Introduction to Uncertainty Analysis and Experimentation
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Module - 06
Uncertainty in a Result
Lecture - 20
Result Uncertainty TSM: Special cases

Welcome to this course Introduction to Uncertainty Analysis and Experimentation. This is a 4th lecture in module 6 which is on Uncertainty in a Result. We will look at result uncertainty from the Taylor series method and look at some special cases. At the end, we will also look at methods I and methods II a comparison.

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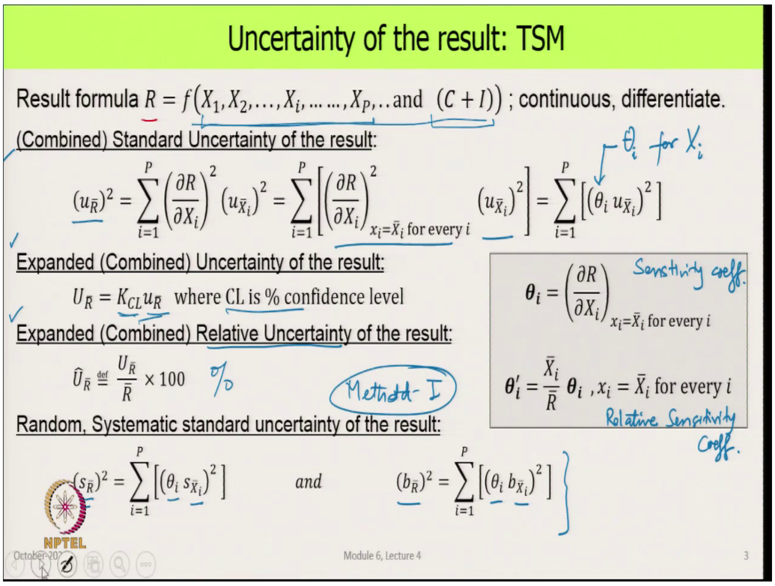

So, what we have done this far is that we started with the result formula R which is a function of various parameters X 1 to X p and of some constants. We put a restriction that this is a continuous function and continuously differentiable and from there, we got the formula for combined standard uncertainty of the result as a function of the result formula and combined standard uncertainties in individual measurements.

And then, we defined theta i which is the sensitivity coefficient for the parameter X i and so the expression became u R bar square is equal to the summation of theta i u X i bar whole squared for all the terms.

So, we have defined theta I, the sensitivity coefficients and the non-dimensional or the relative sensitivity coefficient and with that the various relations that we can use in our calculations, they are expanded combined uncertainty of the result is here U R as a product of the multiplicative factor K CL which depends on the confidence level chosen and the random stand, the combined standard uncertainty in the result.

Then, we can get the expanded relative uncertainty in the result which is U R bar by R bar times 100 so, this becomes a percentage, dimensionless and we can compute the individual random standard uncertainty S R bar from these two inputs and the systematic standard uncertainty in the result from these two inputs.

In method I, we do not calculate this particular part that we will see at the end of this lecture. Method I is only up from this, this and this so, we get the answer that is where method I ends and we do not get information about the random and standard uncertainties or the elemental uncertainties. So, if that is good enough for us, method I is the one to use. If we want more detailed information, we have to go to method II.

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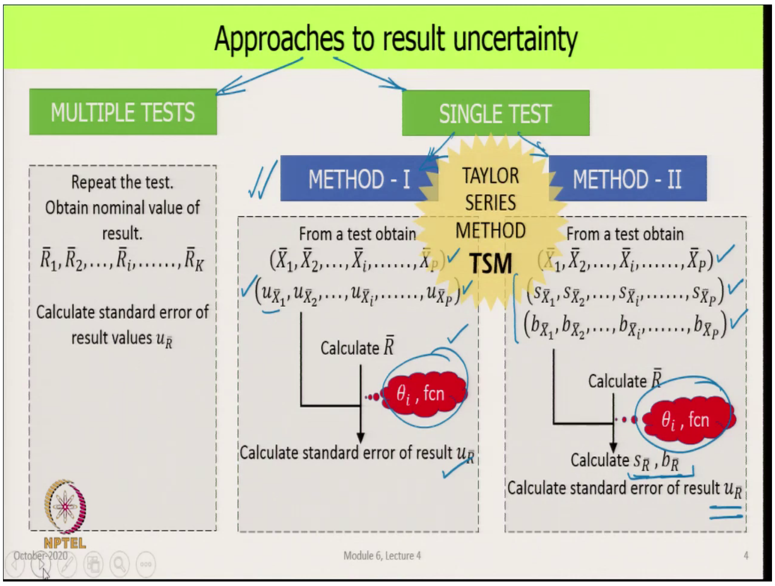

So, this was our big picture of calculating results uncertainty. One was the multiple tests, the other which we have been looking now is a single test, then method I in this and then, method II. So, we have seen that we have all this data, we have this, we got this function that we were looking for now, we are in a position to calculate this so, we have everything about method I.

We will right now look at some special cases which are applicable to both method I and method II. So, just to tell the difference between method I and method II is we calculate this part just like before, our data input is also same as method I, a single stay single test input.

Instead of calculating the combined standard uncertainty in each measurement, we calculate the random standard uncertainty in each measurement and systematic standard uncertainty in each measurement and then, we have the functions that we have already seen, we use those and we calculate the standard the random, standard uncertainty and the systematic standard uncertainty in the result, combine them and then we get the answer.

So, a very small difference; in this case, we would have got these by calculating s X, s and b's and combining them at this stage itself. In this case, we do not combine it at that stage, we keep them separate and because of that we are able to get this information and the end we will see how this information helps us in assessing our experiment and all of this was based on the Taylor series method.

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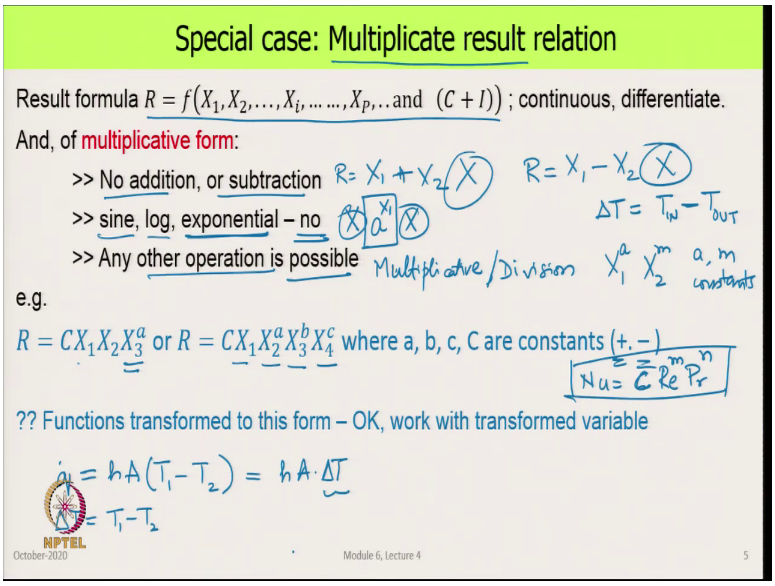

Now, we look at one special case which is the multiplicative result relation. So, let us see what we mean by this term multiplicative result relation or multiplicative result formula. Here, we have the general form of the formula as a function of X 1, X 2, X i, X p with constants. Now, we put a restriction that this formula should be such or if the formula is such that it has no addition and no subtraction.

So, X 1 plus X 2 as a formula, this is out. Similarly, if the result were X 1 minus X 2 this is also out, and it does not mean that we do not encounter such type of relations in experiments. A good example of this is what the fluid being heated where X 1 is inlet temperature and X 2 is the outlet temperature. Then, we have to have the result which is the change in temperature as an additive subtractive formula.

So, this method that we will come out with right now, this will not be applicable here, for this we have to go back to our original equations that we had and solve it from there. Those relations had no restrictions so, they were universally applicable. Now, we are looking at a restrictive use of what we have learnt. We also say that there are no functions involving sines, logarithms, logarithmic functions, sines, cosines, hyperbolic sines, hyperbolic cosines and exponential terms. So, this is also no.

Beside this any other operation is possible. So, what we are saying is essentially only multiplicative operation should be there and this also means that division is and when we said exponential, what we are saying is that some number raised to the power X 1, this is not allowed. What is possible is X; what is allowed is X 1 to the power a or say X 2 to the power m, this is ok where a and m are constants, but this constant to the power parameter, this is not allowed.

So, examples of this are that result is constant times X 1, X 2 times, X 3 to the power a or X 1, X 2 to the power a, X 3 to the power b, X 4 to the power c where a, b, c are constants and they could be positive or negative, they could be less than 1, they could be more than 1, there is no restriction on that.

A good example of where we encounter this all the time is in heat transfer where we say Nusselt number is some constant times Reynold's number to the power m times Prandtl number to the power n, now these are very common formula one would enquire in, and you will get this formula in heat transfer, mass transfer even in drag.

So, this is the restrictive form that we are talking off. What we could do is if we do have addition or multiplication in that formula, we could treat that as one function forget that there are plus and minus signs in it and then, in the result formula, treat it as a multiplicative relation.

So, for example, in if the heat transfer rate is equal to h heat transfer coefficient into area into T 1 minus T 2, this relation by itself will not qualify as a multiplicative relation, but if we say that I will call the T 1 minus T 2 as delta T so, this relation becomes h A delta T where delta T is a parameter now, then this is a completely a multiplicative relation. We can do what we are about to see, we can apply that to this one.

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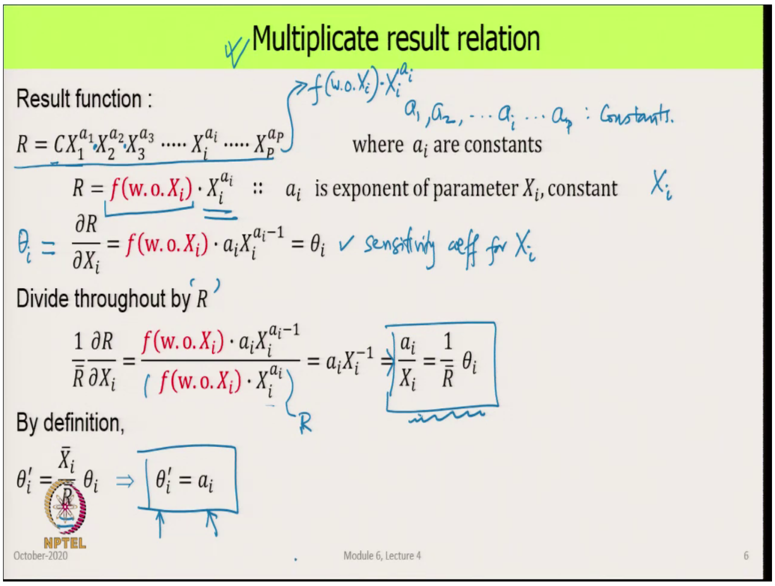

So, here is how we will treat this case. We will say that the result formula is like a constant with X 1 to the power a 1, a 1 is a constant, X 2 to the power a 2, the this is a product there, a 2 is again a constant multiplied by X 3 to the power a 3, again a constant like that X i to the power a i, X p to the power a p. So, we have all these constants so, all of them are constants and of course, the trivial to say that they are non-zero, but they could be positive, or they could be negative.

Now, we want to do is isolate one parameter at a time and so, say well, we will look at X i alone and see what I get for sensitivity coefficient and using that what help I get? So, we re-express this formula here by keeping X i to the power a i as one term and club everything else as another function which I have denoted here as w dot o X i that means, this is a function without the parameter X i, everything else is there in it.

So, now, we differentiate this with respect to X i and remember when you dR by dX i, this is what we define as the sensitivity coefficient of X i, this is function without a X i, it comes out and the differential of the this becomes a i into X i to the power a i minus 1, this is our sensitivity coefficient for X i.

Now, we divide throughout by R, the result. So, this comes 1 by R dR by dX i. On the right side, we have this numerator which we had just now this one so, this is there and we know that R which was the modified form, here we have this as function without X i times X i to the power a i. So, that is what we have written in the denominator on the right side. So, what you see here? This term, this is essentially nothing but R and when you simplify this, we get a i X i to the power minus 1 and which tells us that a i upon X i, this is theta i, this is 1 upon R theta i. So, we have got a very nice simple relation coming up here for theta i.

But by definition, we also have there is a non-dimensional sensitivity coefficient theta i is X i bar upon R bar times theta i and this if we combine it with this relation that we have here, this tells us that the theta i prime is a I that means the sensitivity, the non-dimensional sensitivity coefficient of X i or the relative sensitivity coefficient of X i, this one is nothing but the exponent of X i.

So, by inspection in the formula, you can say that the non-dimensional sensitivity coefficients of X 1 is a 1 of X 2 is a 2 of X 3 is a 3 of X p is a p, this is possible because we have a multiplicative result relation and this immediately tells us that life can be much simpler.

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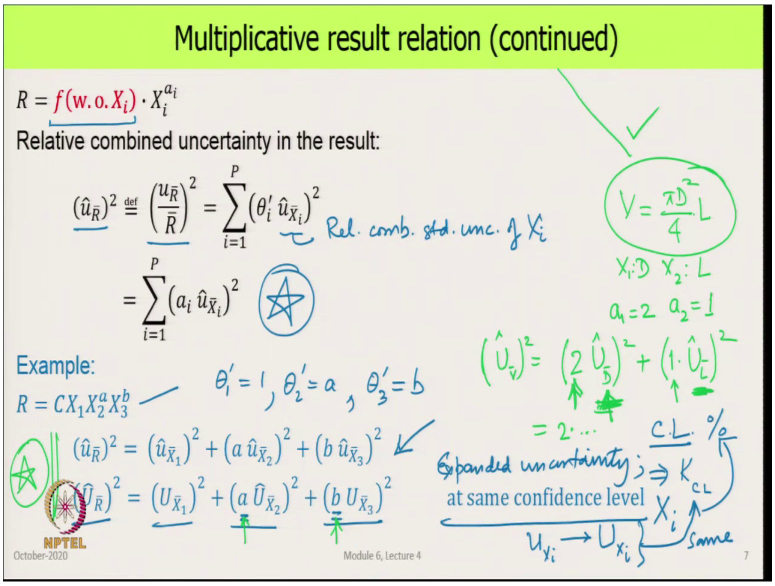

And we can see that happening in the next step here that this was the result formula that we had, everything else clubbed in without X i into X i to the power a i. The relative combined uncertainty of the result u hat R bar square which we defined as combined uncertainty divided by the value of the result, this is theta i prime into u X i prime so, this is again the relative combined uncertainty of X i and using our earlier relation, theta i prime is nothing but a I so, we write this a i times u hat X i bar whole square.

So, this becomes a very nice elegant simple expression and that is usually simplifies life while solving problems and here is an example that if R is C times X 1 with X 2 to the power a X 3 to the b, then by inspection alone, we can say that u hat R bar is equal to u hat X 1 square plus a times which is the exponent of X 2 u hat X 2 whole thing square into b times u of X 3 whole square where a was the exponent of X 2, b is the exponent of X 3. This is a.

So, what we have to what tells you that if you have the result function and you want to write the relative combined standard uncertainty, you do not need to go through all the steps of derivation and calculating every sensitivity coefficient individually, you can straight away say that looking at it that the theta i, theta 1 prime is equal to 1, theta 2 prime is equal to a and theta 3 prime is equal to b. So, straight away we can write this formula, it saves us a lot of intermediate steps and time as well.

Then, we can extend this result to the expanded uncertainty to the expanded relative uncertainty in the result and note that when we talk of expanded uncertainty, it is essential that we define the confidence level so, this confidence level as percentage will tell us what is the multiplicative factor which was K; which was K CL.

And now, a restriction is that for every parameter from u X i going to capital U X i with or without the hat, every one of them must be at the same confidence level. If X 1 is reported at 95 percent confidence level, X 2, X 3, X i everything should be reported at 95 percent confidence level. We cannot have X 1 at 95 percent confidence level, X 2 at 99 percent confidence level and X 3 at 60 percent confidence level, then this is not done.

So, all the measurement uncertainties are reported at the same confidence level which is almost always the case that we will do in uncertainty analysis and then, K CL is the only factor which came in into all these expressions on all sides, we just put it there and we get the relative uncertainty in the result is the sum of the squares of these multiplied by their respective exponents. So, very quickly, we can see what is happening over here.

So, if you have a function like volume, so volume of a cylinder is pi D square upon 4, D is the diameter multiplied by its length, then we see a very simple thing. The exponent of D is 2, exponent of L is 1. So, if we say X 1 is D, X 2 is L, then a 1 is equal to 2 and a 2 is equal to 1, this is the two parameter formula.

And it will tell you that by looking at this that you had the relative uncertainty in the volume squared, this will be relative uncertainty in the X 1 those exponent is 2 so, this will be sorry, so, this will be exponent of X 1 which is exponent of D which is 2 times U hat D bar square plus exponent of L which is 1 times U hat L bar square. This we could do because this satisfies this condition, multiplicative formula.

And by looking at this, it tells you very quickly that uncertainty in diameter propagates twice as rapidly than uncertainty in the length actually, it is more than that because this is squared so, uncertainty in diameter is something that is going to be more important than uncertainty in the length.

And in the measurement, if both these uncertainties are similar say 1 percent, then we know that the result uncertainty here, this will be 1 here which is 4 plus 1, 5 this is square root 5, 2 point something of which a large chunk came from uncertainty in the diameter so, you rather put your energy in reducing this parameter, this uncertainty than worrying about measuring length more accurately then, it tells you what strategy as an experimentalist you should do and not say I want to reduce uncertainty so, I will reduce uncertainty in all parameters. This method tells you, you it is better to focus on the ones which are relatively large.

So, this is what we have been look wanting that for multiplicative relations, the uncertainties becomes very simple, you do not need to worry about doing any calculations or in the sense that calculating the derivatives, you do not need to calculate theta 1, theta i, theta i prime we get that straight away by looking at the formula and of course, if it is not a multiplicative relation, this method is not to be used, you have to go back to the original formulas that we have derived and do everything in detail from that point itself without touching this point.

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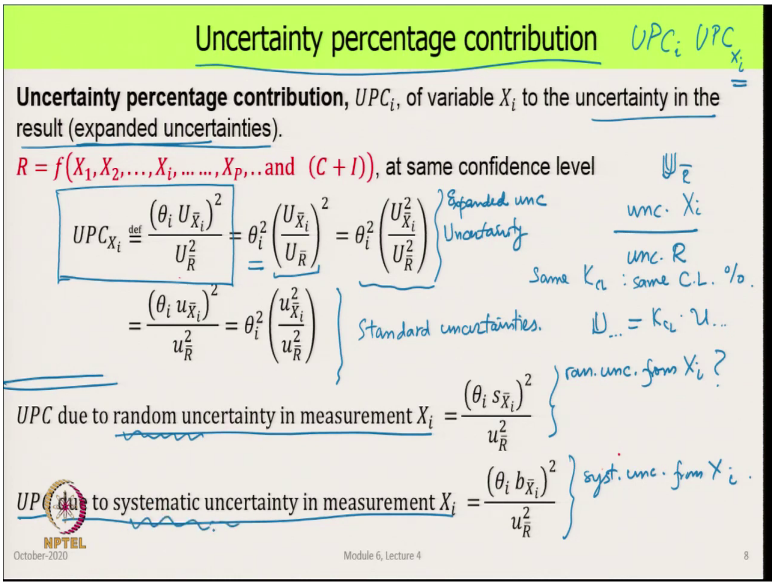

Now, we will look at the uncertainty percentage contribution or UPC i or sometimes we call UPC X i. So, this is a number associated with a particular parameter. Very broadly, UPC, the uncertainty percentage contribution is that fraction of the uncertainty of the result, the expanded uncertainty that comes from the uncertainty in measurement X i.

So, how much come uncertainty came from X i divided by that combined total uncertainty in the result, this is our definition of uncertainty percentage contribution. So, our formula becomes UPC X i is defined as theta i U X i whole square upon U R bar square, U is capital which tells us this is the expanded uncertainty that means, R U R bar is our final answer of the uncertainty in the result, and we are asking what fraction of that came from every individual measurement that we had in the apparatus.

So, the definition is theta i U X i bar square, this whole thing squared upon U R bar square which can be written as the sensitivity coefficient comes out and then, we have a ratio of combined uncertainty in the measurement to combine uncertainty of the result, this whole thing is squared or square of the ratios of these two.

Now, if we report all uncertainties at same multiplication factor which means at the same confidence level which we must always do, then the U whichever way we look at it is K CL time the standard error whether it is the measurement or the result. So, if we do that, we can substitute here, and the case will cancel out in the term here and we will get theta i u X i bar this product square upon u R bar square and this theta i square upon u X i bar square upon u R bar square.

So, that is a definition in terms of the expanded uncertainty or the total uncertainty or we just cause the uncertainty, uncertainty in the measurement and uncertainty in the result where this ratio is the ratio of the standard uncertainties. So, that is the definition of uncertainty percentage contribution.

Now, we can further classify these in terms of their breakup, in terms of random and systematic uncertainties. So, UPC due to random uncertainty in the measurement X i is contribution to the total uncertainty in the result is theta i s X i bar square upon u R bar square. So, this ratio tells us that in the result, what is the fraction of random uncertainty from measurement X i, what is the contribution of the random uncertainty of X i to that combined or the total uncertainty in the result that is what this ratio will tell us.

And similarly, we can qualify UPC that UPC due to systematic uncertainty in measurement X i is theta i b X i bar square upon u R bar square. So, this tells us that what fraction of the uncertainty in the result is coming from systematic uncertainty from the measurement X i. So, when we talk of uncertainty percentage contribution, our default condition is that it is the ratio of the expanded uncertainties of the result to the expanded uncertainty of; expanded uncertainty of the measurement to expanded uncertainty of the result that whole thing squared that is what our prime definition is.

If we want to look at systematic and random uncertainties, then we have to specifically qualify that it UPS; UPC due to this or due to this. So, that gives us all the definitions of uncertainty percentage contribution. And now, let us see how this helps us in the result relation.

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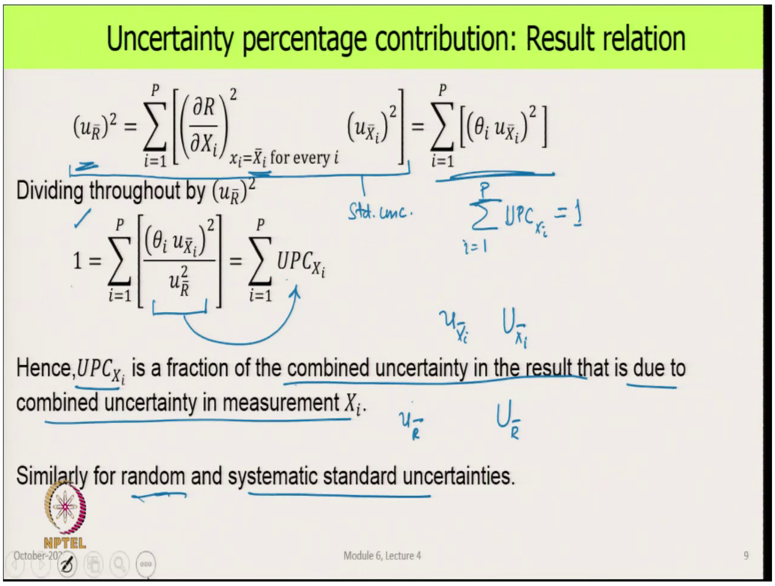

So, first we write here what the result relation is and that is given over here u R bar square is the summation of dR by dX i squared evaluated at the mean value of each measurement multiplied by u X i bar whole square. So, the standard uncertainty in the measurement, this is the standard uncertainty in the result. So, the definition is coming in terms of standard uncertainties and from our definition, we can see that this term here is theta i so, the expression becomes quite elegant that is theta i u X i bar square summation 1 to P.

Now, what we do is we take this relation, this part of the relation and this part of the relation, divide throughout by u R bar square so, the left side becomes 1 which is what we have here, and the u R bar goes into this termand the denominator and you have theta u X i bar square upon u R bar square and this is nothing but as we saw a few minutes back, this is UPC of X i.

So, we have a nice conclusion coming up here that summation of the uncertainty percentage contribution from each measurement this is 1. So, this helps now, gives a quite a tool to understand more about these numbers. So, with that, we can state again that UPC X i, the uncertainty percentage contribution from measurement X i is a fraction of the combined uncertainty in the result that is due to combined uncertainty in the measurement X i. So, it can be other combined standard uncertainty u and X i and u R bar or the combined expanded uncertainty U X i bar and U R bar and you can make the same definition for random and systematic standard uncertainties.

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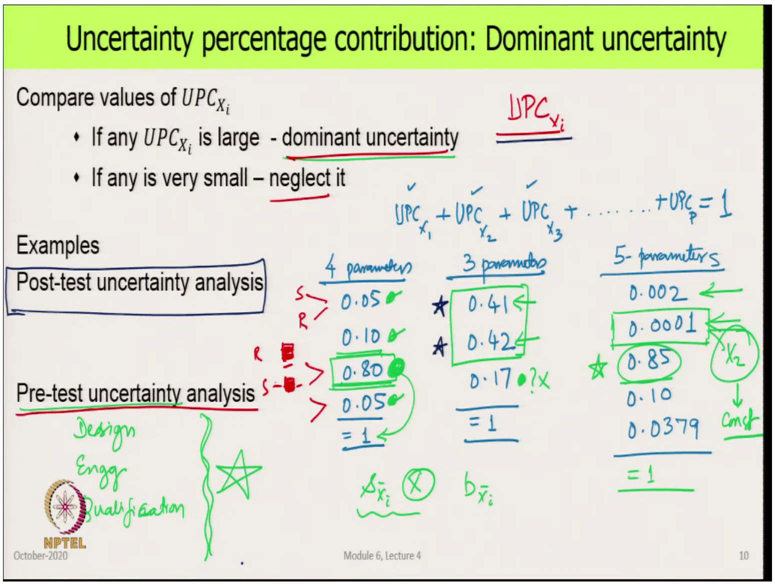

Now, let us see how all of this helps us in the uncertainty analysis process. So, what we have is that UPC X 1 plus UPC from X 2 plus UPC from X 3 and so on, we add up all of them UPC from the last parameter p, this is 1 and we have all that information required to do these individual calculations.

So, what we get is a series of numbers for each UPC whose sum is 1 and now, we can look at these numbers and figure out what is going on. So, we will take a few examples and let us see what happens. So, in an experiment, we have 4 parameters, we do all the calculations and we find that the UPC's that we get are 0.05, 0.10, 0.80 and 0.05. So, we checked the sum is 1.

Now, if this is the way the numbers stack up, just by looking at these numbers, it tells us that here you have a number which is very large in comparison with a large fraction of the total which is 1 and much greater than the others that are there. So, this would be what we will term as the dominant uncertainty and its implication is as follows.

If we do something to reduce this or reduce this or reduce this hoping to do a better experiment, the benefits will not be there, they will be very very small, they may be insignificant. So, whatever we should what these numbers tell us is if you want to reduce the uncertainty in individual measurements, focus on this one, the dominant uncertainty. If you reduce this by half, your total uncertainty will come down say substantially, reducing this by half will have very little effect.

Then, we take another example where say we have 3 parameters and the uncertainties in them are 0.41, say 0.42 and 0.17. As before, we check this has to add up to 1 and now, we say well what do these numbers tell me and what we have what we see here these two are comparable to one another and they are much greater than the third one not as great as in this first example, that was 0.05 and 0.8, but still about factor of two, two and a half times greater.

So, in this situation, our dominant uncertainty becomes these two. So, our objective should now be if you want to reduce the uncertainty in the result, you have to focus on this and this not so much on this. So, figure out which are these measurements and come up with strategies to reduce the systematic uncertainties and the random uncertainties

And you take a third example which has say 5 parameters. So, we have 0.002, 0.0001, 0.85, 0.10 and the remaining is over here so, this is 95 so, this will be something like 4 04 0.04 or 38 something like that 379. This happens in many instances. Without doing much effort, we find that in some measurements, the uncertainty is very very small say like this one even this one.

And you wonder you know what is, am I, have I made a mistake or is it actually the case and if you go back, check what is happening, you find out what this is actually the case. So, you have some cases some numbers which is they are so small relative to of course, this is all adding to 1, they are so small that we say you know I need not even take this uncertainty into consideration in my analysis.

I will make my life simple instead of 5 parameter, I have now I have to deal with 4 parameters that becomes a little easy and I can say that is my result formula, this particular parameter say it was X 2, this is almost exact and I approximate this and treat it as a constant and make my entire calculation little easier without losing any information in the analysis.

And like before, this was to tells us that is you look at this number, this is hugely the dominating uncertainty and if you want to reduce the total uncertainty in the result, concentrate on this measurement and if you go back to the experiment and say well, now I will tried to reduce this and it may so happen that this is a very difficult measurement and to reduce it substantially is actually quite a challenge. So, that is the reality we run into and then, we say what is the best that I can do with this, I will do that and live with it.

We could further break up any of these numbers into their systematic and the random part. So, what we could do say in this first case is look at 0.8 and say well, what are the elemental contributors to this uncertainty? And we will find that there are some random causes and there are some systematic causes. Then by comparing those, you can figure out which one is the main culprit that is making a big contribution here and again like the others, you will find one or two of these or three of these which are large contributors to the total uncertainty of that measurement.

Then, we can say now that is where I need to focus, the systematic uncertainty if that was the issue or we may have to change our instrument, if we have the result, the random uncertainty as an issue we may have to take more measurements and reduce the uncertainty in that or get a better instrument and take more measurements.

So, this is the help, this is what our UPC has come to our rescue. We are able to take quite a few important decisions without even doing the experiment in some cases as would actually be the case with the pre-test uncertainty analysis.

What is happening in the pre-test uncertainty analysis is that we do not have measurements so, we do not have the random uncertainties the s X i's are not there, we can work with b X i's and get these from some other source or make a good estimate of that and if you do that analysis and we come up with these type of numbers at the pre-test stage which means you are in the design stage or you are in the detail engineering stage where you are deciding what instruments to get, what parameters to have or you are in the qualification phase.

Then in all these cases, you have not yet done the experiment, but by doing this analysis, you have got to know what is it that to expect in this type of a setup? Which are the measurements which will have low uncertainty? Which are the measurement which are contributing very large uncertainty and we can at that stage itself make changes in our in this part and get uncertainties to a lower value which we actually want.

So, that is the reason why we did all of this uncertainty analysis so that even before doing the experiment, we are in a position to say what uncertainty I will have and so, it is a situation where you have an apparatus say for measuring thermal conductivity of a material and somebody comes and says please measure the thermal conductivity of this material for me.

The first thing you would do is to figure out the broad parameters are acceptable and if that is ok, then you come and do the pre-test uncertainty analysis and see what sort of uncertainty can you expect and if those are acceptable to that person who has come to you, then you go ahead and do the experiment, if they are not acceptable, the nothing goes forward we says we cannot do that, something else has to be done.

So, we took a very important decision that telling somebody that something is not possible because the uncertainty is not coming in my apparatus. In the post-test uncertainty analysis, we have all the data, and all this analysis helps us in interpreting the result and drawing our conclusions.

If we draw a certain conclusions based on the values of the result, then we can go further qualified and say in this result, this particular factor was the dominant uncertainty and in light of that, the interpretation of that result should be done in a different way. So, that is how valuable our UPC has been in both the pre-test phase and in the post-test phase.

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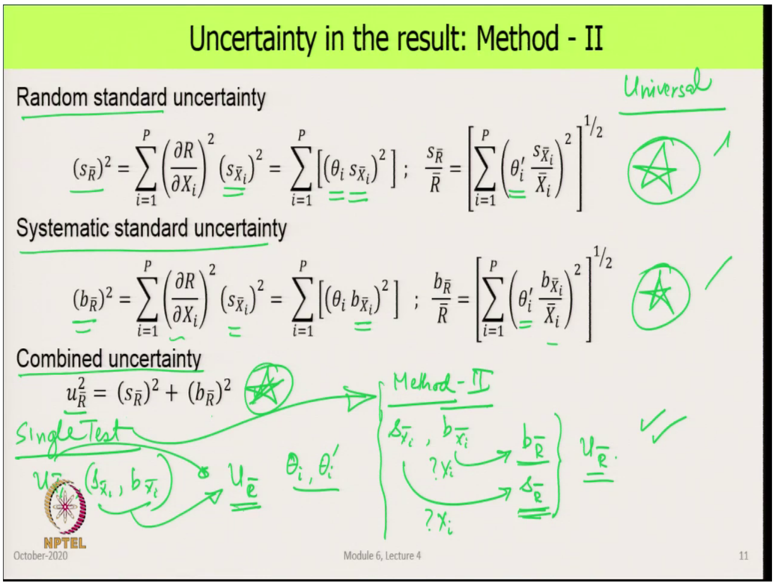

So, what we have done so far is assume that we were looking at method I and in method I, I just mentioned we were looking at u X i bar which we calculated from s X i bar, b X i bar and then, directly from here, we calculated U R bar and all this came in the category of what we had already classified as a single test as against the multiple test.

So, when you got final result in method I, we got contributions of individual uncertainties, the combined individual uncertainties so, theta i's or theta i prime, this we got. So, we know which parameter is contributing how much, this method I tells us. What method I does not tell us is whether that uncertainty contribution that we are looking at is coming from systematic uncertainties in a measurement or random uncertainties errors in a measurement. This is where method II is slightly different from method I.

What we do in method II? We have first calculated X i; s X i bar and b X i bar. Using these, this we calculated b R bar, systematic standard uncertainty in the result, using this, we calculate s R bar the systematics the random and standard uncertainty in the result, then combine these two and get the combined standard uncertainty in the result. So, we have used the same data that was there in the single test. So, this is also a single test, but we did not do the calculation from here to here in the beginning, we let it stay as it is, we calculated the other two parameters and then calculated this one.

So, now, what do we have finally? We have contributions from systematic uncertainties, contribution from random uncertainty. By looking at these two numbers relative to this, we can say well what is it that is really hurting us is it the random uncertainty or the systematic uncertainty and then, within that, we can further break it up and then, go backwards and say you know which parameter is causing most of that.

So, we can then say that go back and say well it is parameter X 2 and it is the systematic standard uncertainty of X 2 which is the big problem, we need to look at that and that tells us that if that were the case, X 2, the instrument we are using or the method of measurement we need to improve that. This is very valuable information and method II gives you all the details from there.

Here are on the top, this and this are the expressions for doing this calculation. So, we wanted s R bar so, this is s R bar square which is del R by del X i square which is nothing but theta i multiplied by s X i square and if it divide throughout by R and take the square root s R by R is the summation of theta i prime s X i bar over X i bar, this whole thing is squared. So, we have calculated a s X i bar, we have the mean here, we got the value of the sensitivity, the relative sensitivity coefficient and you can use this.

Now, remember these formula were universal. They are also good for the multiplicative formula that we may come across, but even that is not the case, these are always good and if you want to be on the safe side, use these. It will take a little more time, little more effort, but you know what you are doing. This was the random standard uncertainty.

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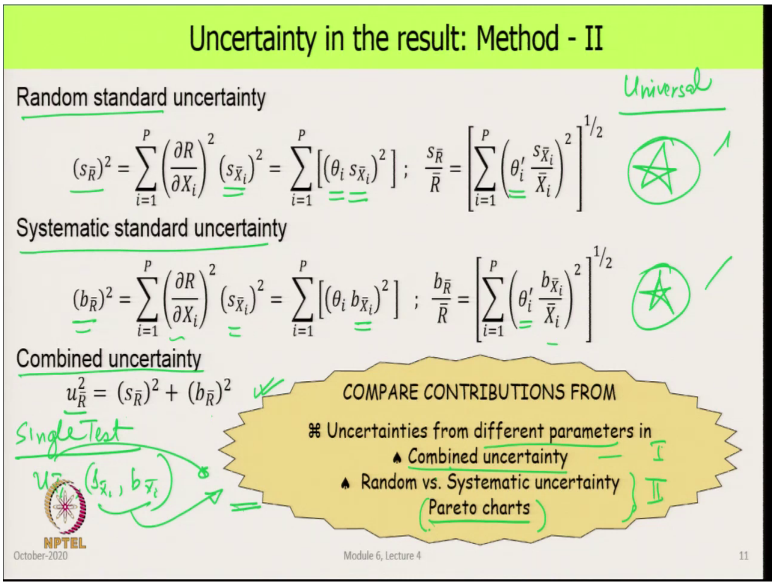

Now, look at the systematic standard uncertainty where b R bar square is this multiplied by s X i bar square theta i times b X i bar square and b R bar by b R is theta i prime into this. So, very similar relation and we can calculate the systematic standard uncertainty in the result here and then finally, we can combine the two and get the combined standard uncertainty of the result as this square plus this square and that gives us the answers we are looking for.

So, what we have seen is so, this what we see here that with this analysis, we can compare contributions to the result uncertainty coming from combined uncertainty which was method I and if you use method II, we could also get the breakup of random and systematic uncertainty from different parameters. So, here, we get as much inputs as there are number of parameters.

In the second case, we get twice as much information because the random and systematic uncertainties are also taken care of and to a graphical representation of that you one can do by using Pareto charts and in one picture, it gives you a complete overview of which has uncertainty which are very significant, which are those that can be neglected and how is the distribution happening.

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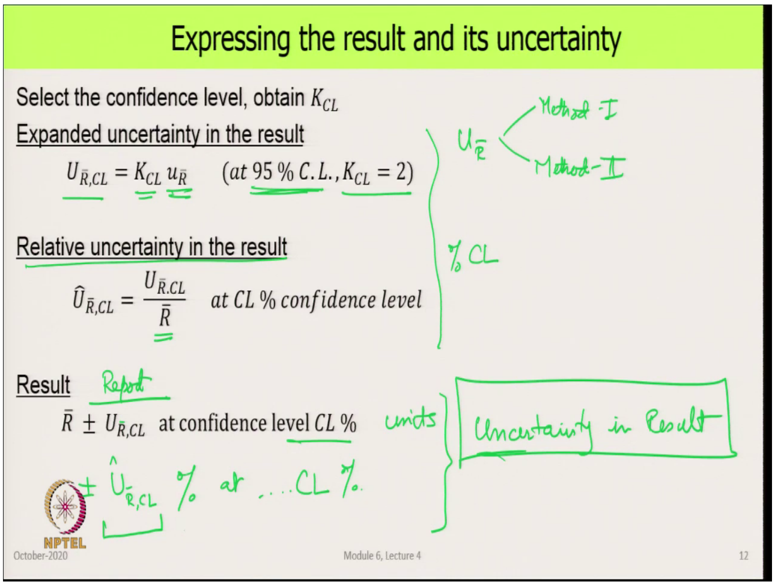

So, finally, whether it was method I or method II, we converge and we say we have calculated the combined uncertainty in the result of either from method I or method II so, u R bar could come from either of these sources and then, we multiply it by the factor which would depend on whatever confidence level we take, we will normally take 95 percent confidence level the K CL becomes 2 is I can round up from 1.96.

And we can get the expanded uncertainty in the result, divide that by the mean value of the result and we get the relative uncertainty in the result, both of these are at a particular confidence level and finally, we report R bar plus minus U R, CL at certain confidence level as in units or as plus minus U hat R, CL, this should be R bar percentage at certain confidence level.

In practice, you will see both of these coming in and this is when people say what is the uncertainty in the result, it is this which is the value that is the uncertainty. So, you may just call it in the end, uncertainty in the result or just uncertainty. Uncertainty could be a little bit confusing, if you are looking at result, it is uncertainty of the result, if you are looking at the measurement, it becomes uncertainty in the measurement. So, that is how we calculate and express finally, the uncertainty in the result.

Method I, method II we have looked at. We also looked earlier at the multiple test, repetitive test technique all of them gave us the answer for this question. So, this was the objective of this module.

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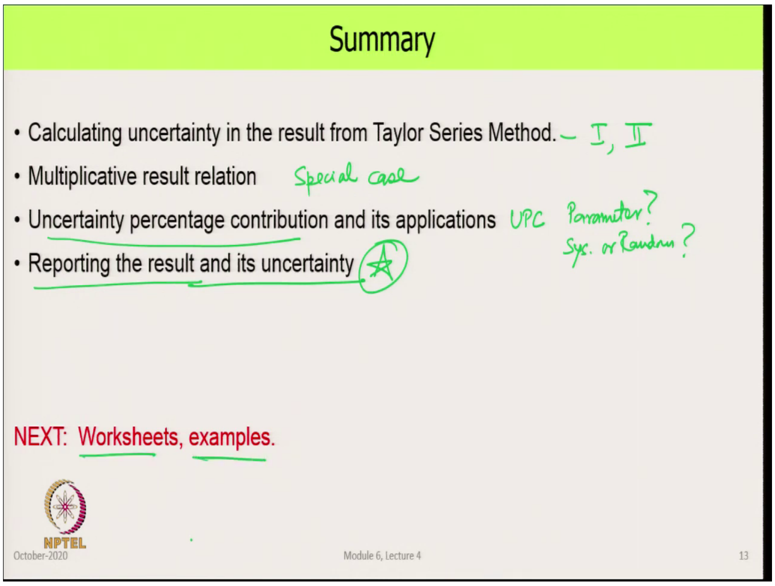

So, we summarize this lecture, what we have learned is calculating uncertainty in the result from Taylor series method and we looked at two methods method I and method II. Then, we looked at a special case of the result formula being purely multiplicative in nature and we saw life becomes much more easier to deal with if we have the multiplicative relation.

Then, this analysis gave us a breakup of uncertainty percentage contributions. So, we defined what is UPC and then, we could see where it came from, which parameter or which measurement and whether systematic or random.

So, we got a lot of information from this analysis and we then combined everything, and we saw how do we report the result and its uncertainty which was central to our objective. Next, we will look at some worksheets and a few examples. With that we conclude this lecture.

Thank you.