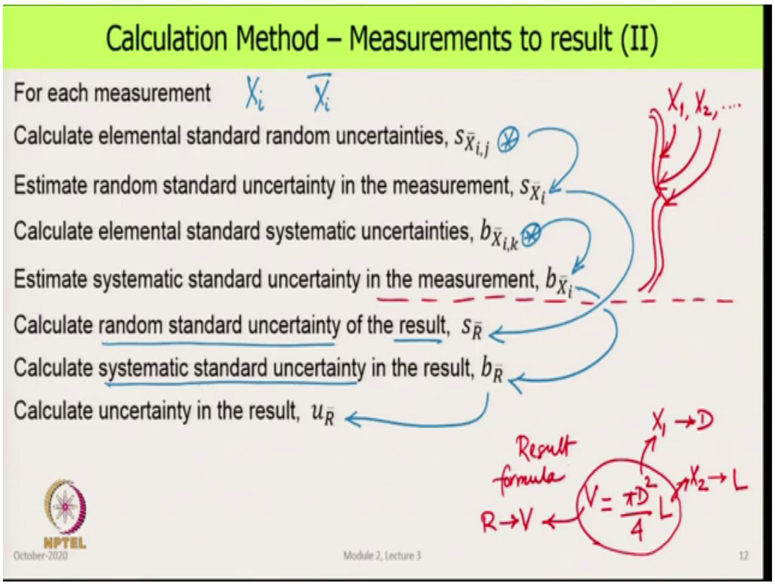

A slightly different variant of this is what we call method number II, therefore; again each parameter X i we first get it is mean value. And, like before we first get the elemental standard random uncertainties, which are these combine all of these we get this get the elemental standard systematic uncertainties, which is over here from that we get this and then we do slightly different thing here now.

We calculate the random standard uncertainty of the result; s R bar which comes from all of these. Then, we calculate the systematic standard uncertainty in the result, which comes from these. And, then we combine these two to get the uncertainty in the result. So, what do we have? Until this point, for this part, the treatment of both methods is the same. And, what we are doing is, we are taking each measurement X 1, X 2, and so on repeating these 4 steps for each one of them.

So, for this we report do all of this then for X 2 we do all of this, X 3 we do all of this until all the parameters have been taken care of. In a simple experiment you may have saved just one parameter, you start to do it for X 1. If, there are 2, let us do it for X 1 and X 2.

To give an example of 2 parameters you say, volume of a cylinder is pi D square upon 4 times L. And, so, are asking the question what I have a cylinder, I am going to measure diameter, I will measure the length and I want to know what is the volume and what is the uncertainty on the volume.

So, in this case our X 1 is D, X 2 is L and we repeat all of these steps that we have just written here. And, then either use method 1 or use method 2 to come for this and this is our result formula, this is our result parameter. So, our R gets mapped into V. And, this whole thing this is our result formula, that is an example of a two parameter measurement.

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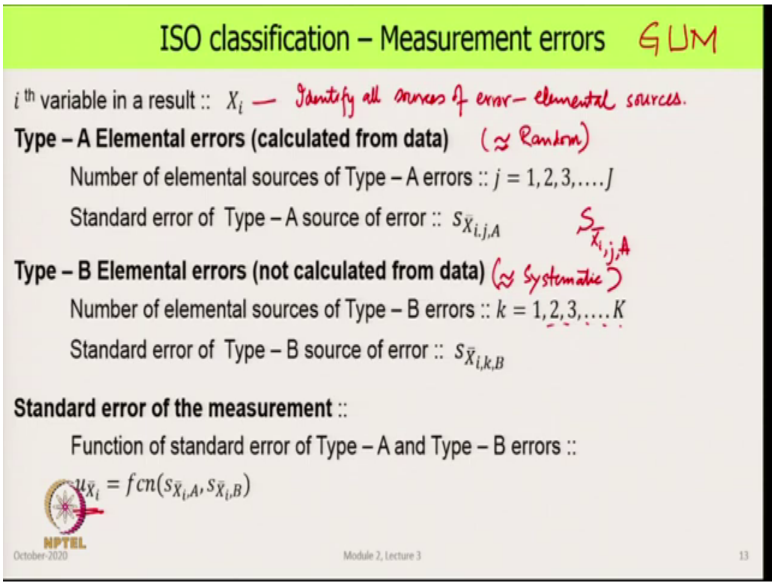

All of that was as per a semi PTC 19.1 we will take a few minutes and just talk about ISO classification and this is as per the gum. So, what we do is the same thing in the first step. For each X i we identify all sources of error, the other elemental sources of error. And, then we do a thing which is slightly different, we classify them as Type-A elemental errors or Type-B elemental errors.

Type-A are the ones that we will calculate from data. Typically, these are likely to be the random errors with few exceptions. So, what we called in PTC SME classification as random here we call it type A. And, these again would have many types. So, that is j 1, 2, 3, all the way to J and we get the standard error for each type of each subtype of type A error which is X i j A.

So, s means this is we are dealing with the database error X i bar tells us that this is a parameter that we are looking at, this is a j th source of type a error and we are thus telling that this is the state explicitly saying this is Type-A errors. Type-B then other errors, which are not based on data, but we calculate from other information including physical behaviour of the system.

These are Type-B elemental errors and you said these would be largely classified as systematic errors. This would be K in number and like before here we call this s X i k B. And, then these are combined together to get the standard error of the measurement u X i bar which is the same symbol as before.

Now, it is a function of the Type-A standard errors and Type-B standard errors. So, that is the difference with the earlier method. This is just to complete the discussion that this is how the other method would also do, in practice you would come across people reporting errors in both ways. So, you should be able to interpret that when they say type error what is it that they are implying. So, that tells us so, that is the complete picture of all the processes.

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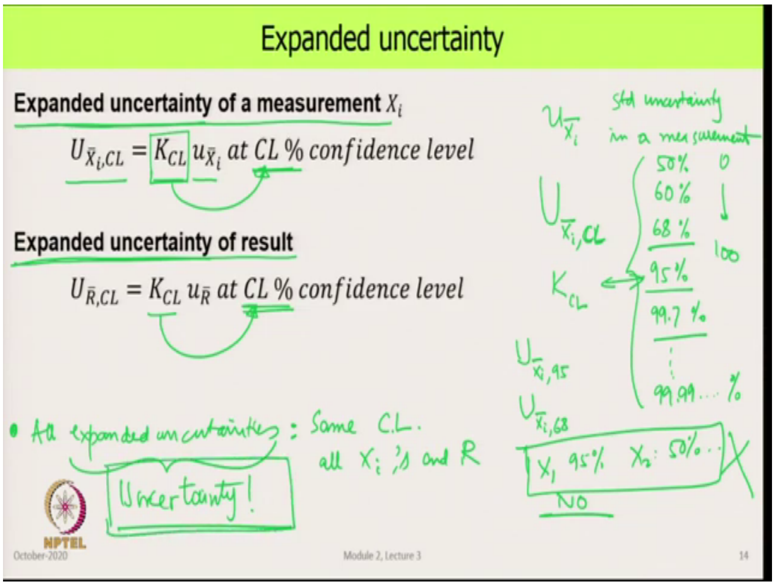

Now, we come to something else, which is how do I am I happy with the standard error value. So, what we had was u X i bar, the standard uncertainty in each measurement that gives us some idea.

But, it is not the final answer, but we are very close to the final answer, because what we are going to report as a result, or use it in a plot, or make a correlation, or use it for making hypothesis testing is what is called the expanded uncertainty of a measurement, a measurement X i.

And, the symbol for that is capital U and we are denoting the parameter, but we have added one more thing here which is CL. And, in a minute this will become clear when we see the right hand side of this equation, that this is K CL this is a constant, multiplied by the standard uncertainty in the measurement U X i bar.

Where this constant depends on the chosen confidence level, confidence level could be 60 percent, 50 percent, any value theoretically you can go any number between 0 and 100. Most common ones we have is 60, 67, 68 percent 95 percent 99.7 percent and then you could go 99.99 something something something percent.

Purely as a statistics exercise there is absolutely no restriction on what these could take, as we have seen in an earlier lecture, in engineering, uncertainty analysis, we stick with 95 percent confidence level generally sometimes we may go here, sometimes we may go there. But, once we select what this confidence level is each one of them has a one to one correspondence with K CL. It is there in the tables is there in the on the web many places, you can just get it in minute now.

So, that is why on the left side this one we have CL. So, what will happen is that, U X i at 95 percent confidence level will be different from U X i at 68 percent confidence level. So, we have to distinguish between them and that is why this additional qualifier has come in this formula. The same happens with the expanded uncertainty of the result this is capital U R bar and CL is the confidence level this is K CL at CL percentage confidence level.

Now, there is one important thing that, one should be consistent with is that in one particular experiment all uncertainties, all expanded uncertainties, this should be always act the same confidence level. That means all X i s and the result should all be reported at the same confidence level.

It could be 95 percent, 99 percent, 68 percent, 50 percent, it does not matter, but you cannot have X 1 being reported at 95 percent, X 2 being reported at 50 percent, this is not done, that is one thing. And, in terms of terminology the full rigorous term is the expanded uncertainty.

But, this is what we practically mean when we report uncertainty. The moment somebody says uncertainty in the result is this much, you know they are talking of the expanded uncertainty, whether it is the result or the measurement that is what being referred to then immediately you have to ask have they specified the confidence level, if not you say is it good enough to say that it is at 95 percent confidence level.

So, this is it what the whole course is about this how to get the uncertainty.

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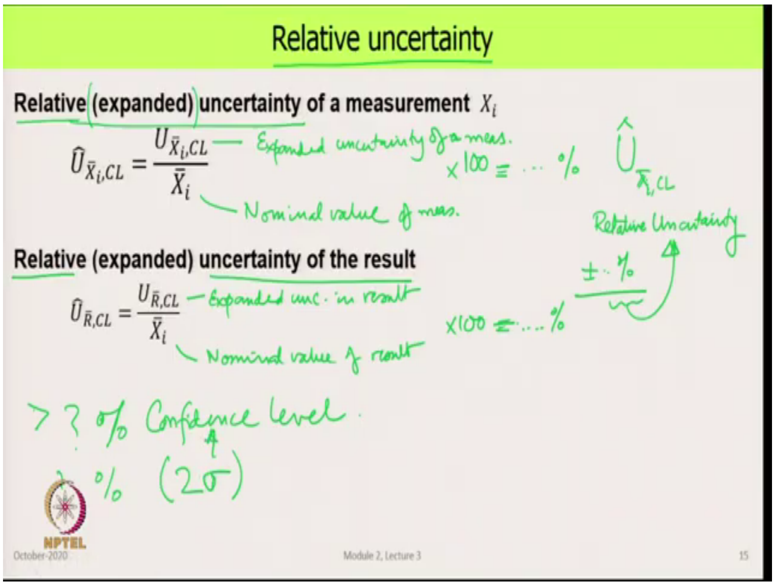

There is one more variant which you often find reported and often makes a lot of sense and a very quick feel for things, which is called the relative uncertainty. The relative or expanded uncertainty of a measurement, they are all relative expanded uncertainty or just the relative uncertainty, we may or may not use this word is the expanded uncertainty, divided by the mean or the nominal value of the parameter.

And, usually this is multiplied by 100 and expressed as a percentage. And, the symbol for that we argue X i bar L CL for the expanded uncertainty we have added one hat on top of it. So, U X i bar CL with this is your relative uncertainty of a measurement. The term the word expanded is quite often dropped and you will see very often in literature people are reporting the relative uncertainty.

Sometimes they may say that the uncertainty is plus minus so, much percent, but by immediately looking at this you would know that, because the percentage has been specified, it is being reported what is being reported is a relative uncertainty. And, then we have this was for a measurement the same thing is the relative uncertainty of the result, U hat R bar CL is U R CL by X i, which is the expanded uncertainty in the result divided by the nominal value of the result.

This was for a measurement, this is for a result, again as before you can multiply this by 100 and express this as a percentage, and because these are dealing with the expanded uncertainties all of these have to be reported at what confidence level. An alternate way that some people may have reported it this or the expanded uncertainty itself instead of putting a percentage confidence level, they may just say this is so, much percentage at in brackets say 2 sigma.

So, there will be some specifications of instruments and devices where they may put this type of nomenclature, we know now what it means. That this is telling you what is the confidence level it is being reported at. And, it assumes that this is on a Gaussian distribution.

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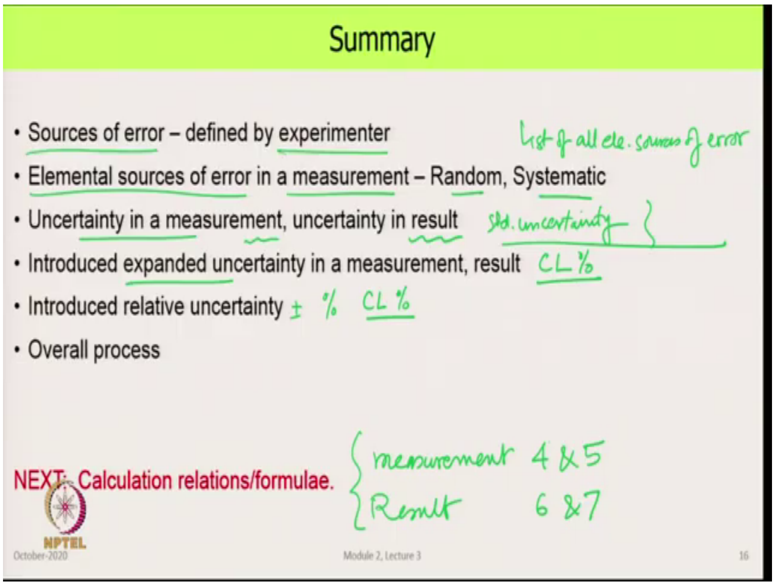

So, with that we have got all our definitions in place and we have a very broad idea of what is it that we want to do in the rest of this course. And, so, here we just summarize that, that we defined sources of error and that is the job of the experimenter. So, you have to know your device your instrument of what they do, what they do not do, why they give errors?

Then for each measurement we make a list of elemental sources of error, which is all those factors that you think cause an error, at this point it should be a super list, a superset. Later on we may not be able to quantify some of them, later on we may argue that some of them are really really very very small we can ignore them, but at this point we make a list of all elemental sources of error.

And, then we go about and say which of these are random, which of these are systematic, then for each of these elemental random and elemental systematic sources, we calculate the standard error associated with them. From that, we get the uncertainty in a measurement which is the standard error of the measurement, either in the measurement or the result and remember up to this point, there is no issue of confidence level.

Then, we come to the next step which is the expanded uncertainty and this is where we have to decide at what confidence level we report and finally, we can even report it as a relative uncertainty against plus minus so, much percent at a certain confidence level this is there. So, this is our overall process.

Our next step is we will look at relations or formulae for calculating element uncertainties and then from there how do I get the uncertainty in a measurement first and then in a result. So, this is weeks 6 and 7, this is weeks 4 and 5, and before we come to that in the next module we will look at this whole idea of what is experimentation.

So, on that note we conclude this lecture.

Thank you.