Introduction to Uncertainty Analysis and Experimentation
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Module - 05
Uncertainty in a Measurement
Lecture - 15
Worksheets for uncertainty in a measurement. Examples

Welcome to the course Introduction to Uncertainty Analysis and Experimentation. Today, we will look at the Worksheets for uncertainty in a measurement and I will discuss a couple of examples.

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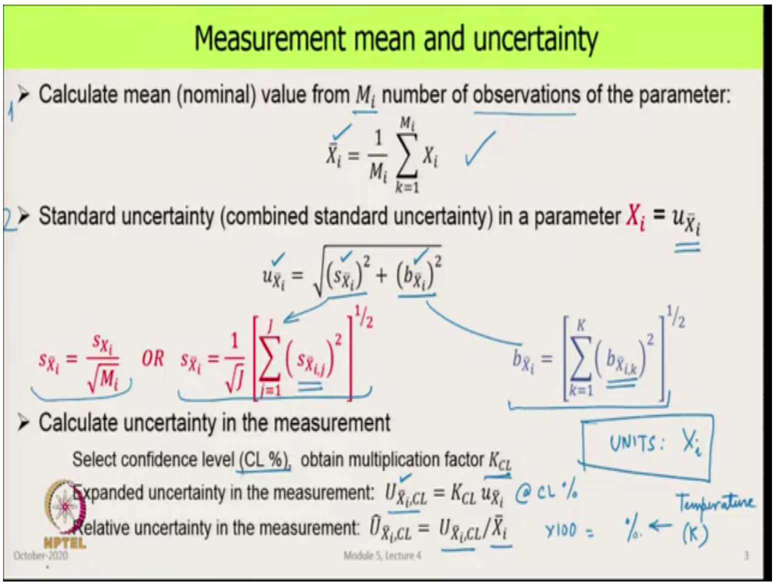

To summarize, the process of estimating measurement, mean and uncertainty takes the following steps. First, we calculate the mean or the nominal value from the various number of observations of the parameter. The simple formula from statistics is what we use. Then, we have combine we calculate the standard uncertainty or the combined standard uncertainty in the parameter which is u of xi bar. Thus, the standard uncertainty or standard error or just yeah.

So, this we calculate a square root of the square of the random standard uncertainty s xi bar and systematic standard uncertainty b X i bar. For the random standard uncertainty, we have two options; we either estimated from the measurements which is here that which we have that many measurements and from there, we did the simple calculation to get as xi bar.

Alternatively, we could identify all the elemental sources of random error, estimate them or by making measurements and then, apply this formula to calculate the random standard deviation; the random standard error. The systematic standard uncertainty, we have to identify every elemental source of systematic error, estimate the standard error associated with it and then, using this formula calculate b X i.

Once that is done, we decide at what confidence level, we want to report the result, obtain the multiplication factor from standard tables. And calculate the expanded uncertainty in the measurement at that confidence level as KCL into u X i bar. So, this is at CL percentage confidence level and then finally, the relative uncertainty in the measurement U hat which is the expanded uncertainty divided by the mean value and this can be multiplied by 100 to be expressed as a percentage.

This is ok with almost all parameters, except you have to take care when looking at temperature. Because depending on the scale that one uses, this value will be different. At best, the safest way is to report all temperatures in Kelvin. Now, here U X i bar, u X is smaller one and all the standard uncertainties have the same units as the measurand which also the same units as Xi bar. So, at every step, we should keep track of units and keep them consistent.

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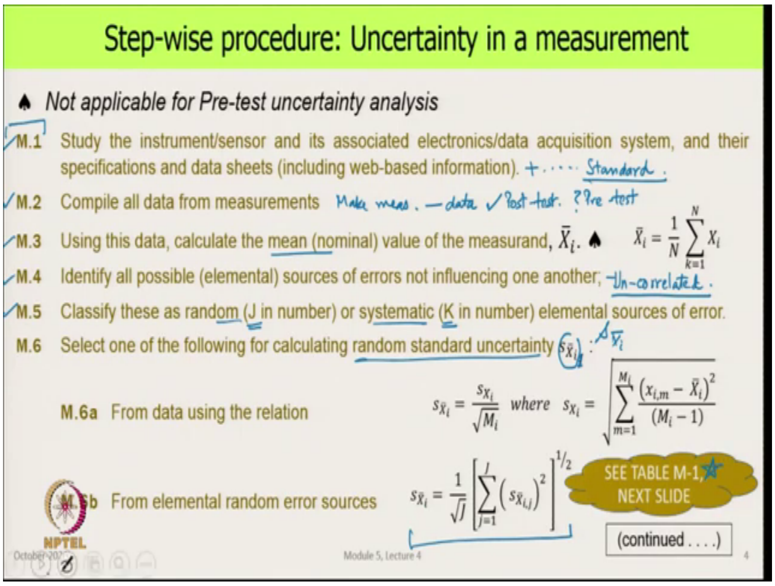

Then, on the same arguments, I have listed here in the step-by-step approach. So, one can follow this procedure, when solving a problem. The step number is indicated over here; M1 denotes measurement first step, study the instrument sensor and its associated electronics or data acquisition system and their specifications and data sheets including web-based information and anything else that you can learn about that measurement.

Among this, you will find that many of these web-based information refers to some standard. So, it is quite helpful to look up the standard because that is what many of these datasheet that we look at from manufacturers, that is how we would like to interpret it. After collecting all this information, this step is done. We move to the next step which is we go and make get all the data that we got from the measurement.

So, we made the measurement and we have data. Now, this is perfectly fine, when we are doing post-test uncertainty analysis. In pre-test uncertainty analysis, we do not collect data; either we say that it is very small and or neglected or if we have data from previous measurements, we can use that estimate straight away for the standard error.

So, we get all data from the measurements and using this data from M 2, we calculate the mean or the nominal value of the measurand which is given by this formula. Then, fourth, we identify all possible elemental sources of error not influencing one another. So, this course, we are only looking at uncorrelated uncertainties or uncorrelated errors.

That means, the source of errors that we have identified, one source of error does not influence the other source of error that is uncorrelated errors. And then, we classify them in the next step as either random or systematic and the number of random errors that we get that is the value of J; the number of systematic errors, we get that is the value of K.

Then, we do one of the following to calculate the random standard uncertainty s X i bar. So, this is s X i bar. From the data we can calculate s X i bar is s X i upon square root M i; where, s X i this standard deviation of the sample or we identified the elemental random error sources. And then, use this formula to get the value of standard random standard uncertainty and this we can do using this table 1 which we will now see next.

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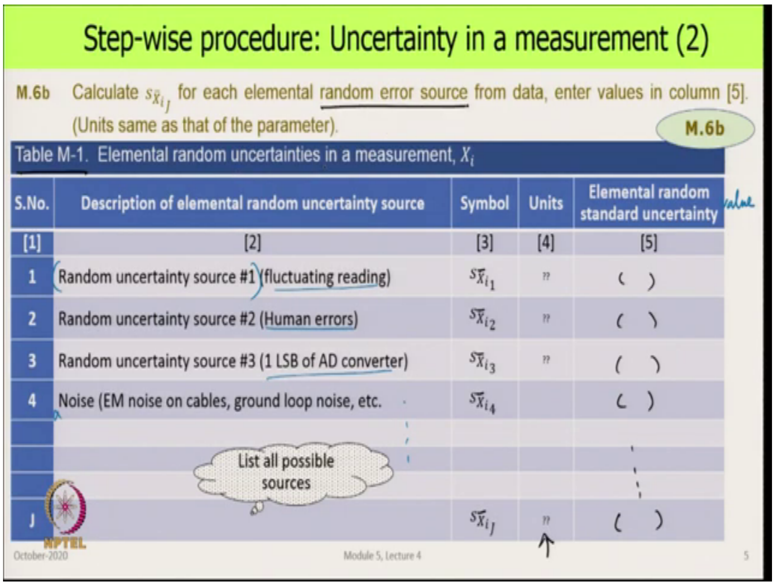

So, this is the table 1 associated with that step, elemental random uncertainties in a measurement X i. So, is the serial number, description of elemental random uncertainty source, symbol, units, and the value. So, elemental random standard uncertainty here we are writing the value. So, what we have? We have to list here as many number of a random uncertainty errors that we have.

Here it is random uncertainty source number 1, number 2, number 3 like that we can keep on adding. And instead of this, we will actually write that there is random error coming in due to something say fluctuating reading or there could be human errors or one least significant bit of the A to D converter, noise, all of that.

Each one of these elemental errors has a symbol s X i bar comma 1, s X i bar 2, s X i bar 3, until the Jth error source. The units will all be same as that of the measurand and for a variable data, we have to estimate what these values are.

So, in some cases, we may get more information, then we have to make a judgmental call; in some cases, we do not have information, we will have to use experience and information in literature to estimate this value. So, like that, we do for every one of these and complete table Mc, this table M 1. So, these were all the elemental random uncertainties in a measurement or sources of error; random sources of error, that also we can say.

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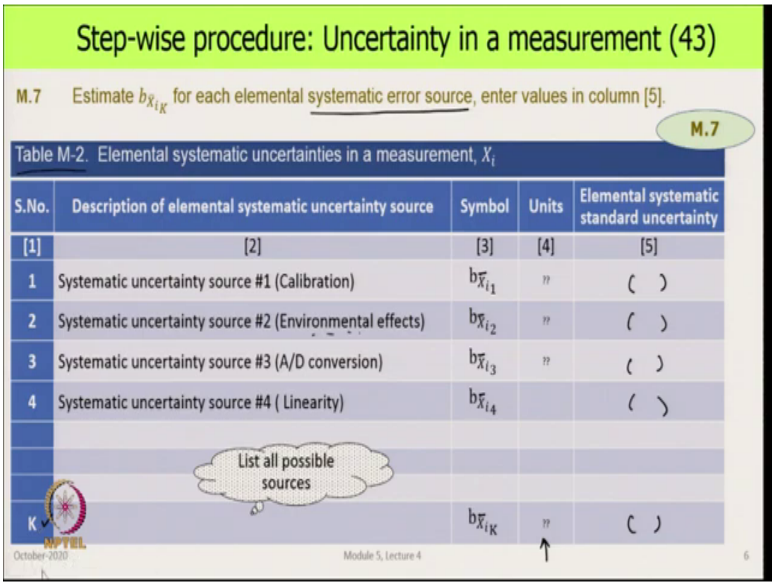

Next, we do the same thing for all the elemental sources of error for the systematic errors. Similar table, this is table M 2. We have the same thing serial number, description of systematic standard uncertainty source, symbol, units and elemental systematic standard uncertainty the value. So, we could say let us systematic standard uncertainty comes from calibration.

So, this will b X 1 X i bar, the first source. The second source could be say environmental effect this is b X i 2 bar; subscript 2. The third could be from the A to D conversion this could be like that and so on.

So, after we have done listed all these things, the units will be of course the same as the measure measurand that we have and then, using that data or experience. We write the value against each row here. So, this is the number of rows in this is our K in the previous case, this was J.

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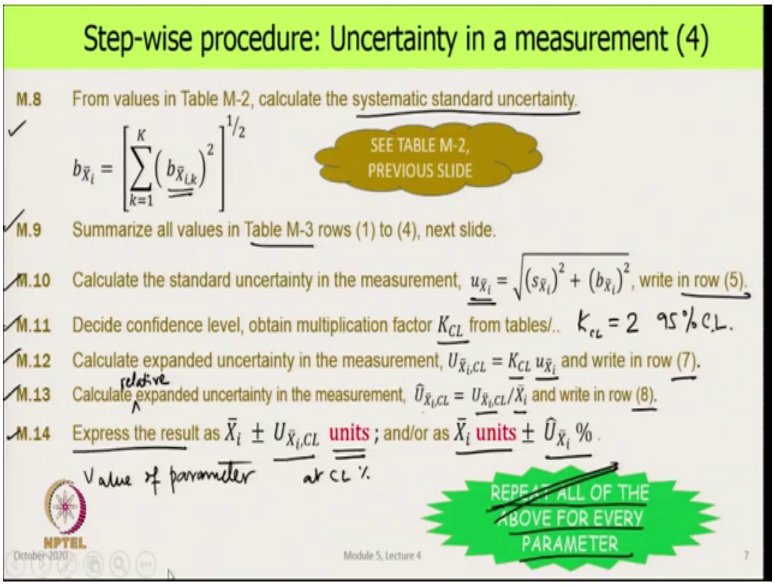

Then, move on to the next step from table M 2 we just saw, we calculate the systematic standard uncertainty, which is b X i bar is equal to the sum of the squares of the elemental standard uncertainties, square root of that. Now, we have all the values that we have, we can proceed to the last step few steps of the calculation that we can do it in either table M-3 which we will see in a minute; but the steps that are there are M-10, M-11. M-12 and M-13.

So, the steps. So, now what we do? We calculate the standard uncertainty in the measurement u X i bar from the two random uncertainty and from the systematic standard uncertainty. So, we got value of u X i bar and we write it in row 5. Then, we decide the confidence level, obtain the multiplication factor from the tables or data or whatever we have and say that KCL is equal to something. In our case for 95 percent confidence level, we will work with KCL is 2.

Then, we calculate the expanded uncertainty in the measurement which is multiplying the standard uncertainty by the multiplication factor and we write the value in row 7. Then, we calculate the expanded uncertainty in the measurement which is the relative, the relative expanded uncertainty in the measurement U hat Xi bar CL which is equal to the expanded uncertainty divided by the mean value, we write in row 8.

And finally, the last step of this whole exercise is to express the result which is the value of the parameter or the value of the measurand, either as the mean value plus minus the expanded uncertainty in units and of course, this is at certain confidence level or as the mean value with its units plus minus the relative uncertainty as the percentage.

So, either of these we can use for reporting our answer. So, with that, we complete the process for one measurand. Like this we repeat this whole thing for every parameter in the experiment. If you have three parameters, we do this whole exercise for the next parameter and then, for the last parameter. So, if you just follow those tables, everything gets summarized and we can then follow the steps and get the answer.

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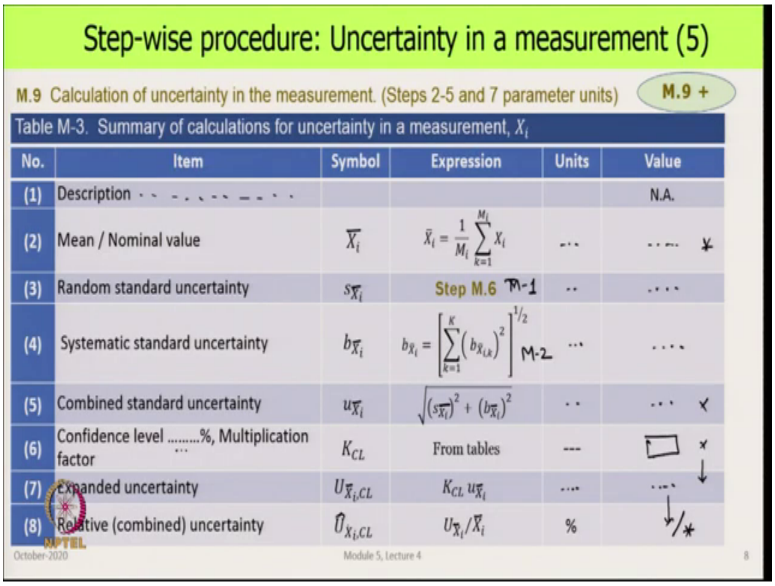

So, this is a table which summarizes the previous steps. The first four, so this first is the item, symbol, expression, units, and the value. We write the description of what this thing is what parameter it is. Mean or nominal value X i bar, we write the units. We already calculated the value, we put it there.

Random standard uncertainty s X i bar, we took it from table M-1, we write that here. Systematic standard uncertainty, we got this from table M-2, we write the units and we will write the values. So, they came from those tables. Now, we calculate the combined standard uncertainty from these two values right, the units and the value.

Then, we say confidence level is something that we want multiplication factor we get. So, we write this multiplication factor is so and so. Then, we write the expanded uncertainty U Xi bar which is K CL into U Xi bar units and write its value. So, we are multiplying this and this to get this and then finally, we get the relative uncertainty which is U Xi bar divided by Xi bar which is this value divided by the mean value which was here.

So, in three short tables, we can summarize everything and we have the value that we are looking for. So, that is a process for calculating and reporting the uncertainty in a measurement.

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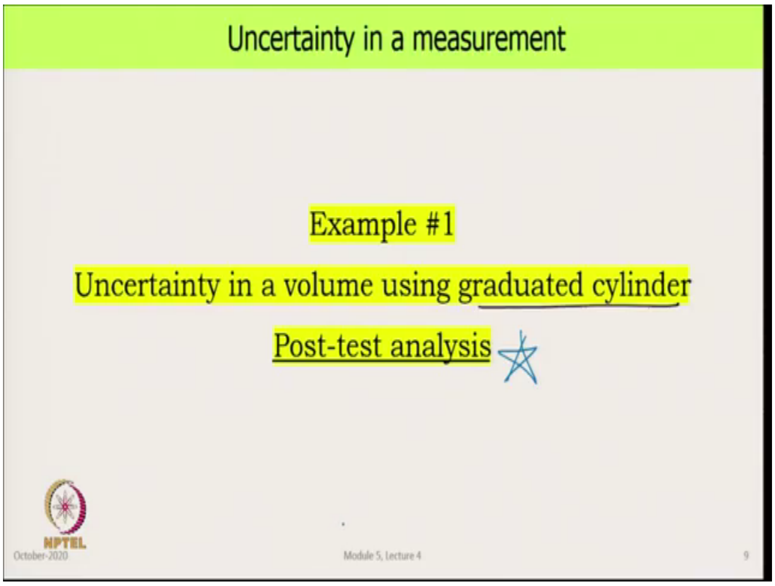

Now, we will take some examples. The example here is something very simple that we have done in school and colleges many times, that we want to measure a volume of water using a graduated cylinder. That means, we are doing the experiment, the water has been poured and now, we are making a measurement.

So, in that sense, here we are in the post-test uncertainty analysis stage. We have data. We have data from the measurement. If we were to say which cylinder to take and we want to estimate the uncertainty, because of that we will not have data and that would qualify as a pre-test analysis.

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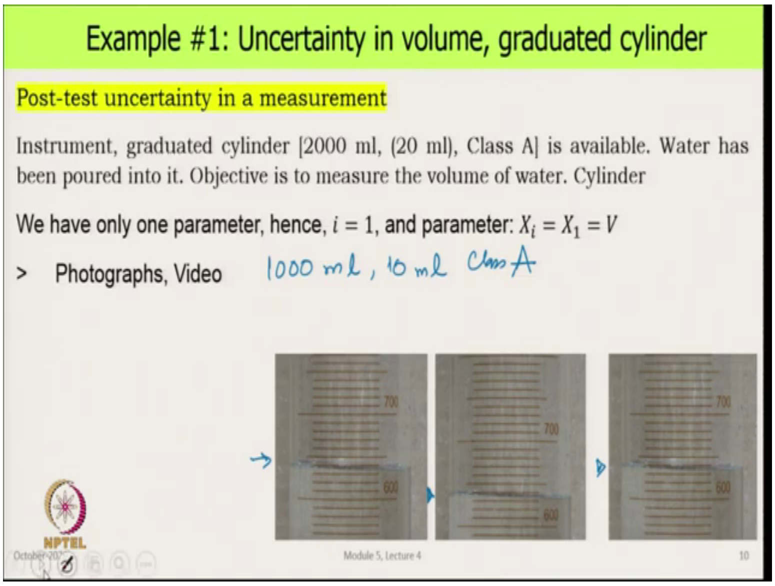

So, we did the experiment in our lab and we use 1000 millilitre measuring cylinder with 10 millilitre markings class A and we have made a video of that and some snaps from that are shown here. So, you can see the water level at this point here, the water level is over here and the water level is somewhere there.

The markings you can see, this is 600 millilitres, 700 millilitres; each marking corresponds to 10 millilitres. Before doing this, we took a spirit level, looked at the table on which the cylinder was placed and made it horizontal and then, we took these photographs. So, like this, one could read the level several times individually or get different people to tell, what they think is this reading.

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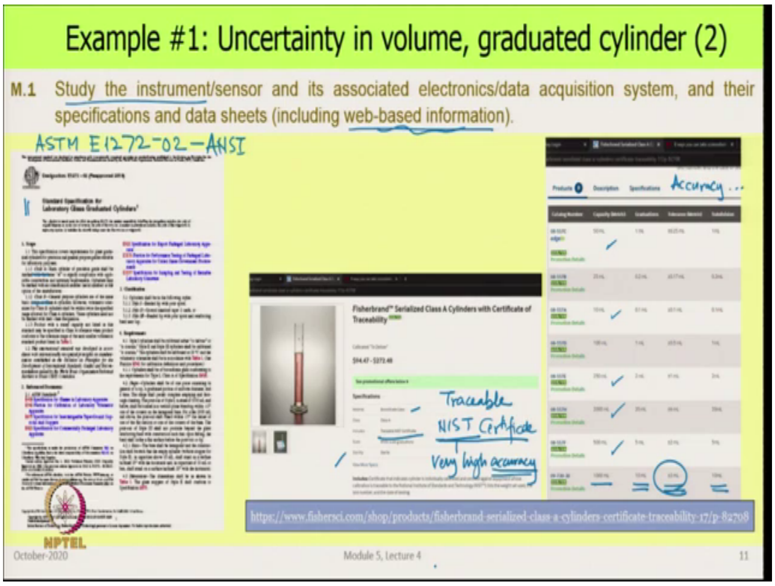

So, now start our step-by-step process to decide what is the uncertainty in the measurement. Our first step is to study this device, the instrument, sensor, electronics and as I mentioned web-based information and even some standards. So, we come across a standard here, which is here ASTEM E1272 dash 02. This is American society for testing and materials.

This standard is what we call an ANSI standard; also, American national standards institute and it would have an equivalent ISO and IS standard as well. There would be very much identical, there might be very few differences between them. But it says Standard Specification for Laboratory Class Graduated Cylinders and unfortunately, this is not very clear; but here it says there are class A, class B and then, they have given various details of what it is.

So, that is one of the points and we could get to this because many of the catalogues that we looked up, they all made a reference to this standard. So, said our standards are made as per the standard class A and so on. So, this is what it is. Now, this is a very widely used device. There are hundreds of suppliers. You can look up on the web and many of them have put their specification sheets from there. Here are a few of them.

So, this is from one make. Here is the specifications. It says it is a borosilicate glass and it says it includes traceable to NIST certificate. NIST is the National Standard Laboratory in the US, like in India, we have National Physical Laboratory and that is the highest level of a standard for any measurement. So, whatever else we use in the daily world, in between there could be other instruments which are of lower and lower quality as from the stop.

But in this case what they have said is that our markings, our procedure for markings, we can trace it and give a certificate that it this is how we did it compared to the next standard. So, this will be a very high accuracy. The procedure is also very involved and very stringent.

So, this is something we cannot use every day in life. So, we have made something that is easily usable, acknowledging the fact that we have sacrificed some of this and then, say well compared to the best how good is it. So, here there is the specification of the, this was the specification here and here is their listing and it says here that for this is a capacity in metric 1000 millilitres, this is graduation 10 millilitres, tolerance metric plus minus 3 millilitres.

This is what we have called a they call it tolerance, somewhere it is called accuracy, we will interpret all these in terms of statistics and give it a name from the way, we have defined names and then, find say the subdivisions.

So, this is what it is, that for a 1000 millilitre cylinder we have 10 millilitre graduations and subdivisions are again 10 millilitres and its comparison to the best measuring device, we may call it how close is it to the exact value is this number here, plus minus 3 millilitres and like this, there are various capacities of graduated cylinders that are listed here; 50 millilitres, 10 millilitres, 250 millilitres, 500 millilitres, 2000 millilitres.

So, that is one piece of information that we said, study the instrument, database web-based information, standards. So, we got. Suddenly, we learned a lot and I am not presenting all the details here; but it tells you how many more aspects are there in just talking about the graduated cylinder, something we think it is so simple in a lab, it is got so much technology to it.

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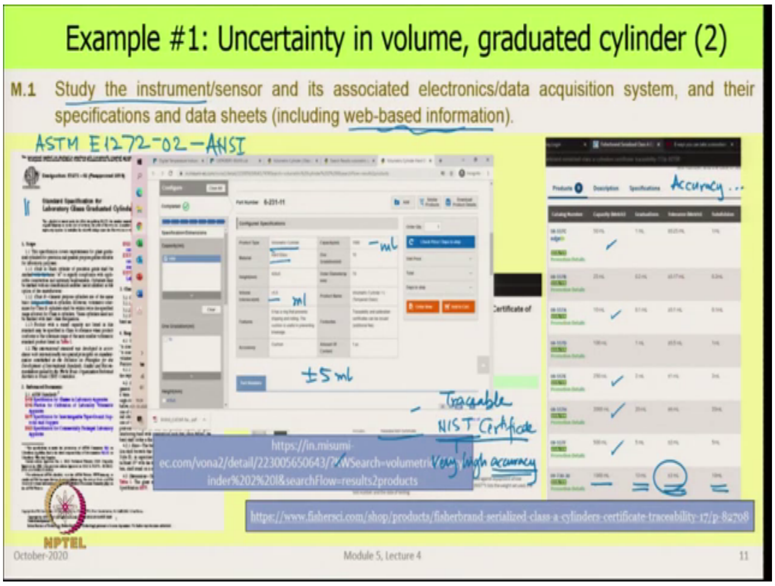

This is from another manufacturer. So, here they have said volumetric cylinder, hard glass, this is a height, then volume tolerance plus minus 5, capacity 1000. So, this is all in millilitres. So, a 1000 millilitre cylinder, their volume tolerance is plus minus 5 millilitres and so on. So, like this, we get lot of data and in this illustration that I am giving; I will pick up one of these numbers to illustrate how we can do the full calculation.

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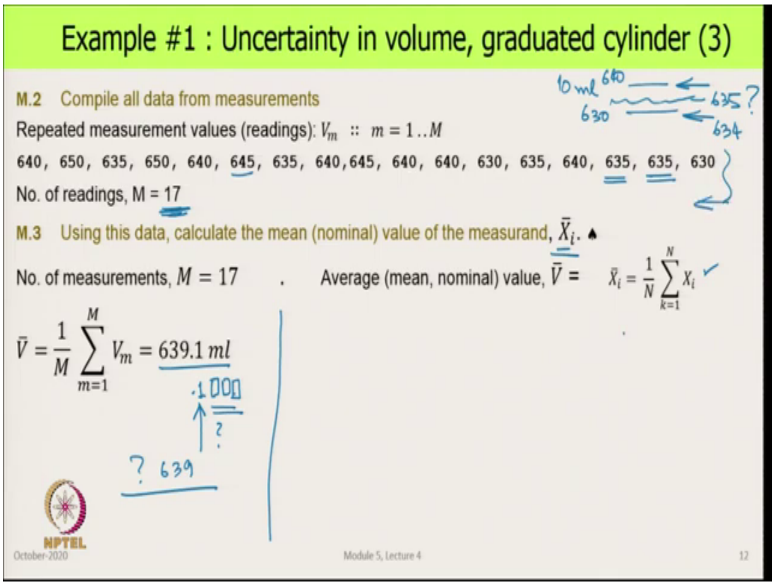

So, that was the first step M.1. Now, M.2 step was compiled all data from measurements. So, what we did? We kept the cylinder some of us read it many times, you got different people to read it and their values are listed over here. We got 17 readings and one can see that there are some readings which we think are in between the divisions.

The division was 10 millilitres. So, we had this mark here, this mark there and say the water level was somewhere there. Question is how does an observer interpret this? One could be a consistent way says that whatever happens, I will always record the lower value; somebody says I will always record the upper value. So, at least we are being consistent.

But to say that the value if this was 630 and this was 640. So, the value is 635 which is the looks like the best estimate from common sense. Well, not full; but we could live with it and see what its implication is. So, we have 17 readings, subdividing between 630 and 640 itself is in doubt.

So, in further subdivision, if somebody says you know this is slightly below the midway point, I will call it 634. Now, this is completely a judgmental thing and we are introducing error, when we make such a number. So, that is our readings. We got 17 readings; the values are over here.

In the next step, we say using this data, calculate the mean nominal value of the measure measurand X i bar. So, we just take all of that take their average and 639.1 millilitres. Now, there were more decimal places beyond this 1 and then something, something, something. The question is should we be reporting these. In fact, one could even question, why I have put 0.1 here at all. Should I not have just said 639? This we can look up and say from statistics to how many decimal places we should report.

The fact is that if you make a measurement many number of times, you can report the answer to a one additional significant digit. So, this is one thing. The second thing, what I have not presented here is that we took this data calculated the mean and then, said in the next step whether all these numbers are expected in such a distribution. So, we will see that in a minute.

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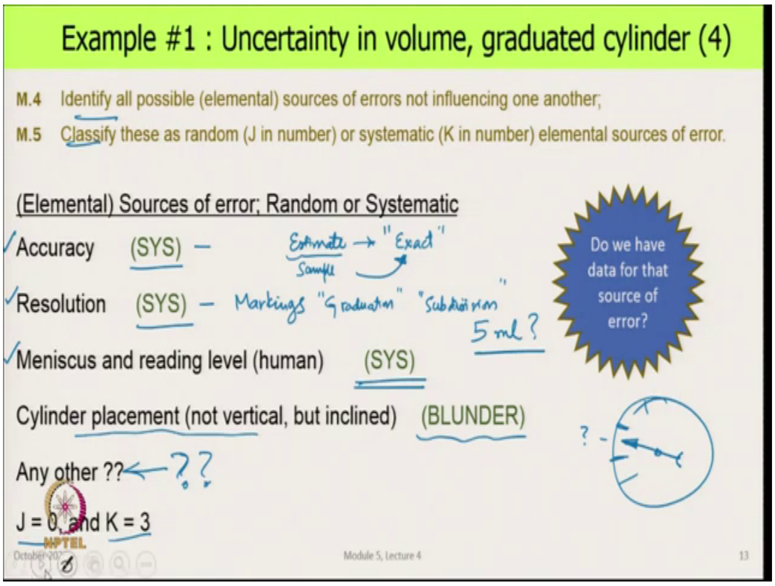

Our next step was identify all possible elemental sources of errors not influencing one another and classify them as random or systematic. So, here, we are I have listed some we said there is accuracy that is one source of error, the resolution which is the markings, meniscus and how the human being reads it that is another one.

And we could say that well, if the what is the cylinder but not vertical, but was inclined. Well, that we will treat as a blender and say look as the experimental, we were supposed to have taken care of it as good as best as we could have done. So, in this particular picture that I showed here, we made the table horizontal, got it vertical and thought that now we are perfect there.

But as the photograph showed, there is slight inclination, we are going to live with it for the time being; but it will be better, you need a table with adjustable screws as his legs only then and see the water level and get it perfectly vertical. That is how you would read it.

And if you can come across with anymore, you can even add those and then, we classify them as systematic or random. Random means we got data from which we can estimate the value. Accuracy is something we are going to rely on from what the manufacturer has given us. So, this is a systematic errors.

What accuracy means is that in this cylinder, if we measure something and we get a value, then if the same amount were measured in a standard laboratory with a much better accuracy instrument, what value it would give that value would be treated as the so called "Exact" value and the value that we get from here, this is an estimate of that. So, this is what we do many times; generate a sample and try to estimate the exact value that was the whole idea of statistics.

Resolution is clearly the markings and we saw from the catalogues that they have called it as "Graduation" and they also called it as "Subdivision". One can ask in that particular cylinder could not the manufacturer have put marks at 5 millilitre divisions? This is a whole involved question. We would not go into that right now and this was this something you will see in many analog devices, where you have a division and the needle points between them.

For instance, like this a pressure gauge has got these markings and say the needle points here, suppose the question is why did not the manufacturer put a mark there also; if they could put a mark there, if they could put a mark there, why not put a mark there. So, we will treat that here as a systematic error.

Then, the meniscus and the reading; we have two options, either we look at it again and again and try to get the value in which case we have got data and then, this would qualify as a random error and we would go in that. If we do not have data, we will use our experience, our common sense and say this is a systematic error and I will estimate how much error I am likely to get. So, we have no random sources of error and 3 elemental sources of systematic error with J is equal to 0, K is equal to 3.

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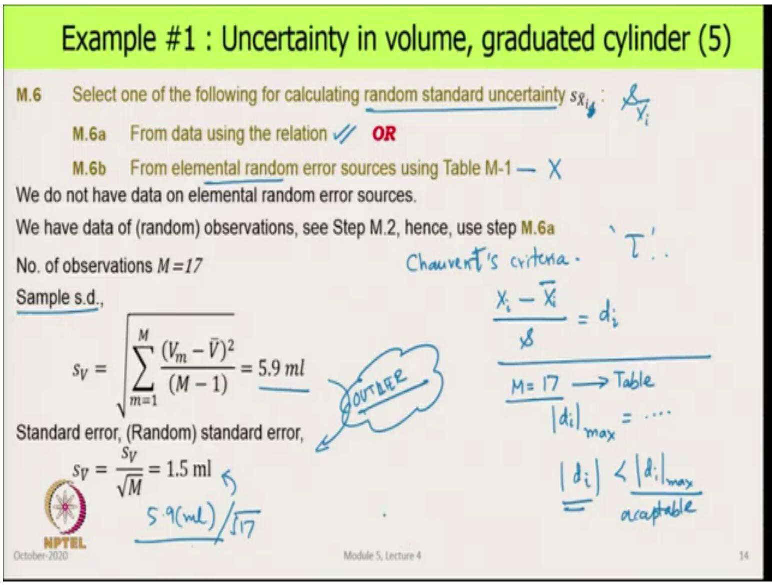

Next, we decide how to calculate the random standard uncertainty Xi bar; J is not here. We had the option of either using data or going via the elemental random source error sources. We do not have this; but we have lots of measurements that we have taken. So, we are here.

So, using that data that we had, we calculate the standard deviation which is the sample standard deviation which comes out to 5.9 millilitres and the standard error which is 5.9 divided by square root of 17 and this is 1.5 millimetres. But in coming from this step to this step, we did something more. We did outlier treatment. So, there are two techniques; one is the Chauvenet's criteria and the other is the Tau test.

We use the Chauvenet's criteria in which what we do? We take every reading Xi, calculate its deviation from the mean Xi bar and divided it by the sample standard deviation. This gives us the non-dimensional standard deviation or we call it di. So, that is half of the story. Then, we say that I have number of readings, in this case is M equal to 17.

We go to the table of Chauvenet's criteria and figure out what is the maximum possible deviation to be expected. So, di max is some value which means that all our di's which are less than di max are acceptable. Those which are greater than di max, they are not expected in this measurement assuming that all the measurements came from a normal distribution and so, we reject those.

We did this calculation in the outline treatment and we found that all the deviations are within the maximum permissible deviation for a sample size of 17. So, we did not we have to reject anything.

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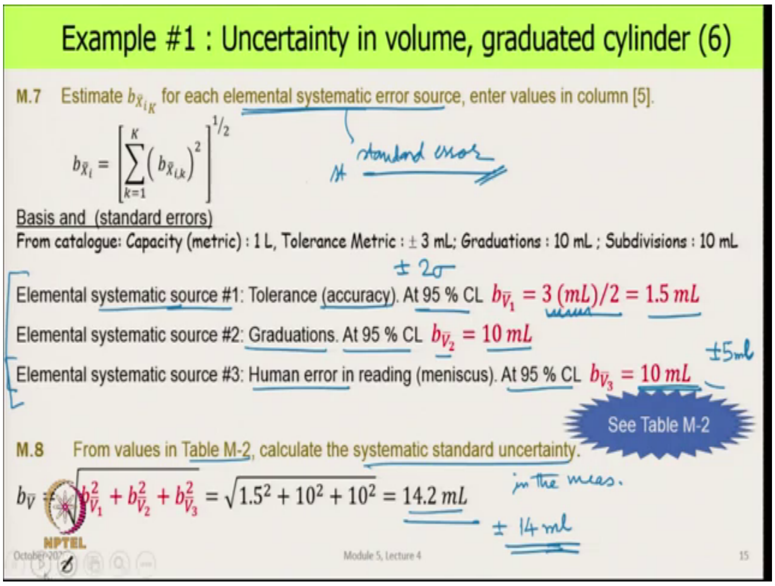

Next, we calculate the elemental systematics errors and since, we have done those three sources. So, we write down here source number 1; systematic error source number 1, tolerance or same as accuracy and here, we go to one of those criteria which is in ISO gum, which says that if the accuracy is reported at 95 percent confidence level, then the standard error associated with that is the value of the accuracy divided by 2 because 95 percent corresponds to plus minus 2 sigma.

So, we took one of those tables from the manufacturers, specifications, who said that the accuracy was plus minus 3 millilitres and using this criteria from ISO gum, we divide it by 2 and we get 1.5 millilitres. Our second elemental source of error was the graduations and say that we do not know how the observer is going to locate the value of the meniscus and assign a reading to it. We could have a maximum error of plus minus 1 graduation itself.

So, at 95 percent confidence level, we say that the standard error associated with graduations is the value of the subdivision which in this case was 10 millilitres. So, b v 2 bar is 10 millilitres; 2 is the second source of elemental systematic standard error. The third thing we have done here is put up a value for the human error in the reading and we say that at 95 percent confidence level, we are look unable to locate anything better than what the graduation itself is and then, depends how we report it.

So, here a maximum deviation would be 10 millilitres. One could argue that I will report it to the nearest possible marking; but the again, there is a judgement recall that if it is somewhere in the middle which one do you report. So, one way to say that well, I will take this as plus minus 5 millilitres, the other one is plus minus 10 millilitres. This is more conservative and represents a more realistic picture.

If you are able to generate data as to what is the human error in reading the meniscus, that would qualify as a random error and it would not appear here; but be in the elemental random sources of errors. We will add it at that point and you can see in the final calculation, this book-keeping here or there, does not make a very big difference. So, next step. So, we got all the values of the standard errors.

So, elemental systematic standard error has been obtained from for all these three sources and which is why the use of the word tolerance or the accuracy is also a somewhat in doubt because these are not statistically defined terms exactly. Standard error is a much more precise term. But since everybody uses accuracy precision, we will continue to use it; but will interpret them as a statistic.

Having got that, we now go to table M-2 and list all of them over there and then, we calculate the systematic standard uncertainty in the measurement as b v bar is going to square root of the squares of each elemental standard error which if we add these three values, we get 14.2 millilitres or you can even say I will just report a plus minus 14 millilitres.

Remember, always this has to have units. If it is if the measurand is a dimensional number parameter and how many decimal places you report, we will have to see in a little later; 14 ml would also be ok.

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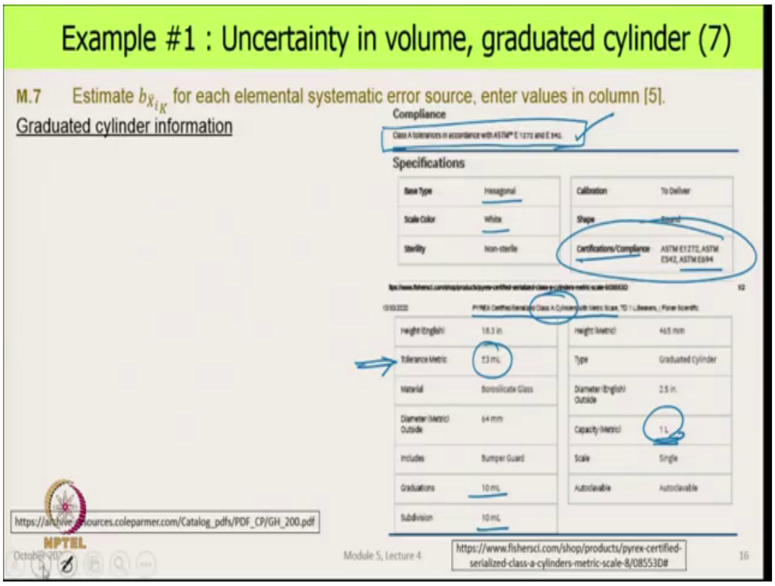

So, this is another example of a graduated cylinder, where it says that it has white round and capacity is 1 litre and tolerance is plus minus 3 millilitres. So, this is what we took in our calculation. Graduations 10 ml, subdivision 10 ml. And it says here Pyrex certified serialized class A cylinders with metric scale. So, question is what is what do they mean by class A cylinders and that is what we looked at this was referring to a standard and that is written over here.

Class A tolerances in accordance with ASTM E1272 and E342. So, this is what they are following and so, we need to understand what this standard was all about. And this class A dictates this value and of course, many more things that go with it. So here certifications and compliance ASTM E1272, ASTM E542, ASTM E694.

So, always try to figure out whenever you look at any instrument, what are the standards to which it is traceable or conforms to them. And one that does not have any we have to be a little more cautious in using those. So, like this, there are lot of information on the web and we have taken one of these.

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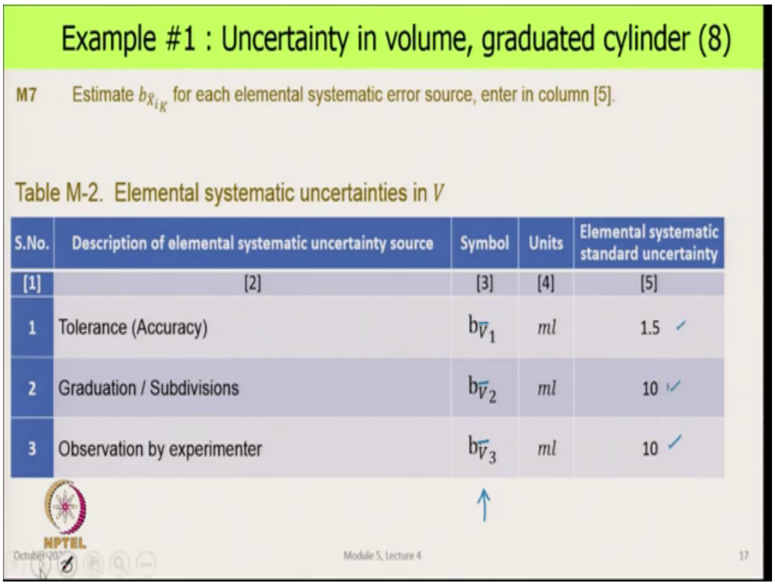

S.No.	Description of elemental systematic uncertainty source	Symbol	Units	Elemental systematic standard uncertainty
[1]	[2]	[3]	[4]	[5]
1	Tolerance (Accuracy)	$b_{\bar{V}_1}$	*ml*	1.5
2	Graduation / Subdivisions	$b_{\bar{V}_2}$	*ml*	10
3	Observation by experimenter	$b_{\bar{V}_3}$	*ml*	10

Here, is the same information compiled in table M-2 that we saw one slide earlier. Here, are the systematic sources of random error and their standard values. And we have just put the numbers that we got from those calculations.

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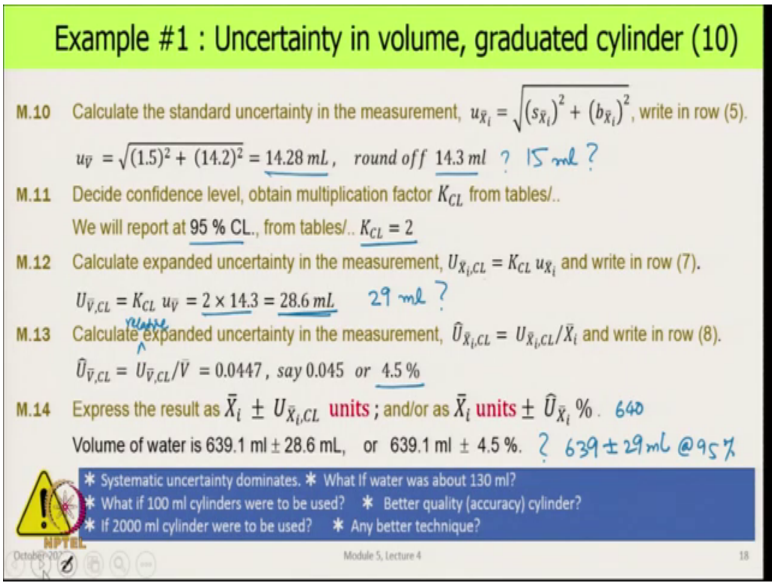

Then, we will do a last sequence of steps which will also take us to table M-3; we will see that in a minute. We calculate the standard uncertainty in the measurement which is square root of this and this which is 14.28, we round it off to 14.3. The question is should I report this as 15? We will come back to it in a minute. Confidence level, we say we will report at 95 percent confidence level, for which our multiplication factor is 2.

So, the expanded uncertainty in the measurement U X i bar CL is 2 into 14.3 which is 28.6 and this also, we can say well can I round it off to 29? Finally, the relative expanded uncertainty in the measurement U hat which is the value which was the mean value in the numerator, it is 28.6 divided by the mean value 0.4477 or plus minus 4.5 percent.

Finally, our answer volume of water is 639.1 millilitre plus minus 28.6 or 639.1 millimetre plus minus 4.5 percent. One can argue whether we should report 639 odds as some say always round it off to the higher side in uncertainty analysis, even 640 plus minus 29 millilitres at 95 percent confidence level, this would also be ok.

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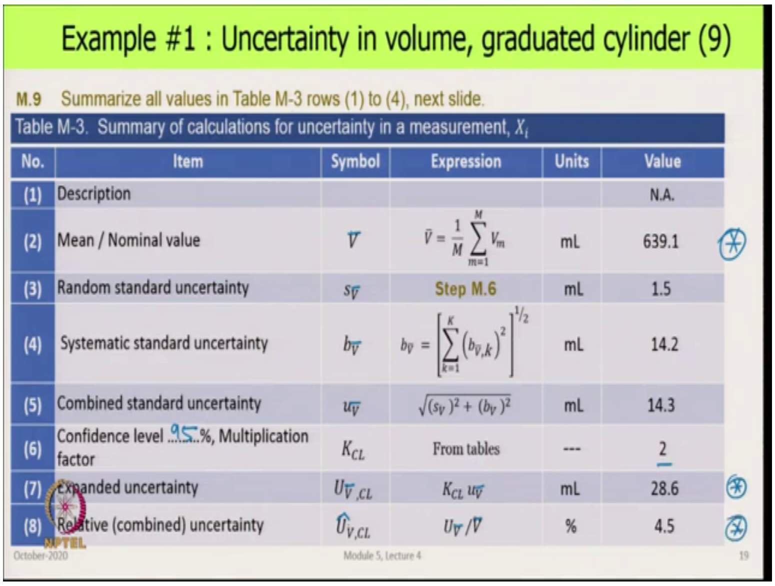

So, that is our answer that we were looking for and here is table M-9 which summarizes all those steps. Instead of writing it step-by-step, we could even put it here and made a spreadsheet that does these calculations or write a program that does it. So, this is our mean value V bar 639.1 millimetres, sample standard the random standard uncertainty we got here as 1.5 millilitres.

Then, systematic standard uncertainty is 14.2, combined standard uncertainty is 14.3, multiplication factor, we took 2 at 95 percent confidence level and expanded uncertainty at 95 percent confidence level 28.6 and the relative uncertainty is 4.5 percent. So, that is what we have. This plus minus this or this, that is our answer.

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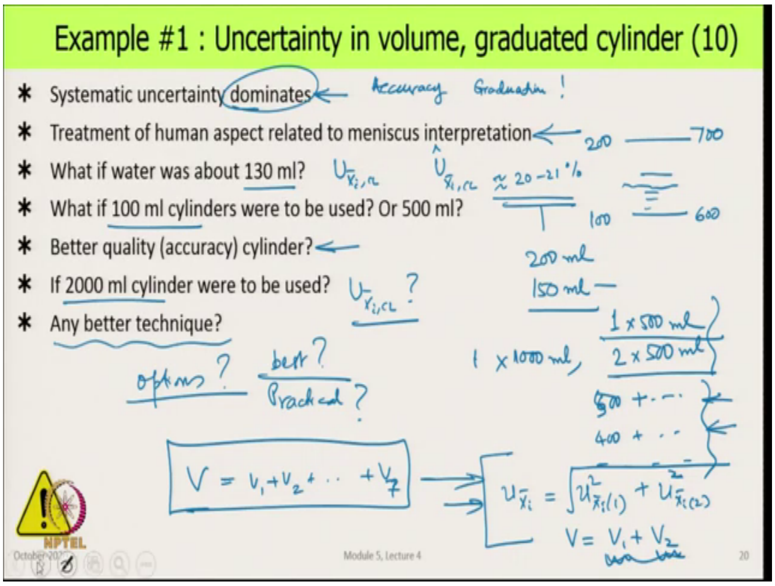

So, now this analysis leaves us with lot of questions. First, we look at what information we got and we saw that compared to the random standard uncertainty, it was the systematic uncertainty which was dominating the measurement. So, it tells us that yes make multiple measurements so that you reduce your random standard uncertainty; but if you really want to improve things, you have to tackle the systematic standard uncertainty which is coming either from accuracy.

But again, we saw in that calculation accuracy was only 1.5 millilitres standard error. Graduation, this was a big issue. So, what the ability to read the meniscus? So, it would mean that if you can get another cylinder with finer graduation, your systematic uncertainty will go down and your overall uncertainty in the result will be better.

The treatment of how we read the device this varies a lot and I said, we could either estimate it and treat it as a systematic uncertainty or try to get lot of people to say you know where is the meniscus, ask that question and what answer they give, that becomes your data for that question. And from there, we can estimate what was the random standard uncertainty in the reading of the meniscus.

Now, what will happen to the uncertainty U Xi bar at the same confidence level. If instead of the markings there as you saw there, the water level was somewhere there this was 600, this was 700. What if this was 100 and this was say 200, will the uncertainty be the same? And the calculation shows that is in the systematic uncertainty dominates and a the random uncertainty would be of the same order of magnitude.

U X bar at 130 millilitre would be comparable to what we have just got. But the issue then becomes the relative uncertainty, this will be much more. Instead of 4.5 percent, we are looking at roughly 5 times larger. I am almost like something like 20, 21 percent uncertainty, that is a big uncertainty, and it tells you that not a nice thing to have.

This sort of uncertainty one generally does not like to have in any measurement; do not use this cylinder, go for some other option and that other option could be that well, I will use a 200 millimetre cylinder. So, cannot I get 150 millimetre cylinder? Well, the answer is no, those are just not made. The world operates on some standard things, we have to make, we have to adjust ourselves to what is there in the market. So, that is one implication coming out.

Now, you say that instead of using one cylinder of 1000 millilitre capacity, I measured the water using say two cylinders of 500 millilitre capacity or just 1 cylinder of 500 millilitre capacity. So, what I do? I put water in one; put some water in the other. It is 640 so. I could put any ratio, I could have put 500 millilitres in one and the balance in the other or I could just put anything, I could put say 400 here. And the remaining here or whatever you feel like, you just pour that water into cylinders and read it.

The question is if you do that, you did improve your graduation because of 500 millilitres the graduations are finer; but because you did the measurement twice, you are compounded the error.

So, assuming that the two cylinders are not in any way connected to each other, these become two independent measurements and we can then, treat that as a result that the uncertainty of the measurement U X i bar. This will be the sum of squares of X i bar from the first cylinder plus U square X i bar from the second cylinder, square root of that.

This is coming to us because volume is equal to V 1 plus V 2 that becomes a result relation and we have to use data from there. So, we have to treat it as uncertainty in the result, not as an uncertainty in a measurement; but there will be two uncertainties in the measurement, one for each one of them.

And I leave it to you to do this calculation and see which is a better method for measuring that much volume and then, you can extend it instead of 500 millilitres, so well I use 100 millimetre, my graduation is much finer. I will have to make say 6 measurements or 7 measurements.

So, I have result formula, where I poured water in 700 millilitres cylinders and I get V 1 plus V 2 plus like that plus V 7, this becomes our result formula and then, calculate which what is the uncertainty in this. The issue of using a better cylinder has come up in the fact that this is a dominating uncertainty, we have to improve that. That is the only option. If we get a 1000 meter cylinder with finer markings, there are some, we could use those.

Now, what if the opposite happened? Instead of 1000 millimetres cylinder, we measured that same water with a 2000 millimetre cylinder, what happens to uncertainty? You can do that calculation on the same lines, see what happens. So, all of this gives us lot of information on what would be various options that we have and first of uncertainty analysis which is the best option and then, we further refine it from practical considerations, what is the best option.

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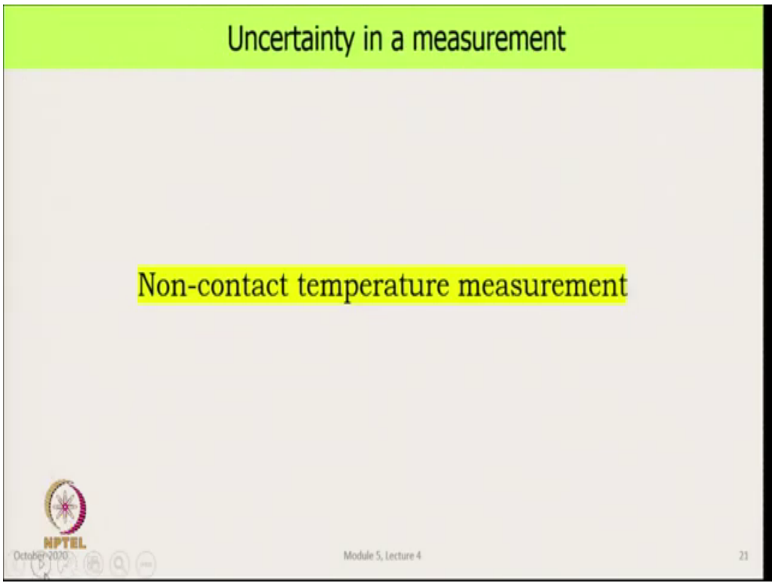

So, that completes our discussion on the graduated cylinder volume measurement. Now, we will what we have seen is something, where we got an instrument we directly gave us the reading. We did not know anything else that was happening in it. In the case of graduated cylinder of course, there is nothing as to worry about; but there are many instruments which directly give you a reading.

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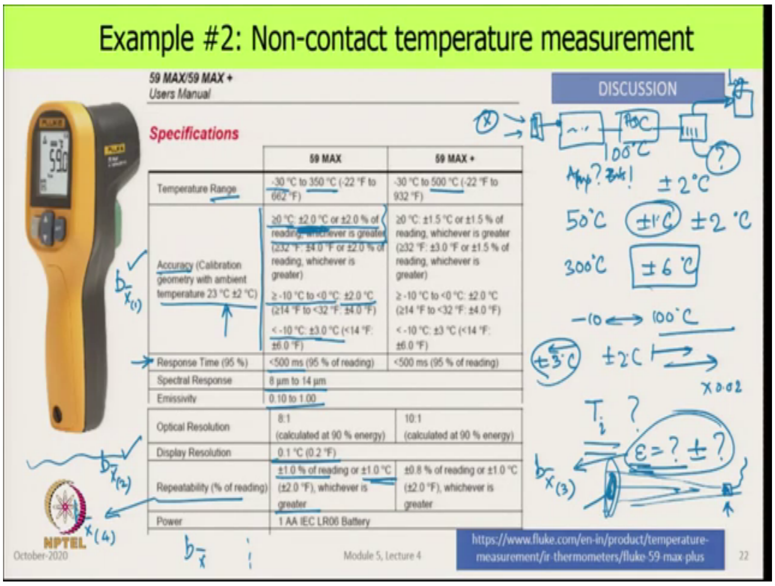

And we will look at one such reading which is a non-contact temperature measurement which in today's COVID times, you go anywhere, they are measuring your temp skin temperature with this instrument. So, let us see what is the uncertainty in that; how would we analyse that.

So, there are literally hundreds of such devices available in the market. I have picked up one which is a little more industry, or laboratory use type rather than the medical use type; the difference is in the range. This one operates from minus 30 to 350 degrees Celsius and a modification of this can go up to 500 degrees Celsius.

The one that I used for our measuring the skin temperature, they are like a clinical thermometer that operate between say about 90 to 94 degrees Fahrenheit, going up to about

108 degree Fahrenheit. These are much larger and we say well what is accuracy and it says calibration geometry with ambient temperature as 23 plus minus 2 degrees Celsius.

Now, things are getting much more tighter; if the ambient were 30 degrees Celsius, accuracy will be different, possibly the reading will also be different slightly. And what it says here is for temperatures less than 10 degrees Celsius, accuracy is plus minus 3 degrees Celsius.

Between minus 10 and 0 degrees Celsius, the accuracy is plus minus 2 degree Celsius and for temperatures greater than 10, 0 degree Celsius, the accuracy is 2 degree Celsius or plus minus 2 percent of the reading whichever is greater. This is something that you will see in many instruments.

So, if you are measuring a temperature of 100 degree Celsius, plus minus 2 percent of that becomes plus minus 2 degree Celsius which is same as what is quoted over there. So, its two values are the same. If you are measuring a temperature of 50 degree Celsius, then plus minus 2 percent of that will be plus minus 1 degree Celsius; but then, this is less than what has been quoted.

So, our uncertainty, this accuracy here is plus minus 2 degree Celsius. But if you are measuring 300 degree Celsius, then 2 percent of that becomes plus minus 6 degrees Celsius which is more than plus minus 2 degree Celsius here. So, our accuracy here will be this much. So, what we are seeing is that we have plus minus 2 degree Celsius in a range which is roughly from minus 10 to 100 degree Celsius, below this it is plus minus 3 degree Celsius; above this it is non-linear and keeps increasing with the temperature.

So, which value to use in our calculation? That will be determined by the temperature that we are measuring. If we are measuring in range less than minus 10, we use this 10 to 100, plus minus 2 degree Celsius; above 100, the value multiplied by 0.02. So, tells you as the higher temperatures, your accuracy is less, your systematic standard uncertainty related to accuracy will go up.

So, the same instrument can result in different values of systematic standard uncertainties; but this is the issue that there are many instruments which will be quoted like this. And it is not just temperature sensors; but also pressure, force, torque, flow rate, many of them will be quoted like this. So, that is one thing that comes up.

So, there is one systematic standard uncertainty because of accuracy which depends on the temperature itself, what is being measured. Then, there is response time. This issue will come up, if we are measurings a body whose temperature is changing with time and it says it takes 500 milliseconds to get to 95 percent of the value of the reading.

Say for the rate of change is much faster than this, this instrument will always lag. If when it is increasing and it will always lead when it is decreasing. Then, there is some more information about the instrument. It is a spectral response. So, it tells you what is the wavelength of infrared radiation that it captures.

The way this instrument works is that there is a body there and from a certain cone, it this is the surface there, on that from this area whatever radiation came onto this, it caught this and this was converted into an electric signal by a sensor here. Then, it says emissivity 0.1 to 1. So, it assumes what the emissivity of this surface and we can adjust it. So, in a way we need to know what is the emissivity of the surface and within that, there will be an uncertainty in that.

So, the final measurement will have an uncertainty arising not just because of accuracy; but also, uncertainty arising from our approximation of the emissivity of a surface because emissivity of a surface is never known exactly and most cases, we do not even measure it. Then, there is optical resolution, display resolution 0.1 degree Celsius. So, that is a displace. So, whatever signal came to the display, it used its own algorithm to decide what number it should display and so, this is the second source of systematic standard error.

Epsilon was another source of error and then, we have repeatability plus minus 1 percent of the reading or plus minus 1 degree Celsius whichever is greater; that means, you measure the same thing again and again, this is what you can expect. Simply because there are the electronics, the sensor, the various random things happening and environmental changes, whatever this is what is causing it so this becomes b X bar 4.

So, that that there are more specifications and here, we have listed already four sources of systematic standard uncertainty. We will have to estimate each value. Accuracy, how to handle it, we have already seen; display resolution how to handle it we have seen in our example. What is new is how to handle emissivity related systematic standard uncertainty and repeatability related this.

From all of that, we will get b X bar in the temperature measurement and then, we do the remaining calculations like what we have seen before. Now, an instrument like this, it takes this signal. So, there is a sensor on which radiation false, it generates a electric signal. There is various types of signal processing being done there, then there is analog to digital conversion and then finally, there is the display.

The point of taking this example was that we do not know anything that is going on inside this. So, we are not we do not even know how many bits were there in the A to D converter that this has; what amplification was used, all these are not known to us.

But we have enough information given to us by the manufacturer to connect the physical world that we are measuring with what we are seeing on the display. And the display also includes that if we were connected wirelessly or with wires, the same value would go to a data logger. So, there will be no error, at least in this last part of this step. So, that is another example of the same process, but is a different type of an instrument.

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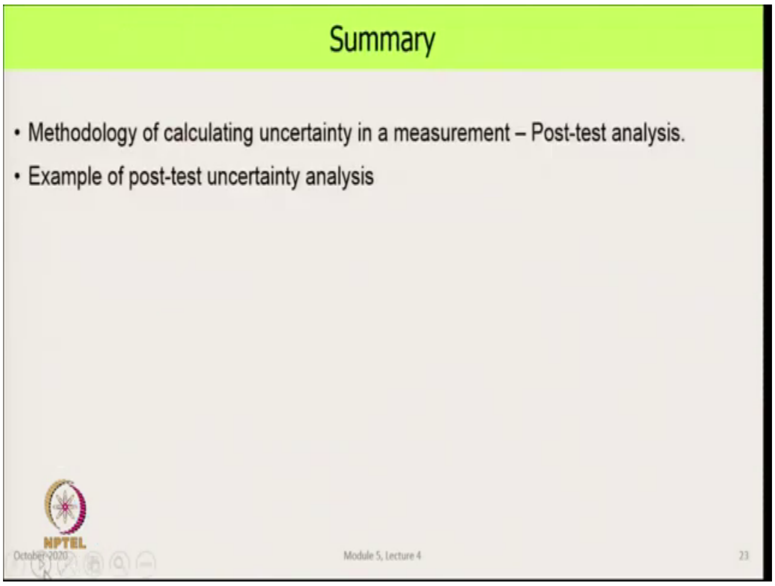

One can take many more examples. In the assignments, I have given more; but we will conclude here. In that what we have done in this lecture is represented the systematic methodology of calculating uncertainty in a measurement largely related to post-test analysis. And we looked at one example of post-test uncertainty analysis and we also looked at in the instrument, whose information we could use to decide how good that measurement is likely to be.

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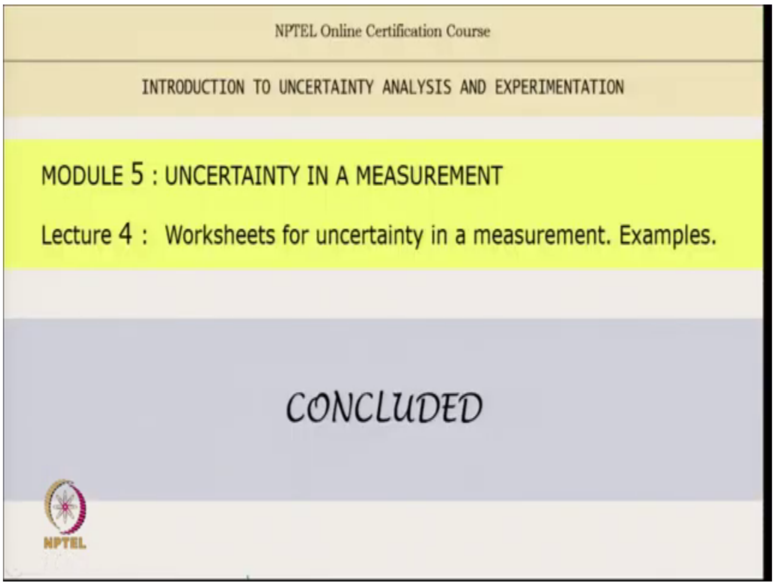

So, with that, we will conclude this lecture, where we have covered work sheets and seen examples for uncertainty in a measurement.

Thank you.