Introduction to Uncertainty Analysis and Experimentation
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Module - 06
Uncertainty in a Result
Lecture - 23
Examples of result uncertainly - 2

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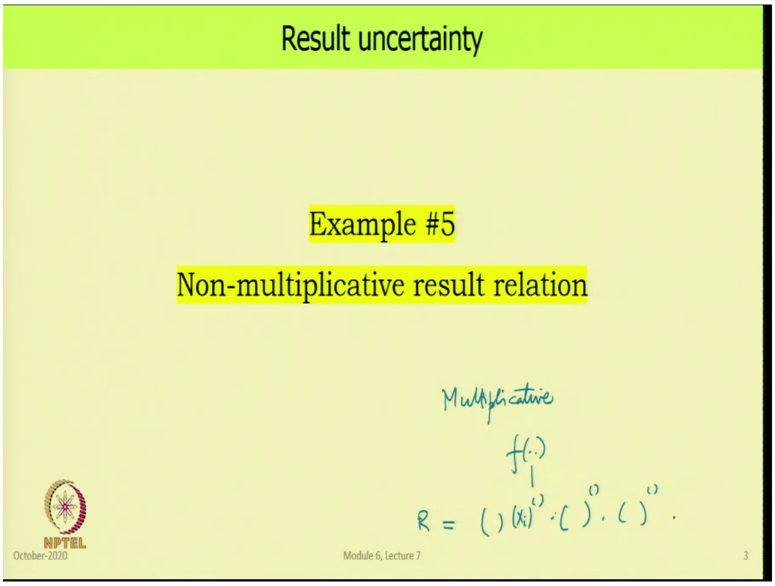

Welcome to the course Introduction to Uncertainty Analysis and Experimentation. In this lecture we will look at examples of uncertainty calculations and we look at uncertainty in a result. So, there are some special forms of the result formula that is what we will look at today and also look at some issues that come up that could lead to possible errors. The first example is on non multiplicative result relation.

So, far what we have done is we have seen multiplicative relation as a function which has only parameters raised to various exponents and some constants there. So, this is the result formula. So, this form of the result with the multiplicative relation, there is no addition or subtraction in this. Now what happens if the formula has addition or subtraction.

So, that is what we will look at. The example taken here is temperature rise when a fluid flows through a tube which is being heated or say which is being cooled does not matter.

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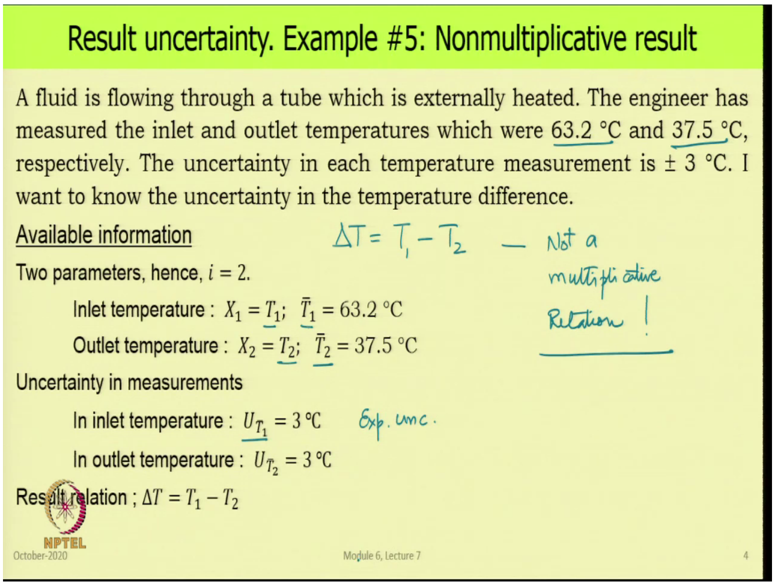

So, here the statement. A fluid is flowing through a tube which is externally heated the engineer has measured the inlet and outlet temperatures which was 63.2 degrees Celsius and 37.5 degrees Celsius respectively. The uncertainty in each temperature measurement is plus minus 3 degree Celsius and the person says I want to know the uncertainty in the temperature difference.

So, let us phrase the problem in our own terms and symbols. First we have our result formula we say that delta T is equal to the difference of two temperatures T 1 and T 2 and we say there are two parameters i and i equal to 2, inlet temperature which is parameter number 1 is T 1 and its mean value is given to us is 63.2 degrees Celsius. This one.

The outlet temperature is the second parameter T 2 and its value given to us the mean value here is 37.5 degree Celsius then we also told the uncertainty in the measurements.

So, in the inlet temperature measurement the uncertainty is 3 degree C which is the expanded uncertainty. So, this is U capital U T 1 bar which is 3 degree C and outlet temperature same thing U T 2 bar is 3 degree C our result relation is delta T equal to T 1 minus T 2 this is not a multiplicative relation. So, the shortcut that we learned for multiplicative relations cannot be applied here and we have to do the full analysis from first principles.

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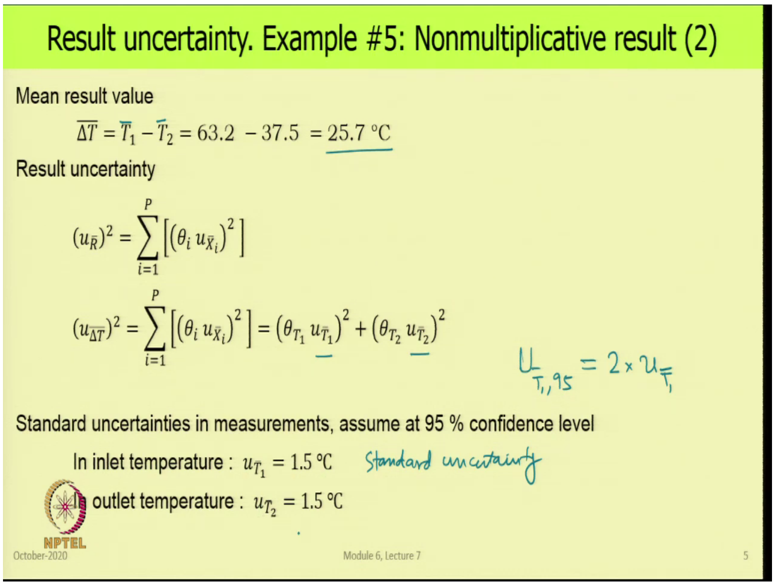

So, first we calculate the mean value of the result delta T bar which is a difference of these two temperatures T 1 bar minus T 2 bar and this is 25.7 degrees Celsius. The result uncertainty relation is that u R bar square if the sum of theta i u X i bar whole square added over for all the parameters which in our case becomes u delta T bar is theta T 1 u T 1 bar theta T 2 u T 2 bar both of them are squared.

Now, we were given U as the uncertainty U T 1 bar and this technically is what we have been using the symbol T 1 95. So, this will be that constant factor which is for 95 percent it is 2 multiplied by the standard uncertainty in the measurement u T 1 bar. And when you do that we get the standard uncertainty or you can call it standard combined uncertainty in each temperature measurement is 1.5 degree Celsius.

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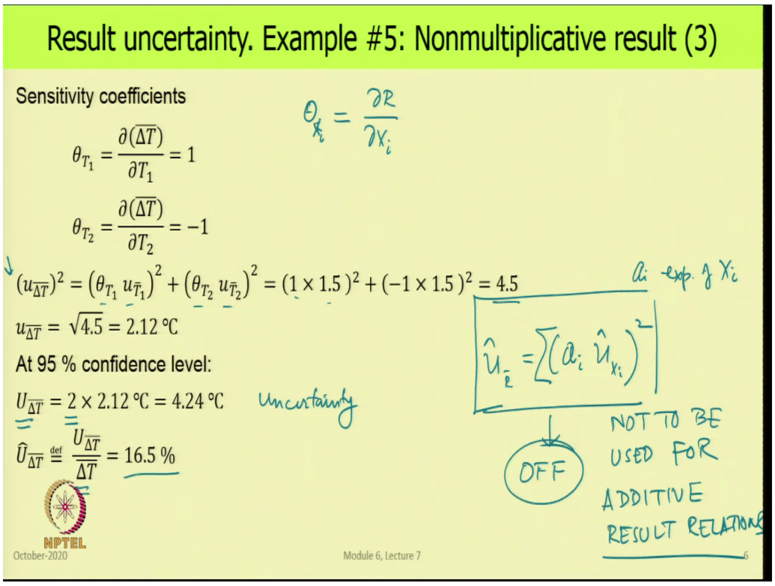

Now, we calculate the sensitivity coefficients, now theta i is defined as del R upon del X i or theta X i if you want to call it. So, theta T 1 is del T by delta 1 this is 1, theta T 2 is del delta T by delta by d T 2 which is minus 1. So, in our uncertainty expression here we have all the values of all these 4 terms we substitute those and we get the sum of squares as 4.5.

And so, u delta T bar is equal to 2.12 degree Celsius which at 95 percent confidence level we multiply it by 2 to get the uncertainty as 4.24 degree Celsius and U hat delta T which is the percentage uncertainty is 4.24 divided by our mean value which was that mean value of temperature difference this turns out to be 16.5 percent.

So, that is the way to handle additive and subtractive relations. If we thought that we could do it by saying that u hat R bar is the exponent of each into u hat of that expression which we could do for multiplicative relations. Where a i is the exponent of X i, then if you use this formula with our expression delta T equal to T 1 minus T 2, you will find that the result is completely off, this cannot be used for additive relations.

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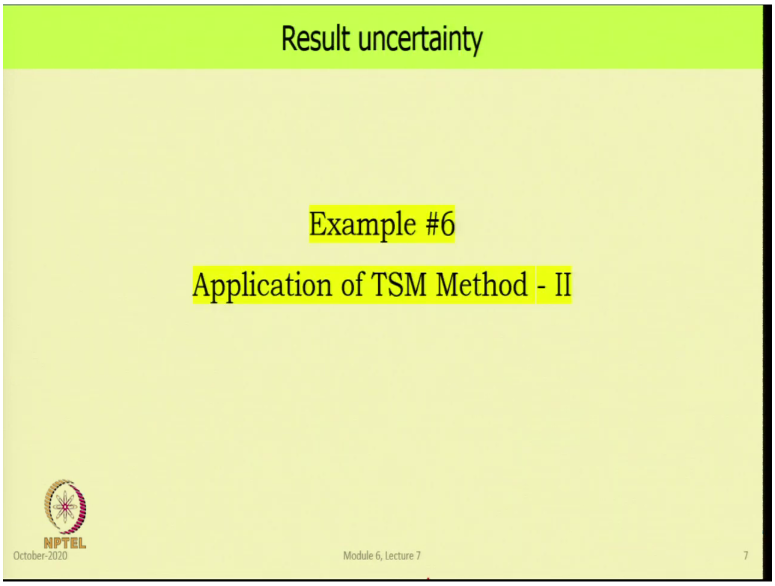

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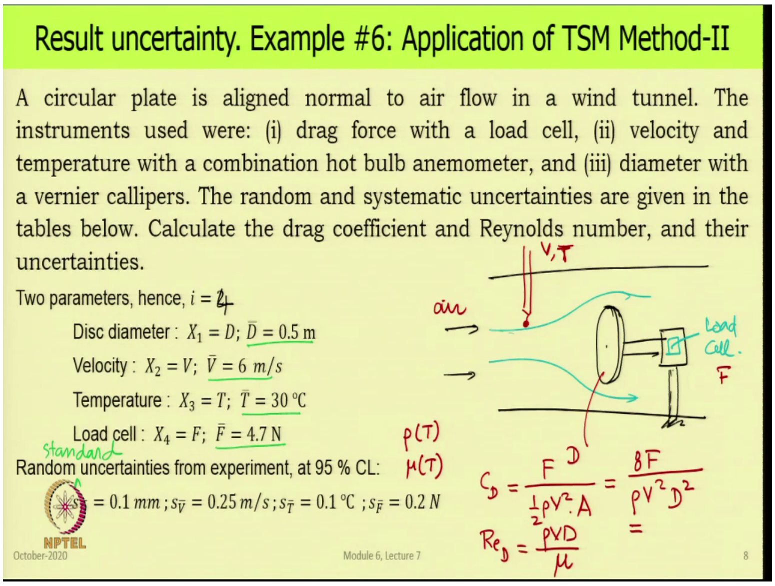

So, that is the uniqueness of this problem and difference between two values we come across very frequently in applications. We will now look at an example where we have to use the Taylor series method the method number II which gives details about the breakups of elemental and random uncertainties. So, here is the problem statement.

A circular plate is aligned normal to air flow in a wind tunnel the instrument used were a drag force with a load cell, velocity was measured with temperature velocity and temperature were measured with the combination bulb anemometer. So, this is an instrument which has a sensor which gives both velocity and temperature and the diameter was measured with a vernier callipers.

The random and systematic uncertainties are given in the tables in the information below. We have to calculate the drag coefficient and Reynolds number and their uncertainties. So, here we have 4 parameters, diameter, velocity, temperature and force. Their nominal values that are given in this problem they are like this.

Diameter 0.5 meters, velocity 6 meter per second, temperature 30 degree Celsius and force 4.7 Newton’s and the random standard uncertainties. So, s means these are the random standard uncertainties they are 0.1 millimetre in diameter, 0.25 meter per second in velocity, 0.1 degree Celsius in temperature and 0.2 Newton’s in the force.

So, this sketch of this setup would look something like this that we make a channel; these are two walls which are on the top and bottom there is airflow established in there and what we have here is a circular disc which is fixed to a load cell which is anchored to the floor of the wind tunnel.

So, air flows around this, exerts a force which is measured by the pressure transducer the force transducer here the load cell, the velocity is measured by putting an instrument. And what we do is we have this instrument which has been put here it has a sensor and this gives both velocity of the air. So, this is air going in and temperature.

So, we have these two measurements the diameter of this disc we are measured separately as D, then load cell force is measures the force. So, the way we will calculate the drag coefficient the definition of drag coefficient is force divided by half rho V square times the projected area normal to the flow which in our case is the force divided by this is pi V square by 4. So, this will become 8 F rho V square D square.

Now, when you see this problem we got rho here and if you look at the other result formula Reynolds number this is rho V D divided by mu. Rho and mu have not been measured, but we have measured the temperature and that is a basis on which we will get rho as a function of temperature, mu as a function of temperature and then we can go ahead with this calculation.

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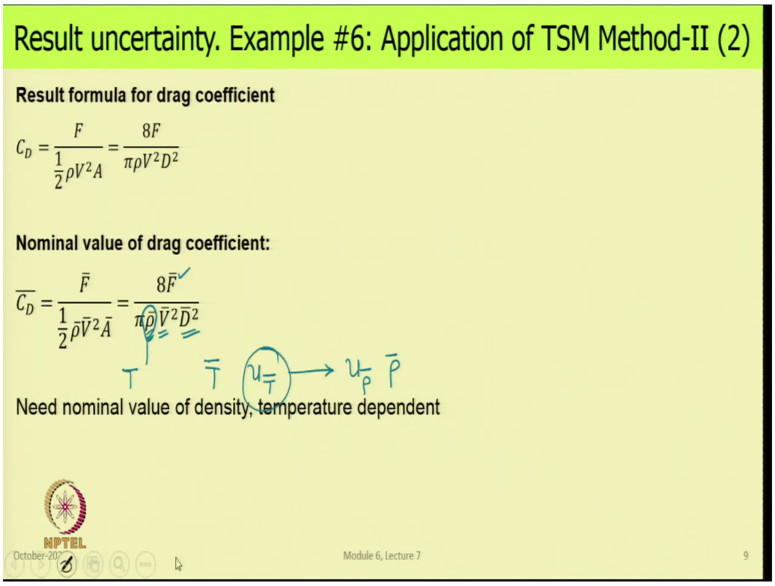

So, let us see how to solve this problem. So, first we have our result formula for drag coefficient and the nominal value is that we just put a bar. So, these were all the values which were measured. So, if you look at what we had just now in the previous slide we had this value, we had this value, we have this value and we have this value, this value is not directly available, but as a temperature.

So, we first need to do a small calculation to get the temperature and then from there the density. So, temperature when we have the nominal value of temperature, we need to get the standard uncertainty in the temperature using that information we will calculate standard uncertainty in the density and also the density itself.

So, that is like deviation and a strategy for solving has to become slightly different that I first need to calculate this and for that purpose what we will do first is to calculate and tabulate all the random uncertainties and the systematic uncertainties for each measurement. So, there will be 4 such tables coming up this is the first one where we are measuring this is about the disc diameter D, we are given that the random standard uncertainty is this much.

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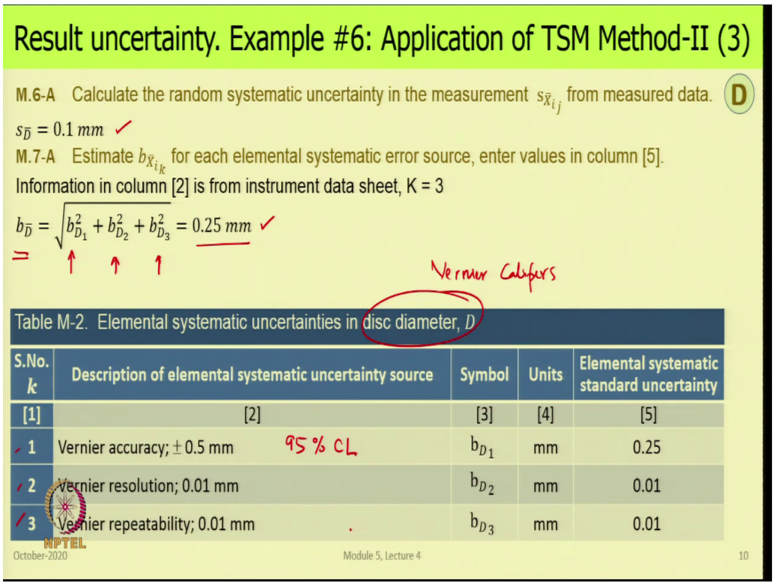

So, we have that value. For the elemental systematic uncertainty we look up what instrument was there and this was a vernier callipers. And from information from the web about different manufacturers and what they have reported and what their instruments do.

Here are some typical values that the 1 year accuracy is plus minus 0.5 millimetres and the symbol for that is b D 1 this is an elemental systematic uncertainty and this is 0.25 which is half of this value because we are going to take these values at 95 percent confidence level.

Then vernier resolution is 0.01 millimetre which we take as it is and vernier repeatability is 0.01 millimetres that also we take as it is. And then we go back to the formula here this is the systematic standard uncertainty in diameter. These are the elemental systematic standard uncertainties 3 error sources were there case 1, 2, 3 here 1, 2 and 3 we put them and this is what we get. So, that is our b D bar. So, that takes care of the diameter.

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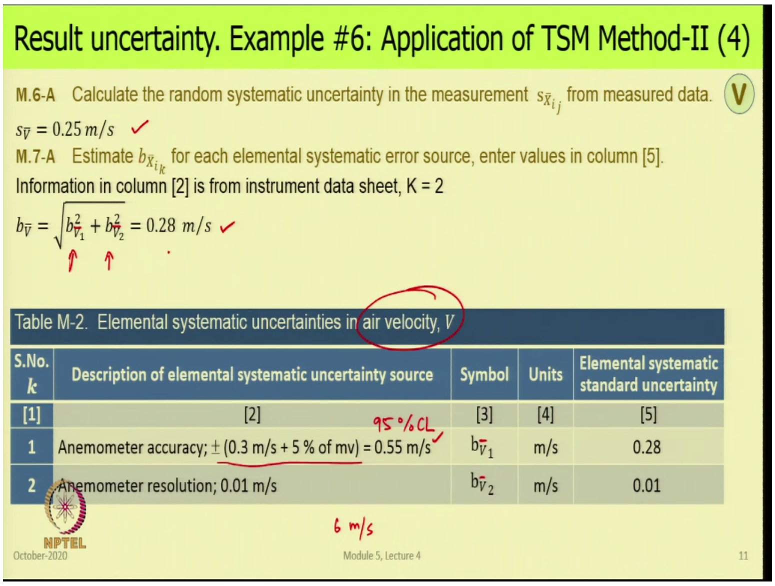

Now, let us go to the next parameter and here we are looking at air velocity V. From the information that is given to us the random standard uncertainty in velocity is 0.25 meters per second. We now have to estimate the b V bar which is the systematic standard uncertainty in velocity from the elemental standard uncertainties.

So, again we can go to the web look up what our instruments are there and see what the manufacturers tell us and from there we are able to identify two sources of systematic error.

One is anemometer accuracy which in one case is specified as 0.3 meters per second plus 5 percent of measured value.

So, measured value is nothing, but what we have been given that is 6 meters per second. So, measured value is 6 meter per second multiply that by 0.05 add 0.3 to it and that is the value we get. So, this is the accuracy and divide this by 2 we get 0.28 again we are operating at 95 percent confidence level. So, this is b V 1 b V 1 bar the anemometer resolution is 0.01 meter per second. So, b V 2 bar is 0.01.

So, we put these values over there and our systematic standard uncertainty in velocity b V bar is 0.28 meter per second. So, we got both random and systematic standard uncertainties for velocity. We next to get air temperature and remember this is something we want to calculate density and viscosity. We are given that the random standard uncertainty there is 0.1 degree Celsius. We need to get the systematic standard uncertainty.

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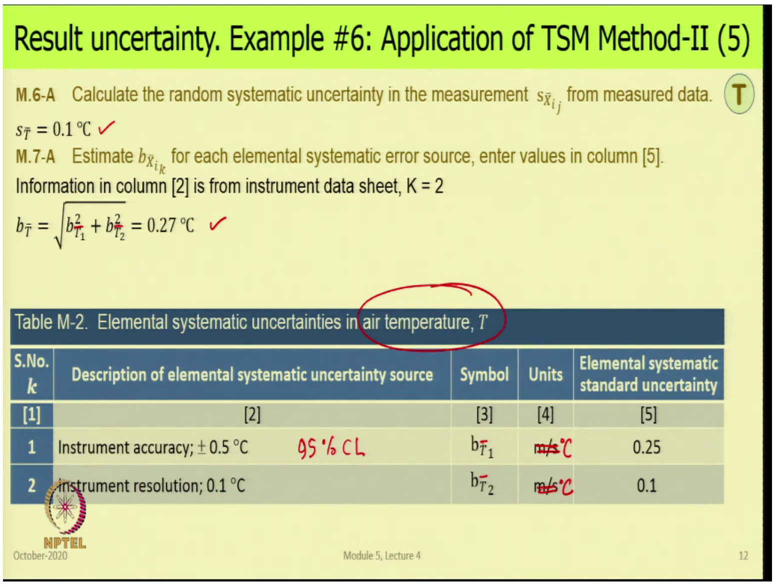

So, again we do the same thing look at what instruments tell us and your instrument accuracy is 0.9 0.05 degree Celsius and add 95 percent confidence level this becomes 0.25 half of that. So, the elemental systematic standard uncertainty due to accuracy is 0.25 and similarly because of instrument resolution which we take as 0.1 degree C. So, the units here are degree Celsius this is 0.1.

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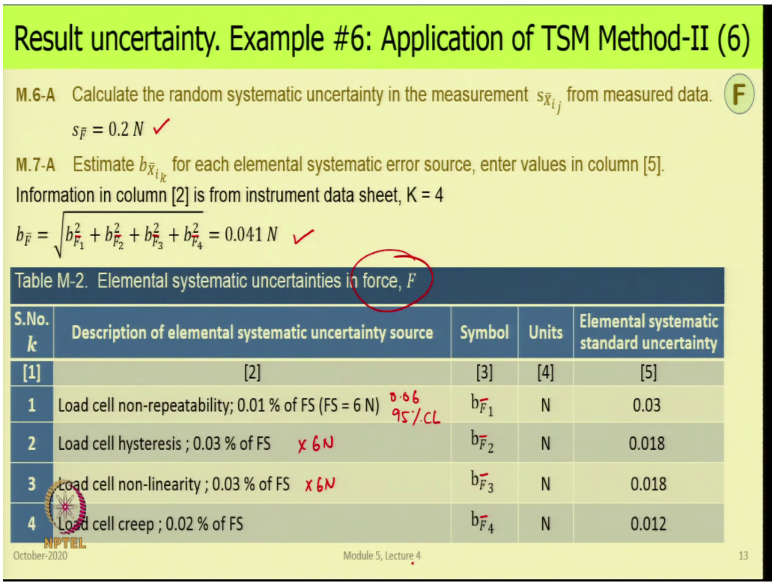

And we go back and add in this formula and we get that the systematic standard uncertainty in temperature is 0.27 degrees Celsius and then the fourth 3 measurement is the force, we are given that this random standard uncertainty in the force is 0.2 Newton's we look up.

So, this is the force a similar table and we are able to identify 4 sources of elemental systematic uncertainties which is non repeatability 0.1 percent of full scale. Full scale we have taken a 6 Newton's and we are measuring about four point something Newton's. If you had another instrument which was of 10 Newton's rate this would have been 10.

So, at 6 Newton's which is the closest that it captures 4.7 Newton's, we get this is 0.06 and at 95 percent confidence level, this becomes half of this. So, the elemental systematic standard uncertainty in force is due to is non repeatability is 0.03 Newton's. Then there is hysteresis associated with the load cell with this quoted as 0.03 percent of full scale.

So, if we take that multiplied by 6 and we take that as 0.018 Newton's. Then there is non-linearity and error due to that is 0.03 percent of full scale. So, b F bar F 3 bar is 0.01 8 which is this into 6 Newton's and the 4th one is due to creep which is 0.02 percent of full scale and this we get as 0.012 Newton's so this is b F bar 4. So, we put all these values in this relation and our value is 0.041 Newton's that is the systematic standard uncertainty in the force measurement due to the load cell.

So, in doing this we did for all 4 measurements we got the systematic standard uncertainty and the random standard uncertainty. Now we will go back to get doing the calculation for density. So, the first thing is we look at what is uncertainty in the temperature, the mean value of temperature is 30 degree Celsius and from standard property data we get the mean value of density at this temperature as this, we are assuming this is at 1 atmosphere.

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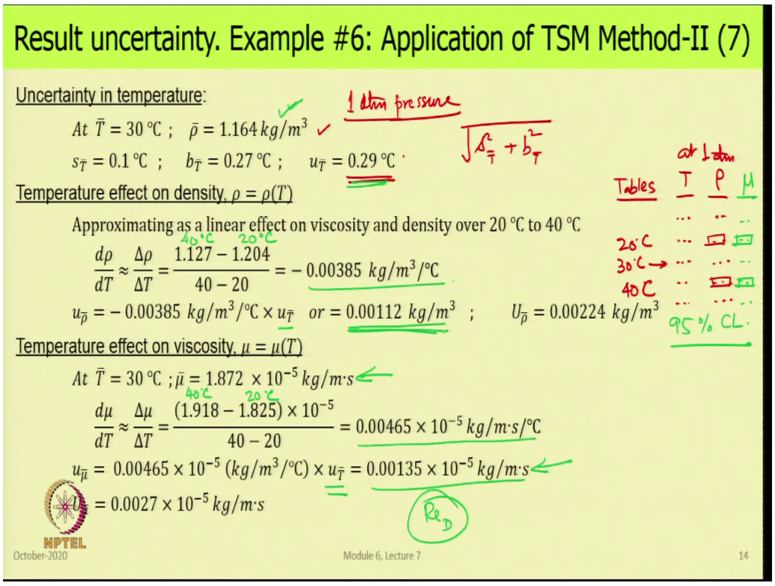

We are given s T bar is 0.1 degree C from the table we got b T bar is 0.27 degree Celsius and we calculate the standard uncertainty in temperature at this sum of the square of these two s T bar square plus b T bar square under root. When you do that this is the value we get u T bar is 0.29 degree Celsius. So, now, we can proceed in two ways either use the result formula for an ideal gas or we use the tables data.

And what I have done here is to use data from property tables where it is listed temperature and density at 1 atmospheric pressure. So, there is a temperature there is a density. So, these are the columns which are there and these are discrete columns maybe at 5 degree Celsius interval or 10 degree Celsius and from there I have taken data. So, our temperature is 30 degree Celsius.

So, say well this is 30 degree Celsius and the next few entries are there and we have taken that we will take one lower temperature which is a 10 degree below this 20 degree Celsius and another 10 degrees about this which is 40 degree Celsius and then we read out the corresponding values over there. So, there we have d rho by dT we approximate this as delta rho by delta T so is a numerical's technique of calculating the sensitivity coefficient this is at 40 degree Celsius it is 1.127.

So, this is the value we got at 40 degree Celsius minus 1.204 this is at 20 degree Celsius, this is 40 minus 20 and we get d rho by dT as minus 0.00385 kg per meter cube per degree Celsius. And then being a single parameter property the result formula gives that U rho bar is this value which is the sensitivity coefficient multiplied by u T bar which is this value here and the answer is 0.00112 kg per meter cube and the expanded uncertainty it double this value at 95 percent confidence level.

For the drag coefficient calculation, we do not require the viscosity part, but I followed the same process that we looked at another table where instead of rho it gave the value of mu again like that and we picked up the value at 20 degree C and 40 degree C. So, this is dynamic viscosity at 40 degree Celsius and this is dynamic viscosity at 20 degree Celsius 40 minus 20. And this is the answer we get the 0.00465 to 10 to power minus 5 kg per meter second per degree Celsius.

And again so, you mu bar we multiply that by u T bar which is this much. So, if you have to do the Reynolds number calculation, then we will require both these properties one is the mean value and the second is the standard uncertainty these two values we carry forward. For now we are only looking at density. So, this is what we need we have the nominal density as this value here and the uncertainty as this value over here.

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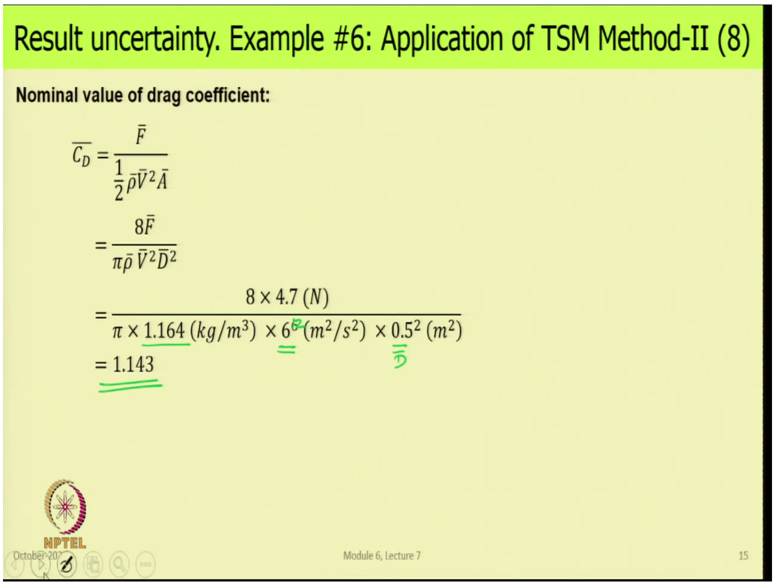

So, now we can go back and say now I have all the information to calculate the nominal value of the drag coefficient and we will do that calculation add the mean values that are reported. So, is F bar upon rho bar V bar square A bar or a pi rho bar V square bar D square bar and we substitute all the numbers that we had earlier our drag force which is given to us in the problem statement was 4.7 Newton's, density we just calculated the velocity is 6 meter per second.

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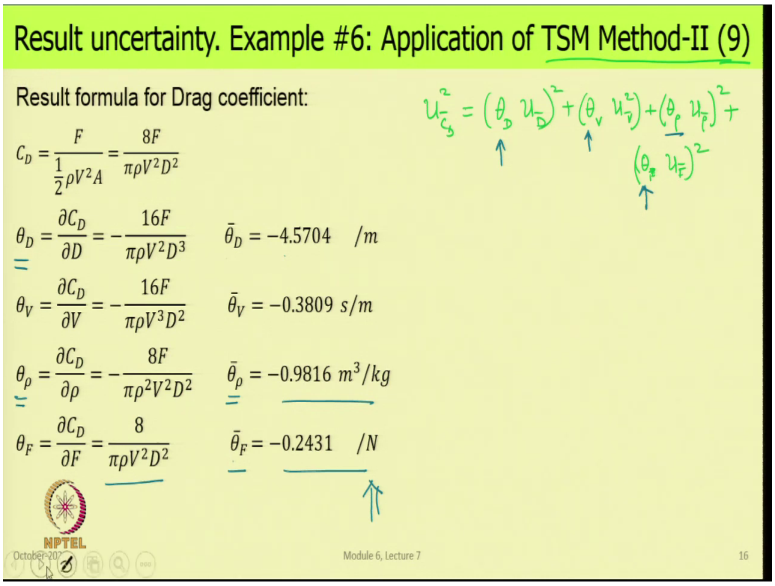

So, this is sorry this is not 6, this is square into 0.5 square meter, this is the diameter when you do that calculation we get 1.143. So, the nominal value of drag coefficient is 1.143. Now we have to calculate the uncertainty that is the next thing and for uncertainty calculation we have two options; one is we know that by looking at this relation this is a multiplicative relation.

So, we can take the shortcut or we can go the long way to get all the sensitivity coefficients, but because we are fully following TSM method number II, we will take the long method. It gives us lot more information which the simple multiplier formula does not give. So, this is what we have and we will ultimately we are working with the fact that u C D bars square this is equal to theta D u D bar square plus theta v u v bar square plus theta rho u rho bar square plus theta F u F bar square.

So, that is the formula that we ultimately use, but the before that what we need to do is we need to calculate all these sensitivity coefficients and that is what we have done here we start with theta D, d C D by d V we differentiate it, we put all the values at the nominal value. So, theta D bar which is the nominal value of the sensitivity coefficient is this much.

Similarly, theta V we differentiate with respect to V and we get this relation and that gives us minus 0.3809 seconds per meter square. Then sensitivity coefficient for density this is 8 F upon pi rho square V square D square with a minus sign and that is what we get and finally, sensitivity coefficient for the force and we get this formula substitute the numbers into each one of these and this is the number we get and the units of each one of them are written here.

This is per meter, this is seconds per meter, this is meter cube per kg, this is per Newton. The thetas are all dimensional, but their dimensions are not the same as the result or the measure angles this is something different.

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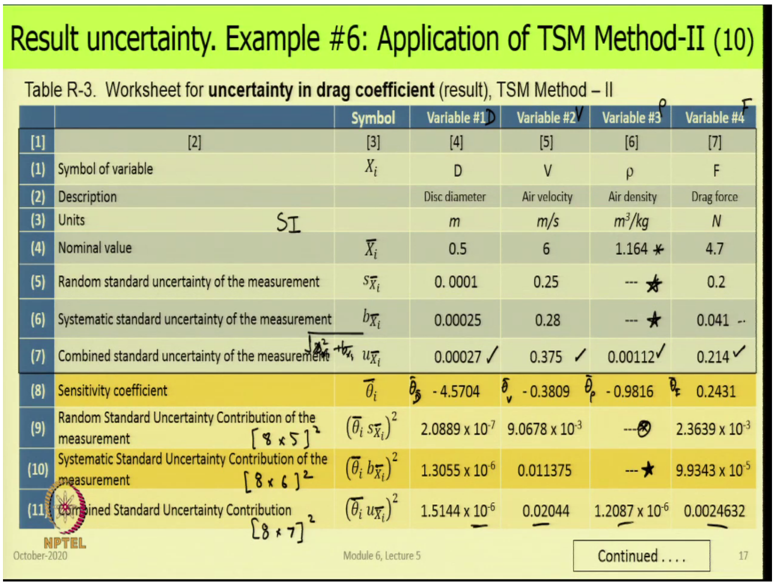

Result uncertainty. Example #6: Application of TSM Method-II (10)

Table R-3. Worksheet for **uncertainty in drag coefficient** (result), TSM Method – II

		Symbol	Variable #1	Variable #2	Variable #3	Variable #4
[1]	[2]	[3]	[4]	[5]	[6]	[7]
(1)	Symbol of variable	X_i	D	V	ρ	F
(2)	Description		Disc diameter	Air velocity	Air density	Drag force
(3)	Units		m	m/s	m^3/kg	N
(4)	Nominal value	$\bar{X}_i$	0.5	6	1.164	4.7
(5)	Random standard uncertainty of the measurement	$s_{\bar{X}_i}$	0. 0001	0.25	---	0.2
(6)	Systematic standard uncertainty of the measurement	$b_{\bar{X}_i}$	0.00025	0.28	---	0.041
(7)	Combined standard uncertainty of the measurement	$u_{\bar{X}_i}$	0.00027	0.375	0.00112	0.214
(8)	Sensitivity coefficient	$\bar{\theta}_i$	- 4.5704	- 0.3809	- 0.9816	0.2431
(9)	Random Standard Uncertainty Contribution of the measurement	$(\bar{\theta}_i s_{\bar{X}_i})^2$	2.0889 x 10^{-7}	9.0678 x 10^{-3}	---	2.3639 x 10^{-3}
(10)	Systematic Standard Uncertainty Contribution of the measurement	$(\bar{\theta}_i b_{\bar{X}_i})^2$	1.3055 x 10^{-6}	0.011375	---	9.9343 x 10^{-5}
(11)	Combined Standard Uncertainty Contribution	$(\bar{\theta}_i u_{\bar{X}_i})^2$	1.5144 x 10^{-6}	0.02044	1.2087 x 10^{-6}	0.0024632

So, thetas are calculated now, now we can go to the next step and let us start listing all these in a table. So, in this table we will first make the first 7 rows which is just taking data from what we already have and putting it over here. So, this is variable number 1 which we will see is diameter, number 2 is the velocity, number 3 is the density and number 4 is the force.

So, that is what here in the first row symbol of the variable d V rho F. Then description, this is this diameter, this air velocity air density and this is drag force. And the units; units for these parameters meters meter per second meter per kg and Newton's. So, we are going to be sticking is best that we always stick with SI system and even though we use other numbers come in millimetres come in kilo Pascal, mega Pascal will come ultimately we will write everything in all calculations in SI.

The nominal value we have been given. So, SI bar is 0.5 meters, here 6 1.164 we got from our earlier calculation. So, this is not in that sense a measured value, but from temperature we got this value and this is 4.7 Newton's. Now we list what we have already got random standard uncertainty of the measurement has been given to us, we have just put it down here in the same units this is 0.0001 meters 0.25 meter per second.

We do not have this value for density. If we wanted we could have calculated that, but we did not we could not we did not do that one. So, we skipped this part and this is 0.2. So, this is all given to us then we like the systematic standard uncertainty of the measurement b X i bar this is this much has been be calculated. This we just calculated as 0.28 meter per second we just reproduce it here this we have not calculated.

And this is coming here 0.041. In a minute we will see that even if we did not calculate these in the ultimate analysis this does not matter. Because contribution of density to the uncertainty is quite negligible and we would have been quite happy to say that density is constant and we go ahead with it we could have solved the problem that we also.

So, the combined standard uncertainty of the measurement u X i bar. This is square root of the sum of the squares of these two. So, this is square root X i bar square plus b X i bar square whole square root. So, when you do that this square plus the square root you get this value, similarly here this, this one and then this one. So, all of these have units of the measurand so, rows 1 to 7 we have got whatever we wanted there. Now we have already calculated a sensitivity coefficient.

So, we will start writing that down here. So, this is theta bar D, this is theta bar V, this is theta bar rho and this is theta bar F. So, same values that we just calculated in the previous slide we have just put it here. Now we start combining and say random standard uncertainty contribution of the measurement theta i bar into s X i bar whole squared. So, this row 9 is theta i bar which is row number 8 multiplied by s X i bar which is row number 5.

So, this is what we got and we squared this. So, this into this squared comes here similarly we are doing for every one of them and get these numbers. We did not have these two values. So, again here this value is not there. Next systematic standard uncertainty contribution of the measurement we can get theta i bar b X i bar there and we do this time instead of row number 8 and 5. We do the product of 8 and 6 square this and these are the numbers we get here these are so, this is again like before not there.

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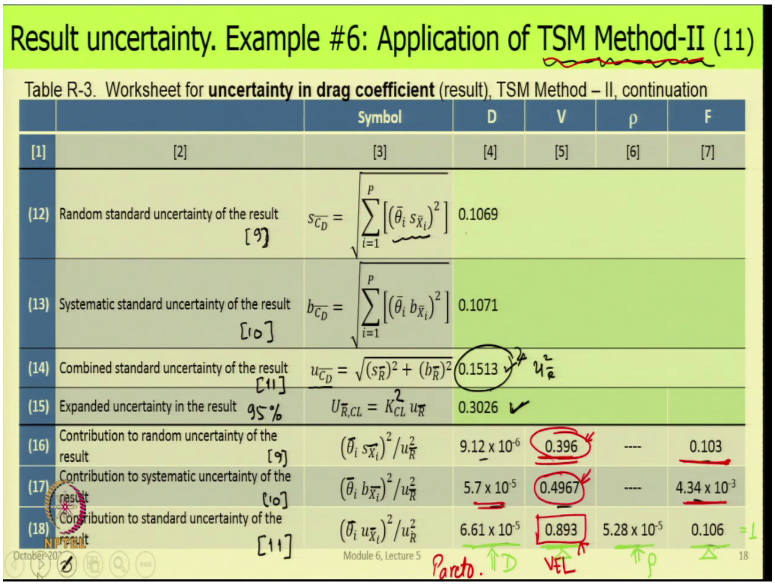

And finally combined standard uncertainty contribution which is theta bar into u X i bar a whole thing square. So, here we are multiplying row 8 multiplied by rows 7 and taking a square and listing the numbers over there. So, this is one part of the table is done. We can continue now and do the remaining part of this table we add up all the values in one by one in these rows these 3 rows 9, 10 and 11.

So, this is adding all the elements in row 9, we get and square root of this one this already we have put in each table, we just had to add them up and take the square root and this is the answer we get here. This is coming from row 10 we added all the other numbers in that row and took their square root that is what you get here. And the combined standard uncertainty of the result again we do the same thing we go back we have those elemental value listed in row number 11.

So, there is 11, this one and let us take a square root and this is 0.1513. So, this is what we were looking for standard uncertainty in the drag coefficient this is 0.1513 and of course, all of these have the same units as the drag which will be Newton's. Then we calculate the expanded uncertainty of the result which is U R bar CL and we will take it at 95 percent confidence level.

So, this case CL is 2 and we multiply this by 2 and we get this value this so, many sorry this is drag coefficient. So, this is got no units. So, these are the two things we got then we look at that individual contributions. So, what we will do is we take the 3 rows there 8, 9, 10 and 11 and normalize them on the value of this which is your u R bar square the result. So, we will normalize it on the square of this.

So, this raise to be power 2. So, this is 9 rho divided by the square of this and this these are the numbers that are listed over there when you do the calculation. This is rho 10 divided by the square of this which will be this and this and this is rho 11 divided by this squared which is these values.

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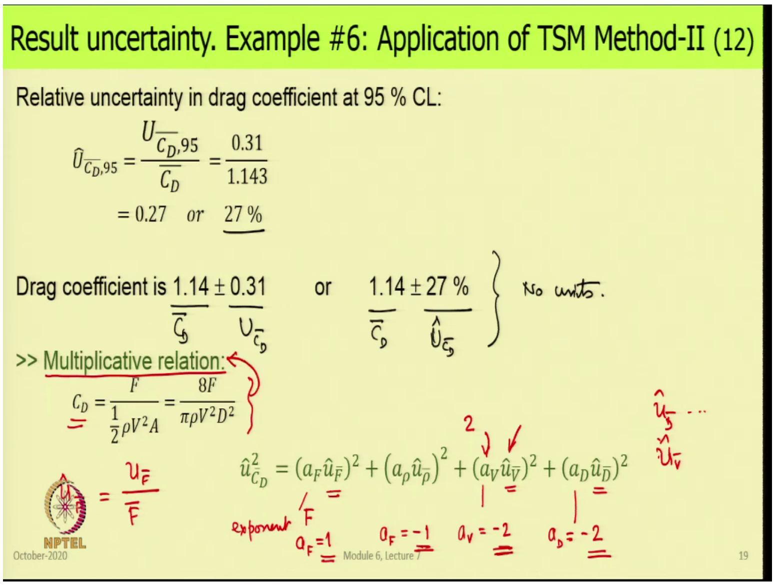

So, having done all of that we can then pick up this number and express our result. We say that the drag coefficient was the nominal value 1.14 and what we got as U CD bar this is 0.31 this was CD bar or as a percentage it is this. So, this is U hat CD bar and this is CD bar. Being drag coefficient there are no units here it is a dimensionless and this relative value came from here were 0.13 divided by 1.14 which is rho 27 percent.

So, as far as what is the answer was there this is gives us the full information this is our answer. But the point of doing method II which is what we are looking at here the Taylor series method II is that we got a lot more information here in this table. So, let us see what it is telling us. First let us look at the last row contribution to standard uncertainty of the result. So, here is what we have?

This value, this, this and this then all add up to 1, but you look at their magnitudes this is something into 10 to power minus 5 this is 5 point something into 10 to power minus 6, this is 0.89 0.106. So, both of these are like at least 4 to 5 orders of magnitude smaller than these two terms.

And this means that the diameter this is what is for that and this is density the density uncertainty came because of uncertainty in the temperature and it is telling us that uncertainties in these two parameters have almost no contribution to the drag coefficient.

So, we could be quite justifiable neglected, the answer will not be very much different. Now the other two numbers that is what we are looking over here this one and this one they are still therefore, both of them are much bigger than the other two, but amongst them this is about 9 times bigger than this one 0.893, 0.106.

So, what it tells us that the single biggest contributor to uncertainty in the drag coefficient is coming from the velocity measurement. So, a strategy to reduce uncertainty in the drag coefficient should begin by tackling the velocity measurement and then further lookup into the contributing elements and what we see is here the random uncertainty in the velocity measurement is 0.396 and the systematic one is 0.4967.

So, about 40 percent is coming from here and 50 percent is coming from there roughly. So, what do we see? Both of them are equally important. So, to reduce this we got to tackle both of these. So, here we have to get a better instrument here we decided to see more see if there is anything else happening in the experiment take much more readings and reduce the uncertainty as much as possible.

And now within this also we see that in this row the random uncertainty contribution this is the biggest one, then the force and similarly here again this is very small this is also small and on the systematic uncertainties the biggest contributor is in velocity measurement. This information you can also show graphically in a Pareto chart and in one picture you got to know what is doing what.

So, that is lot of information that came out because we have Taylor series method II, Taylor series method I would not have given us this much detail. We also have another option if we look at the drag coefficient formula this is 8F by pi rho V squared D square this is a multiplicative relation there is no addition, no subtraction. So, we could have taken a shortcut and the shortcut goes like this.

That u hat which is a relative standard uncertainty in the drag coefficient the square of that is a F into u hat a F this is relative standard uncertainty in force a F is the exponent of force. In this case exponent of F which as you can see from this relation it is 8F. So, a F is equal to 1 and similarly we add three more terms one for density, one for velocity, one for diameter and in this case we can see a F is equal to minus 1.

In this case a V is equal to minus 2 and a D is equal to minus 2. So, something nice comes up quite quickly here is that because of these tools this uncertainties will get magnified more than the ones where the exponent is one that one thing comes out. The other thing if you are compared when you make do the calculations for u hats for all these 4 values we will find that u hat D is very small whereas, u hat v is not that small.

And we will immediately see why uncertainty in velocity is having such a big impact because firstly, its value is reasonably large and second its exponent is 2. So, in this formula when you square it this will become 4 and then you take the square root.

So, the contribution coming from velocity is getting further amplified and that is what we saw in the Pareto earlier analysis. So, we could have quite comfortably done this method, remember the definition of u hat so u hat in any one of these parameters.

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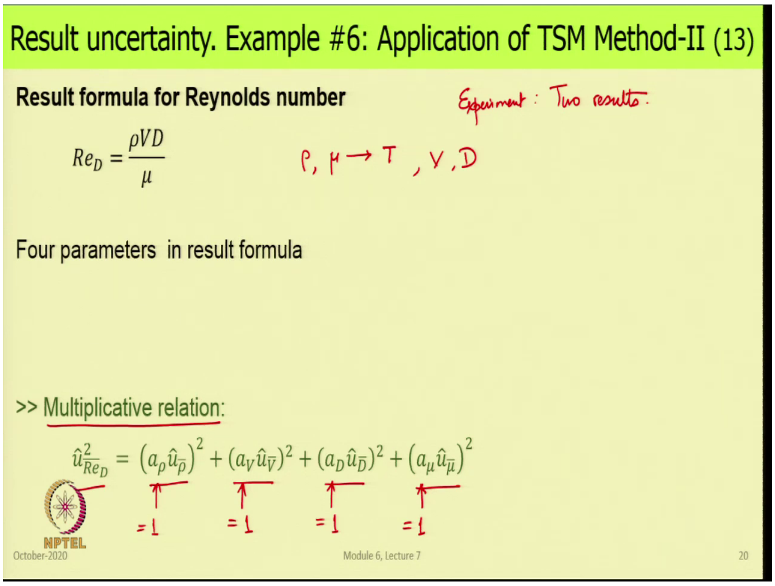

Let us say force, this is u F bar upon F bar the nominal value of the first. So, it is the same relation for all these relative standard uncertainties and we could have done it because the formula happens to be a multiplicative relation. So, that completes our discussion on the drag coefficient in this experiment we will calculate one more result.

So, this is an experiment where there are two results and here the results Reynolds number is rho V D upon mu, rho and mu we just saw they came from temperature and we had calculated their uncertainties in addition we have V and D in this case the force is not a parameter in this formula.

So, there are 4 parameters in this result formula you can follow the same procedure that we have done there. In fact, many 3 of the columns will be common only mu is the new one coming in and you will see that rho and mu are both very small contributors.

But diameter is also relatively small contributor then dominant thing is coming from V; however, V as an exponent of one when we look at it as a multiplicative relation. So, this is a multiplicative relation. So, we can apply this that the relative standard uncertainty in Reynolds number is related to these 4 terms where all these coefficients or the exponents they are 1.

So, I will leave this as an exercise for you to do and with that we conclude this example. Now we look at another case of a multivariate relation, but we see how we could go off in the procedure and do something which is very wrong.

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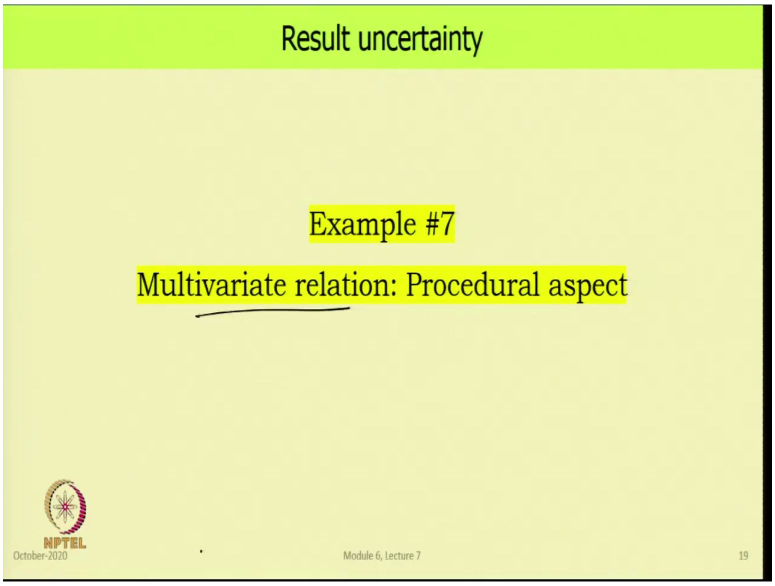

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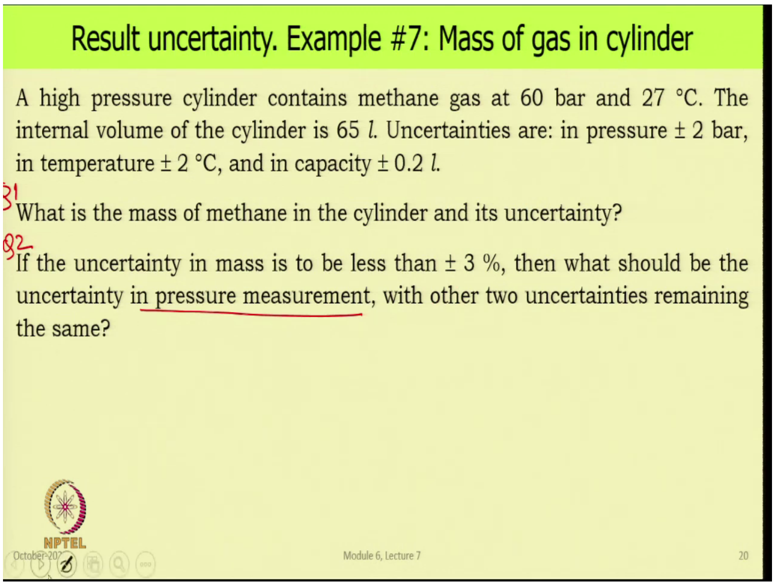

So, the example here is as follows. This is an example of CNG cylinder which you see in many vehicles. So, it says a high pressure cylinder contains methane gas at 60 bar and 27 degrees Celsius, the internal volume of the cylinder is 65 litres, uncertainties are in pressure plus minus 2 bar in temperature plus minus 2 degrees C and incapacity plus minus 0.2 litres. What is the mass of methane in the cylinder and its uncertainty?

So, this is question number 1 and then there is a 2nd question. If the uncertainty in mass is to be less than plus minus 3 percent then what should be the uncertainty in pressure measurement with the other two uncertainties remaining the same? So, we are just doing a calculation here we are not saying how we are going to reduce that uncertainty or in the measurement, but it just tells us that if you want this is our criteria what do we expect? So, here is how we will do this.

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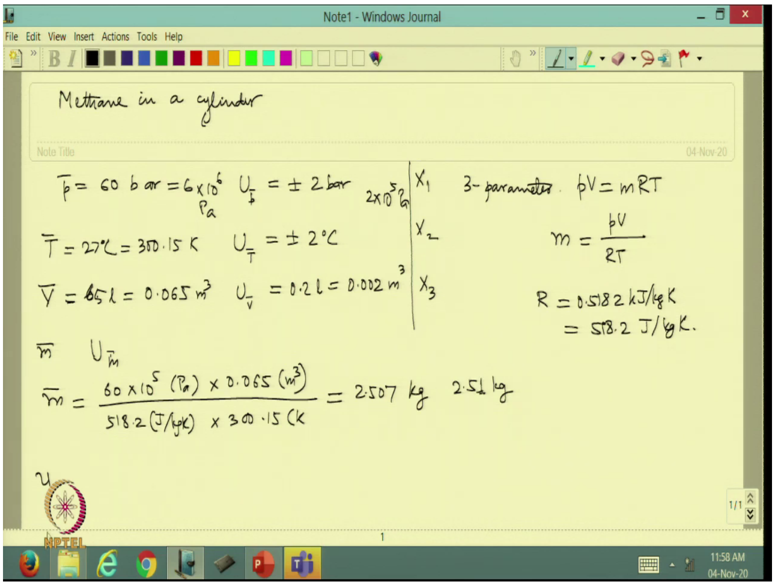

So, this is methane in a cylinder and we have been given that the mean pressure is 60 bar and its uncertainty U p bar is given us plus minus 2 bar then we are given that T bar. The temperature is 27 degree Celsius which is 300.15 Kelvin and U T bar is equal to 2 degree Celsius, then we have volume of the cylinder mean volume is given as point is given as 65 litres which is 0.065 meter cube.

And uncertainty in the volume this is given as 0.2 litres which is 0.002 cubic meters. So, this is all the information we have and we are asked what is uncertainty, what is the mean value of the mass? And what is the expanded uncertainty in the mass. So, we first write we recognize that we have 3 variables we can call this X 1, this is our X 2, this is our X 3.

So, 3 parameter result formula has 3 parameters and what is the result formula? That we have assumed that this is an ideal gas behaviour and so, we have pV is equal to mRT and m is equal to pV upon RT. So, to calculate the mean value of the mass we just say this is equal to we substitute these numbers there R for methane we get from the tables as 0.5182 kilo Joules per kg Kelvin or we will use with SI units consistently we will use Joules per kg Kelvin.

And of course, of pressure this will become 6 into 10 to the power 6 Pascal this is 2 into 10 to the power 5 Pascal. So, we put all these things here in mass which is 60 into 10 to the power 5 Pascal multiplied by 0.065 cubic meters, 518.2 Joules per kg Kelvin multiplied by 300.15 Kelvin and when you do all of this we get the answer as 2.507 kg we can even round this off and say this is 2.51 kg or just 2.5 that we will see.

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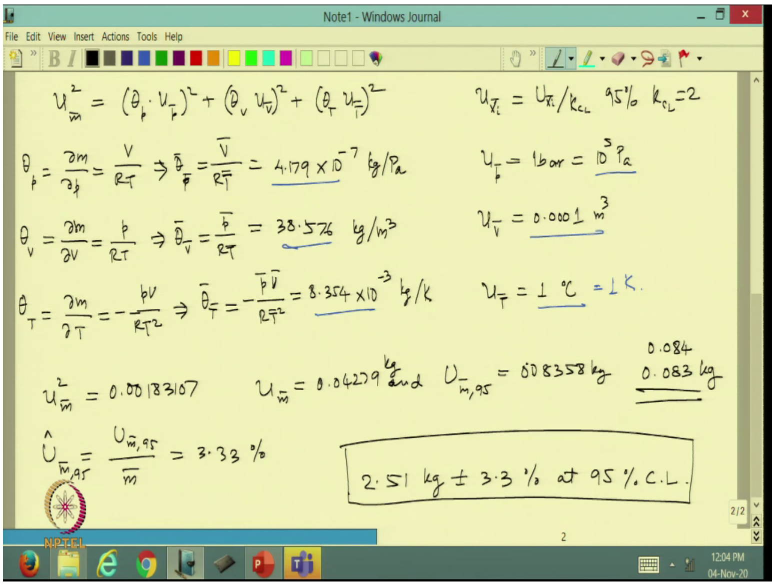

So, that is half of the story done now for the uncertainty calculation we will calculate we have two options; one we go the long way calculate the sensitivity coefficients or we take a shortcut knowing that this is a multiplicative relation. So, we will do it both ways in the first case the full wave our relation will become u m bar square is equal to theta p into up bar square plus theta v into uv bar square plus the same thing for temperature we need to calculate three sensitivity coefficients.

So, we do that theta p is equal to d m by dp and this is v upon RT and when you do this whole calculation by substituting that the mean value of p, this will be V bar upon RT bar this is 4.179 into 10 to the power minus 7 and the units are there kg per Pascal.

So, thetas have units generally they need not be the same drag there may not be the same at all. Now theta v is dm by dv this becomes p upon RT and you do this whole calculation at the mean values theta bar v bar this is equal to p bar upon RT bar and this is 38.576 kilograms per meter cube.

And similar is third one dm by dT this is minus pV upon RT square. So, we get substitute all these values into that at the mean point minus p bar V bar upon RT bar square put the value and you get 8.354 into 10 to the power minus 3 kg per Kelvin. So, there we are we got all our sensitivity coefficients. Now we need the standard uncertainty in each.

So, we realize that the standard uncertainty in a variable will be expanded uncertainty in that variable divided by the K CL. We assume that it is uncertainties reported at 95 percent confidence level. So, K CL is equal to 2. So, what we get here is that u p bar is equal to 1 bar which is equal to 10 to the power 5 Pascal u v bar this will be 0.0001 meter cube half the value of the uncertainty quoted and u T bar is 1 degree Celsius.

Now, you put all these values in this we have all the thetas which is given here. So, these are the values that goes with this, this value goes with this, this value goes with this. Or you even say this is 1 Kelvin we put it in this relation do the whole calculation and the final answer is that u m square is 0.00183107, I just kept so, many decimals we will round it off in a minute and with that we get u m bar as 0.04279 and U m bar 95 percent confidence level, this is twice this value which will be 0.08358 kg this is also kg.

Now we can round this off and we say this is 0.083 kg nothing more is really ok we would even have said 0.084. So, that is what we wanted one part we could have calculated the relative uncertainty U hat m bar, this is U m bar at 95 divided by the mean value of the mass and when you substitute the values.

And calculate it this comes out to be 3.33 percent. So, we did the long method got the answer that we were looking for this is the answer for the first part we got the mass earlier this is the. So, the result answer becomes it is 2.51 kg plus minus 3.3 percent at 95 percent confidence level that is our answer. Now let us look at the other method the shortcut method we can say.

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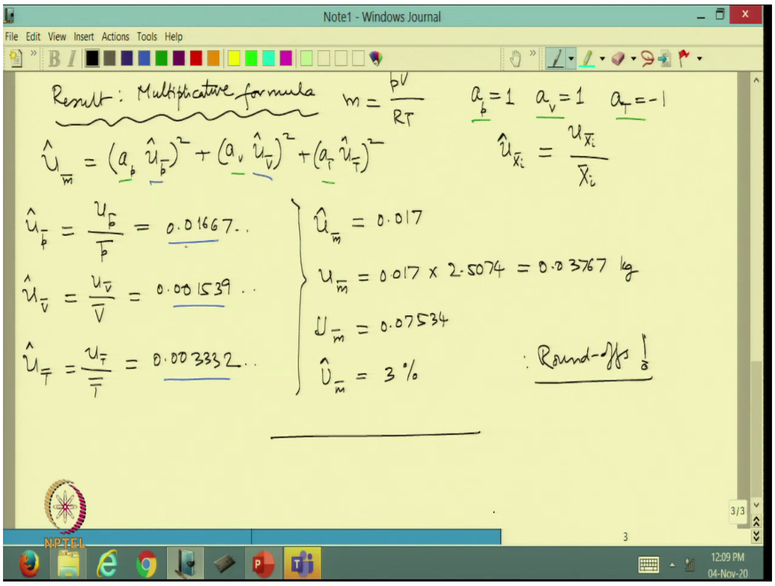

We recognize that the result is a multiplicative formula. So, we will get m is pV by RT then we write the exponents so pV and TR the 3 parameters the exponent of p a p is 1, the exponent of v is 1, exponent of T is minus 1. So, then we know that for such a relation you can take a shortcut and say that U hat or maybe capital U hat which is a non-dimensional standard uncertainty. This is the sum of the individual coefficients multiplied by their relative uncertainties.

Where, u hat for any parameter is defined as u divided by X i bar. So now we need to calculate these values. So, that we can do that u hat p bar this is up bar upon p bar and when you do the calculation this comes out to be 0.01667, u v bar hat this is u v bar upon v bar and when you do that calculation this will come out to be 0.001539. And similarly u hat T bar this is equal to u T bar upon T bar and this is 0.003332.

I just kept decimals just because they come to show you that somewhere on the line we have to do the round offs. Now that we have this we can substitute this values in the. So, we have the u hat bar which is coming here and we have the ap which are coming from here. We just substitute all of that and when you do the calculation we get u hat m bar this comes out to be 0.017.

I am using this formula here this one we get u m bar is 0.017 into the mean value of the mass which we calculated earlier as 2.5074, this comes out to be 0.03767 kg. This is this has got no units u hat is non dimensional you can look at this. So, this is that and finally, U m bar this is the double the value of this which is 0.07534 and when you calculate the relative uncertainty, this is 3 percent.

Earlier we got 3 point 3 percent now we are getting 3 percent and all of this is because of the various round offs that we have done on the way. So if we had kept more decimals over here more significant places we will got a slightly different answers. So, that is another feature of uncertainty analysis that we are doing so, many calculations often involving very small numbers of very big numbers. So, if you round off in between it could lead to error. So, this is the second method.

So, we did the calculation by the multiplicative formula and what you can see here this is much less effort, much less calculation than what was there in the first one. Of course, we are doing it on a spreadsheet or you have a program written for it I any of them will be the same effort. Now we come to the second question that it says that U m bar has to be less than or equal to plus minus 3 percent and the question is that temperature and volume uncertainties are same.

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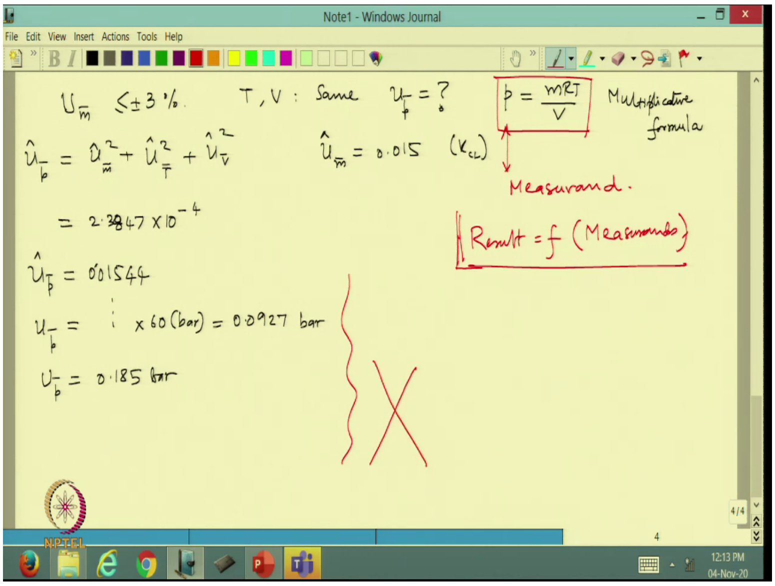

What should be uncertainty in the pressure measurement? This is a question. So, we will be tempted to think that we can right now I want to calculate p. So, p equal to mRT upon V, we look at this as a multiplicative formula and when you do that we can then write u hat p bar which is what we want to calculate as u hat m bar square plus u hat T bar square plus u hat v bar square; u m bar is given as plus minus 3 percent.

So, we can get that standard error in mass this is we can calculate that ratio. So, this is 0.3 percent. So, u hat m bar this is 0.015 multiplied divided by 2 the K CL factor and so, this number comes here we already have the other two numbers. And when you put all these things here this comes out to be 2.3847 into 10 to the power minus 4 or u hat p this is equal to 0.15 0.01544 and then you can calculate u p bar this into the nominal value of the pressure which is 60 bar.

So, it is this multiplied by 60 bar and this becomes 0.09265 or you can call it 2 7 bar. And so, U p bar is 0.185 bar. So, you may think we got the answer, but we have done a big mistake. We use this relation and in this what we have done is that p is on the left side, but remember p is a measurand and in uncertainty analysis we have always written that the result is a function of the various measureands or we call the parameters or variables.

So, the entire analysis rests on this logic to recast the equation in this form distorts this basic requirement of the uncertainty analysis and so, this is wrong. So, this entire answer that we have done here is wrong. So, then what is the correct way to do it? We go back to our result formula that measurands on the right side.

So, m is equal to pV upon RT and we get the same relation earlier that u hat m bar square is equal to u hat p bar square plus u hat v bar square plus u hat T bar square. And now we recast this because we want the uncertainty in the pressure, we say u hat p bar square is equal to u hat m bar square minus the other two uncertainties.

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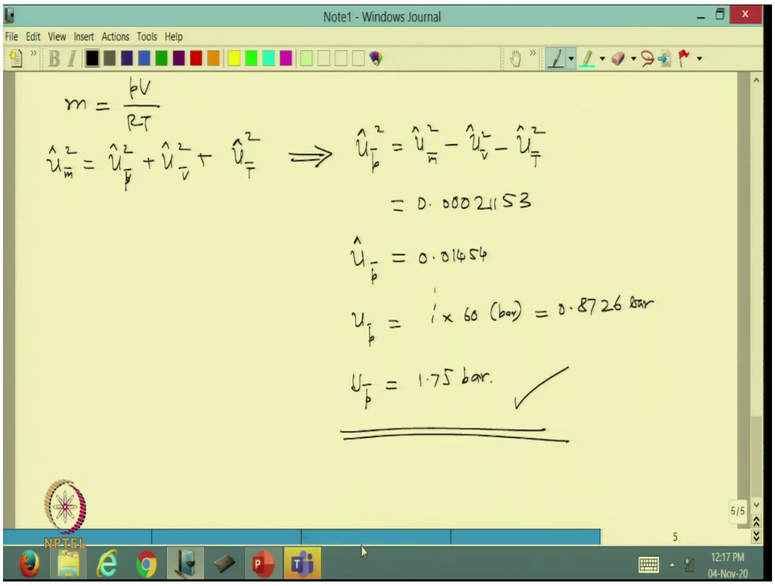

And then we put all our values that we had earlier and do this calculation and this will give us the value of 0.01454 which means that u hat p bar is the square root of this. And then multiply this by 60 bar; and the answer you get so this is square root 0.01454 this value comes here and this final value is 0.8726 bar no ok there is some this is not right.

You take the square root of this will be about 0.2 roughly 0.2; something 0.12 yeah no sorry ok. We go back we do this entire calculation and we do this calculation and we get the answer here as 0.00021153. And we take the square root of that and we get u hat t bar is equal to 0.01454 and from there we get u p bar as this value multiplied by 60 bar which is equal to 0.8726 bar and the expanded uncertainty in pressure should then be twice this value which is about 1.75 bar.

So, this is the right answer, it tells you that earlier we were measuring the uncertainty of 2 bar we reduced it to 1.75 bar we can reduce the uncertainty from 6 percent something to 3 percent. So, that is another example where you are require to calculate expected uncertainty in the measurement.

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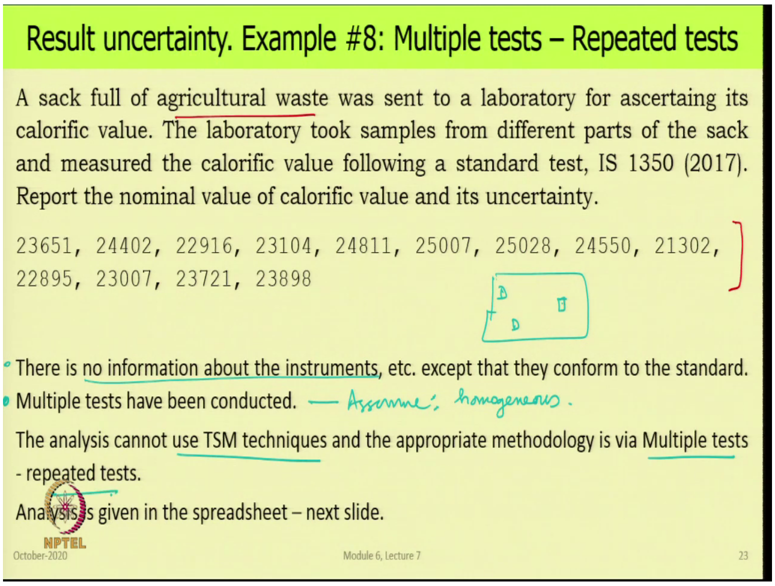

If you want to achieve a certain uncertainty in the result. So, that is how we have done this, we saw the entire process and we got the answer. So, we solved this problem. The next example is of multiple tests repeated tests. So, here is the problem statement, that somebody got a sack full of agricultural waste was sent to a laboratory for ascertaining its calorific value. The laboratory took samples from different parts of the sack and measured the calorific value following a standard test IS 1350 2017.

The following other data of the nominal calorific value. So, we have got all these numbers which came out. Now in this there is no information about the instruments we do not know what the elemental contributions random or systematic uncertainties are. We assume the experiment has been conducted in the same way and we also assume in this particular case.

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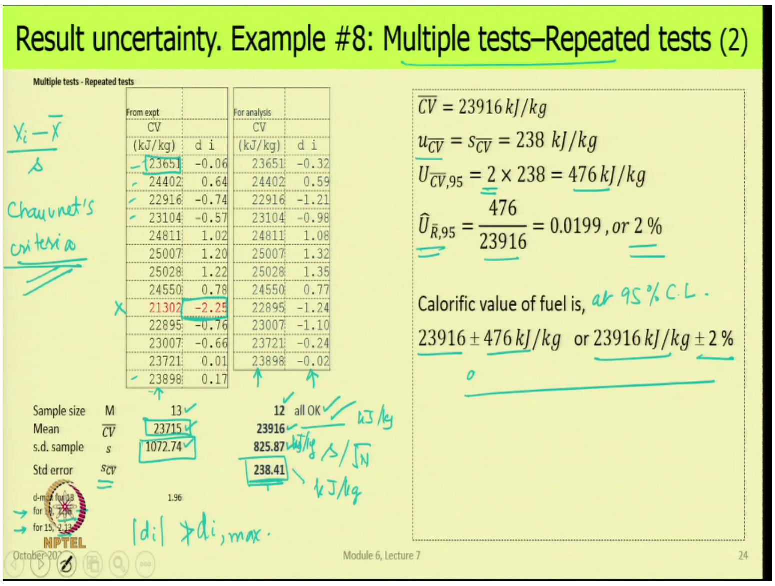

That if we have a big bundle of some material, we take a sample from there, we take a sample from there, we take a sample from there, there could be variations from sample to sample that could be one of the objectives of such an experiment to begin with, but we are assuming that the sample does not vary, it is homogeneous. We cannot use the TSM techniques and the data that is given the appropriate approach is multiple tests repeated tests.

So, let us relook at the analysis and this we have done in a spreadsheet you can do it in many other ways. So, here are in this column all these, this one these are all the values that we have been given this the calorific value in kilo Joules per kg. Using these values we got the sample size is 13, we calculated the average which came out to be this and we calculated the standard deviation of the sample which came out to be this.

Next we applied Chauvenet's criteria that should be reject any reading and to do that we calculate the deviation of each reading which is X i minus X bar upon s. So, X i is say this value X bar is this value s is this value and we do this calculation for every value we have in this data set and this is a deviations given here. Then we ask the question that if 13 samples are so, that 13 is our sample size what is the maximum permissible deviation.

According to this criteria and we look up the tables and we say that it says that 4 sample size 10 you should have no deviation greater than 1.96, for 15 the deviation should not exceed 2.13. So, let us take a little more Chauvenet's criteria of 10 which is 1.96 and we look at all these numbers here and we find that 2.25 its value exceeds 1.96 the modulus of this value; that means, mode d i that we got should not exceed the d i maximum given by this criteria the Chauvenet's criteria.

So, this reading we have to reject. So, we do that take all the remaining readings into this column, again now our sample size is 12, mean this is our standard deviation and again we calculate the deviations for each measurement and we get these values. And again our limiting number is 1.96 none of these are more than 1.96.

So, all are ok then we calculate our sample standard error which is s by square root N which is 8 point 825.87 divided by square root 12 which is this and this has got the units kilo Joules per kg. So, does the mean value and the standard deviation. So, that is a standard error standard uncertainty. So, our now result becomes a mean value of the calorific value is this much u CV which is s CV which was what we got here 238.4. Or you can write 239 also.

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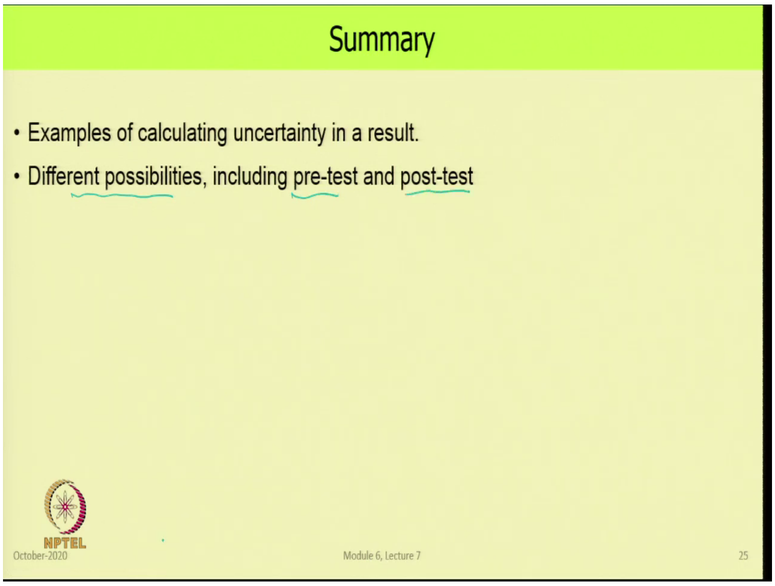

At 95 percent confidence level, this is 2 times this value which is 476 and the relative uncertainty is 476 by the mean value or 2 percent. So, our report calorific value is this much plus minus this much at 95 percent confidence level or it is this much plus minus 2 percent at 95 percent confidence level.

So, that is the simple way by which we can get the result uncertainty from multiple tests and repeated tests. With those examples we will conclude and we have seen some examples of calculating uncertainty in the result. We saw that there are different ways in which the problem can be posed and in some cases it is a pre-test analysis and then there is the post-test analysis with that we conclude this lecture.

Thank you.