Introduction to Uncertainty Analysis and Experimentation
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Module - 06
Uncertainty in a Result
Lecture - 21
Method selection. Worksheets for result uncertainty

Welcome to the course, Introduction to Uncertainty Analysis and Experimentation. We are studying Uncertainty in a Result. We have done all the mathematics, the calculus, the statistics, and seen what are the arguments by, which we come up with a relation to calculate uncertainty in a result.

In this lecture, we will look at two things, one we will see the selection of the method, what criteria should be there. And then we will put together all that we have learned in a form of a worksheet and a step by step procedure by which we can straight away put numbers and do calculations or write the program and get our analysis done.

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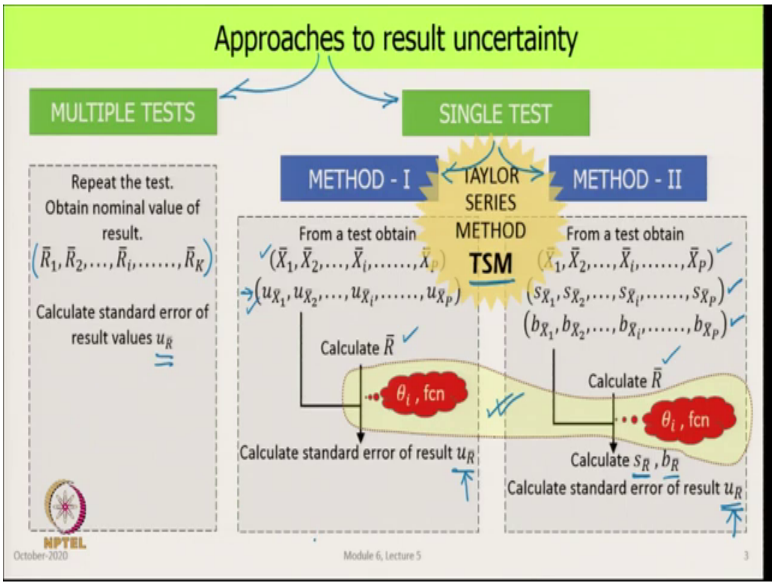

So, to just revisit what we have done in the previous few lectures. We have said that there are two broad approaches to uncertainty analysis, particularly uncertainty in a result we have multiple tests possibility and the single test. Whereas, in the multiple test all we do is we just do the experiment many times and get the result value. We do not look at the individual measurement uncertainties or their contribution to the result. And then using these result data as a sample we estimate the standard uncertainty of the result. So, that is multiple tests.

The other option was doing a single test, which we said that we could follow method 1 or method 2. Both these tests are based on the Taylor Series Method or TSM. In method 1, we calculate the standard uncertainty in each measurement and then use that to calculate standard uncertainty of the result.

Method 2 is a slight modification of method 1, in that we keep and from our calculations, values of individual systematic and random uncertainties and using that we calculate the systematic, this systematic and the random standard uncertainties in the result from which we calculate the standard uncertainty of the result.

And we started off this whole exercise by asking that we got all this data, we know how to get it, we learned this first part in the measurements uncertainty part, and we got the mean value of the result. Similarly, here we got all these things and the mean value of the result. The question we were asking is what is this function by which I can connect all of these numbers and get this?

Now, we have this result, the result relation for uncertainty and we can now see how to use it to complete our analysis. So, we have achieved the objective that we set out with. On the way in using the Taylor series method we made some a very important assumptions. So, under those assumptions we can do this. For many experiments one does, those assumptions are reasonably good.

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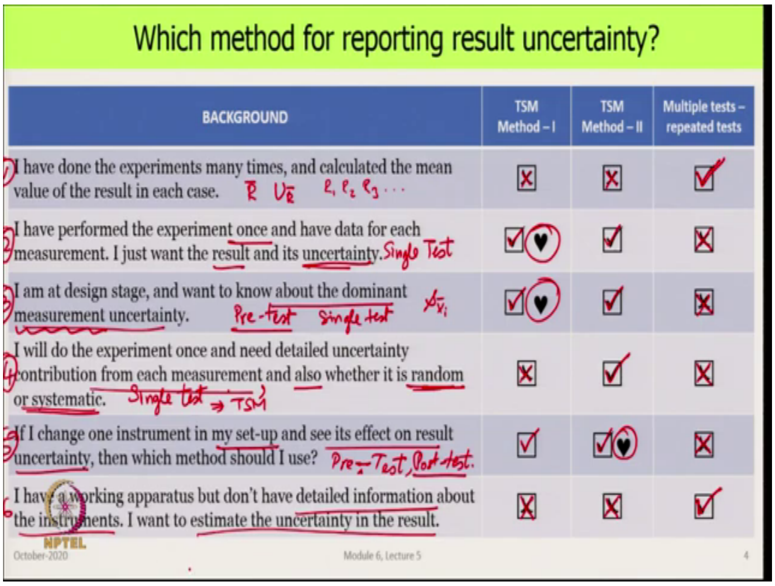

The first thing we will tackle today is under what conditions should I pick up TSM method - 1 or TSM method - 2 or the multiple test - repeated test method. So, on the slide, I have put some scenarios and we will go through them one by one to see which are the appropriate method and why. So, the 1st one, then somebody says I have done the experiments many times and calculated the mean value of the result in each case.

Now, I want to calculate R bar and u R bar. So, R bar is fine. We just take the mean of those measurements. How do I get u R bar? Well, that what do you have done, is you have only got the experiments and done many times, so you have calculated the mean value in each case. So, what we have basically got is u R 1; sorry R 1. So, what we have basically got is R 1, R 2, R 3 and like that.

So, this is clearly the case where you did an experiment many times over and the correct method to use would be the multiple test repeated test technique. And obviously, because we did not get details about uncertainties in every measurement these two methods, the Taylor series method either by method 1 or 2, they are just not possible.

Now, 2nd example: Somebody says, I have performed the experiment once and have data for each measurement. I just want the result and its uncertainty. So, it is telling us here that the experiment has been done once. So, this is clearly a case of a single test situation.

And we can use either method 1 or method 2, and definitely not the multiple tests repeated tests. And if you say that I just want the result and it is uncertainty, they are not interested in the breakup of the contributions to the uncertainty, then either method 1 or method 2 would be both applicable, but since your objective is only to calculate the uncertainty in the result we can say that we should prefer method 1.

Now, the 3rd example: Somebody is designing our setup and at the design stage they said they want to know about the dominant measurement uncertainty. So, they are asking I have made this setup, I have conceived the setup, I am not physically made it or done any experiments, but by looking at this can I get an idea as to which measurement is likely to be a bottleneck in improving the quality of my result.

So, we are looking at a pre-test scenario condition. In the pre-test scenario, the multiple test, repeated test is just not there because you do not have data. So, cannot be used. We could use either of the single test method, the technique, although we are not we have not done the test, but we are using the technique for the single test method and say I am applying this for the pre-test case and I can use either method 2 or method 1.

And if you just want to know the dominant uncertainty from a measurement as to which measurement is causing this big problem then method 1 will do the job quite adequately. So, you would go for Taylor series method 1 and being a pre-test case of course, we do not have data on s X i bars. This will either have to come from our experience or we just simply neglect it. So, that is the 3rd example.

The 4th example: The person says, I will do the experiment once and I need detailed uncertainty contributions from each measurement and also whether it is random or systematic. So, this person has already defined lot of things for us. Since, we are doing the experiment once we are quite clear this is going to be a single test process, so either of the TSM methods we will be used. And they want detailed break up of contribution from each measurement and also its contributions from systematic and random uncertainties.

So, the breakup of random and systematic uncertainties to the result. So, in a first the multiple tests, repeated test is just out because it does not tell us anything about measurement uncertainties. Taylor series method 2 gives us all the details, contribution from the measurement and contribution from the standard errors in the measurement, and systematic errors in the measurement, the random errors in the measurement. So, we can use this and we cannot use method 1 because it does not give us the breakups of the random and systematic uncertainties. So, we have to use Taylor series method 1 from a single test.

Now, next example: Somebody say that I have a set-up and it is operating and I will replace one instrument by say a more accurate one or less accurate one or a new one and say I want to see its effect on result uncertainty. Which method should I choose? So, here is a case where you have a certain uncertainty coming in at this point and you have changed one instrument.

So, you can look at the contribution of uncertainty from that measurement and in particular that instrument and see what is the change that is happening.

So, you are doing the experiment once or it if it could be a pre-test case also where you are deciding that if instead of this instrument, I use this instrument, what will happen? So, it could be a pre-test case. This multiple test option is gone. It does not give us a break up. This could be ok, because it tells you what is the contribution from individual measurement.

But it will be even more helpful to see look at the contributions coming from random and systematic uncertainty errors sources and so, we might be better off using Taylor series method 2 or the Taylor series method 1 would also be adequate it to some extent. So, this is useful both pre-test and post-test. Post-test in the sense that what we have done here is you have done everything, you have taken to run some experiments and then after that you say now I will change an instrument and take more data.

So, that is a slightly modified version of what we are looking at is the post-test. And such things happen in real life, when instrument goes bad and you have to replace it or you go to a better instrument comes in the market and you want to replace it. These are all real-life situations.

The 6th option that I put here is this is I have a working apparatus, but do not have detailed information about the instruments. They said the instruments are old I do not know where they came from, I do not know whether the calibration was done or not, I just want to estimate the uncertainty in the result.

So the simplest thing to do is that if you do not have details about the instruments just do the experiment many times and get the answer, so the multiple test, repeated test is your option. Do not worry about the Taylor series method, the single test methods is there. So, here were some examples of how to decide which method to apply to calculate uncertainty of the result or maybe something else which is the contributions of uncertainty is a measurement.

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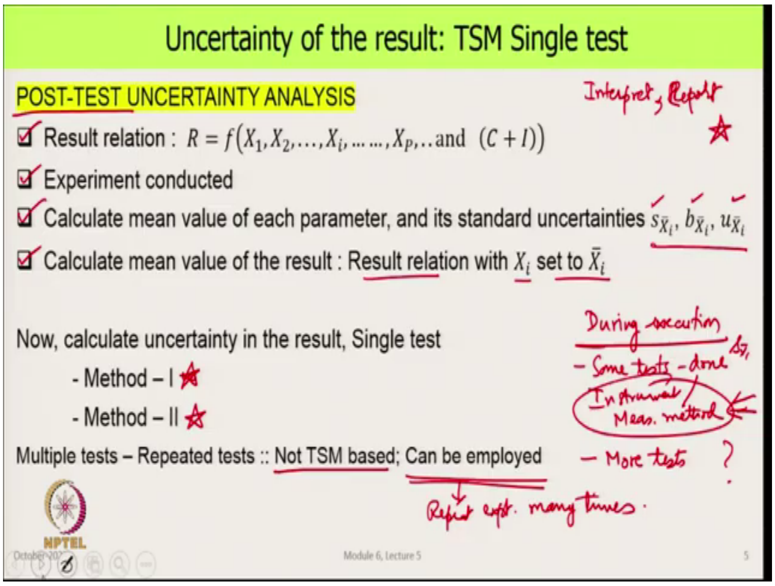

Now, let us look at these two broad categories, post-test uncertainty analysis and pre-test uncertainty analysis. And one of the biggest applications of uncertainty analysis is in this case. Here we have to finally, interpret the result draw conclusions and finally, report it in papers, theses, journals, papers, like that. So, this is very very important.

Now, what we do in the post-test uncertainty analysis is that we have the result relation there. We have conducted the experiment. We calculate the mean value of each parameter and its standard uncertainties which are the combined one as well as the random and systematic. And then, we calculate the mean value of the result by using the result relation and setting X i equal to X i bar, so that is done.

We have all this data from the experiment because we are in the post-test phase, and then we can use either method 1 or method 2 whichever way we want and estimate the uncertainty in the result. The multiple test, repeated test method is not based on Taylor series method. It does not give us the breakups. It could be employed simply that you repeat that experiment many times.

And we saw one other modification of this that if during execution. Some testing have been done. And now, we change some instrument or the measurement technique, and then we continue to do more tests. Then also, the impact of this change on our result uncertainty can be calculated by this way. But here we would have some s X i bar and so, we are not as handicapped as we were in the pre-test case.

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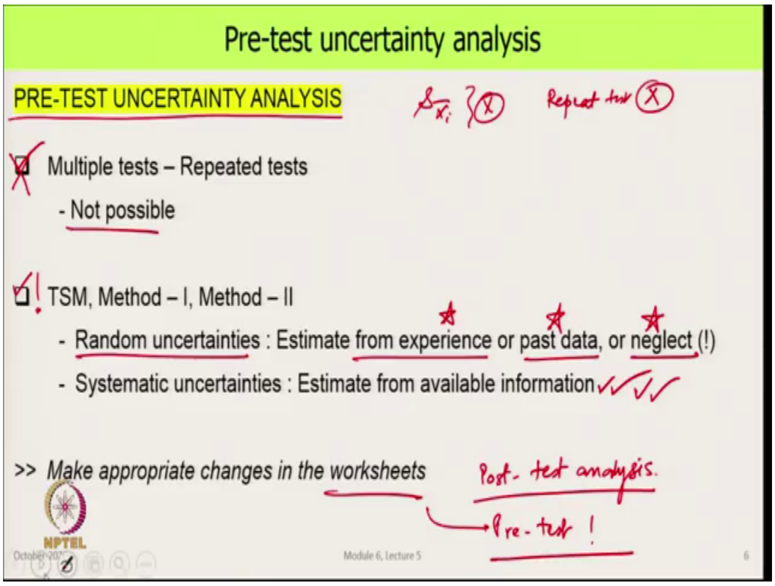

Now, we did the post-test uncertainty analysis and everything we did was fully applicable for the post-test scenario. The question now is which part of that I will have to skip or missed or modify to do the pre-test uncertainty analysis. And this reason is simply we have not done the experiments, X i are not known, we did not, we have no idea of repeating any test.

So, that option is also gone. So, none of the three methods for uncertainty analysis strictly in the whole sense are applicable in this case. So, that is what we are put here; multiple tests just not possible, forget about it. We could use this with caution, the caution being that random uncertainties need to be estimated from experience or past data or neglect.

So, you have either this option or this option or this option. And then, we can proceed exactly the same way as we have done systematic uncertainties by and large everything is available over here. And being pre-test, we could even say, I use this instrument if I things do not work out or I do not get that instrument I use some other instrument, we can pick up information about that instrument continue doing the analysis.

So, this largely this systematic uncertainties can be estimated very well even in the pre-test case. So, what this tells us is that when the worksheet that we will now see they are fully applicable to post-test scenario. Certain items will have to be modified if we are using them for the pre-test scenario, but the worksheets can be used as they are.

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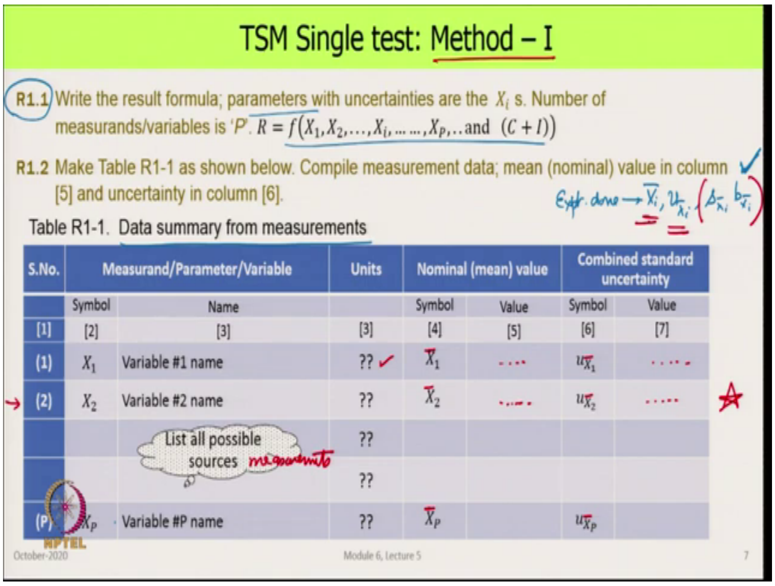

So, let us see the step by step procedure for the Taylor series method, the single test method 1. These details are also given in the notes, each step has been given a number and that number you can see here. This is step in a sequence or one tells us this is method number 1 for the result.

So, we write the result formula the parameters with uncertainties and that is what we have. Then, we make this first table or one dash one and this is a data summary from measurements. So, a starting point in result calculation is that we have got the analyzed data from the measurement part which came from doing the experiments, so experiments done. X i bar and its various uncertainties u X i bar, s X i bar and b X i bar. All these are available with us. And now we are proceeding for the result analysis.

So, the first table R1 dash 1 is a data summary from the measurements, which is here column wise, serial number, then measured parameter, measurand or the variable whichever way we want to call it. It is units; it is nominal mean value and the combined standard uncertainty.

In method 1, we only require these two so, this and this. This is the information required in method 1. This we can then forget at the for the time being. So, we write down first variable number 1, X 1 its whatever the diameter, velocity concentration whatever it may be, name of the variable we write it is units, and then say this is the symbol for it, X 1 bar and we write the value, which came from here.

Similarly, for the combined standard uncertainty we write u X 1 bar and write it is value. We then repeat the same thing for the 2nd variable, X 2 bar write the value there, u X 2 bar write all the values there. And that way we list all possible sources or other all the instruments, all measurements and complete this table for every variable.

Final one, being X p bar u X p bar. The reason for making this table is that we will use this information in the next stages, and this table in one step tells us everything about what we measured. There is a very compact way of looking at what came out of the experiment.

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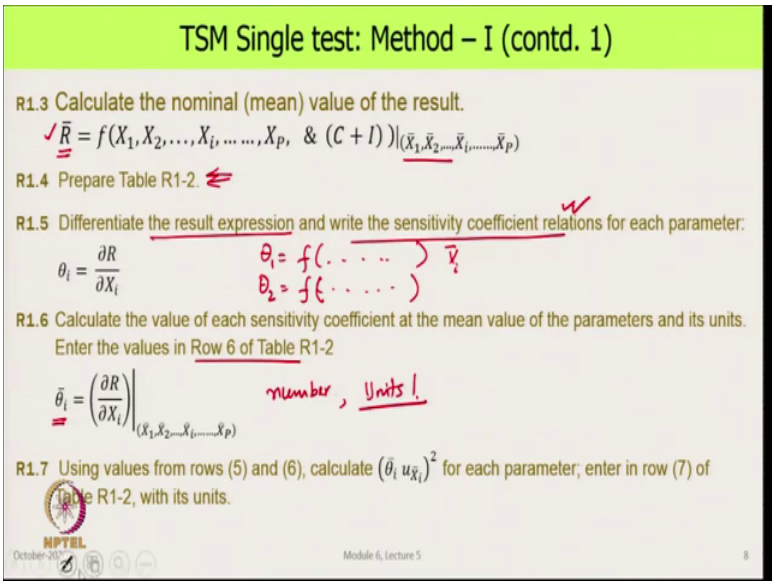

Then we continue. We put the mean values into the result formula and we calculate R bar. So, that comes up. Then we make another table. And what I will do is instead of looking at these steps, these steps are integral to the table. So, I will explain the steps in the table itself.

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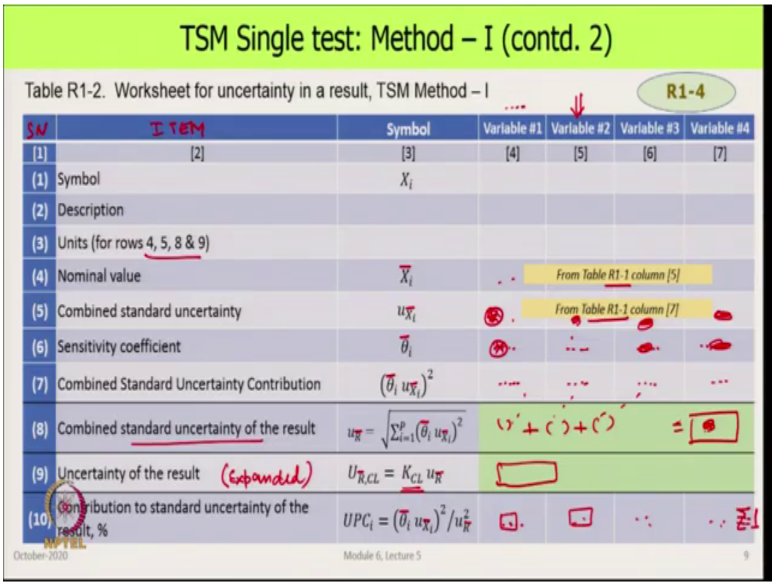

So, here is the table. We write here what item it is. This is serial number. Symbol, variable 1, variable 2, variable number 3, variable number 4, and this corresponds to the step R 1 dash 4. So for, we fill column wise for each variable. So, this could be a length measurement of some type, this could be say the velocity measurement like that. So, we write down the symbol, description, units.

Now, these units are applicable for some of these next steps only. Then we write in the 4th line the nominal value of that parameter, which came from the first table. We just copied from there. We write the combined standard uncertainty of the first variable, this also came from the previous table just copied it, so the corresponding numbers came over there.

Then, we calculate the sensitivity coefficient and for that we first differentiate the result expression. And write the formula or the mathematical relation for every sensitivity coefficient. So, we will have theta 1 as a function of all the parameters there, theta 2 as a function of all the variables in the formula and so on.

So, here we just got the mathematical relation written up. Then, we put the mean values into the right side of this, which is all the X i bars and we get the value of theta i which we have called here as theta i bar. So, these are now numerals, numbers. And remember this has generally has units sometimes it may not have units and this we write in row 6 of the table.

So, that is what we do here? This row 6 is sensitivity coefficient, theta i bar, this is how we got these numbers here. Then we do a column wise operation. We multiply theta i bar for variable 1 which is this with X i, u X i bar which is this, square it and write it over here.

Same thing we do for this and this we write it over here. Then for this and this, and write it over here. Then this and this, we write it over there. So, this tells us the combined standard uncertainty contribution that numerator part of the we may see the UPC. Then, we do an operation where we add up all these terms in this one, which is a combined standard uncertainty of the result u R bar is equal to summation of theta i bar u X i bar square.

So, we add this is equal to the first one here plus second one there plus third one there and all of this we get one number in this row. So, that is why this cell we coloured differently. There is only one number. So, this is a standard uncertainty of the result or the combined standard uncertainty of the result. The next we compute the expanded uncertainty of the result or just the uncertainty of the result U R bar CL is equal to K CL times u R bar, which is that we have to decide first.

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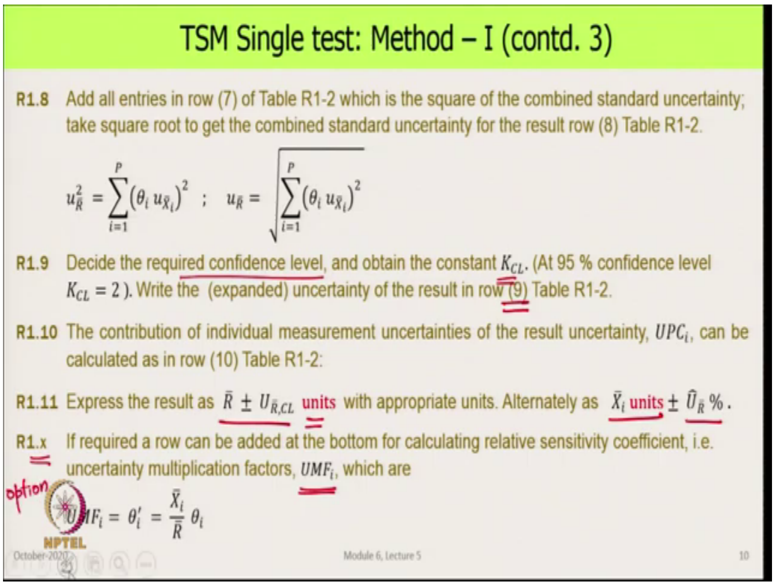

So, we did this part. We have to decide what is the confidence level at which we want our result K CL noting that at 95 percent confidence level K CL is equal to 2 and we calculate the expanded uncertainty by multiplying that. So, we have the relation there, K CL which is from our confidence level multiplied by u R bar and you just get one number here, which is the expanded uncertainty of the result that is what we were looking for.

And then if you want to look at the contribution of individual measurements, we result uncertainty. In the last row, we can say contribution to the standard uncertainty of the result in percentage, UPC i is equal to this square upon u R square. And so, what do we do is now that we have calculated u R bar here, this value, we divide this, we divide this by this, and square it up and put it over here.

Similarly, we divide this by this square it put it here and like that. So, this data you can then begin to compare and see which one is the dominant uncertainty, which uncertainty can be neglected, and of course, the sum of all of this in this row the sum has to be equal to 1. So, this completes method 1.

Finally, of course, we just express the result as R plus U R bar CL in terms with its units or X i in units plus minus some percentage either way. And if need be if you want to calculate the theta i primes we can add one more row and calculate the uncertainty multiplication factor theta or the non-dimensional sensitivity coefficient as X i bar R bar upon into theta i. So, that is why it is left as an option.

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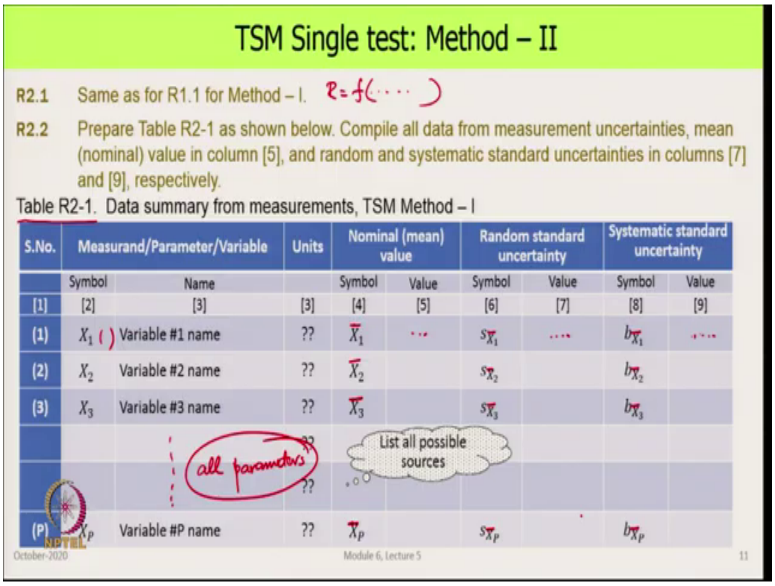

Now, we come to the 2nd method. Our first step is the same as for method 1 which is we write the result formula. Then, we prepare table R2-1 which is given over here and like the first table in the earlier method.

This is also a compilation of all measure data in one snapshot. So, you have measurand or parameter units, nominal value which is there, we have that. The random standard uncertainty in the result we put it in these two columns and systematic standard uncertainty over here.

So, we go row by row, one row for each parameter or each variable. So, this is X 1 variable, we here put it is name, units, then we can put it is symbol, there the X 1, which is the symbol for this. We can write the mean value, this is the random standard uncertainty, and this is the systemic standard uncertainty in X 1.

We do the same thing for each of the other variables. So, we complete this. So, we will do this for all the parameters in the experiment. And so, we have in this one the mean value of X 2, s X 2, b X 2 bar, X 3, s X 3 bar, b X 3 bar and finally, the Pth variable X bar P this and b X bar P. So, now, we have all the data that we calculated and we can proceed for the result calculation process.

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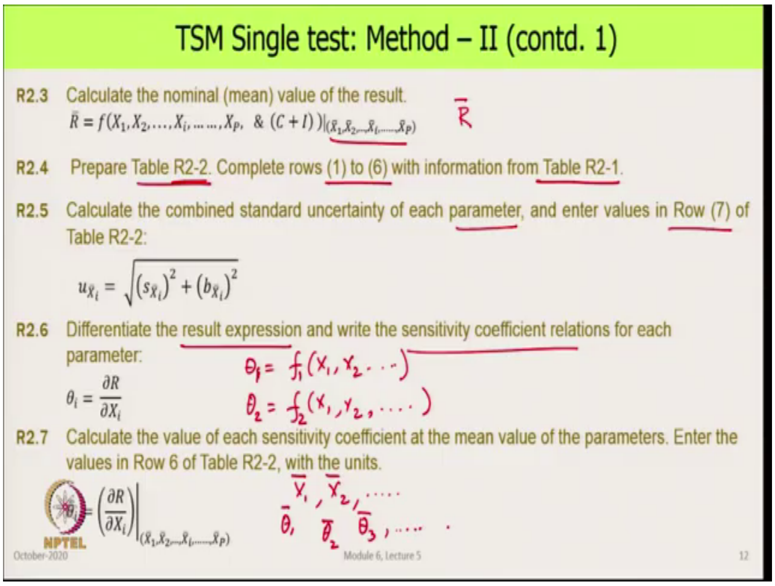

So, in the next step, we go back to the result formula, put the mean values, and calculate the value of the mean value of the result R bar. So, this is R bar. Then, we go off and make another table R2 dash 2, and let us look at that and it says complete rows 1 to 6 with information from table R2-1.

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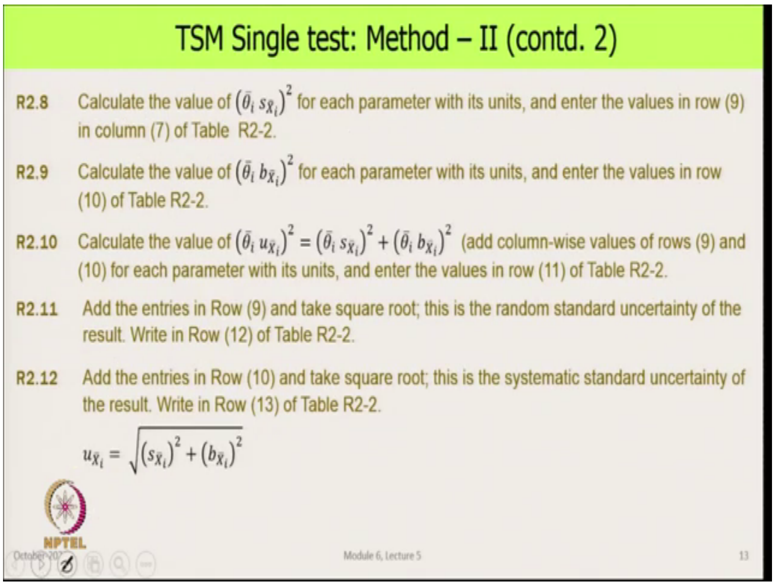

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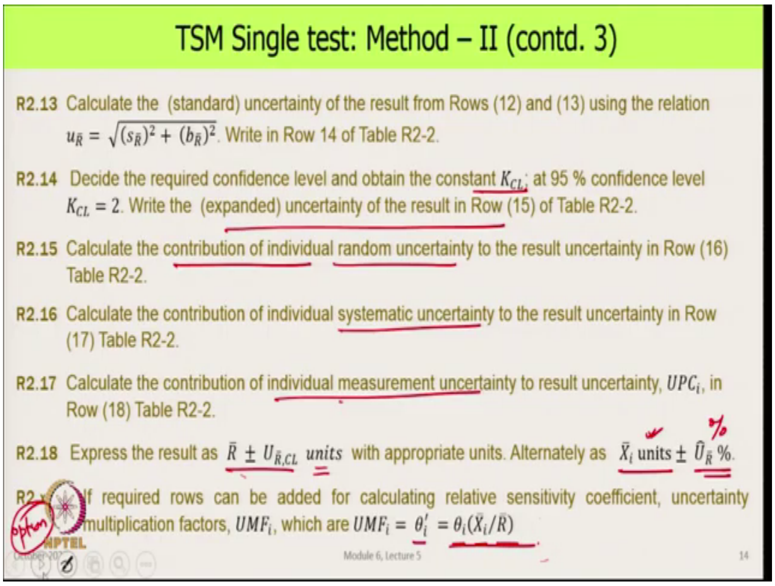

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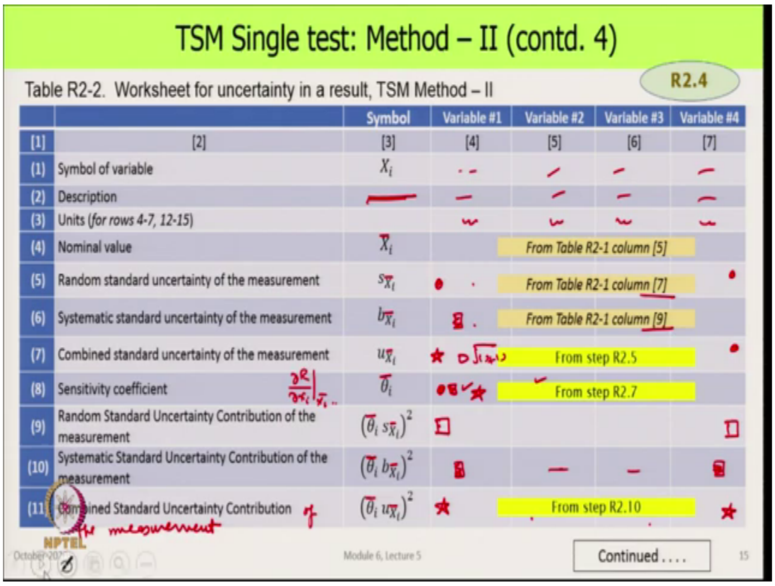

So, we are transporting data from the first table to the second table. So, here is the table and this is a longest table, so it continues. Name of the variable, so we can write down the name of the variable each one of these; description, we can put a small description here there is no description, we can write that.

Then units for various rows not for some of the terms, we can write we must write those units over here. Nominal, value we just copied from the first table column 5. The random standard uncertainty of the measurement, s X i bar we got it from the previous table column 7, systematic standard uncertainty of the measurement we got it from the previous table column 9. So, rows 1 to 6 got populated.

Now, let us see what to do next. Calculate the combined standard uncertainty of each parameter and enter in row 7. So, now, the u X i bar is equal to square root s X i bar square plus b X i bar square, we will write it there. Then, we will do a differentiation of the result expression and write the sensitivity coefficient relations. So, theta 1 as a function of X 1, X 2 like that. Theta 2 do as a function of X 1, X 2 some other function, so this is function 1, this is function 2 like that and we write all these functions.

And then in the next step, we take these functions and substitute for X 1, X 2, and all of that their nominal or the mean values. And from there we calculate theta 1 which we call theta 1 hat, theta 1 bar, theta 2 bar, theta 3 bar and so on. So, let us go back to the table and see what where we reached.

So, we got combined uncertainty of the measurement over there which we just saw came from this value came from the square root of the sum of squares of this and this. So, like that from every one of them. Then, we got the sensitivity coefficient, we differentiated it, dR by dX i and calculated it at the mean values of X i. So, we got these values.

The next step we do the random standard uncertainty contribution of the measurement. So, what we do is we multiply and take make the square of theta i bar which is this one and this one and calculate this one. So, this into this squared is this. Similarly, for the last one this into this squared will be this.

And remember these are parameters for which the units of X i are not applicable. Then, we do the systematic standard uncertainty contribution, this is now this, no, not this. This multiplied by this and we get this number square it this into this squared is this, similarly we go for each one of these boxes.

And then we compute the combined standard uncertainty contribution from each measurement, of the measurement. And here what we do is we multiply u X i bar which we had here this we multiply it by this, square it and we get this. Similarly, for this one, this one, and for this one.

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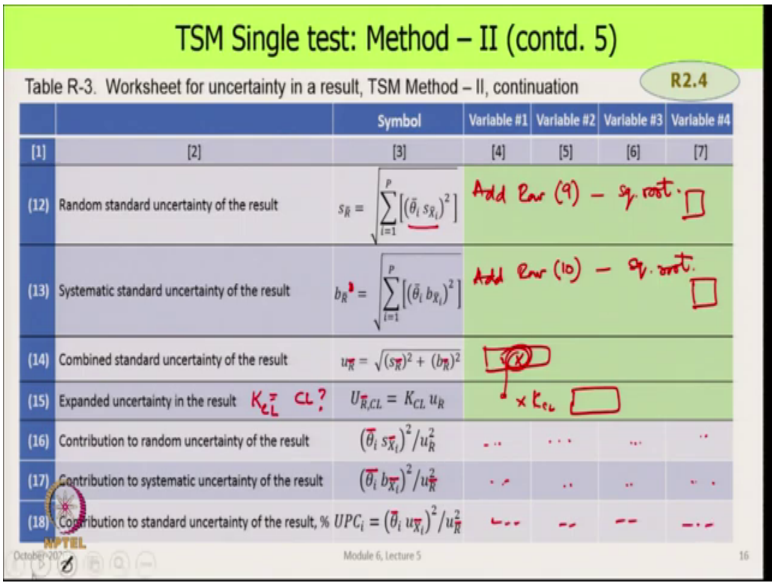

Then, we continue on the table and say what is the random standard uncertainty of the result. This is the summation of these terms the square root. Where did these terms come? In an earlier row, see theta s X i bar terms, there were here, this is row 9.

So, in the next one, we say we add numbers in row 9 and take their square root. Then, for the next one systematic standard uncertainty of the result, we add all the elements in row 10 and take the square root. And then, we make the combined standard uncertainty of the result, which is u R bar in terms of s R bar and b R bar square. So, here we have one number. In this row also, we have one number that is why these are coloured separately.

We square them up, add them and take the square root, we get this answer, sorry, here these 2 is not here, ok. Then, expanded uncertainty of the result, we decide first some value of K CL.

So, we say what is CL that we want for that we get the value of K CL, put it here, multiply it by the number that was here.

So, this into K CL gives us the value on this row. Then, we can calculate contributions and uncertainty percentage contributions. So, contribution to random uncertainty of the result from the system at the random standard uncertainty we can take this number which was in some other row earlier divided it by this number here, square it, and we can put the number here like that here and so on.

Then, measure contributions from systematic uncertainty we can do the whole thing again, but this time instead of s X i we have b X i each term in the earlier row and then we got all these. And finally, contribution to standard uncertainty of the result which is our UPC Uncertainty Percentage Contribution of the ith measurement this is u X i bar by u R bar. So, you can take the value of u X i earlier or take the sum from here and calculate these numbers.

So, what we got? We decided K CL and got the expanded uncertainty in the result. Then we looked at the contribution of individual random uncertainties, systematic uncertainties, and contribution to the individual measurement uncertainties. All these calculations done, we are ready to express the result, which is R bar plus minus U R CL with its appropriate units or this with its appropriate units and something as a percentage.

And then finally, if needed, this is an optional thing here. We can add a row for calculating relative sensitivity coefficient, which is the theta i prime theta i into X i bar upon R bar. So, that brings us to the completion of this analysis using the Taylor series method number 2.

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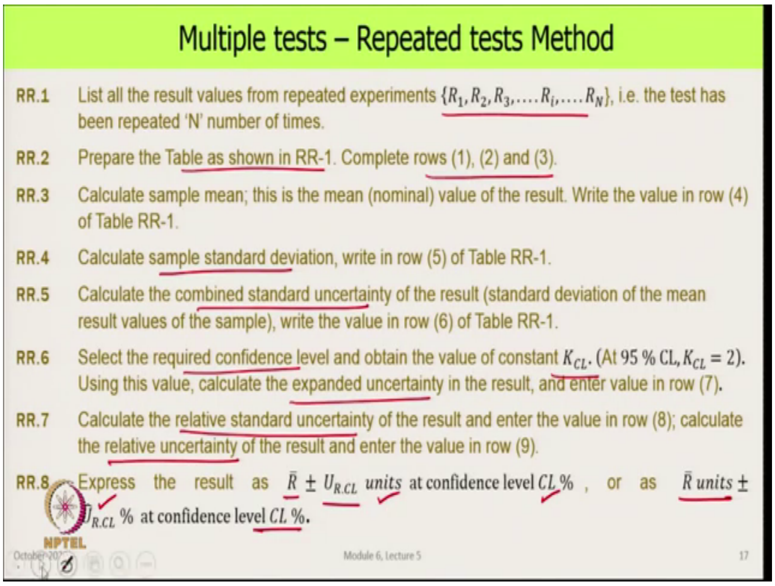

Now, we come to the other option, which is the multiple tests repeated tests method. So, the steps have been denoted by RR dot 1, RR dot 2 result first R for result second R for repeated tests. So, what we do is we list all values from the measurement which is R 1 bar, R 2, R 3, like that. Then, we make the table as shown there and we complete rows 1, 2, 3. So, let us see what this is.

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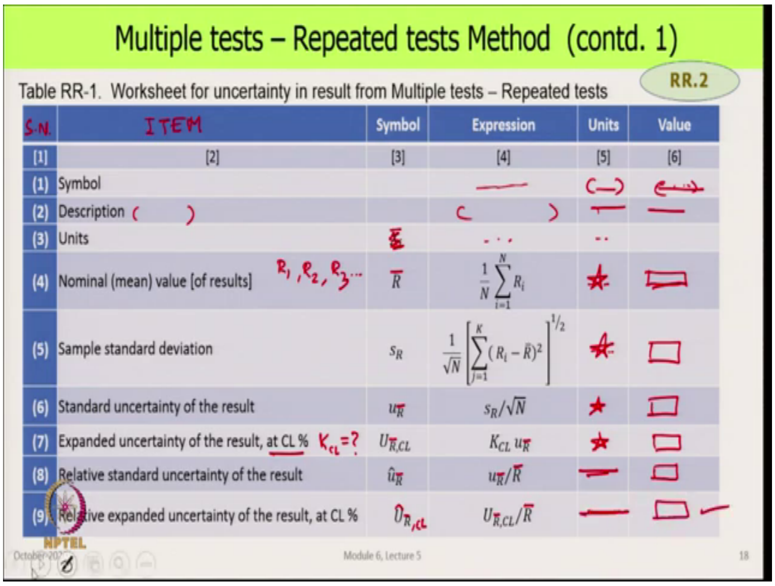

So, this table is slightly different from the others one, in that here we have the symbol, the expression, units and value. So, this one here is what is the item we are looking at and here what is the serial number. So, the first row we have the symbol. So, here there is you can write down what is the symbol for that parameter length would be l, diameter would be d, whatever way.

Then, we write down if there is any expression for it with there is none must write the units and write down there is no question of a value at this point. Description, we write down what it means. So, we can either write down here itself and get done with it. It has got no units, no value, we are just writing something about it.

Then, we are looking at the units that we have for the symbol. So, we can put those over here. Then, from all our resultant data R's which was R 1, R 2, all of that that we had from multiple tests we take the average the units of the result we know we write the value over there.

So, this is R bar. Then, we write about the sample standard deviation s R, which is this is not s R bar, which is the classical formula for calculating standard deviation that we have learnt we can write the units here. So, units of this will be same as units of this and we can write the answer over there.

Then, we write the standard uncertainty of the result u R bar which s R upon square root N, same units as before we can write the value over there. We pick up what confidence level we want and from there we get K CL is equal to what? And the expanded uncertainty of the result at that confidence level is this multiplication factor multiplied by u R bar, it has the same units and you can put the number over there.

And then finally, relative standard uncertainty of the result u hat R bar, which is u R bar upon R bar. This has got no units and we can write that as a number over there. And finally, the relative expanded uncertainty of the result at the desired confidence level u hat R bar CL which is U R bar CL divided by R bar. Again, this is not dimensional, so it has got no units and we get the number over there.

So, that is what these methods are we calculate the sample standard deviation combined standard uncertainty and the required confidence level we get K CL, and compute the expanded uncertainty, calculate the relative standard uncertainty, and calculate the relative expanded uncertainty.

And finally, expressed the result as R bar plus minus U R CL with units at confidence level this much or R bar with units plus minus this much percentage at confidence level this match. And that completes the objective for which we did all this exercise. That was for multiple tests, repeated tests.

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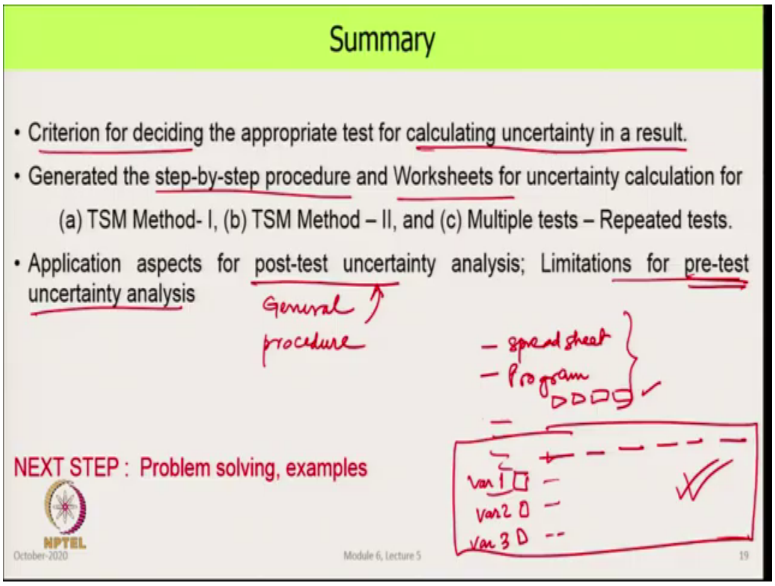

So, now we will summarize what we have done. We looked at criteria for deciding the appropriate test to use for a particular application. And the objective was to estimate the uncertainty in a result. And we saw how at different stages under different scenarios we could use either TSM 1, TSM 2 or the multiple test repeated tests.

Then, we have developed a step-by-step procedure and along with that a worksheet for doing all these calculations. So, we had three procedures, three worksheets, we could either do that manually or we could put it in a spreadsheet or you can make a small program on this, and automated in either of these cases, and that will that gives just a quick way of doing these calculations.

There is other possibility the tables that, I had shown on the worksheet tables, the parameters were there and the variables were in this side. One can flip it and put all these parameters as columns, and the variables as rows. Both do the same thing, the only difficulty with this one is that when the number of columns becomes large, doing it manually becomes cumbersome.

Doing it on a spreadsheet is quite easily manageable, but manually on a paper pen type of a scenario we find that we are constrained about how much width of paper we have. So, one can do it either way. Both are possible. If you look up PTC 19.1, the examples are given in this format. Same thing as doing it in the way I have put here.

And finally, we looked at the application of the uncertainty analysis which was largely a general procedure which de-facto assumed post-test uncertainty analysis, and then we said that with some slight changes in the way we evaluate certain parameters. We could use it for pre-test uncertainty analysis and of course, for pre-test uncertainty analysis the multiple test method just cannot be used.

So, the method we have is robust of course, for post-test and also to very large degree sound and good for pre-test uncertainty analysis. So, on that note, we will conclude this discussion on result uncertainty in a result. We have achieved, what we set out to do in the beginning. We have the expressions, we have the processes, we can calculate uncertainty in the result and make many decisions on that. So, our only thing remaining is to give some examples and do some problem solving.

So, in a subsequent lecture, we will look at some representative problems and you will gain more experience when you solve the assignment problems. So, on that note, we conclude this lecture on uncertainty in the result, where we saw method selection and develop the worksheets.

Thank you.