Introduction to Uncertainty Analysis and Experimentation

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Module - 08
Reporting and Archiving
Lecture - 27
Reporting uncertainties

Welcome to the course Introduction to Uncertainty Analysis and Experimentation. We are now starting module 8, which is about reporting and archiving. In this lecture, we will look at reporting in general with specifics related to Reporting of uncertainties. To begin with let us look at the issue of reporting. So, traditionally, we think of reporting as writing a report, but there are many additional features in the professional world that we have to consider.

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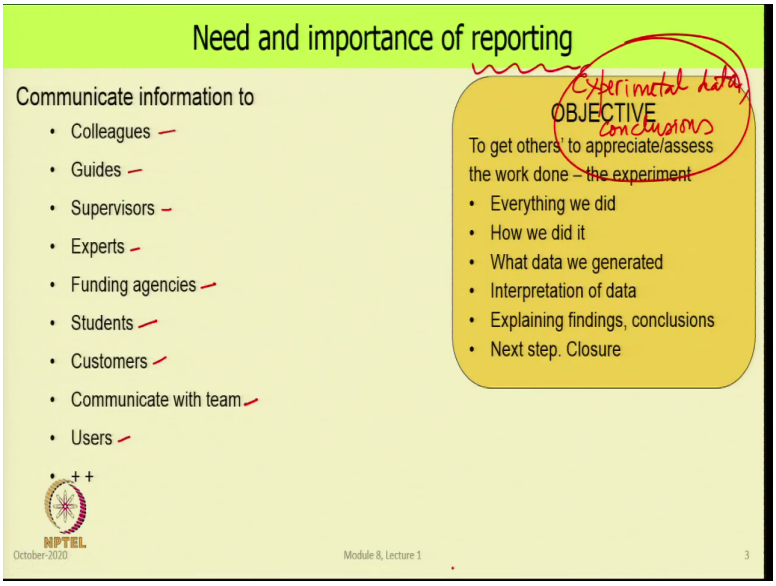

So, the first question is whom do we want to communicate with or what the recipients of the information that we want to give? So, here I listed some of the possibilities, it could be colleagues, coworkers with us with whom we want to share what the experiment has given us, it could be guides or even in industry supervisors, your immediate seniors or their seniors to whom you want to communicate what the findings of the work were.

These could be experts, who would appreciate that fine points of the work. Could be funding agencies, we could be communicating with students, either our own peers or if you are a teacher you are communicating with students in a college or maybe even in a school.

Then, there could be customers to whom you are giving the information or like all engineering activities which are done in big teams we could we regularly have to communicate our findings with our team and finally, there could be users of whom you have to advise something.

So, what we are doing going to do now is we are going to be looking at all these people who would be receiving information in general about reporting and there is no restriction on field or at what level it is done. Reporting is extremely frequent and very important in one's professional life all the way through.

We will restrict ourselves to reporting experimental data findings or conclusions, that what this course has been about. We have looked at experimentation, we had looked at data, we have looked at uncertainty, then we looked at generating regressions and now we say how do I communicate all of this to someone else.

So, our objective in doing the communication in the first place is that we want others to appreciate the work that we have done. In particular, we have done an experiment we want others to appreciate what we have done present our values and data to them and together draw some conclusions out of it. So, what do we need? What does it take to fully communicate and appreciate what data we have?

So, at the least we need to communicate everything that we have done, everything means every aspect of the experimentation process. Then, how we did it, what difficulties we had, what compromises we made, what shortcuts we took, where did we could not do anything that was we wanted to, but we had to make a compromise all of that.

Then, we after the experiment what data did we generate, it may be volumes of data, but we summarized that [FL] this is all the data that we now have. We can work with this from that data we draw interpretations and conclusions. And finally, present our findings based on this data and then finally, after everything is done, we take a decision.

We go for closure where everything that we have done we archive it, so that in future if you have to use it examine it again or somebody else has to examine it, it is there available and in an unstable form.

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So, why are we going to be doing all of this? Because we have some motivation to do it. We have some expectation from others that after we present our data to them, they will do something. So, with colleagues, guide, supervisors, here we would take various types of decisions. Supervisors especially in industrial setting could take a much larger call on maybe a process.

Experts could guide us and we want them to appreciate that we have done something new and something better and they can then qualify the data and make it available to many more people. We may be presenting our findings to funding agencies. So, we want to support our request for maybe a project proposal or funding. For students this could be that we are presenting our data in a thesis, so that we complete our degree requirements.

To customers we could present data and say well this is a decision that we recommend or you change an earlier thing that you were doing and so on. So, we are not only communicating data we are saying that who are the recipients and what is our motivation in telling them what is it that we have done all of this here and what do we expect out of that.

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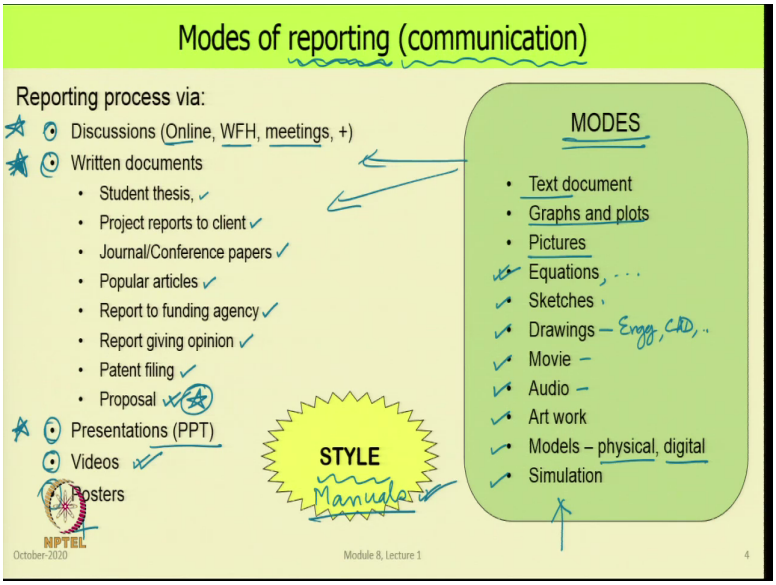

So, what are the modes of reporting? So, while you are saying in general is reporting which is classically means a technical report, a written document, but we are looking at a larger picture in general communicating, findings from an experiment you can look at it that way.

So, there are various ways in which this communication can happen. The first one that I have listed here is discussions, and in this age of COVID where there is work from home in a big way, these are largely talking. So, basically it could be online or it could be person to person

meetings or a mixture of person and virtual. In a way this is the easiest and the most effective way of communicating.

Time was when we could discuss and that would be the end of it and the information would be lost on everyone. Now, we can record it, we can make a text out of it, and archive it, so that in future we can go back and say well what did we discuss last time and we can move forward from that. So, discussion was the first one.

The second is written documents, and this is what we traditionally mean by report. In a school or a college, we do an experiment or we do a project we ended up with a writing a report. And, but this is not the only place where we write the report there are many professional situations where also we have to write the report.

So, here are some examples of writing, this could be a student thesis or a dissertation. If somebody is doing a project for a client, we are writing a report to communicate the results to the client, we could be writing it for the sake of publication in a journal or conference paper, we could even write something for the popular media, the general public to read about it. It could be the end of a work or maybe intermediate milestone to a funding agency.

A report could be even that we are giving our opinion to someone and we could also be writing a report for patent filing. And last, but not the least one of the most important things we write is a proposal. This is a somewhat understated aspect in our education, but proposal writing is the key to all professional work not just experimentation, but in general doing any project or anything else. So, this is very very important.

Besides written documents, we have the next most frequent mode which is a presentation which these days goes through some sort of a power point presentation. Now, this could be a standalone PPT type of a thing or it could be a mixture of video or just pdf file that we are putting out and supplementing it with data voice video everything.

Then, there is these days the mode of communication through videos, like this lecture. We have made a video made it available to everyone and we are communicating it through this

medium. So, there are various ways with in which we can communicate with videos, including putting things in on YouTube and things like that.

Then, there is also been a way of communicating with posters where we just make a flyer or a large sheet just put all the work done in a nutshell and that poster becomes a way of communicating with others. So, these are the most frequently used common techniques of reporting, the modes of communication, and depending on the type of work we do we should appropriately choose this. But discussions of course will always be there. Reports is the most frequent form of communicating and accompanied by presentations.

So, we can communicate this way, but then what do we communicate, what are the various you can say classification by which the information goes out. So, I have listed here modes, this could be a text document, it could be graphs and plots, we make that very frequently. These could be pictures, photographs, or sketches, which is sketches is here.

Then, there are equations, equations, regressions, formula, all of that will come here. There are drawings, and drawings also include engineering drawings, like CAD models. Then, there could be a movie, which is what we are saying this could be a video format or it could be just audio. For example, if you are communicating about the sounds produced by something or a commentary about something you get the audio form.

Then, there is a artwork. So, this could be like paintings, sketchings, and model making things like that and that is what is coming also up in the next item which is models which could be physical models, probably make a scale model in the case of say an experiment or making a building things like that and also digital models.

So, you can make everything in some software and visualize everything and lastly there are simulations. So, from solving equations or from other ways we create how something is likely to play out.

So, these are the tools available to us to selectively pick and choose to put together to make any of these, and we usually use a combination of these in each one of these activities. In all

of these, we are somehow constrained to work within certain styles. And style would mean, that if it is a thesis, we have a certain expectation of how a thesis should be put together. Whether we are using text or pictures or equations there are certain styles and usually available as style manuals.

So, we will not go into the details of these, but I recommend you look up these on the web or if your organization already has style manuals, use that, if it is not there develop one.

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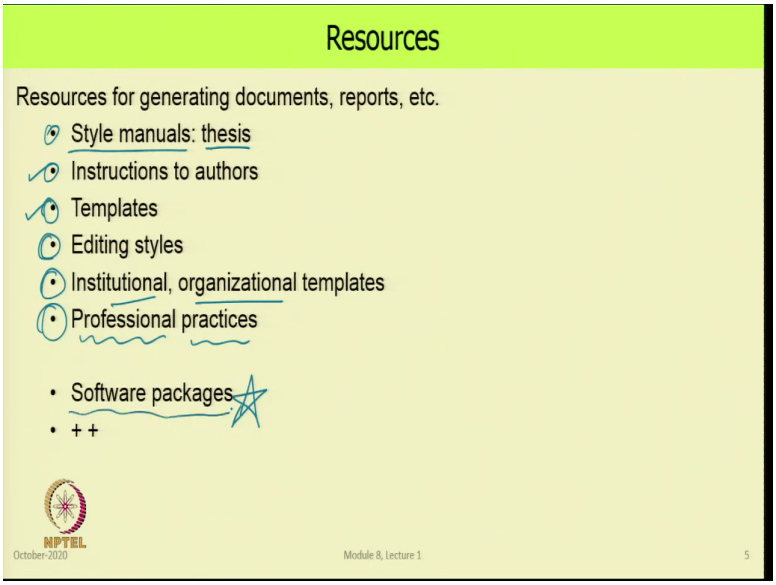

So, what are the resources at our disposal for creating these reporting processes? I just mentioned style manuals. For example, there are style manuals for thesis. Then there are instructions to authors. So, if you are sending an article to a paper or a conference or for a patent there are instructions on how to do it you, generally you have to follow it otherwise it will come back and tell you have to change it, we will not accept it.

So, we generally do not have a choice there or we could have templates which could be either software or a hard copy. The software is there everything has been set for you just have to fill in everything that is required the document becomes ready. So, those are templates, this is also quite frequently used. Most organizations would have for example, a presentation power point they would have a template for the background style of a slide.

Then, there are editing styles. So, this would include how do you put the heading styles, how do you do styling of the letters for pictures, figures, tables all of that. And I mentioned this as a part of templates, institutions and organizations are likely to have their own templates for various work. And finally, there will be lot of things which may not be written anywhere, but these are understood to be, I have done.

So, that you are expected to follow this and that is what I have listed here as professional practices. And for doing all of this, we have a whole variety of software packages available to us. It is our own choice, what we want to use, what we are comfortable with and these days they give you a lot of flexibility to do various things.

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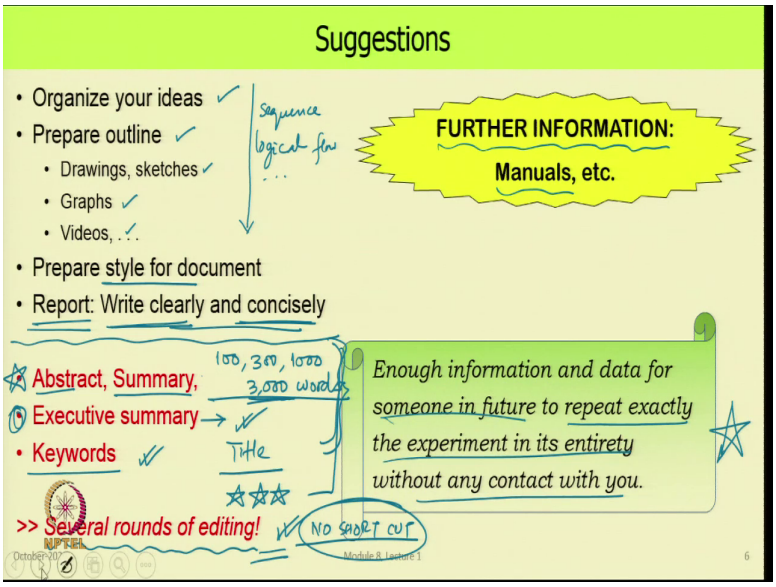

So, we will not go further into more details of report writing per se, but here are some very broad suggestions, for report writing in general and we will look at then writing a report of an experiment in particular some of the issues on that.

So, in general, writing should begin by organizing one's ideas and then prepare an outline of whatever form in which we want to communicate. This would be just [vocalized-noise] line items. We could then populate it with some drawing, sketches, graphs and videos, so the works the information flow gets put together.

So, all of this also ends up giving us a sequence or sequential flow or a logical flow to that report. And this is very important. If others have to appreciate your document they must feel

like reading it. And the best way to make them feel like reading it that it becomes easy to read, easily comprehensible and grips their attention.

Having done that if we do not have a style, make our own style. If there is already a style available use the style. So, we will all be using some sort of a software within that software set up the style. So, where we will put the page numbers, what will be heading level 1 look like, heading level 2 look like, all of that gets set up right in the beginning.

And then we start writing the document, whether it is a report, presentation, whatever. With very few simple things that I can suggest here details there are in many other places I will not go into it is write clearly and concisely. Figure should be very clear and readable. Those are very simple important dos and do nots.

So, we leave report writing at that point, but then I will emphasize these next 3 items. The abstract or the summary, where in a few words there could be some abstract which are just 100 words, some abstract which go 300 words, some abstract would go 1000 words, some abstract which are extended abstract which will go to 3000 words.

And irrespective of what document is being written, the first thing that anybody will do is they will read the abstract or the summary. If that seems interesting then they will read the report. So, if you want somebody to read your report and appreciate it this is where the key lies. This should be written very thoroughly.

Another form is what I have listed here is the executive summary. So, you are writing it generally for your seniors and maybe 3000 words or even less maybe 2 pages 1 page something like that we have to tell everything. So, both these things are such that all the work that has been done has to be described in this many words, and quite often people will say I have done so much work how can I write it in just so many, so few words.

Well, that is the whole idea; that you should be able to express everything that is done, in most important part of what everything has been done in this including your motivation, your

planning, how you did, what you did, and what your findings were, everything has to be done and reported over here. So, this is an extremely crucial aspect of a report.

And then for every report or whatever document we make there have to be keywords, a few keyword may be 3, 4, 5, 6, no more than that.

Because in today's world when people go about doing searching, the first thing they will look at is keywords then they will look for some words which appear in the summary or the abstract or the title and that will decide that when you do a search which documents will show up, and which documents will show up high in the list, which will show low in the list, and which documents may not show up at all.

So, if you want your document to be seen then only your work will begin to get appreciated, this part is absolutely the most important part. And we will wind up this part by saying that whether it is writing a report making a presentation or write definitely abstract summary or executive summary, it will invariably require many rounds and iterations, many rounds and rounds of editing.

Maybe 5, 10, 15 only then you will get a document of a quality that will probably go out. There is no shortcut. Writing it once is absolutely the just the beginning step and not the end by in any way. So, this is something which is very very important in one's education and they should learn how to write present all that.

So, we leave these ideas here. There is lot more information everywhere else on the web in your institutions, organizations, etcetera. See what it is like. Read others works, so you get an idea of what good writing presentation is all about, and there are manuals on how to do it. So, we will leave this over there.

And finally, some very important piece of advice that in order for your work to be appreciated one of the underlyings important things is that we should give enough information and data for someone in future to repeat exactly the experiment in its entirety without any contact with you. This is the key to scientific and engineering work that if you report some finding, if

someone else wants to check it they should be able to do the whole experiment exactly the same way as you did and be able to verify what you have done.

Without that your experimental findings are likely to be suspect. And you would have seen instances in the recent past where somebody reported a result, when others tried to replicate it they could not replicate it, and then the original author admitted to having committed some unethical practices. That is not a thing to be done.

So, on that note, we will leave this issue of report, report writing. But now we will concentrate specifically on those issues about what we have learnt in this course which is uncertainty analysis and experimentation. So, let us see what are some typical issues on reporting related to these topics.

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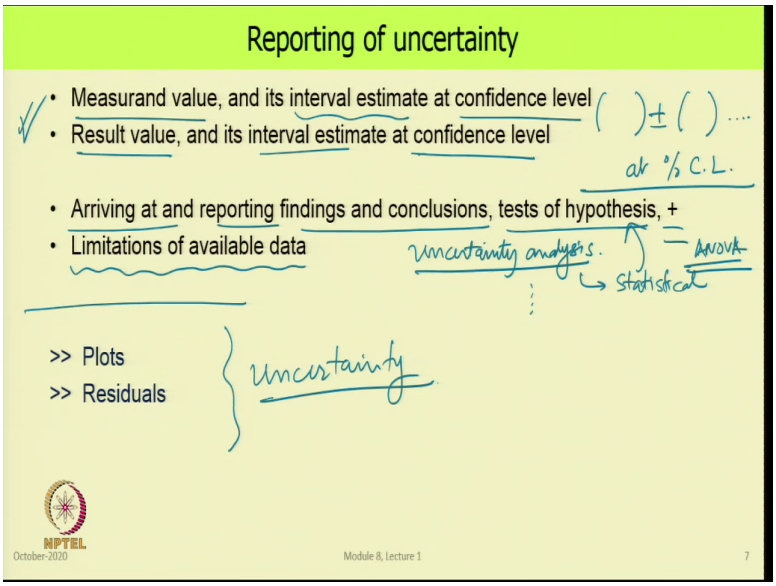

So, as we have said whenever we report data, report the result, report the measurement, so whether it is a measured value or a result value, we must always give the interval estimate at certain confidence level. So, it is both a mean value or a nominal value plus minus something with its units at some confidence level. So, whenever we report a data we should not say there is a number, we would always say there is an interval. This is our first thing that we have learnt in this course.

Same thing go continues, that we use this information in arriving at and then finally, also reporting findings and conclusions or test of hypothesis and other findings. That means, that whatever conclusions we are drawing, whatever we are recommending, whatever what you call critique we are making of others data it is supported by our uncertainty analysis.

And then of course, what we have not done is that this will also lead to many more statistical techniques, such as test of hypothesis, analysis of variance, all of those things which we have not done in this course, but they are part of any standard course on statistics.

But that can be done only when we have this done at this level. We are also put uncertainty analysis which let us others appreciate what are the limitations of the data that we have, only then people get a realistic and an honest picture of what all these data and what all the meaning of this experiment was all about.

Now, we will look at two issues, one is plots and residuals, and all of this in the context of uncertainty. So, these were the ways in which you put uncertainty in numbers in a text. Now, let us see what it happens in graphs.

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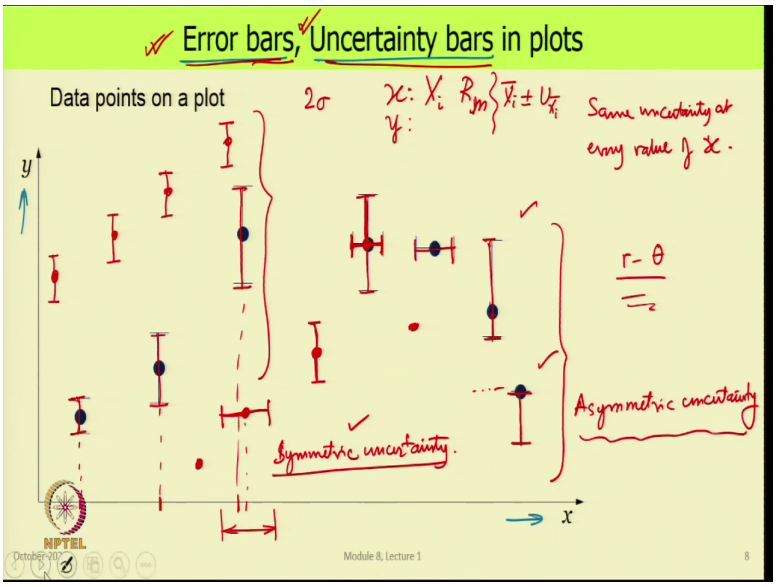

So, the way we represent uncertainty in a graph is by showing what would be called as error bars or uncertainty bars or uncertainty limits. So, a typical graph would look like this that on this axis we have x, this we have y, and we have put our data points over this. Say this.

Most graphs that you would see would just have these things on, and this information is incomplete because the uncertainty has not been reported. And plots where uncertainty is not shown should be taken with a healthy dose of skepticism. So, how do we show uncertainty in this plot? So, on these plots that I have shown here there are various forms of showing the error bars or the uncertainty bars.

So, the most basic way of showing the uncertainty limit on this one is that if in the y variable this was the uncertainty, we can show this magnitude each at say 2 sigma level or maybe 1 sigma level and we have to clarify that that is 95 percent confidence level. So, this tells you

that this point has this much variability in the value of y. If we want to show the uncertainty in x, then we would draw a line like that and it says that this is the mean value, but 95 percent confidence interval is this.

In all these cases, we are showing equal signs here and here, equal up and equal down. So, these are examples of symmetric uncertainty. And in this course, we have only looked at symmetric uncertainty, in advanced course we would can look at asymmetric uncertainty limits.

So, now once we have drawn this, as we have seen, in most in if in our analysis usually both x and y whether this x is coming as one of the measurands or as one of the results, and same thing with y, all of these have uncertainties. So, if both have uncertainties then this is the way we show it like over here. This is the uncertainty in x and this is the uncertainty in y.

So, that fully tells us the goodness of this value. So, this is the mean value that we have got and on that, so you have X i bar and then you have put plus minus U x i bar which is the expanded uncertainty at a certain confidence level. So, this part is capital U x i bar, y i bar. So, same thing with x. So, this would be a typical complete depiction of a point where both x and y have uncertainties. If only x had uncertainty and y had no uncertainty then the picture is here there is no vertical band, it is just one horizontal band.

Again, in all these cases, I am showing symmetric uncertainties. Then, in a plot we could have many points and two situations can arise that all the points have the same uncertainty and let us say for the time being that as we have done in regressions uncertainty in x is very small, so we will not show it. So, we will only show uncertainty in y which becomes this.

Now, it could so happen that uncertainty in every one of these points is same as this, so the magnitude of these bars will be the same. So, this is same uncertainty at every value. This is one scenario. But then there could be other scenarios where that uncertainty band varies with the value of x and that is what these 3 points are showing here. So, here the uncertainty is

small for this value of x, for this value of x uncertainty is larger, for this value of x the uncertainty is even bigger.

So, then we have to show uncertainty bands for quite a large number of points in the plot. In this case, if uncertainty was same for every point we could show the uncertainty at one point and that would imply that uncertainty at all the points is also the same and in our discussion we would explicitly say so. So, this is the most common way of showing uncertainty, typically in y values with very little uncertainty in x values.

Two case is shown here, this one and this one, these show asymmetric uncertainty bands. So, in this case, this is the band, and it shows that here it is smaller on the top here it is much bigger. And in this case, the case where it is showing like this; that means, the mean value that we have shown is the upper limit of the uncertainty band and not the midpoint.

So, all probability the values will be less than this value of y that we are reporting. So, those are two cases of asymmetric uncertainty, and as I mentioned this is left for an advanced course.

So, in almost every package, statistical package or plotting package or any scientific packing plotting package, you will always see these options of error bars, uncertainty bars and you must always show this. So, whether it is a x y plot like this or r theta plot or maybe any other plot everywhere we have to show uncertainty in one or both.

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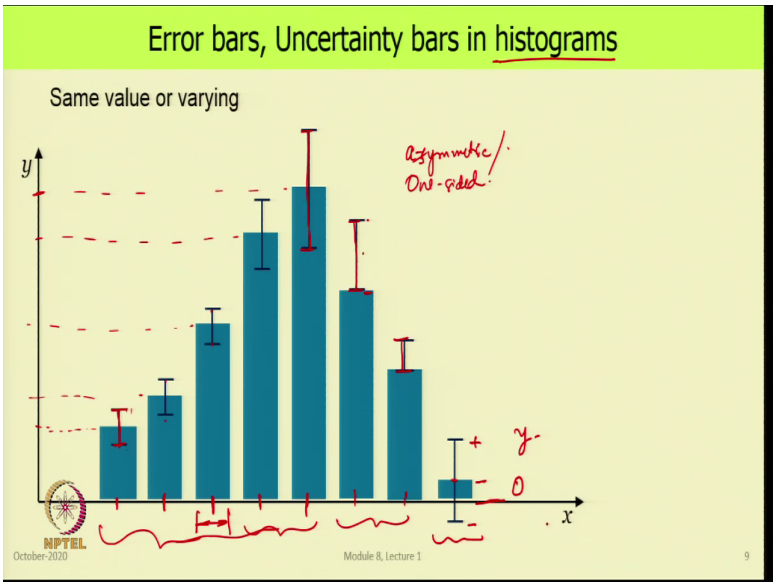

Another thing is how do we show these uncertainty bars in histograms. So, here is [vocalized-noise] the example of that what we have shown here is that at this value y value is this much, at this value y value is this much, and at this mean value it is this much and this is the width of the histogram. And we have placed this error bar on the top showing that about the mean value of this, this is the uncertainty in this.

So, we put this and this appears over here. We are saying that this is a symmetric uncertainty on both sides. And if this value was same at all values of x this same thing would appear here and it appears here, but if it increases with the value of x, as we have shown here this band becomes bigger for this value of x and for this mean value of y and for this mean value of x this becomes this mean value of y and the uncertainty limit now is shown as being this, which means that it is much bigger than what it was at the smaller values.

So, we have seen both cases, where in the histogram uncertainties are the same and in these two cases or we can say even all these cases put together the uncertainty increases with increasing value of x. It does not matter whether it is increasing or decreasing, we could show it that way.

The next two plots they show that this is the mean value uncertainty is only one sided or so, this is asymmetric or one sided. So, we have put a error band like that which tells you that in all probability the value lies between this and this. So, our measured value or reported result value is always on the lower end of the estimate.

And this is the same thing, but the limit here is less than the limit over here. And finally, we have shown one case where we have made a reading and the error is quite large compared to the mean value itself.

So, this tells you that if this line were a 0 line, the uncertainty in this value is plus something and minus something. So, of course, mathematically this will be so physically you will have to argue yourself whether this parameter that is being looked at the y parameter, can it take negative values.

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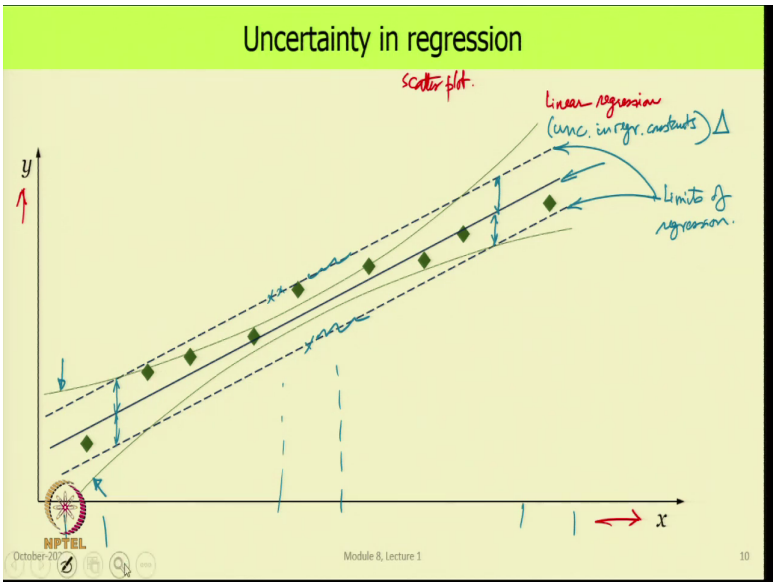

So, now let us look at the uncertainty in regression, and is a typical plot a scatter plot as we have called this. On this axis is x, this side is y, and as we have said either of these could be any of the measurands or could be either of the results. And say, this is that data that we got.

Now, we do a regression and using that option in the software we said show me the regression line and the regression line will come up. So, this is our linear regression. So, now, I want to show the uncertainty on the regression no, so far we have looked at uncertainty of every point. So, one thing is we do not give the uncertainty on the regression, but we do put the uncertainty bars on every point.

So, whatever they may be if they are all the same, we could put that and that would be say good enough, but in our discussion, we will have to give what is uncertainty on the regression constants. This is important. However, if we do not want to do that the way of showing the

uncertainty in the regression that shows then we have to show two lines, one a lower limit line and second an upper limit line.

So, this is the regression, and these two lines are the limits of the regression. The other ways of showing the regression is to show it as a coloured band, and either it could be uniform colour or it could be decreasing in colour as we go outside. So, it tells you that the probability goes down as you go away from the mean. So, this is how we would show the limits of regression.

It could so happen that as in this case these values are identicals, the up the positive side and the negative side, plus minus. At any value of x, they are the same and they are symmetric. But as we have seen in regressions this may not be the case. It could be that the lower regression line is like this and the upper regression line is like that. So, you have this line and this line in which case we will not show these lines, and we will just show these lines to depict what is the uncertainty with different values of x.

And as we have seen in regression, at the middle values of the regression the uncertainty is the minimum going to the either end the lower end or the upper end the uncertainty increases. So, that is how we would show uncertainties in regression.

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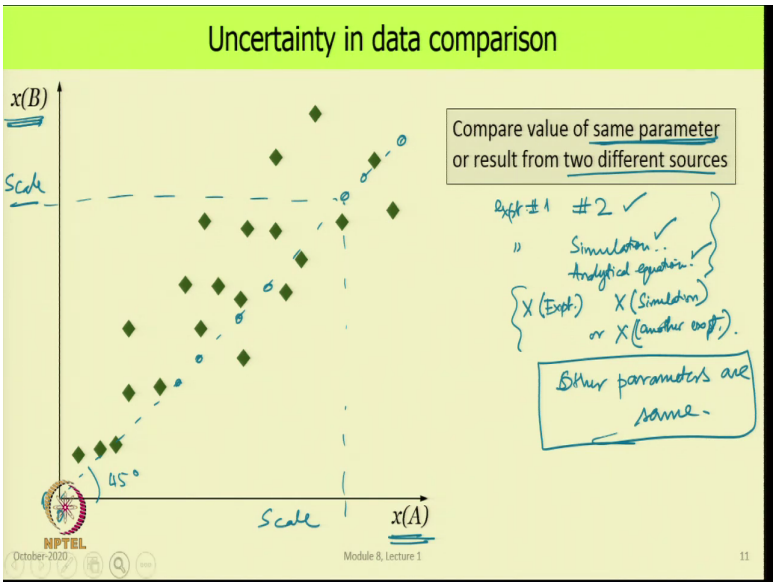

Now, yet another situation that arises is that we want to compare a parameter from two different sources. So, it is the same parameter from two different sources, say one could be experiment done today and maybe same experiment done some days ago or same experiment done by somebody else. Or, it could be that we have an experiment on one side and for the same conditions we have got a result from a computer simulation or solution from an analytical equation if possible.

And we want to compare the same parameters. So, here we are not comparing two parameters. So, what we are saying is I have one parameter which is x, but that came from an experiment and I am comparing with x, but that came from a simulation or that x came from another experiment.

So, how do we show these? So, our axis are now still two, but one is x from source A, the value of x the parameter. It could be any of the measurands in the experiment or it could be any of the results of the experiment, and x from source B, and that B as we said it could be either an experiment or a simulation or from an analytical equation.

And we plot the points assuming that all of these data that has been collected where all the other parameters are same. No variation in that. This is very important. Otherwise, we will end up comparing apples and oranges. So, when we put all these points, they [vocalized-noise] must look like this. Now, ideally if we draw a 45 degree line over here, it would represent those locals of locus of points and the scales are the same, this scale here is same as this scale here, both are from 0.

So, it is a 45 degree line on if both of these are linear scales, then lines all points on that if this were 45 degrees all these points that would lie here they would tell us that there is a perfect match whatever was the value of x, we got exactly the same value of y. So, this all the points falling on the line which is like this 45 degree line would tell you that you have a very good match between x from source A and x from source B. In practical world, this happens very rarely. So, let us see what we can do about it.

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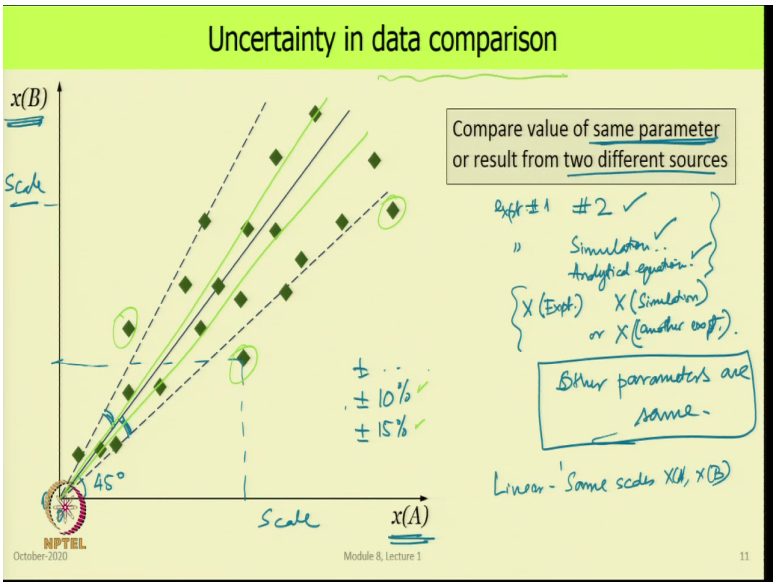

So, this is the first line we put up, and this is a line where the two are equal. On a linear scale this will be a 45 degree line. Linear scale and in any case x A and x B scales are same. It could be the case that if x A is a log scale, then well x B should also be shown on a log scale, in which case again we would get a line like this, but if one is linear the other is log the comparison can be made, but this line will not be straight you will then have some other curve. It just makes the interpretation little more tricky that is all. This is the easiest way to do it.

Now, as we see if both were linear we got this as a 45 degree line, and the points are scattered, some are above this line, some are below this line. What it tells us is that for this value of x from A, we got this value of x from source B. That is why this is not falling on this

line; it means that there is a deviation. So, in this case, what we do is we draw two lines like this which represent a certain percentage variation in the value of the variable.

So, this could be say 10 percent variation line plus minus 10 percent or it could be plus minus 15 percent variation line or any other value. So, we draw these two lines, this angle is equal to this angle and now we say how many points lie within this line. So, we can see that there are some points which are going out, say this point is outside this limit, so is this, so is this, but all the others have been captured within this.

So, that tells us that whatever these lines represented whether it was 10 percent, 15 percent or 5 percent, our result is within that much, the agreement between x from source A and x from source B is like this. If we put about title limit, a line like that and a second line like that then; obviously, the number of points lying within that is less.

So, we will say that the number of points that are in agreement is this lower fraction at a higher at a much smaller tighter margin. So, this gives us a technique for comparing data from two different sources.

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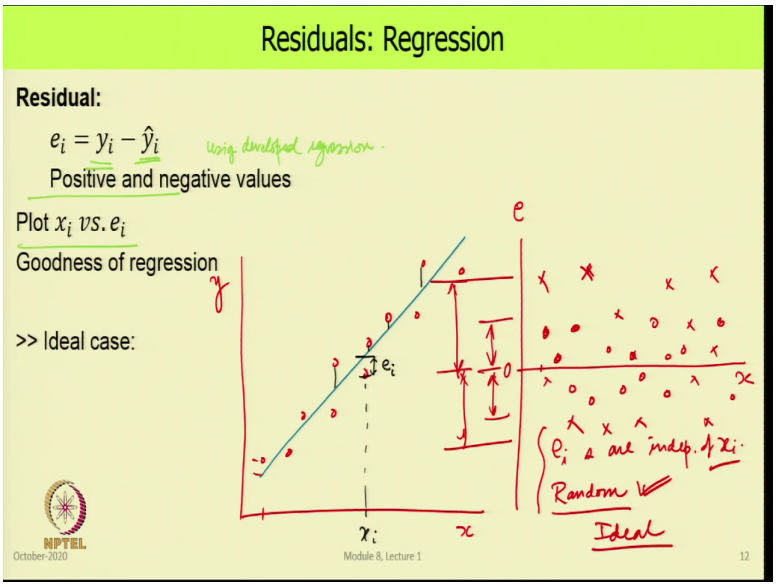

Now, we come to the idea of residuals. In the lecture on regression, we have defined the residual as the value measured value of y and the predicted value of y using the regression that we developed, and e i could have positive and negative numbers. So, now, what we do is we plot e i as a function of x i and that gives us a good idea of how good our regression is.

So, in the ideal case, this would happen. We will make two plots, this is x, and this is y, and we would get some points. And when we draw made the regression, we would get a line, and then when we plot the residuals, we would be plotting this as e of i. So, at this value of x, this is the value of e i. And that is what we will plot now here. But here what we do is we change the scale, we put the scale over there, where this is x, this is e and this is 0 for this axis; x, may not be 0, there does not matter.

And now we pick up each one of these x and plot the residual. So, in this case, we could see something like this. So, if these points are spread out equally on the positive and negative sides, within a certain limit and there is no pattern that you can make out by looking at these points. Then, it tells you that the e i s are spread randomly and they have no connection with x i, there is no correlation, e i is not correlated with x and that because they are completely random.

So, this is a good situation to have. This is what the ideal situation should be. So, you could have one set of data which was like this and another set of data that could have been like this shown by the crosses. The only difference between the cross and the circle is that all the crosses lie in a larger range, than all the circles, and it only tells you that yes, the second case also e i s are independent of x i they are also random.

So, we are in good shape, but when you compare it with the other one you say that the limit within which the circles lie is smaller than the limits in which the crosses lie which tells you that this data which was taken here this is of a better quality than this data. So, this is the ideal case. And it is not that we cannot achieve in reality, in many cases, you will get these things, provided of course you took all the precautions in designing the experiment.

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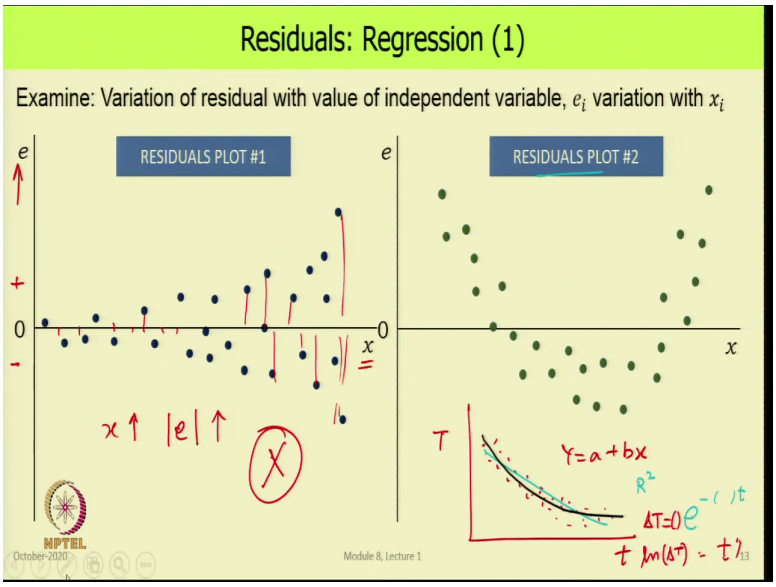

But there are just enough cases where that may not be the case. And we will look at 4 cases. So, in this case here there is x on this axis the residual is plotted on e and this is 0 this is positive, this is negative. And what do we see? At small values of x over here the residual values are small, as x keeps increasing these residuals their magnitude is increasing. So, as x increases mod e is increasing.

So, here also it has increased even more. So, this tells you that either the regression is not the right regression or we have to reexamine the data and see whether everything is ok in that. So, this is something we do not want to see. We cannot report a data like this, even though we may see a nice regression, but if we see values coming like that, we should be skeptical and say well no something is not right in the regression.

Similarly, the situation over here and this arises when you have data like this. A good example of this is time versus temperature. So, you have a pot of hot water or a hot body sitting in air and cooling, and then what one does is that they say well I will form a linear regression. If you compute the r square, it may even look nice.

So, you may actually believe that this is the case, but that is not the case. And this is a good example where you will go back to the physics of the problem and say Newton's law of cooling does not go like this, it is e to the power minus something times time.

But the way to check whether our regression was ok or not is to plot the residuals. And here is what we see, in this case, the residuals will look like this and it clearly tells you that there is some real serious issue in this whole regression, maybe it is not the right one at all.

So, if we change our regression from being a linear y equal to a plus b x, but take something that delta T is equal to something times e to the power minus something t and take log of that then we will get a linear regression. So, on a log linear scale this will look like a straight line. But on a linear scale this regression technically should be like that. So, that is the second case where our regressions could mislead us.

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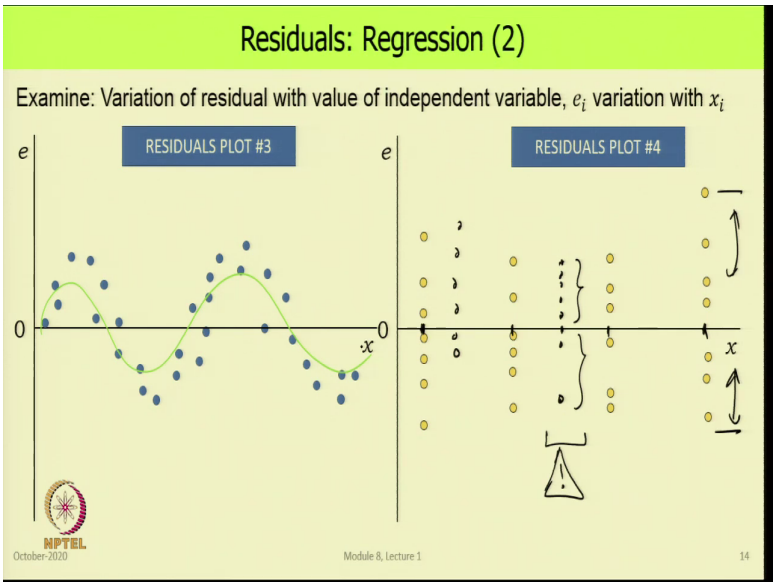

The third case is shown here, where you can see that this regression seems to be following a pattern like that. That means, there is something systematic happening there and the linear regression that we have used is not the best thing.

And this last plot is something that one sees in our experiments most of the time because we do experiment at some fixed values of x say this, this, this, and this, and maybe we repeat the experiment at the same value 2, 3, 4 times and then we plot the residuals for some of the related parameters. And here is what we would typically see, that for this value of x we got these residuals, here we got this, here we got this, and here we got these.

So, the first thing is the band, if the upper limit and the lower limit are comparable that is nice. If they happen to be that all of them lie like this, then we know we have a problem. The

second is if you look at within these if we find that at some, they are all equally distributed and randomly there is no order from this to this to this to this then also we are doing well.

But if we see this that there is one here, one here, and then here, so the band are the same this here is same as this here, but you see that there are lot more on this side than on this side. So, this will also be a caution that you know something is a miss, go back, re-look at the experiment or we look at the regression. So, that is one of the things we must report, is the residual plots to convince ourselves and others that how good the data is that we have got.

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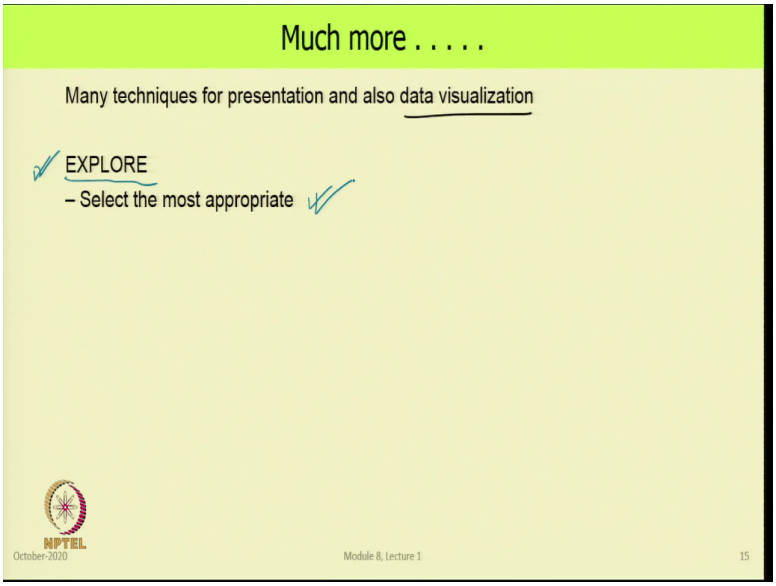

So, those were some of the important things about uncertainty and its reporting. There are many other techniques and many other ways of presentation and also techniques for data visualization that we will not go into the details over here. I leave it for you to explore. There are many good references, many examples, and there are many ways of presenting data.

So, go ahead, explore, and then make your own judgment to decide which is the most appropriate one. With that discussion on Reporting of Uncertainties, we come to the end of this lecture where we have looked at reporting in general and its importance.

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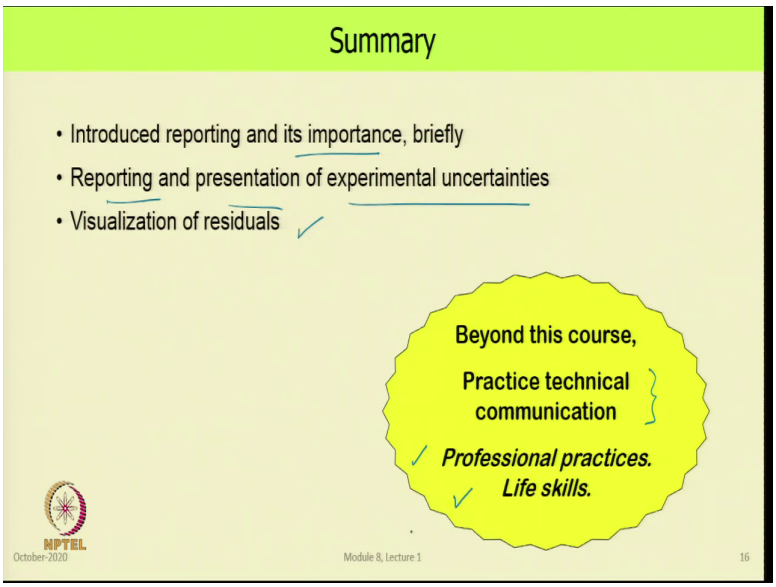

We have also looked at reporting and presentation of experimental uncertainties. And we have seen an important aspect of regression which is visualization of residuals.

So, beyond this course there are various aspects related to the practice of technical communication. These are important things in one's professional life, and are generally referred to as professional practices and life skills. So, I encourage you to practice technical writing and pass it on to others. On that note we conclude this lecture.

Thank you.