Introduction to Uncertainty Analysis and Experimentation
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Module - 06
Uncertainty in a Result
Lecture - 01
Methodologies. Multiple tests method

Welcome to the course Introduction to Uncertainty Analysis and Experimentation. We now look at Uncertainty in a Result; we will begin in this lecture by looking at the Methodologies and we will look at one of the methods which is the Multiple test method.

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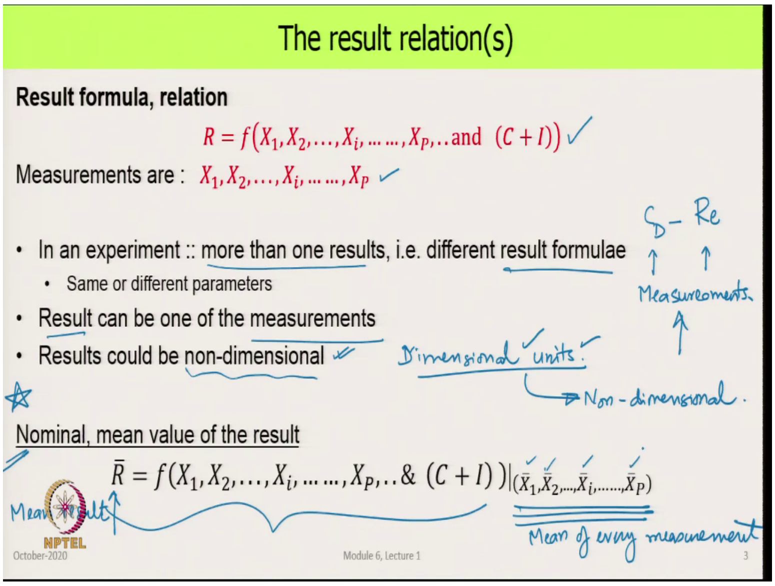

So, to recap, in all result analysis, all the uncertainty calculations for a result are primary inputs are the result formula and from the experiment, we have got measured values for each of the parameters. Now, in an experiment, there can be more than one result or what we call result formulae; for example, it could be like in a drag on a object, you could have the drag coefficient as one and the other result could be the Reynolds number, both are counted from a set of measured values.

So, we have an example here of a two result experiment; it is also possible that the result could be one of the measurements itself. So, if you are trying to measure something and we say that look this is my final requirement, that becomes the result. The other thing could is that results could be non-dimensional; for example, in this here, both drag coefficient and Reynolds number are non-dimensional.

So, typically all measurements that we make in the laboratory, those are dimensional measurements; their units are there, they have units. And using these, we convert them into non dimensional numbers; the results of non-dimensionalizing are many, we will not go into those details.

The first step in the result uncertainty analysis is to calculate the nominal or the mean value of the result, where we use the result formula which is there and evaluated at the mean values of every measurement.

And that gives us R bar, which is the mean of the result or the nominal value of the result, either way we can look at it. So, this is a simple straightforward calculation, we need to have the result formula and values of every parameter the mean values.

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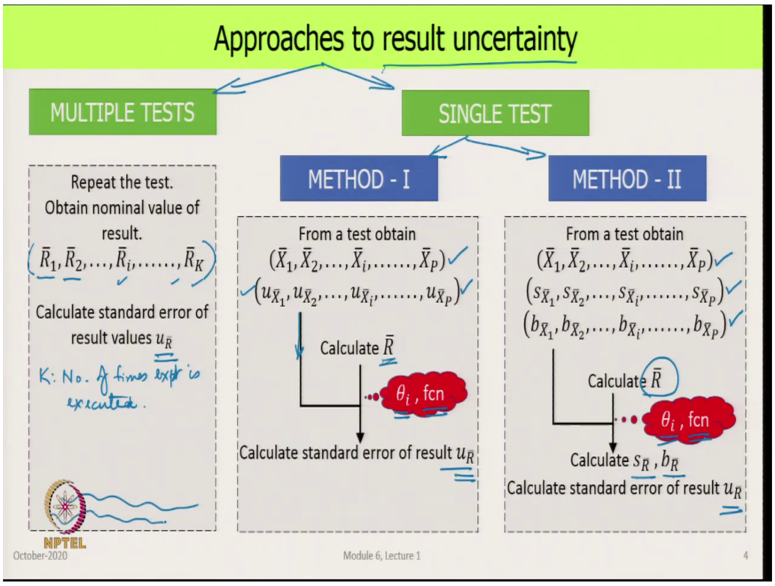

So, now we are ready to look at the approaches by which we can get the result uncertainty, we just got the mean value of the result. Now, the question is, how do I calculate the uncertainty of the result? And for that we have two broad types of tests, either we could go this way which is the multiple tests method or this way which is a single test method.

In this lecture, we will look at multiple test method in more detail; but the idea is like this, we do an experiment once and calculate the result and that is what gives us R 1 bar, that means the result from the first experiment. For that we had all the mean values of the parameters.

Then we go back, do the experiment exactly the same way again and calculate the result that is R 2 bar, the result mean value of the result from the second experiment. And like that we continue as many times as we want. So, K here is the number of times we do the experiment. So, now what we have is a bunch of numbers here, which themselves form a sample and using that data as a sample, we can then calculate the standard uncertainty of the result u R bar.

So, what happened was that we bypassed calculating uncertainty in every individual measurement; the when either we could not do it or we did not want to do it, this is one technique by which we can establish the uncertainty in the result. We will look at this method in a little more detail in a few minutes.

Our second option is to do the experiment once, which is called the single test and then to calculate the uncertainty in the result, we have two methods; we can go this way, or we can go that way, method I or method II. In method I, we do the experiment once and calculate the mean values of every parameter which is the first line; then following the procedure that we have learnt in the uncertainty of a measurement, we calculate the standard uncertainty in every measurement u X 1 bar, u X 2 bar like that.

Then with the result formula and these data, we calculate the standard error or the standard uncertainty of the result u R bar. But the question here is, what is the function? That is what we will learn here and we use something here called theta subscript i which is called the sensitivity coefficient. So, this is method I, we calculated u X 1, bar u X 2 bar and from there we calculated u R bar.

Method II is slightly different from method I, although the data we start with is exactly the same as what we would have in method I. We have all the measurements from every parameter from an experiment that has been done once, from there we calculate the mean value of every parameter.

And now we do something in little more detail that we have learnt earlier; we calculate the random standard uncertainties in every measurement and we calculate the systematic standard uncertainty in every measurement. And then again using some function and the sensitivity coefficients using the result formula, we calculate two parameters; the random standard uncertainty in the result and the systematic standard uncertainty in the result.

And then combining them, we calculate the standard uncertainty of the result u R bar. So, we will now look at multiple tests technique, after that we will look at the single test technique.

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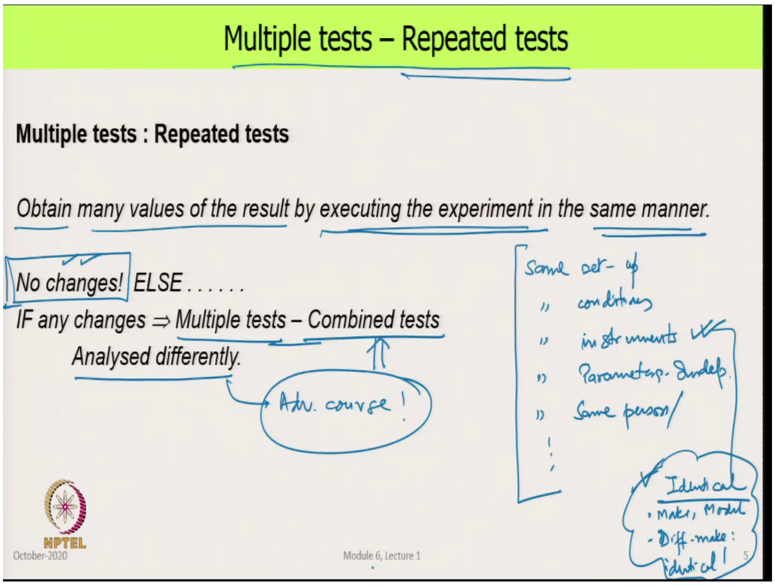

As said in the multiple test technique; we specifically further qualify it by saying that, this is multiple tests repeated tests. And in a minute it will become clear, what are the implication of the second part of the qualification which is repeated tests. So, what we have said earlier is now put up here that, we obtain many values of the result by executing the experiment in the same manner; which means from experiment to experiment, there was absolutely no change done at all.

It was the same setup, same conditions, same instruments, same all the other parameters, especially the independent parameters and done by the same person or same experimenter.

So, anything else that has to be there, that is also exactly the same. So, this is what we say as no changes; this is central to the idea of multiple tests, repeated tests.

But now consider a case where what we did, we did a few experiments and then we went away and replaced one of the instruments; say we replaced a pressure transducer or we replaced one of the temperature sensors. By an identical one and by identical there are many practical implications, it could be from the same make, same model or from a different make, but identical in all ways.

Now, what happens? The experiment that we are doing now is not exactly getting the same set of uncertainties coming in from all the instruments. The first few set experiment which were done with one instrument and the next few experiments which were done with a replacement instrument; they got uncertainties coming from different sources, the experiment no longer qualifies as being repeated tests.

It qualifies as multiple tests, we did many tests again and again; but this is called combined tests. And in this series of lectures, we will not study combined tests; their analysis is quite different and this is topic for an advanced course in uncertainty analysis. But it is important to remember that in what we are going to learn now; what are the if's and but's for executing the experiment and that nothing should change.

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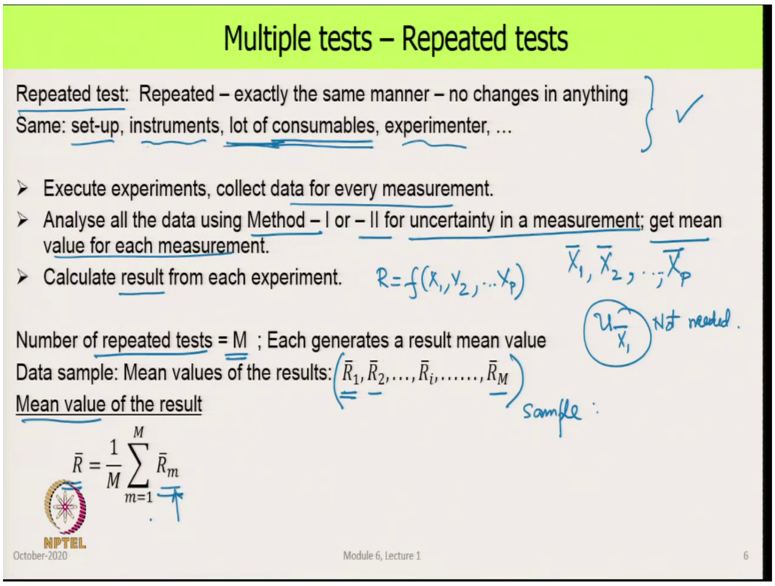

So, this is what we have said here now that, in the repeated test nothing changes; one of the issues could be of consumables. For example, if you are working in an experiment where you are burning a fuel, you may have the same categorization of fuel; but it could have come from either different sources or one could have been lying outside for a long time and other not lying for a long time, in which case the consumable source itself is likely to render the repeated test into a combined test.

So, we have to be careful on this. The rest of the issues of setup, instruments and experimenter should be the same as we have just mentioned. Having taken care of that, we execute the experiments, collect data for every measurement, analyse all that data of the measurements either by method I or by method II for the uncertainty in a measurement analysis that we have learnt earlier. And then we can get the mean value for each measurement.

So, that is all what we are looking at, we are only getting the mean values; X 1 parameter we got X 1 bar from the measurement, X 2 parameter we got X 2 bar from the measurement. And finally, if there are p number of parameters, we got X p bar. We do not do the calculation for the standard error in each one of these, it is not needed in this method.

Then we use the result formula R is equal to function of X 1, X 2 like that till X p and from there we calculate the result or rather the mean or the nominal value of each result which is R 1 bar, R 2 bar, like that R M bar, M is the number of times that the experiments have been repeated.

So, this for us is now a sample in the statistical sense and then we use all the techniques of statistics on this sample. So, what we do is, we take all these things and calculate the mean. So, the mean value of the result is R bar 1 upon M times the summation of every result value that we had. So, we get the mean value from the experiment, this was the individual means.

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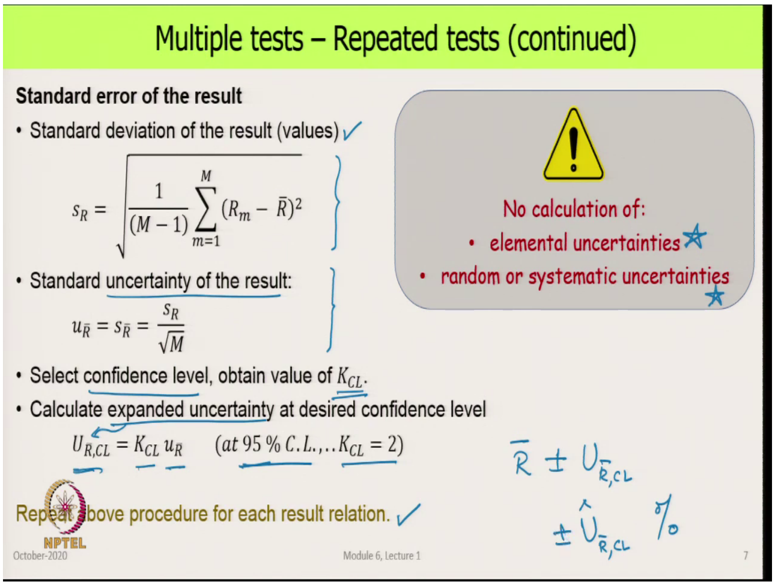

Then we calculate the standard error of the result, for which we first calculate the standard deviation which is the same formula that we have in statistics. And using that we calculate the standard uncertainty of the result or the standard error, which is U R bar. We then select a particular confidence level that we are interested in and from the tables, we calculate or obtain; we do not calculate it, we just obtain the multiplier multiplication constant or the multiplier.

And then using this relation, the multiplier into the standard uncertainty gives us the expanded uncertainty of the result at whatever confidence level that we have chosen; if we chose 95 percent confidence level, this constant is 2. And then if there are more than one results in the experiment; we repeat this procedure for every result in the experiment, every result relation.

So, a word of caution here is that as we have mentioned that, we are not calculating the elemental uncertainties; we are not calculating the random or systematic uncertainties. In a way this makes life easy, we did not get into the nitty gritty details of that; but by just doing the experiment over and over again and getting just the mean value in each case, we were able to establish the mean value of the result and its uncertainty.

So, finally, we would express the result as R bar plus minus U R bar at some confidence level; this is in terms of units or we could give a percentage value by dividing it by the mean value of the result and expressing this as so much percentage.

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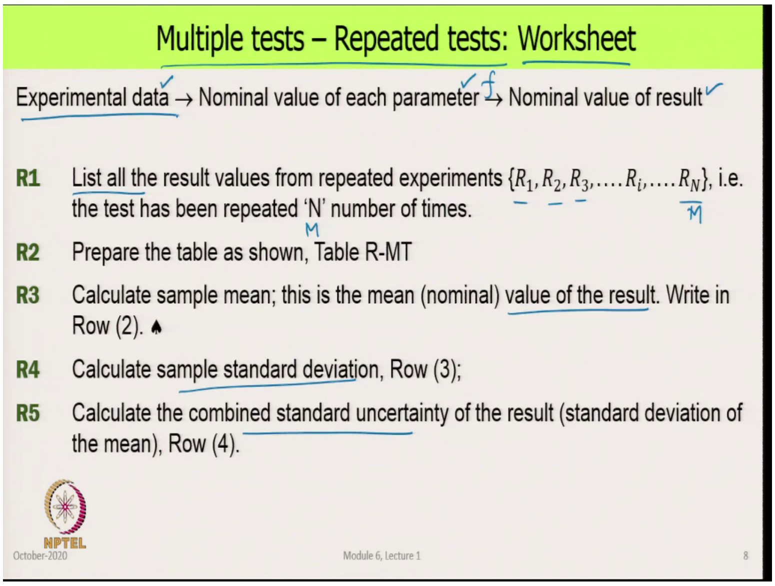

So, now we will give a step by step approach as to how to do this. So, just summarize now this in a very systematic way; we have seen what is the statistical basis, logical basis, now let us see what is the step by step basis. So, this is the worksheet for multiple tests, repeated tests.

We have done the experiments; we got lots of data for each parameter, from which we calculated the nominal value of each parameter. And using the result formula, we calculated the nominal value of the result and this we did for every experiment. So, in the first step, now what we will do is; we will list all the result values from the repeated experiments, which is R 1, R 2, R 3 and R N. Here we have put N, earlier we had put M; we can put that if we have to use that as M.

And then we take the table as we will see in a few minutes; we calculate the sample mean, which is the mean of the values of the results, then we calculate the sample standard deviation, then the combined standard uncertainty.

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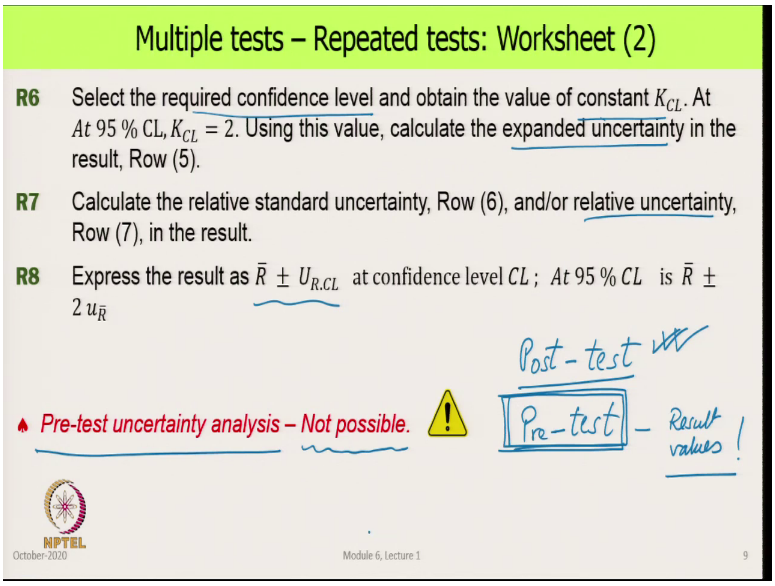

Then we select the confidence level and obtain the constant K CL; from that we get the expanded uncertainty and finally, the relative uncertainty. And then put all things together and make a means answer of the result. Now, a word of caution here that, if we had; this is all fine, because we had data.

We did the experiment many times; that means we were looking at a post test scenario. The very first step that we had we said, we get the experimental data, which means we did the experiment. Now, what happens if we were in the pre test stage? We have not done in any experiments in the pre-test uncertainty analysis, we do not have any data; we have all the information about the instruments if it be at the design stage, but then that is not enough, because we need the result values.

And we do not have any of these, for that we have to do the experiment. And so, since we do not have result values, we cannot even get started in this technique for doing the pre-test uncertainty analysis. And so, this method of multiple tests, repeated test is simply not possible for the pre-test requirement; we cannot do it, it is good for the post test analysis.

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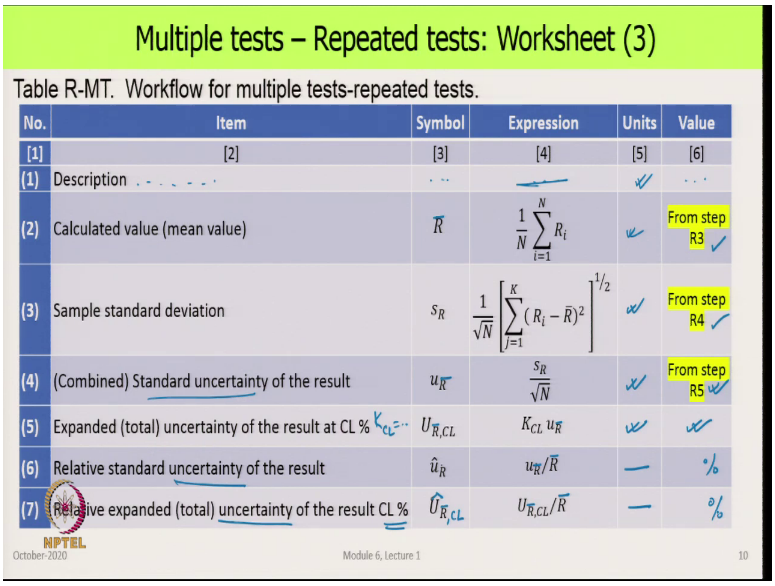

Table R-MT. Workflow for multiple tests-repeated tests.

No.	Item	Symbol	Expression	Units	Value
[1]	[2]	[3]	[4]	[5]	[6]
(1)	Description				
(2)	Calculated value (mean value)	$\bar{R}$	$\frac{1}{N}\sum_{i=1}^{N} R_i$		From step R3
(3)	Sample standard deviation	s_R	$\frac{1}{\sqrt{N}}\left[\sum_{j=1}^{K}(R_i-\bar{R})^2\right]^{1/2}$		From step R4
(4)	(Combined) Standard uncertainty of the result	$u_{\bar{R}}$	$\frac{s_R}{\sqrt{N}}$		From step R5
(5)	Expanded (total) uncertainty of the result at CL %	$U_{\bar{R},CL}$	$K_{CL}\, u_{\bar{R}}$		
(6)	Relative standard uncertainty of the result	$\hat{u}_R$	$u_{\bar{R}}/\bar{R}$	—	%
(7)	Relative expanded (total) uncertainty of the result CL %	$\hat{U}_{\bar{R},CL}$	$U_{\bar{R},CL}/\bar{R}$	—	%

Now, we see the table, in this there are columns which is serial number, item; then there is symbol, expression, units and the value. We write down the description of the parameter over there, what it is symbol is and expression is not applicable at this point; we must always write the units, this is very important. And from our mean value calculations, we get the value over there.

In the next line, we write the calculated value of the mean; so this is R bar 1 upon N for all the results or M if we are using M, the units. And then we can using this formula, we can write the value here. Then the sample standard deviation S R; this is nothing but 1 upon square root N and square root of all of this sorry, not this is not right, yeah this is standard deviation.

This is what we have in statistics 1 upon square root N summation all of this; we can write the units, it has the same units as the parameter we are looking at and we write the result over there. Then we look at the, the standard uncertainty or the combined standard uncertainty of the result u R bar.

And this is the standard deviation of the sample S R divided by square root of N, same units it should have and we can enter the value over there. Then we decide what is the value of K CL we want and we make the expanded uncertainty; this is R bar and this is U R bar, multiply the two, units are same and we can write the value over there. Then the relative standard uncertainty is u R bar divided by R bar and we can, this has got no units and we can write the value over there and express it as the percentage if we want that.

And finally, the relative expanded uncertainty or the total uncertainty or just the uncertainty of the result add certain confidence level; this is U hat this thing, this is equal to U R bar CL divided by R bar, no units, again this is the percentage. So, this completes our analysis of multiple tests, repeated tests.

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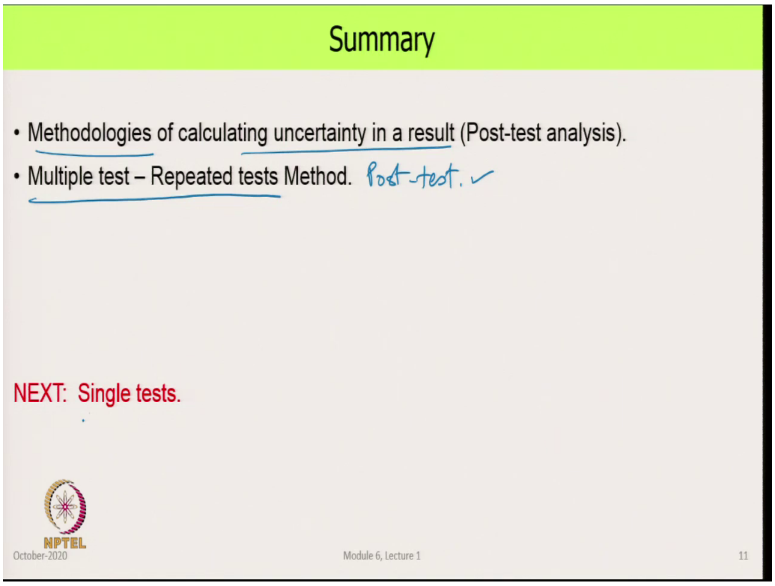

So, what we have done in this lecture, is we have looked at what are the various options in terms of methodologies that we have for calculating uncertainty in the result.

Right now we looked at one method, which was good for post test uncertainty analysis only. And that is what we looked at; we saw what is the technique, what are the mathematical formula that we need and that was the multiple test-repeated test method, where we said we do the experiment again and again, get a value of the result from each experiment and finally, do the calculations.

In the next lecture, we will look at the single test methods. So, on that note, we conclude this first part of the lecture on uncertainty in a result.

Thank you.