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Module - 02 Error, Uncertainty Lecture - 04 Errors, their causes and classification

Welcome to this lecture in the course Introduction to Uncertainty Analysis and Experimentation. Today, we will start exploring errors their causes and classification. As a part of module 2 in which we will establish the methods of Error and Uncertainty determination in an earlier lecture.

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We had defined the result in an context of uncertainty analysis as a function of constants and parameters. Parameters are denoted by X subscript i and there could be 1, 2, 3, 4 P number of parameters. So, it is a function of constants and parameters. The constants have no uncertainty associated with them the parameters are measured and each one of them has an individual uncertainty associated with it.

And our objective is to give an answer, which is the result as a nominal value which is the first part of this expression, plus minus the uncertainty of the result at a certain confidence level. This is the end point of this uncertainty analysis using this data, we can do many other operations, but for now as far as uncertainty analysis goes this is where we will be stopping that is our objective.

So, we have to determine two things. The nominal value of the result is a function of the constants and the parameters, and in this result formula we represent X i and substitute that by the nominal values of each of the parameters. So, whatever was the mean or average values of each parameter, we put it in the formula do a calculation and the number that comes out is the nominal value of the result.

This we have done very often in our school and college experiments and that is where we are generally stopped. In this course we go one step further and say, now I have to establish the result uncertainty for that I will get the result standard uncertainty, multiplied by it some factor which depends on the confidence level and that will give us the result uncertainty.

So, what we have to calculate is the result standard uncertainty of the result, or the result standard uncertainty, what it means we will come to that in a little while. And, this is a function a different type of a function not the same one that we had here of the constants the mean values of the parameters.

And the standard uncertainties in each of the parameters all evaluated at the nominal values of the parameter X i. So, what we want is how do I get the nominal values of the parameters?

That is question number 1 and second how do I establish the standard uncertainties in each parameter?

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So, this is our big picture this is what we want to do and so, for that we start by saying what is the methodology for measurement uncertainty, or the measurement error. Once we have the measurement uncertainties and the mean values in place, as we saw in the earlier slide we can go back to two formulas and get the answer that we are looking for.

So, what we want to do now? Is for each measurement we want to calculate the mean value of each parameter, which we shall also call at measurand and we need to calculate the standard uncertainty in each parameter.

And for this we use two pieces of information one is data that came from readings and observations, when we do the experiment. And second information or data about the measurement technique, which includes the instrument or the sensor and the sensor and the electronics that goes with it and the data acquisition process. So, what we are saying is that we have all these things together, we have not calling it an instrument although sometimes we will say that this is an instrument.

But, to differentiate it there are different elements in that instrument that is what we call it as the measurement technique, that this whole thing is a measurement technique comprising of the instrument, some electronics and data acquisition and initially there is the sensor. All of this is packaged in many cases and comes as a single device that to us in some way is the instrument or what we call as the measurement technique.

So, that is the two pieces of information we have, and the methodology for doing that is that we go back to using statistics on these above two pieces of information this and this. And using various relations, we estimate the standard error and the standard uncertainty in every measurement instrument combination.

That means, in each measurement that what we will call or the measurand or the parameter, we estimate the standard error rate. So, that is what we want now, what are the formula the relations and their scientific basis by which we can take this data over here and get, what we want over there. So, what is that statistics and mathematics that helps us to do that.

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So, to begin with let us look at something very fundamental which is causing this whole business of uncertainty, we touched upon it in the earlier lecture, and said that error is defined as a measured value minus the exact value, that is something that is very basic.

The difficulty arises because the exact or the true value is never known. We always are trying to estimate it and trying to get a number, which is a good indicator of the estimate of that value. The true value, but the true value itself is never known this is fact number 1.

And, because of that since this value in the second part here this is not known. So, the error also is never known the only thing we have in this is what we measured from the instrument. The same is true of the result that we may have calculated a result. So, we have a measured value of the result, but we do not know the exact value of the result. So, we do not know the error in the result, this is where we are.

And then, even more fundamental thing is that we talked about this error because, every measurement or observation or reading is always in error. This is a very very fundamental thing which drives the whole science of uncertainty, which also tells us every result is in error. So, what does it mean? That we are trying to we cannot get the error.

So, we will accepts that this is something inexact that we have, we will represent it by an uncertainty which again by definition cannot be exact and so, we will express it as a range. That range is what we call an interval estimate and that range that we specified depends on the confidence level that we have chosen here. So, you specify a confidence level some value, we will get a certain range this is our uncertainty.

You specify some other value of the confidence level, we will get some other range. So, what we are saying is that uncertainty is not one answer one number or one range, but depending on the confidence level chosen you will get different ranges for the uncertainty. So, this is all what we want to learn now.

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But, you come back to the fact that every measurement is in error, every result is in error, then the question arises is there nothing in the world which is exact are there no exceptions. And the answer to that is that there are some things, which are called universal physical constants. These are considered as exact and the definition of which of these universal physical, which of these physical parameters are constant.

And exact was decided as recently as 2018 by a large body of scientists and engineers, who looked at all types of data about all types of constants and asked which is it that is exact and what is their exact value, and this is what we get. These are the 7 constants which are known as the universal physical constants. First is the Avogadro constant symbol is N A the value is given here and the units are there.

Then, there is a Boltzmann constant k or k b, elementary charge e in Coulombs, Planck constant h its value is here and units Joule seconds. Speed of light in vacuum this is the value units caesium hyper fine frequency, this is the value per second or Hertz. And luminous efficacy of a defined visible radiation K c d 683 lumens per Watt at 540 into 10 to the power 12 Hertz this is the units.

Just 2 years back scientist decided that these are the parameters of the physical constants, which are exact nothing else is exact that is one second, they said well what is the value of these numbers if these are exact. So, you can see here in this table it says exact value, there is no uncertainty in these so, they are exact. And exact their value is given and what you have to note from this column here is the number of significant places.

So, what its tells you is that this is the exact value, the next digit in this case say that decimal and something after that there is no doubt that all of that is 0, this is the exact value and to this many decimal places. So, the world body of scientists all across the world decided, these are things which are exact.

And they are classified as universal physical constants, everything else is in error nothing is exact beyond these. And, what it ends up telling you is that even in what we have learnt in school and colleges as something as being constant in the context of this understanding of science nothing there is also exact.

And the reason they said that these are exact is that using these, we can calculate as accurately as we want or measure and calculate as accurately as we want all other parameters. And the example of this will come in the next slide.

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And the closest thing that comes to that exactness is what we have called here as physical constants. And, I have listed here two examples taken from the website of NIST all these are available in the open literature; you can go to the website and see everything else. A very large number of physical constants have been listed and their values have been given. I picked up two constants which many of you have come across in your school and in college education.

The first one here is the Newtonian constant of gravitation the symbol G, we have been told that this has got a exact value of 6.74 into 10 to the power minus 11 these are the units. But, this table tells us something else, it says that the numerical value of G is as given here 6.7430 into 10 to the power minus 11 meter cube per kg per second square.

Note that this gap which is given between these two things, is something that the world body of scientists has decided that this is the standard way of expressing numbers. So, if we had to express this number some of us would say it is 6.67430 and we may put a comma, there are some places where they put a comma here. And then, they put a decimal both are in use you will come across both depending on where that information is coming from.

But, the standard method is that instead of using these it is best that, we write 6.674 leave a little space and write 30 that is the meaning of this number. So, this is the numerical value of G which we have also called as so, far as the nominal value, or we also refer to this as the mean value or sometimes also as average value.

So, this has been given to this many decimal places, the significant places is here 5 after the decimal. The 0 tells us that 0 is not in doubt, but anything that will come after this we do not know what it is it, could be 1, it could be 3, it could be 5, it could be 4, it could be 9 there is doubt in the next significant place. So, that is where you have uncertainty coming in.

And that uncertainty is characterized in this the next statement, it says that the standard uncertainty is this much into 10 to the power minus 11 this is the units. We will learn in a few lectures, what standard uncertainty means and how do we calculate it right. Now, we just accept that this is a standard uncertainty.

And, the next line it is reported what is the relative standard uncertainty? Which is the standard uncertainty divided by the numerical value. And this as you can see is 2.2 into 10 to the power minus 5. And, if you have to express this as a percentage we multiply it by 100 that gives you 10 to the power minus 3.

And this tells you that the relative standard uncertainty is 0.0022 percent, which by all practical means everything that we see in daily life compared to anything in that this is negligibly small very very small. Whether it is specific heat of water refractive index of glass, specific heat of a gas whatever we may look at the uncertainties in all of those are much much greater, and when as a much much greater means at least 10 times if not 100 times greater than this uncertainty. So, that is one example.

Another example is on the second picture here, the molar gas constant R which you have used so, many times in many subjects. Its value is 8.314 4598 Joules per mole per Kelvin, this is the same thing as what we have said over here. Its standard uncertainty is given here, it is 5 0s and then 48 Joules per mole per Kelvin. Relative standard uncertainty 5.7 into 10 to the power minus 7, this is a ratio of these 2 so no units.

And what it tells us is that in if you make it as a percentage, this will become 0.000057 percent, very small compared to anything we measure in our practical life. It also gives us one more line here, which was there also it says concise form which says that this is the first part is the value or the nominal value, whatever you want to call it or the numerical value.

And in bracket it says 4 8, which means that the last two digits of these this has this much uncertainty. So, that is another way of expressing what we had been saying earlier as a mean value plus minus something, all of that is put here as just one number here, with a bracket of something and of course, the units are there.

So, these are two examples of physical constants which we have used every day in life, we have treated them to be exact, but as we have seen here they are not universal physical constants. They have uncertainty associated with them, the uncertainty associated with them is very very small compared to uncertainties that we have in practical life.

And so, in most of the work we do we assume these physical constants like these to be exact. Exact because, their uncertainty is much less than all the measurement uncertainty that we do or measuring any parameter in the lab. So, they are not universal physical constants, but we will treat them as being exact.

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So, that leaves us with another thing of what are the other types of constants that come in, and these are what we see here as empirical constants. These are constants that come about with lot of experimentation; people do data fitting to it and start assigning values to it, which are in turn used by practicing engineers in many many things.

So, empirical values are what determined from an experiment or many experiments. And in some cases from numerical simulations, which is that we solve the governing equations of that problem and get those values that we are looking for. So, examples of these one is here, drag coefficient as a function of Reynolds number which we are saying is we take some object I have just drawn an object, there it is experiencing some fluid flowing over it at a certain velocity v.

And we are saying what is the force, that is keeping the object in place which is the equivalent opposite to the force exerted by the fluid on this body, and we define that force divided by the dynamic head, multiplied by the projected area as the drag coefficient. And, if you go and look up in charts you will find that for say flow in a pipe or flow over a sphere, we get lots of such correlations of this drag coefficient as a function of Reynolds number, which in our case is another result.

So, that correlation that we get in this case C D as a function of Reynolds number, this has got constants in it and those are called empirical constants. They are not exact they are only as good as the experiment that has been done. And uncertainty associated with them could be significant and cannot be said to be 0. A second example is from heat transfer, where we express commonly the heat transfer coefficient as a non dimensional number as a Nusselt number, as a function of Reynolds number and Prandtl number.

Some of these relations have recently been obtained from numerical simulations; most of these correlations that we use have come from experiments. And, what the scientific and engineering community has done is we have taken those experiments, which we regard as the best researchers their best results their best experiments.

And the constants C m and n that came out of their experiments, we treat that as the best possible value of these empirical constants. We treat that as a constant in the relations that we are looking at here, but practically there is uncertainty in them because, this is not an exact relation.

Similarly, in chemistry and chemical engineering, you will come across reaction rate constants. These are again determined from experiments from physics and physical phenomena, and they all have a certain plus minus value, but in our calculations we take that value treat it as almost being like a constant if justified and proceed with the analysis.

So, that is what we mean by empirical constants, they have uncertainty associated with that and depending on the problem, we may or may not neglect it. In fact, we could be actually doing an experiment, which could be superior to any of the existing experiments to precisely determine the empirical constants in the first place that is also one of the objectives of the experimentation. So, that is the other type of constant that come in the result formula.

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Then, the result formula has the other thing which is parameters and variables that are there varying in an experiment. So, these are quantities that are measured in an experiment. We call them parameters, we could also call them as variables and you can also call them measurands. All three are used in various texts and standards, sometimes we will use this, sometimes we use this sometimes we will use this, but we know what we are meaning by that quantity that is measured in an experiment.

And, there are two type within that we can have two sub classes, one which is called the independent parameter or the independent variable. This is a parameter whose value the experimenter will set is that ok, I want to measure something at this value. So, you set that measure it and say this is what I have set it at and say, now because of that parameter what is the change in the other parameters that I am measuring, those are the dependent parameters.

So, both values are coming from measurements from the experiment, except that one is set by the experimenter. The other is an output from the apparatus. Both are measured this is measured by its instrument, this is measured by its instrument, both have errors and we have to treat them accordingly. So, the two are related to one another say this is the system, or the apparatus to this we give some settings, which are the input settings and we see what is the output.

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So, that is the parameters coming in the result formula and these are the ones whose uncertainty we want to estimate. So, we use some terms have already come in that earlier slide that I showed and let us see what they are. The first there was value which is we have said with the nominal value mean value or the average value a number, this is a specific value a numerical value.

Then, we have standard uncertainty, we will define what it is and we will also learn the process of how to calculate it from data. Or in the absence of data, if you have some information say accuracy of an instrument, how can I calculate the standard uncertainty from that type of information we will learn that also. And the third term we came across was the relative standard uncertainty, which is the ratio of standard uncertainty to the value multiplied by 100 and often expressed as a percentage.

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So, we have come across three terms, we will come back to it later on, but first what is it the reason why there are errors in a measurement. And, there are many reasons for it a partial listing of those reasons is given on this slide and we will go through this one by one. First I have listed the human factor. So, every place we are reading what the value of the instrument display is, and that could change depending on the instrument, its own characteristics and also sometimes it depends on the human response.

A good example of that where you can get different numbers while looking at it, is something you have studied in schools as parallax. Sometimes there is an issue with that the pointer or the value lies between two markings then what do we do. So, there is a reason why error is coming up from that point. Then, there could be error depending on who is looking at it, one person may look at the instrument and say this is the value in some cases another person may look at it and so, no this is something else.

For example if there are markings which are like this and say the pointer is coming here, or the level if a mercury thermometer is over here. Then, one person may say the value is this another person may say value is this, some person may even try to interpolate between the two ends and the value is something else that is a source of error.

Then, depending on the instrument you could get different values. Suppose you have gone to fill pressure air into your tire of your vehicle, the reading there is coming as say 1.7 bar, or there is some others place where the instrument displays 1.75 bar. Then, the same thing being measured with two different instruments is giving us two different readings.

How do we handle this, but that is a source of error. Then, that could be variations from setup to setup. You may want to make the same measurement, but once you use one apparatus the other time you use another apparatus and you find you get two different values. An example of that is that you want to take your weight, you take one weighing balance you make your measure your weight on that, you get some value you go to another balance you get some other value.

Another example of this could be measuring the pollution of a car or the emissions from a vehicle. If you try to go and get the pollution under control PUC certificate one instrument will give you some value, another instrument will give you another value, one may say that you are the other said no your car is polluting do something up to it.

Then, there is issue of limitations of instruments themselves. One example of that I just showed over here. Even digital instruments have this issue that when you look at it sometimes the number keeps fluctuating, sometimes the number fluctuates a lot and very frequently. Then, you wonder you know what is the value that I am reading and if you make some compromise and take a number and seven that is you take that is the value, but then there is error associated with that.

Then, there is error associated with digitization of an analogue signal to a digital signal. These days in the worlds almost everything looks a digital display. But, many things that our sensors and instruments would give or the world itself the way it is an analogue world it is a continuous world there is no discrete change in the state of say the water of the temperature of the air.

But, when you measure it with a digital instrument it gives you discrete values of the air temperature. So, what we have done there is taken analogue signal, and converted it into a digital signal electronically. And in doing that we introduced some error then there are errors due to environmental variations.

The ambient temperature changes for instance, the relative humidity could affect the instrument. So, many other things we do not have control on them in some cases very small changes could make very large change in the output, what we read some cases it may not be. So, large depending on the experiment we may even ignore some of these things.

For example, if you want to say that I want to do an experiment with constant air temperature. Then, several between 12 noon and 1 pm the variation in the ambient air temperature is less than plus minus 0.5 degree Celsius, I am quite happy this is constant temperature for me. But, if you want to do an experiment over a longer time and you start at 8 am and continue the experiment till 4 pm assuming that the air temperature is constant.

Then, you could be do you know big error that at 8 am the temperature may be say 28 degree Celsius and at 4 pm, it might be say 32 degree Celsius or 40 degree Celsius in between it has gone through a much bigger variation. So, the error is coming in.

The next source of error I have listed is noise, signal from electronic signal from a transducer or from any electronic device always has noise in it. Simply because at the fundamental level what is happening in the material is not exact, it always produces ambiguous signals as well this noise will also lead to errors.

Then, there is error associated with calibration of an instrument. And we will learn little bit about this that calibration means, we are saying what is my for a given input what does my instrument give, and what is the true value true value we do not know. So, the next best estimate of true value is from a much better quality instrument. And this we say is a calibration standard.

In day to day measurements we do not use a calibration standard, its too expensive its too difficult to use and its sometimes just not practical. So, we choose another instrument which is a lower grade or a lower quality instrument than the calibration instrument, and say we will look I am going to map my reading with the reading from this and say how good am I. So, that is calibration and inherently it ends up introducing an error.

And finally, there is always error in the underlying physical phenomena, which ultimately if you keep asking why it ends up at Heisenberg's uncertainty principle. So, that is where things are and to this list you can add many more things depending on the type of measurement you are making, this list can get bigger some of them may be relevant, some new things may be relevant that varies from instrument to instrument measurement to measurement.

So, what we see here, is that there are many reasons why errors will always be there in a measurement. And our idea is how best to find out what is their magnitude and maybe in some cases compensate for them.

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So, do that we need to know what type of errors are there? And here we have classification done, and on this slide I have listed the first type of error which is called a blunder.

A blunder means a very big mistake so, big that by just looking at the value or of the measurement, measurand or by looking at the value of the result that you got by using that you find, that something is drastically off. And purely by one's own intuition, and one's own knowledge we should be able to say this is not right something is gone terribly wrong, I have to go back and find out where I did the mistake.

We cannot correct for this type of an error, we cannot treat it as a random error or a systematic error or a statistical error it is a big mistake. And, also sometimes this is referred to as an accidental error. For example, in doing an experiment you are supposed to set something at some value, but by mistake you left it at the value which was something different, you did the whole experiment and find no I put it at the wrong place.

So, there what are the some of the obvious reasons for it, there could be a malfunctioning instrument, it quite happens that if you are using a thermocouple and measuring ambient temperature is not showing plus 25 degree Celsius, it may show minus 25 degree Celsius. There is a some mistake in the wiring that was done or the program that you wrote to do the analysis.

Another source of blender is wrong units, quite often when we are converting a pressure difference into a velocity. We forget that this has to be in the units of Pascal and this will come out in meters per second. But, we rarely have any instrument which measures anything in Pascal sometimes; we measure it in bar sometimes in kilo Pascal sometimes in units which we do not use. Now, so much but pounds per square inch.

And, we forget to convert it and we end up making a big mistake. Then, there could be spurious signals. For example, if you are measuring electric signal coming from a wire and we get a very nice signal which has got a frequency something looks like this, with a frequency close to 50 Hertz you can be pretty much sure that you have picked up noise from the electric power cable, this is not a signal that we are looking at it all.

And finally, I have put here data integrity as to how good is the data being taken in, in conversion as well as in its interpretation. You may take a data multiplied by some factor and get the number that you will work with, but if that number is itself is in a big mistake was done. You got a big mistake happening there. There are many other examples where blunders happen, and whenever you see whether it maybe you yourself or as a student when you experiment you got an answer which is terribly off.

You know that there was a blunder in the experiment; it is not at all uncommon that blunders happen this is very much a part of life ok. The second thing that we have to work with that when we do uncertainty analysis, result analysis reporting of results writing of reports, we assume one that we did not commit any blunder.

So, everything we will run learn later on in this course assumes that there were no blunders that is one assumption. A second assumption that we make and this is very important is that there has been no manipulation of data; that means, we did no cooking of data. We have been honest, if at the end of the calculation we get a very drastically off result, it looks like a blunder or it does not make sense, well this will tell you that you need to go back redo the experiment and figure out what's happened.

We will not cook up the data to get the result that we are looking for, or the conclusion that we want to draw, this we will not do. So, this is a fundamental requirement of all experimental work and in this course we assume, that all the data that was there collected was not manipulated and the experimenter has been honest. This is a very important precondition.

In either case as I mentioned if there is a doubt, we revisit the raw data that we got. If you find that there is something off in it, we go back and if it can be mathematically corrected we correct it. Otherwise, if its cannot be done we have to go back do the experiment all over again and do it faithfully. So, that the first type of error blunder and how do we handle that.

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Assuming there is no blunder, we can classify errors in two types. And I have written here two methods by which we can classify. The first one is as per ISO GUM. And the second one is ASME PTC 19 1 ISO GUM says that errors can be of two types and we should classify them as such, one type A error the other is a type B error.

Type A errors are those which are estimated from physical reasoning, without recourse to data; that means, we are not using data that came out from any experiment measurement or anything. But, we are looking at the phenomena reasoning it out, getting a number or we say that the instrument itself has been specified to be like this, we use that as an input to get the value of the error.

Type B error is the error that is estimated from data. So; that means, we went collected data we got lot of numbers, from there we did some analysis and we said what is the magnitude of the error of the uncertainty in this. This is the definition as per the ISO GUM. ASME PTC 19.1 defines errors in a slightly different way. The terminology that is used is that there is something which is a systematic error, and there is something which is called a random error.

A systematic error is one where the value of the error is known, there is no randomness and it has been estimated from information that is already available with us not necessarily data. Random error is caused by fact that the value was changing randomly we could not predict, it we could measure it, but we cannot predict it and from measuring it, we did some calculations on it and predicted its value. So, give one example systematic errors and type A errors could include say inferences from accuracy of an instrument.

Linearity, resolution, we look the definition of all these is given in the various standards that I mentioned, but these are various things you would have come across when you looked at the instrument or did an experiment. These are the things which we have cannot do an experiment to estimate what is the uncertainty in that. We have to look at some other data, do some physical reasoning logical reasoning then use statistics to say that this is the standard error because of these reasons.

Random error or type B error is something which you have to get from data. So, you make a repeated measurement, or use different instruments to make a measurement. And from there you say well this is the random error in this instrument or this measurement. Important thing here, is that in an experiment in a measurement what are the sources of error and what are the types of those sources.

The causes the sources and the definition is the responsibility of the person, who is doing the experiment. So, it is entirely the prerogative of the experimenter to say what are the errors that are coming in this particular case, how do I classify them and then I proceed how do I quantify them? All of this is the job of the experimenter, in this course I have given some examples in the notes of measurements, and what type of errors that come in and which of them are type A, type B or systematic or random.

So, it is your the experimenters responsibility to decide. What are the sources of error? What are the causes that error are there? Classify them and later on as we will learn in this course quantify them. That is what we got to learn in this course.

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So, we will conclude this lecture here, and say that what have we learnt today is every measurement is in error we can remove that. And, because of that we cannot give an exact value to anything that we measure, we represent that parameter or the measurand with an uncertainty, which is an interval estimate that is a range at a certain confidence level.

Then, we learnt that there are some things in the world which are absolutely exact and those we said are universal physical constants, and we got their values. Everything else in the world is either measured or used make measurements and use the universal physical constants to calculate the values, we saw some examples of that those were the physical constants. And then we said that as engineers and scientists, we make some other measurements make our own constants and formula those are empirical constants.

Both of these have uncertainty associated with them, some of them can be neglected some of these cannot be neglected. Then, we looked at the classification of errors according to both the standards. And, we said finally, that there is a default expectation for uncertainty analysis, which is that there are no blunders and that the experimenter is being honest.

So, with those preconditions we will next move to the fact that, how do I take data which is about errors, use statistical analysis and get various values. We will pick this up in the next lecture, on that note we conclude this lecture and.

Thank you.