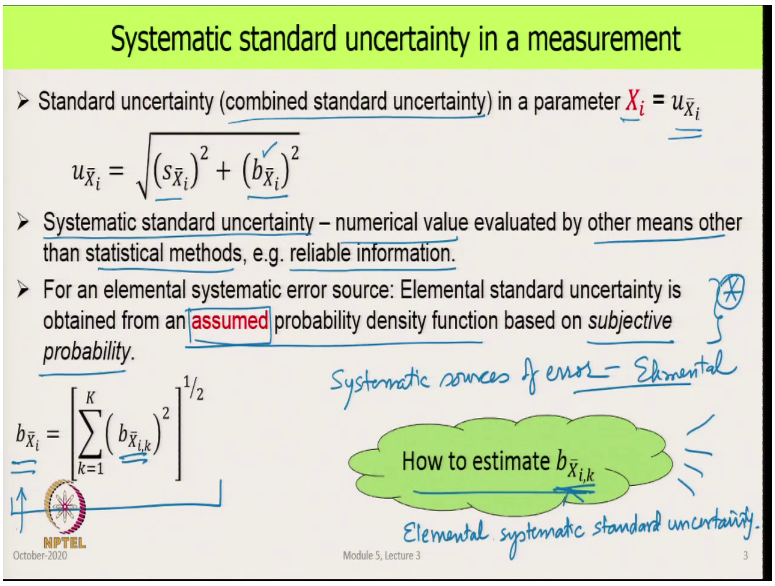
Introduction to Uncertainty Analysis and Experimentation
Prof. Sunil R. Kale
Department of Mechanical Engineering
Indian Institute of Technology, Delhi

Module - 05
Uncertainty in a Measurement
Lecture - 14
Evaluating systematic uncertainties

Welcome to the course, Introduction to Uncertainty Analysis and Experimentation. We are looking at module 5, which is Uncertainty in a Measurement. And today, we will look at how we evaluate systematic uncertainties.

(Refer Slide Time: 00:34)



To recap, we have said that the standard uncertainty or combined standard uncertainty in a parameter X i, which is u X i bar is the square root of the sum of squares of the random and systematic standard uncertainties. We have already seen how to calculate s X i bar. Now, what remains is to find out techniques by which we can calculate b X i bar.

Now, recall that systematic standard uncertainty, which corresponds to type b uncertainty in ISO GUM, the numerical value of this uncertainty is evaluated by means other than statistical methods. That means, we do not deal with numbers, we do not deal with a distribution, we do not deal with a sample, but we rely on some other information which we call reliable information, which we believe to be trustworthy.

And from that information, we will calculate b X i bar, assuming that the elemental errors values are coming from some assumed probability distribution function. We do not know this function; we do not have data to make this function and so, we are subjective in our analysis.

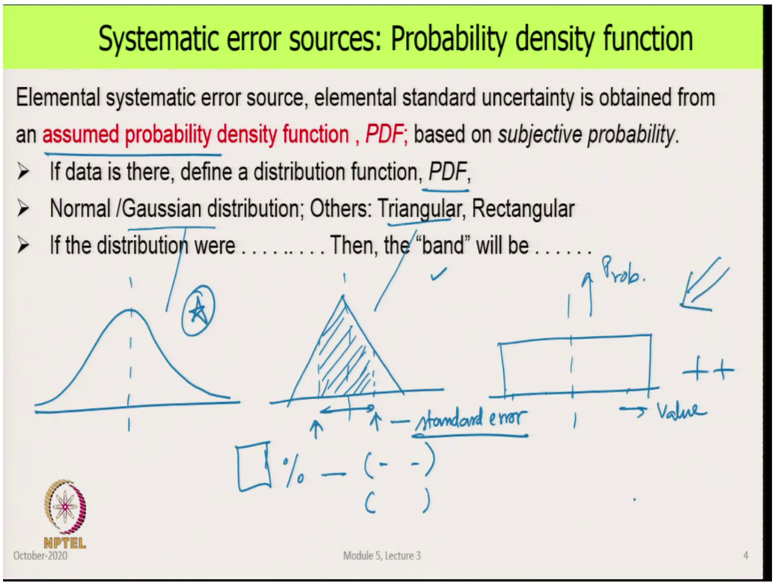
And we say, well, this particular error is coming from such and such a type of a distribution. And if that were the case and this is the data that I have. How can I associate a standard uncertainty or a standard error for that distribution? So, this is our large strategy on how we will estimate b X i, k. So, what we do is for each elemental systematic standard uncertainty, which is what this is.

So, there is systematic sources of error. This we said right in the beginning, that the experimenter has to list all sources of error, classified them as being random or based on data, or systematic, as not being based on data and statistics. Those are all the elemental sources of error. And each elemental source of systematic error has this elemental standard uncertainty, which is our b X i, k.

So, that is what it is we want elemental systematic standard uncertainty [FL]. So, that is what we are looking for b X i, k as many elemental sources of systematic uncertainty are there in that particular measurement, for each one of them we do what we have written over here, get these individual values.

And then finally, use this relationship based on the right side we have the elemental standard systematic standard uncertainties and this side we get the systematic standard uncertainty of the measurement. So, now, our question is how do we execute this part?

(Refer Slide Time: 04:54)



That is what this lecture is about. To go back and clarify a little bit more about what we mean by this probability density function is that if data were there we could have made a PDF, Probability Density Function and this could take many forms. So, we could have say, what we have done so far in many cases, a Gaussian or a normal distribution.

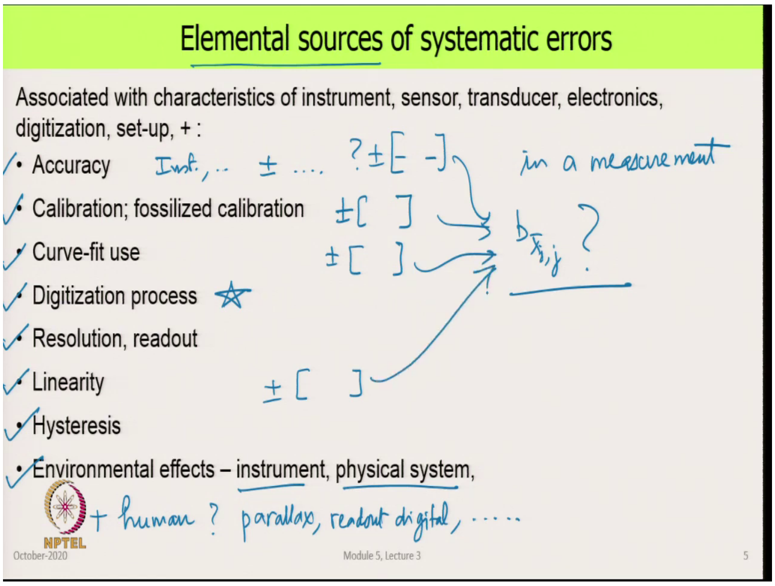
Other thing about something we could have had a triangular distribution. So, this is a Gaussian distribution, that is a triangular and it could also be say a third one, which is a triangular distribution. So, this side in all the cases, this axis is the value and this axis is the probability of that value. So, much of what we have learnt has been based on this one. But reality may be different in that different errors may have different distribution functions. There could be some which could be like this.

That means that irrespective of the value, the probability of that value coming up is the same. Here it could be triangular. And we could add more such distribution functions. So, now, what we are saying is, if somehow we would have known this distribution function which we actually do not so, we are assuming it, and we say ok this particular systematic error source has a triangular distribution function.

And then we say, I have been told by the instrument catalogue, or by the instrument manufacturer or some other data that at such and such a confidence level, the accuracy is this much or the linearity is this much. In that case, what we do is we say; well to get that much of uncertainty here, this percentage confidence level. We need to find out what are the limits within which that much of the area lies.

These limits would be the values that we are looking at here. And these limits are related to the standard error. So, that is our strategy, how we will establish certain standard error for information that we got from some source, assuming one of these were the case, which we in reality we do not have this data.

(Refer Slide Time: 07:52)



So, that was the strategy or the statistical basis of converting data that we have from different sources into a standard error. Now, here we are listed, something we are done earlier elemental sources of systematic errors. So, in a measurement, what are the different ways, which can cause a systematic error?

This is what this slide is all about. We will just list these, and later on we will see how to use this to convert it into a standard error, and that we will do some of it in this lecture, others while doing problems. So, the first one I have listed is accuracy. So, this is about every instrument, transducer; that manufacturer will tell us what is the accuracy, we will not at this time going to details of how that accuracy gets established.

But we want to know that if the manufacturer has told us that the accuracy is plus minus this much. How do I interpret that? That is our idea. Then we have issues with calibration; that means, we compare the signal of the instrument with the standard instrument, and also related to that is fossilized calibration, something which is fairly outdated or not correct, but we continue to use it.

Then you have the use of curve fits, where we do not do calibration, but we just correlate the input and output and see, and use that information to get a measured values. Then there are systematic errors coming from the digitization process. This we will look at in some detail today.

Then there is error coming in, because of the resolution and readout. Then there is error coming due to linearity of the instrument. Then hysteresis effects and environmental effects which affect both the instrument performance and also the system that we are measuring.

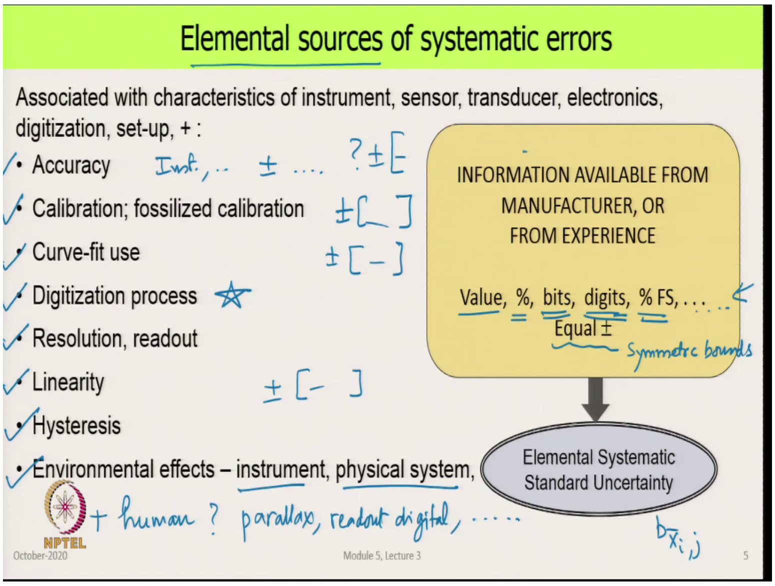
So, this means that if the system is ok, but the environment is affecting our instrument we could get an error. This could also mean that, that ambient conditions are affecting the physical domain that we are trying to measure, but the instrument we assume is not affected, or the third possibly possibility is that both are being affected.

And we can add some more things to this for instance, the human aspect, and you can decide whether some of the human aspects should be random errors, or they should be systematic errors. For example, parallax, or we have a digital display, in which the digits are fluctuating. So, issues with readout from a digital output, and so on.

So, what we will have is many of these are specified as some limit plus minus this much plus minus this much.

And there is a certain probability associated with those numbers. And our question is, how do I convert these plus minus values, a range, which has been given plus minus 3 percent say plus minus 1 percent plus minus say, one bar like that. How do I convert this into a corresponding b X i, j bar? So, this is what we want to do. And that is what we do in the, in this in the next few examples

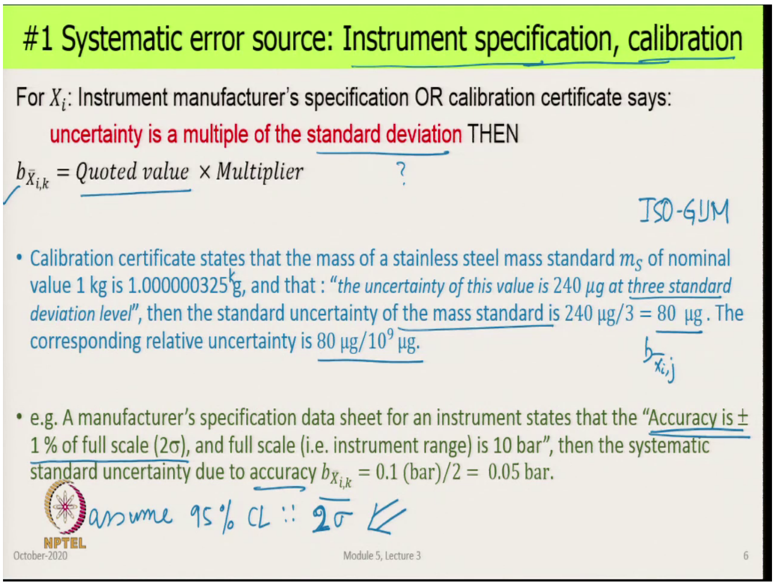
(Refer Slide Time: 11:52)



So, what we have is that we have information available from the manufacturer, which is coming to us as I mentioned these limits here; this could be as a plus minus value, absolute value. It could be a plus minus percentage value, or it could be as number of bits, digital bits, or it could be in terms of digits, digits of the display, or it could be as a percentage of the full scale range of the instrument or the device, or some other way.

So, that is how this data is coming to us, from the manufacturer or from somebody else. We assume that these are equal on plus minus side; that means, these are symmetric. We call it symmetric bounds. And our job now, is to use this information to calculate the elemental systematic standard uncertainty, b X i bar comma j. So, there are many ways of doing this, depending on how this information is available. And we will look at these in the next few slides.

(Refer Slide Time: 13:31)



The first one we say that, we have information about instrument specification, and its or its calibration. That means, the manufacturer specification or the calibration certificate says, uncertainty is a multiple of the standard is a certain multiple of the standard deviation. Which means that, in this case, whatever is the quoted value of the calibration error or the accuracy, that multiplied by the multiplier corresponding to this standard deviation gives us b X i bar comma k right.

So, here is an example. The blue ones are directly taken from the ISO GUM and the other ones are those which I put myself. So, here it says calibration certificate state that the mass of a stainless steel, mass standard m s of nominal value 1 kg, is this many grams. Sorry, this will be yeah, this will be kg. And that uncertainty of this value is 240 micrograms at 3 standard deviations level.

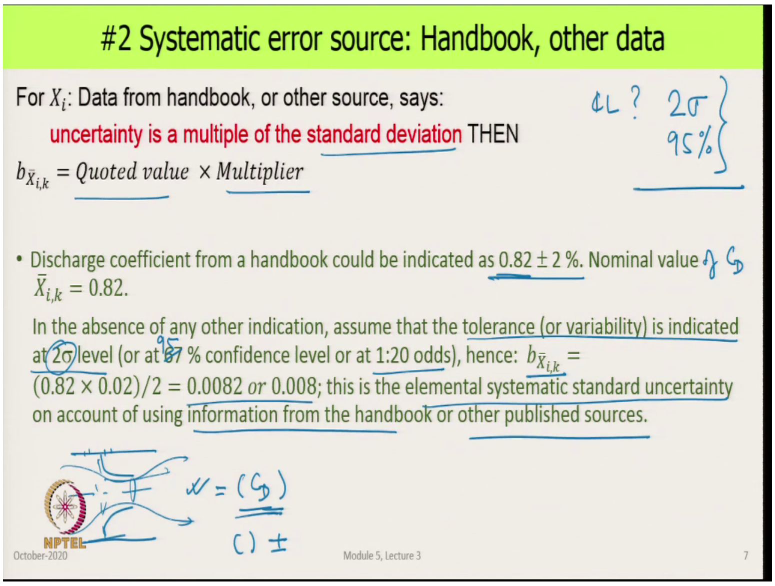
That means, the standard deviation of the mass standard is 240 micrograms, divided by 3 standard deviation level 3 sigma level has been told to us, we assume this is a normal distribution so, we divided by that, and we get 80 micrograms. So, this is b X i comma j, if such a statement were given to us by the manufacturer.

And it says the corresponding relative uncertainty is this much, so you can divide this 80 micrograms by the mean value which is 1 kilogram, and that is our ratio, or a percentage. The other example, a manufacturer's specification datasheet for an instrument states that, the accuracy is plus minus 1 percent of full scale in bracket 2 sigma. And full scale instrument range is 10 bar.

So, in the instrument datasheet there is an item which says range, which says say 0 to 10 bar or 1 to 10 bar. There is another item which say, accuracy and I guess that it says, plus minus 1 percent or just 1 percent of full scale in bracket, 2 sigma. That means, that the systematic standard uncertainty due to accuracy issue so, the accuracy is our elemental source of systematic uncertainty, b X i, k is 0.1 bar by 2 which is 0.05 bar.

So, this is one way to interpret information which is quite often given to us in this form, sometimes it does not say 2 sigma explicitly in which case, we assume that in all the analysis that we are doing, in the information that we are interpreting from other sources, all of this is reported at 95 percent confidence level, which is same as 2 sigma level. So, this is what we do. So, that is the first type of a information about the instrument or the calibration that we may have.

(Refer Slide Time: 17:23)



Now, we have got the next one that you could have this information not from a manufacturer, but from handbook or some other data source. So, in many of those cases, the confidence level may not be specified neither would they say whether it is 2 sigma 3 sigma.

We again assume at everything is at 2 sigma or at 95 percent confidence level under the assumption, that if it is not this it would have been explicitly specified that way. So, the same trick or the same criteria that we just saw in the previous slide, applies that uncertainty is a multiple of the standard deviation, then b X i, k is coded value multiplied by the multiplier.

For instance, in many flow measurement issues, we use what is called a nozzle flow nozzle; that means, we have a pipe or a duct in which we put this flow nozzle it is symmetric, the fluid flows through this and we measure the pressure here and here. And from that pressure we can calculate using a formula, what is the flow rate. And that calculation is of from the real calculation by a constant, which is called the discharge coefficient.

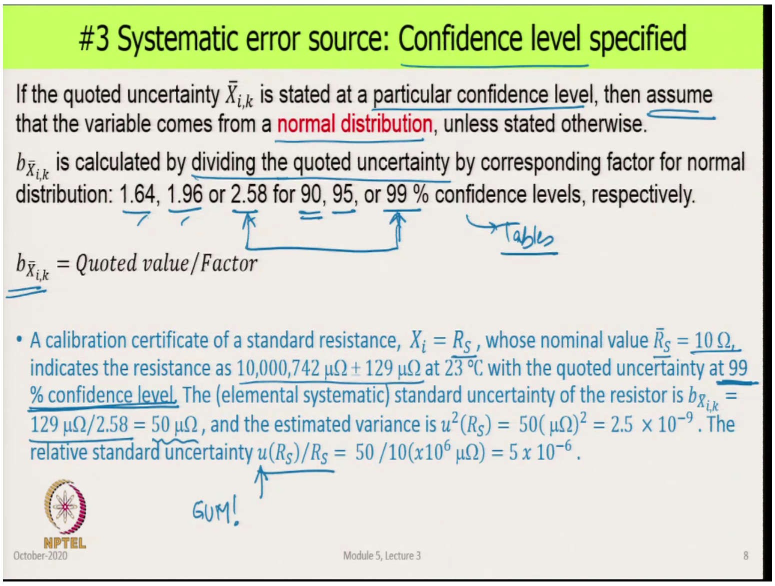
So, we multiplied that calculation by C D, and we get the actual value of the flow rate. Now, this C D, how did we get it? There are some standards will tell you what should be the geometry of the nozzle or the orifice or the venturi meter, either of them. And they say, if you make the device like this for this particular application, then your nominal value of C D will be this much, and the uncertainty in that will be this much.

Now, we get that information while we are using this type of device in our experiment. And so, well, what is the uncertainty that the use of this nozzle has caused us? So, we go back to the handbook and say that the handbook says that, the discharge coefficient is 0.82 plus minus 2 percent now, we going to find out the uncertainty related issues.

So, the first thing that this information is telling us is that, the nominal value of the discharge coefficient C D is 0.82 which is this number. And since there is no other information that we have, we assume that this tolerance or variability is indicated at 2 sigma level or of course, this is not 67 percent, this is 95 percent or at 1 is to 20 odds.

Assuming that, then the systematic elemental systematic standard uncertainty due to the discharge coefficient, will be 0.82 into 0.02 which is 2 percent of this divided by 2, because of the 2 sigma level assumption that we have made or this is the elemental systematic standard uncertainty, on account of the discharge coefficient, which we got information from a handbook or some other published source. So, that is the second type of systematic error source.

(Refer Slide Time: 20:54)



Now, the third one where we have been given the confidence level. So, if the quoted uncertainty is stated at a particular confidence level, then we assume that it is coming from normal distribution unless otherwise stated. And all that we need to do is divide that value; divide the quoted uncertainty by the corresponding factor for normal distribution which is 1.64 for 90 percent confidence level.

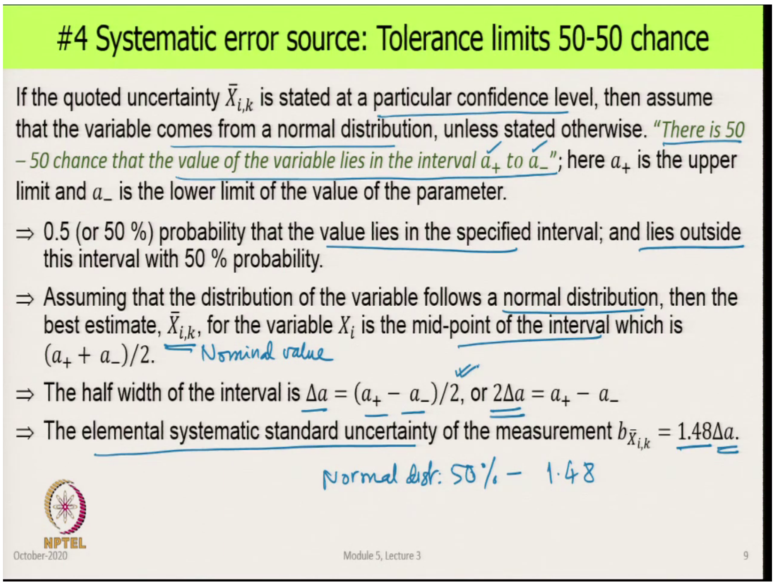
So, at this is not 1 point yeah 1.96 for 95 percent and 2.58 for 99 percent confidence level or if there any other value, we can look up the standard tables and pick that number from there. So, then the elemental standard uncertainty on account of this is quoted value divided by this factor, this, this or any other one that comes out of this.

And the example here is like this. A calibration certificate of a standard resistance, the variable is called R subscript s, whose nominal value R s bar is 10 ohms indicates the resistance as this much micro ohms, plus minus 129 micro ohms at 23 degrees Celsius, when quoted with an uncertainty of 99 percent confidence level.

So, here we are. Confidence level has been explicitly stated. That means, the elemental standard uncertainty of the resistor b X i, k is 129 by 2.58, 2.58 came from here at 99 percent confidence level we have 2.58 as the factor. So, we divided by 2.58 and the answer is 50 micro ohms and the of course, you can calculate the variance which is the square of that, and the relative standard uncertainty which is the ratio of these.

You can see this, u being used here this is what ISO GUM uses is a slight different from the symbols that we are using here, but the meaning is the same. Let be a little careful, when looking up either of those standards.

(Refer Slide Time: 23:18)

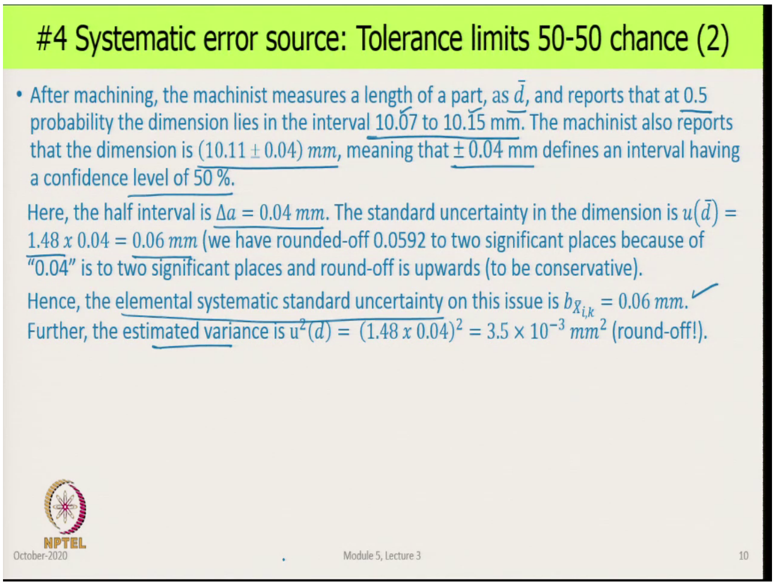


Our 4th systematic source of error, is when someone says, that the particular confidence level is such; that the variable comes from a normal distribution and there is a 50 50 chance, that the value of the variable lies in an interval a plus to a minus. So, they are we are told, that I have certain value plus minus something that plus minus something you add and subtract will give you this and this and you say, that half the values lie in that interval, and that half the values lie outside the interval.

So, that is what this is telling us. That there is 50 percent probability that the value lies in the specified interval and 50 percent probability that it lies outside that interval. So, assuming the distribution follows a normal distribution, then the best estimate of X i, k is the midpoint of the interval, which is a plus plus a minus by 2.

So, this is the nominal value or the mean value. The half width of the interval delta a is the value of these divided by 2, or 2 a is a plus minus a minus. And being 50 50 probability the normal distribution the factor is 1.48. And so, the elemental systematic standard uncertainty on account of this particular factor, will be 1.48 times delta a; delta a is defined from here.

(Refer Slide Time: 25:13)

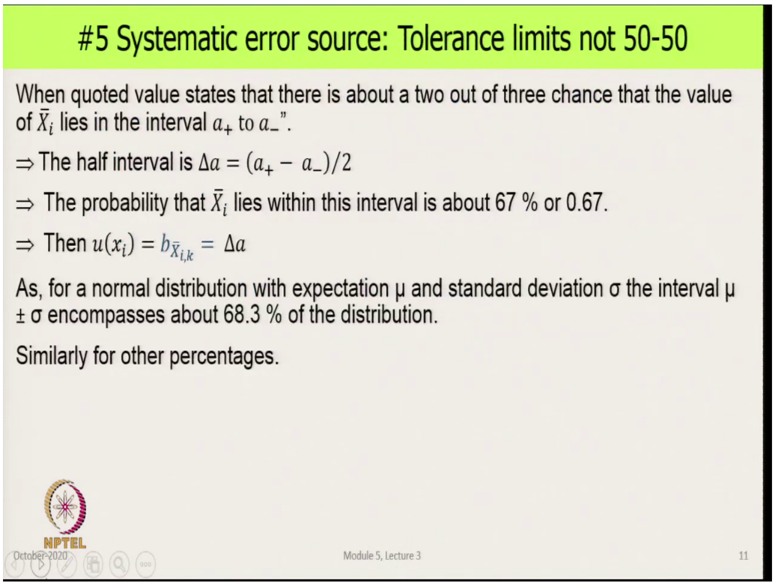


So, here the example, it says, after machining, the machine is measured the length of the part, as d bar, and reports that at 0.5 probability the dimension lies in the interval of this to this. The machinist also reports that the dimension is 11 10.11 plus minus 0.04 millimeters, meaning that, plus minus 0.04 millimeter defines an interval having a confidence level of 50 percent.

So, you can see that from here, we are getting the same one there. This is 10.11 minus 0.04 is this value plus 0.04 is this value. So, here the half interval delta a is 0.04 millimeters. The standard uncertainty is the dimension is 1.48 which is, because it is a 50 percent thing, it is this much and we round it off from 0.05992 to 0.06.

And the elemental systematic standard uncertainty on this issue; so, this becomes 0.06 millimeters and of course, we can then calculate the variance which is the square of this. So, that was an example of 50 50 chance. Now, what if it is not 50 50?

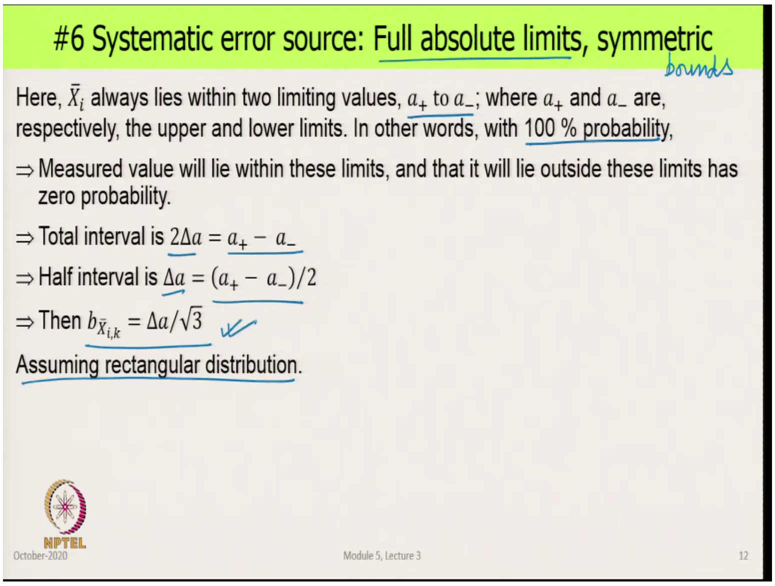
(Refer Slide Time: 26:36)



This is our 5th case, which says that for example, instead of 50 50 there is a 2 out of 3 chance that the value of X i lies in this given interval. So, now, we have 2 out of 3. We could have any other number from the normal distribution we will get that factor.

So, in this case, the probability that X i lies within this interval for is 67 percent, 2 out of 3 and if you look at the normal distribution, the multiplication factor for this is 1, 1 sigma we want to look at it that way. So, then the elemental systematic standard uncertainty on discount, is 1 times the half width of the interval 1 times delta a.

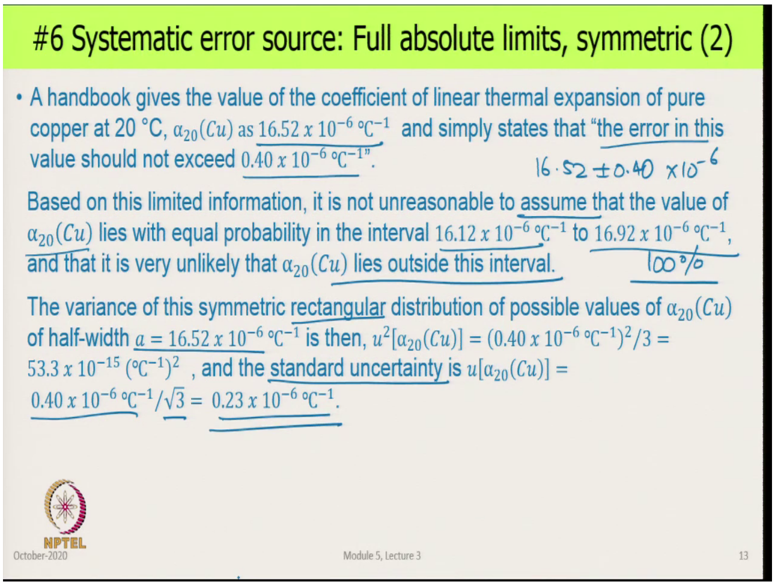
(Refer Slide Time: 27:35)



Now, we look at the 6th option, where we say, that the value of the measurand lies in an interval a plus to a minus with 100 percent probability. This we call full absolute limits and we assume that, these are symmetric bounds. That means, there is nothing going to be that is lie outside this, we are absolutely sure.

In which case? Now, the total interval 2 a is a plus minus a minus, half width of the interval is just half of that, and if we again take a normal distribution and do the checking or in, we assume a rectangular distribution, then we get b X i, k is delta a by square root 3. So, this we can see the arguments, these are given in the standard and also in the notes.

(Refer Slide Time: 28:44)

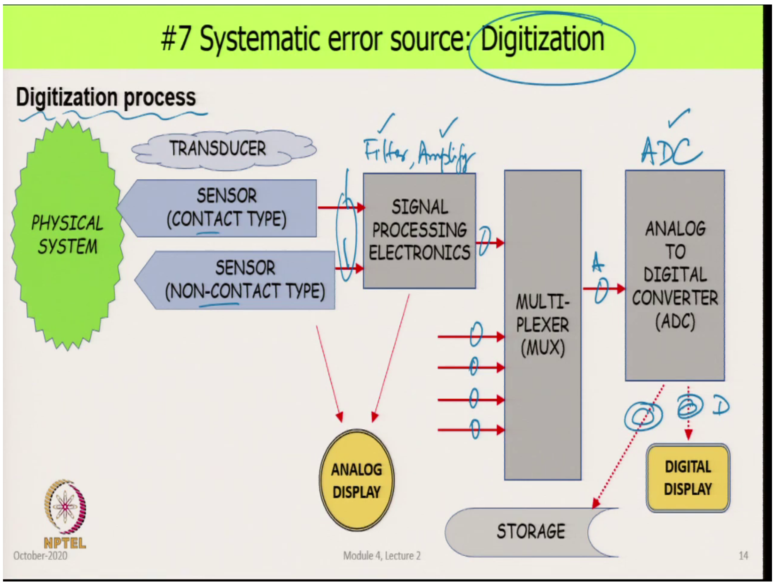


So, here is an example of that, a handbook gives the value of the coefficient of linear thermal expansion of pure copper at 20 degrees C as this much and simply states that, the error in this value should not exceed this much which means that, nothing lies outside this limits.

So, it is we can assume, that the value of, the thermal conductivity of copper, lies in this interval which is this plus minus of this, what we have done is 16.52 plus minus 0.40 into 10 to the power minus 6 and these are the numbers we get. And it is very unlikely, that any value lies outside this interval. So, we are looking at 100 percent probability that the value lies within this interval.

So, the variance of the symmetric distribution, we assume this to be a rectangular distribution, we can calculate from this half width is this much and the variance the standard uncertainty is that value divided by square root 3, which is this much.

(Refer Slide Time: 29:58)

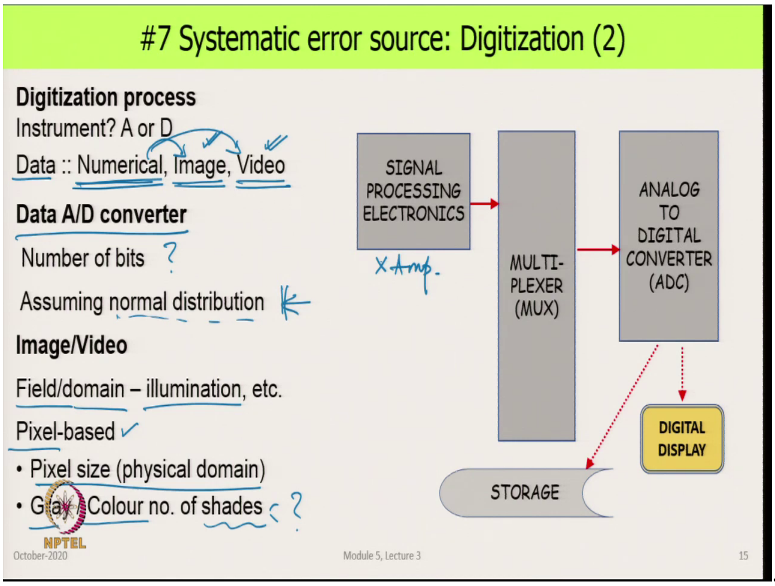


So, those were some of the instances of calculating systematic standard error. Now, we go to a slightly different type of a systematic standard error, we spend some time on this. And this is what is the systematic error that is introduced by digitization. We live in a digital world and so, we cannot escape uncertainties of the digital world.

So, this picture has been taken from an earlier lecture, which depicts the digitization process. So, very quickly, we have a physical system from which we have a sensor which could be contact or non contact type, it generates an analog signal and this signal goes into processing electronics, where it might be filtered and or amplified and again comes out as an analog signal. This goes to a multiplexer, if there are more than one inputs. So, these are all different inputs coming in which switches one input at a time to this.

It does not do any change to this; it does not change to the signal. And then, we have our device which is the a to d converter, ADC which convert this analog data into a digital data. This can be either displayed or stored. Now, we are going to we look at this thing, and say look there are be uncertain coming, because of this, because of this and what is it that A to D conversion is causing uncertainty, in the measurement.

(Refer Slide Time: 31:54)



So, we only look at this part, we can leave that other one. And as I said all of this may even be integral to the instrument itself, in which case, the analysis is still the same except that we are not looking at the items in a different as individual items. The first thing to note is that we are looking; we will be talking here largely, about data which is a single particular number, whether a number or a numerical value. But these days, we also do a lot of calculations assuming based on images.

For example, if we want to take the size of something, we take a photograph, and from that photograph we try to interpret what is its size or video, which is nothing but a lot of images in a time sequence, and we say that in this frame of the video, this particular object was at this position it has moved and the next frame the object is at some other position, I will want to analyse these frames and calculate velocity, acceleration, position, size so, many other things.

There we will leave for an advanced course, although these days they are very important especially, considering that there will be so, many advances in digital technologies. For the time being we will look at numerical values, a number or a data that comes. In some sense, this also has implications on this and this that is important to know this first part.

In looking at the standard error, because of the A to D converter, the issue is how many number of bits, does the A to D converter have, and we assume that the data coming into it follows the normal distribution. So, the error analysis in the discretization process is assume, that it has a normal distribution.

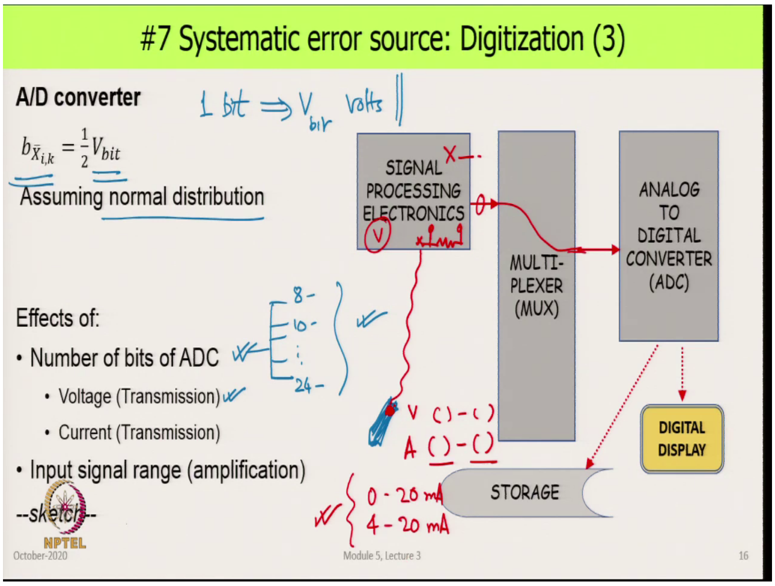
The signal that we get is influenced by, the amplification that takes place here, for the moment we will not worry too much about filtering, we say that the noise has been removed, we are only looking at the useful signal that we are; that we are wanting to measure.

So, that is what is coming. So, that is one issue coming amplification. In the case of image and video there are many other issues, that affect the picture, if have you use the camera anytime or then you will see, how all of those get affected. One is the field of the domain itself, what is the size of it, how much is it in focus and what is the illumination on it?

The same object in low light versus bright light looks very different; the digital image does very different things. And these are pixel based, but each pixel has a physical size on the screen, which corresponds to a physical size in the real thing, and each pixel in turn has a several set of numbers associated with it; one of course, is exposition.

And if it is a grey or a colour one, then in each one of these there are many shades, 256 shades or maybe even million shades and that tells you the richness of the colour, that is nice to see on what happens to a television, or a cell phone camera in for image analysis purposes, we have to look at what is the difference between those two shades, and how does that affect our quantification of data.

(Refer Slide Time: 35:57)



So, that is what this thing is about. We will look at the details in a minute, but the final answer is that V bit is the voltage associated with 1 bit of the A to D converter; that means, we say that when there is a change of 1 bit, this corresponds to a change of V bit some number, so many volts. Volts or milli volts, whichever way we want to look at it.

So, this is our fundamental thing that we will look at it in a minute, but be connected to the elements systematic standard uncertainty due to digitization, as half the value of this bit. And there we have assumed that we are working with the normal distribution.

So, before we go there a couple of things, the number of bits of the A to D converter this is an important thing; in the sense that, when we buy an A to D converter or we make one or the instrument already has one, this could take lots of possible values.

The A to D converter could start at 8 bit converter, 10 and go all the way to be a 24 bit converter. They do very very different things, the output is very different and we have this is the first thing, we must know, in understanding digitization. The second thing is, almost all data that we deal with in the digital world, comes to us as a voltage.

So, the thing is, that when we were looking at the earlier picture, that we have an instruments over there or a sensor sitting over there; this is your sensor. And it send a signal, let us assume let us hardwired to the processing electronics, in the laboratory these could be quite a ways apart, in an industry say like a petrochemical complex, or a refinery, or these could be 100s of meters if not 1000s of meters.

In a smaller thing like say, a milk processing plant, this could be 10s of meters. So, what happens? We have two options; the instrument can either output a voltage signal, over a certain range and this could be the analog signal or we could even output a current signal. Both are possible.

Now, what happens with the voltage signal? If this puts out a certain value of the voltage, by the time it gets here, because of line drop the voltage here, will be slightly lower than the voltage here, and you have introduced an error, a transmission error. If these were small distances, we could live with it that error is not very significant, but for long distances this is not an option.

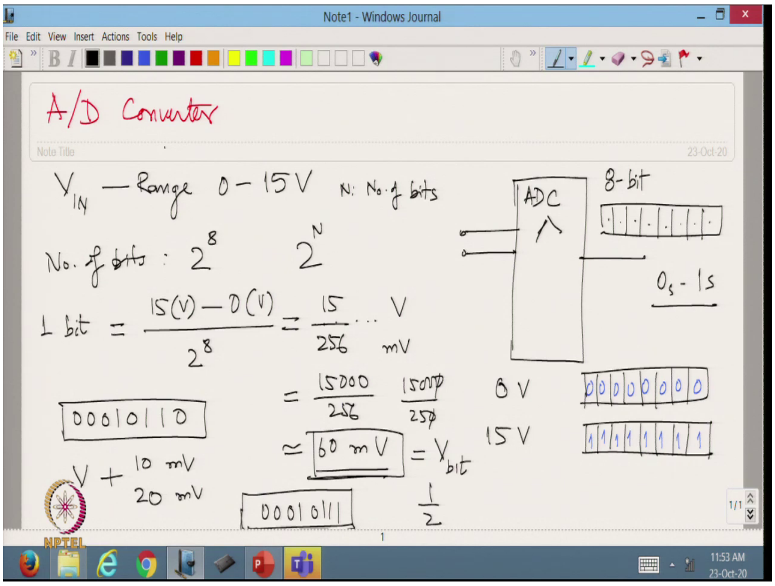
So, if you look at instruments that are used in industry, most of them will say, that we output a current in a certain range, it could be analog or in some cases, it could be digital. And typical ranges here will be 0 to 20 milli amperes, or 4 to 20 milli amperes. So, instrument itself has certain electronics in there, where it may take a voltage as a sensing signal, convert it into a corresponding current signal over a range and output it in the wire.

And so, what happens is the same current is coming to the processing electronics, where you pass that current over resistor and give then input the voltage value to the signal processing. In this process the length of the cable, which is this length here, becomes immaterial, because the current device has been designed for a certain length of the transmission cable. That means, in this case, there is no attenuation of the signal on account of the transmission wire.

So, that is what you will see, when you look at instruments for process industry or you want to buy an instrument invariably, it will say that the output is either 0 to 20 milli amperes or 4 to 20 milli amperes. We have to be able to interpret what that does for us, and why it is done.

The second thing is once it comes here, this voltage signal is what you always process irrespective of the transmission of the current signal and this gets multiplied by certain values and that is what we get into the analog to digital converter. So, now let us see, what happens, what are the issues with the analog to digital conversion process.

(Refer Slide Time: 41:36)



So, that we are looking at A to D converter. So, what it does is there is electronics in this thing, hardware stuff into which we input a signal. So, this is the V input signal to the A to D converter, and each A to D converter is designed to take V input signal, over a certain range. This range could be adjustable or it could have different A to D converters with different range.

For instance, say this could say it takes input 0 to 15 volts; that means, your input signal has to be between 0 to 15 volts, if it is more than 15 volts the input will to this will still count as 15 volts only, a 16 volts signal will be interpreted by the A to D converter as a 15 volt signal. Then there is electronics in this and there are two types of those electronics, we will not go into the details, you can look up any book on instrumentation and figure out what they do.

What we are looking at is that, what you get is an output, which is a number of bits. And when we say, it is a 8 bit converter; that means, it measured this value within this range and converted it into a system of 0s and 1s. And what the A to D converter does is that when the input signal is 0 volts that say for an 8 bit converter, all these digits will take the value of 0. So, it will be 0000 like that.

And in the second case, when it is taking a value of 15 volts, which is the upper limit of its range it will put out a number which is all 1s. So, we interpret the other way, around that if my signal has all 1s, we say it is 15 volts, but with caution that I just mentioned; it could also mean 16 volts.

So, we have to be careful, when we set the ranging of the A to D converter by, matching it with what the instrument and the amplifier are doing. So, what is doing is that, 1 bit change, the question is that; 1 bit change, how much volts does it correspond to? So, what we do is here, we have for 8 bit converter, the number of bits possible, this is simply 2 to the power 8 also, we can say, this is 2 to the power N, where N is number of bits.

And then, what we do? We say that 1 bit is equal to the range that is 15 volts which is the maximum value, 0 volts which is the minimum value divided by 2 to the power 8, this many volts or we call it in so many milli volts, this is the change for 1 bit. That means, if the input signal changes by these many milli volts, this number will go up by 1.

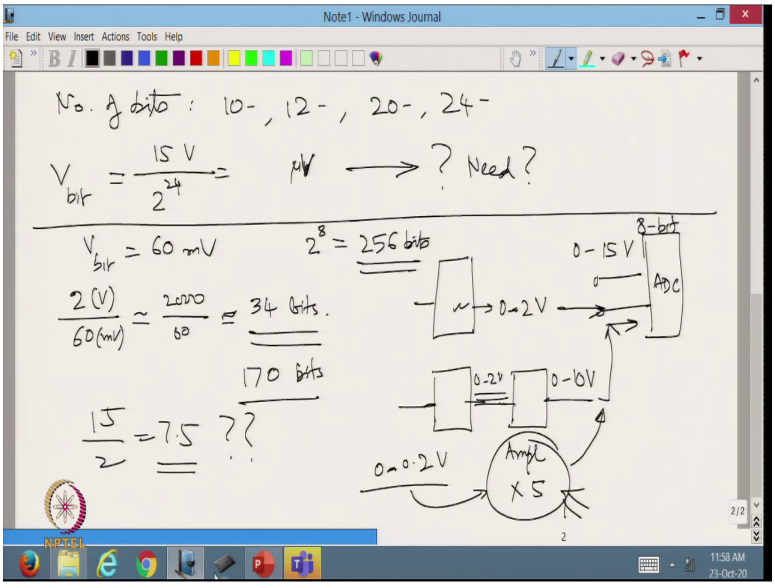
So, we can calculate that, this upon 2 to the power 8 this is 256 and we can say, that this is roughly about; in terms of milli volts, we can say this is 15000 divided by 256 and I just make it 250, and we can see that this is 15000 by 250 and this is 60 milli volts. So, let us say is around off, I have taken the 60 milli volts. So, what it tells us is that, if the signal has some value and we can say that this is 00010110, I am just taking some digital number.

So, you can calculate and see, how many volts it corresponds to, and if in the next instance, the signal input signal changes by less than 60 milli volts, the number will not change. So, this corresponded to a certain value of the voltage V and if we say this signal has increased by

10 milli volts, nothing will change; we still get the same reading. If it increases by 20 milli volts, we will still get the same reading.

Unless of course, it increases by 60 milli volts, then this number will go up by 1 and it will become 0010111. So, every number that we are getting in the digital world, has got something which is off by, about this and that is why, we set the standard error of this is half of this. So, this is what we call earlier as V bit.

(Refer Slide Time: 47:46)



Now, we have the option of number of bits. And in fact, if you see, cameras or screens or televisions or anything else, it is quite likely, that the cost of that if it is very high, it is probably using a very high end A to D converter; that means, the number of bits is more.

So, if you go to 10 bit converter or maybe 12 bit converter these are typical that we get, we can even buy 20 bit converter and 24 bit converter, then we can see, that is the V bit in these cases, say in the last case, this will be 15 volts divided by 2 to the power 24, which becomes a few micro volts.

This becomes very very sensitive, but then the question is for an experimenter we have to answer, do I really need this? If my input signal is not varying by that much and I cannot discriminate it, why do I need a 24 bit converter?

I may have be quite happy working with a 20 bit converter or a 16 bit converter. So, that is the thing. So, this is one issue about the importance of the number of bits in to the A to D converter. Now, we look at another issue, which is the theory of the A to D converter and we are said that, this will take an input signal 0 to 15 volts, but what happens, if I have an instrument which is sensing something and giving out a signal, which is only 0 to 2 volts and a directly input the signal into the A to D converter?

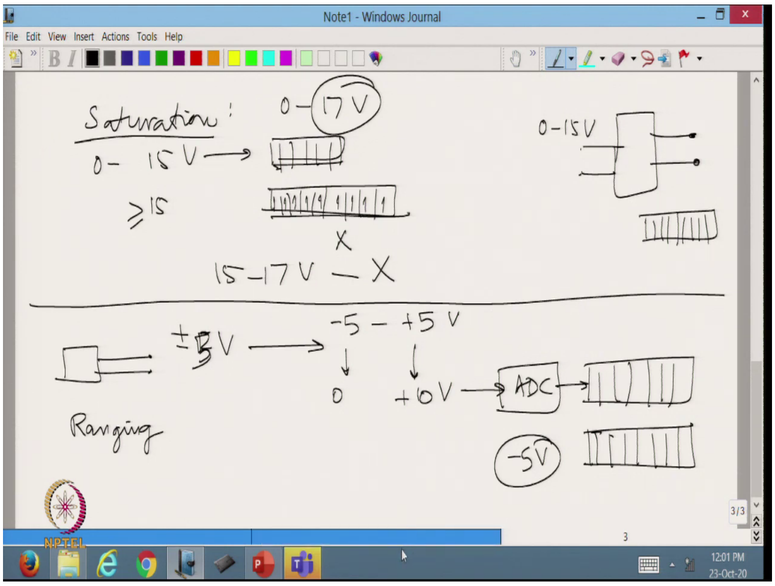
So, here is what happens. First our ability to discriminate this signal or to refinely and get the accuracy on this is limited, by the fact that say, if this were 8 bit converter we just saw V bit to be 60 millivolts. And so, what we are doing is; we have taken a signal which is 2 volts if we divide this by 60 milli volts, this is 2000 by 60 which is equal to about 33, 34 bits. So, we had a full option of using 2 to the power 8 bits which was 256 bits.

But we are using only the first 34 bits of that capability. So, we are losing something. So, what we could do? We could say well, yes, my instrument is still giving 0 to 2 volts, but now, I will multiply it and amplify it. And say I amplified by a factor of 5. So, the output signal is now, 0 to 10 volts and this now, goes into the A to D converter. So, now, what we have done? Instead of using 34 bits, we are now using 5 times as many more bits, which is 170 bits.

So, our ability to discriminate has improved so, that decision we have to make. There are instances like, when you are using a thermocouple the signal would not even be in 0 to 2 volts, it may be 0 to say 0.2 volts 200 millivolts say, in that case, you need much more amplification, before you can convert it and put it into the A to D converter.

So, this is one option that we have in trying to get the best out of this system from the measurement, for the measurement process. Can we match it exactly to 256 bits? Well this would mean that you would have to have an amplification of 15 by 2 or 7.5. If this is possible in the digital world we can do it, otherwise it is best that we remain slightly below 7.5 so, that we do not end up getting into the problem of the saturation.

(Refer Slide Time: 52:36)



So, what we mean here is that we got the A to D converter, which can take 0 to 15 volts. We amplified it so much or maybe we did not amplify it so much, but the parents signal itself was 0 to say 17 volts. And then we started getting numbers over here. These are all your digital bits.

And what the A to D converter does? It just keeps functioning and functioning nicely, for 0 to 15 volt range, it will give you the corresponding number in digital form which will be correct, but 15 or anything greater than or equal to 15, it will always end up putting all 1s. So, you would not know, by looking at this which is the data, which is what you will analyse later on, after the data has been collected; you will find that here you have a bunch of readings, which are all 15, 15, 15, 15, 15, 15, 15 volts.

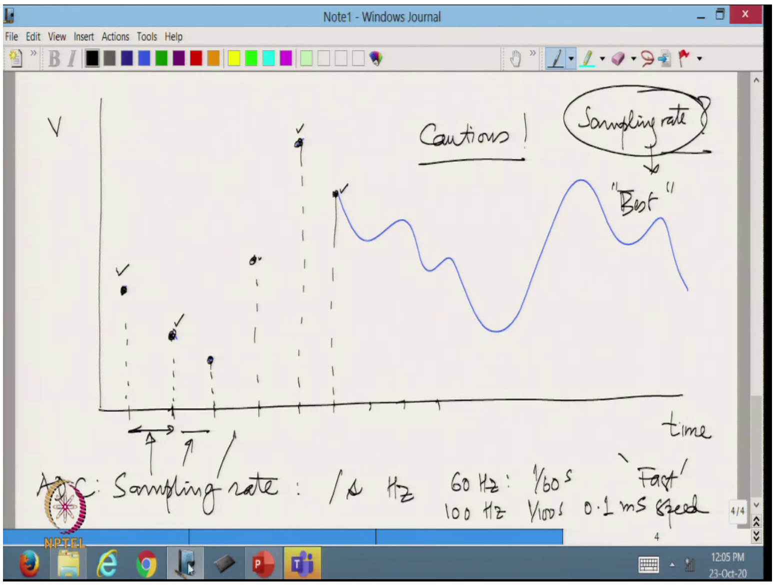
Now, you should be very cautious in interpreting that type of an information, because it could either had been a genuinely, 15 volt signal, but chances are it ended up having a problem of saturation and this was missing out on something that was actually happening, and what was missing out is that all the signal in the 15 to 17 volt range has been knocked out. We have lost information this particular data collection is in is useless, we got to completely discarded do that whole measurement all over again.

And possibly, the whole experiment all over again. So, this is one issue that we are seeing here. Then we have a case, where there are certain devices, which will give out to signal in a plus minus range. So, that is this signal could be plus minus say, 5 volts. Now, A to D converter does not like, minus volt signals.

So, what we do is we do a pre processing and map minus 5 volts to plus 5 volts to 0 and plus 10 volts. So, just do a simple addition, put this into the A to D converter and after that, interpret that digital data by saying, that I added 5 volts to it, and now I subtracted to get my real number.

So, that is the other thing we do. So, this is another aspect of ranging, which the other one what we saw was the multiplication part of it. Then we also saw that, we should not be restricting only a few bits of the A to D converter we should try to use as much of the range as possible. So, now, let us have a picture as to what actually, is happening in the real world.

(Refer Slide Time: 56:07)



We saw this in an earlier lecture, but now, this will make more sense, this side is time and this side is the signal value in volts. So, in the our digital signal, this as we said this was something like that and we digitized it at a certain rate, which for the A to D converter comes up with yet another thing, what is the sampling rate?

That means, how many inputs does it take per second or it could be in Hertz a 60 Hertz sampling rate means, it takes one sample 1 by 60th of a second, 100 Hertz sampling rate, 1 by 100th of a second or 0.1 millisecond. Higher the sampling rate faster is the A to D converter. So, in a way, this is some sort of an indication of the speed of the A to D converter.

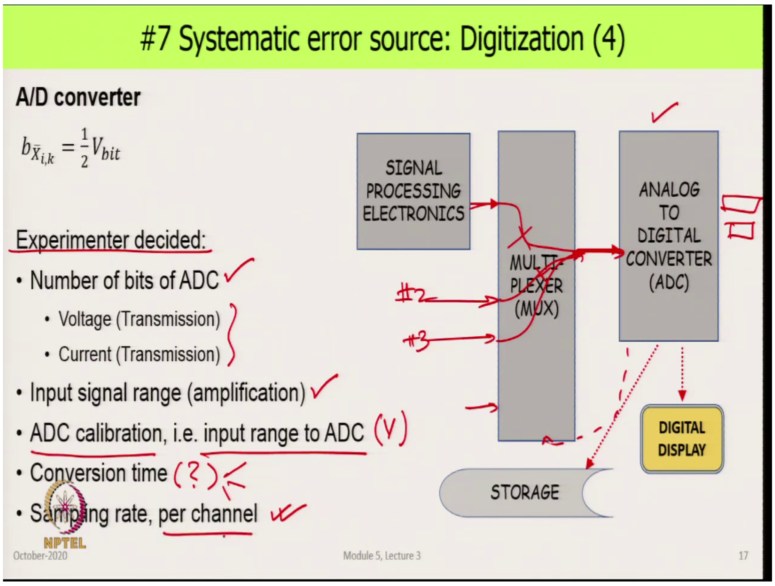
Now, what happens? This sampling rate will tell you, what is this time. And so, at regular intervals which is all of them are exactly the same here; the A to D converter will go to the input and pick up a data. So, at the goes in sees this signal, an analog value converts into a digital value and stores it. Then it waits after some time it does this, and like that, it does with each and every one of these things.

So, in the output file, we have a number corresponding to this, a number corresponding to this, a number corresponding to this, a number corresponding to this, a number corresponding to this. What we have missed out is our original signal is completely gone; that means, all this information which was there, we have missed it out. All we are left is this, this, this, this, this, this. And since, we do not know what the original signal was; we have to be extremely, cautious and careful in interpreting what the original data was.

So, we have to decide that depending on the rate of change of this signal, what should be my sampling rate so, that I can faithfully reproduce my analog signal. What you may call the best digitization.

This is an another important setting that the experimenter has to do; for this unique knowledge of the parameter being measured, how fast it changes, what range is the change, what is the range in which it changes and then match this with both one the amplifier A to D converter, its number of bits and its sampling rate. So, those are all the important things about the A to D converter. And what we now, have here?

(Refer Slide Time: 60:14)



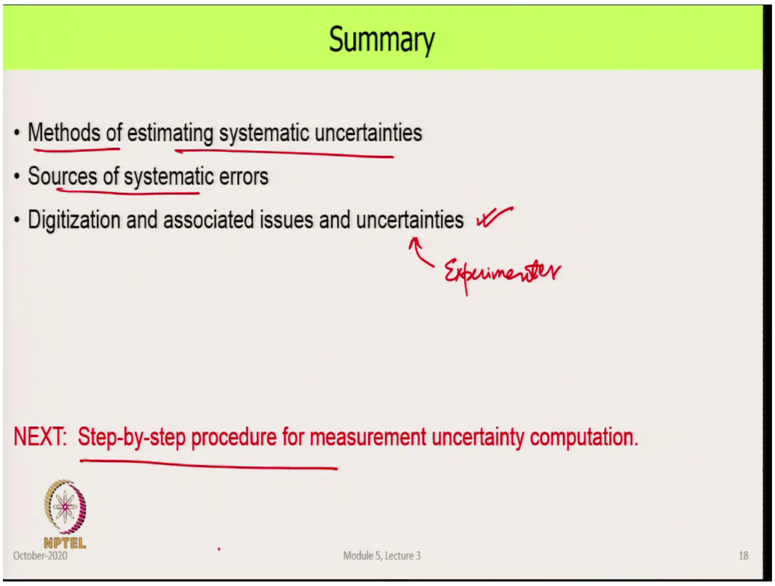
We decided all these issues, the experimenter will decide, what is the number of bits, whether or not we have voltage or current transmission, what is the input signal range that the A to D takes, what the instrument gives and how much amplification we should do, the input signal to the A to D converter could be adjusted; this is called A to D calibration, which is the input signal range to AD converter typically in volts.

How fast it converts, is that it takes a sample does some electronics on it and puts a number. It may be in microseconds, but that is the question; how much time does it take? Again in the market you will find a range of converters, we have to decide; what is good enough for us, we do not want a very high end which is expensive, very low end which does not serve the purpose. And of course, what is the sampling rate and this is sampling rate per channel, if we were looking at a multiplexer, what it will do is the A to D converter will take this signal digitize it and put it out.

Then it will give a signal, back to the multiplexer to say that, I am ready to take channel number 2 send it. So, there it will then disconnect this, connect this and convert it into a digital signal, that will send a signal and say, I am ready for channel number 3 this gets disconnected, number 3 gets connected and now, this gets digitized. So, like this it goes through all the channels on this, and then the cycle is repeated.

So, which means that, you may have a single channel being sampled at a certain frequency by the A to D converter, but because of the multiplexer; the actual rate which say 8 of them will be one eighth the rate at which it actually, does the sampling. So, per channel rate then goes down. So, all of this is what we need to understand and carefully, select in getting the best digitization.

(Refer Slide Time: 62:12)



So, on that note, we end this lecture. And we say, that we have and looked at, what are the various methods of estimating systematic uncertainties, we also looked at some of the sources of systematic errors. And we have looked at what is digitization, what are the issues associated with it as far as uncertainties goes and what are also the issues uncertainty associated with digitization, which as an experimenter, we not only have to understand so, that we can select the right instruments, but also we need to understand it so, the output digital data put out should be correctly interpreted.

So, that brings us to the end of this discussion on systematic standard uncertainties. In the next lecture, we will summarize this with a step by step procedure for calculation of measurement uncertainty. So, this concludes the 3rd lecture, on uncertainty in a measurement, this was about systematic uncertainties and we stop at this point.

Thank you.