Introduction to Uncertainty Analysis and Experimentation
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Module - 05
Uncertainty in a Measurement
Lecture - 02
Basic procedure - II

Welcome to the course Introduction to Uncertainty Analysis and Experimentation. We are on module 5 which is uncertainty in a measurement and in this lecture we will continue where we left off in the first lecture on Basic Procedures and look at the different ways in which standard uncertainty in a measurement can be established.

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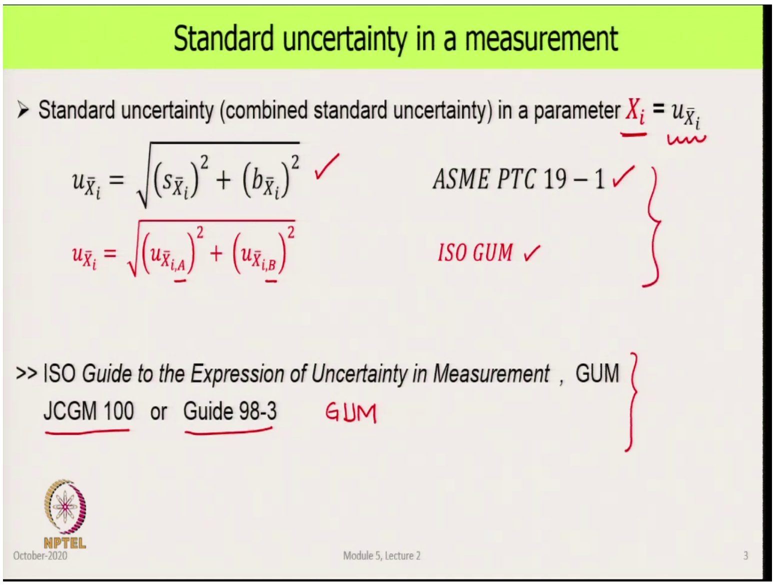

In the previous lecture we saw the full detailed method for calculating the standard uncertainty in a parameter measurement X i which is given by the symbol u X i. And we followed this first relation, this was as per ASME PTC 19 dash 1. And we said that we need some readings and some information and from that combination we can estimate the uncertainty in the measurement.

The alternate to this method is what is specified in the ISO standard GUM, where uncertainties are classified as Type A uncertainties and Type B uncertainties.

So, I encourage you to get this document it is free available on the web and you can search on key words JCGM 100 Guide 98 dash 3 or you can just put ISO GUM and you can download the PDF. This is a fairly exhaustive document as I have mentioned earlier and now we will look at those aspects which are related to what we are learning in this course.

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Now look at the vast sweep of the standard, it was prepared primarily by BIPM International Bureau of Weights and Measures which is the international body governing everything related to measurement and its applications. So, their document was put together by people from all these professional organizations representatives which are listed here and taken from the standard.

We will just go through this quickly to get an idea how wide spread is the use of uncertainty analysis in science and engineering. So, besides BIPM the other organized body that was with the in the formation of the standard was IEC which is the International Electro Technical commission a body that makes standards on all types of electrical, electronics and communication related standards.

So, whether it is our power supply or whether it is a Wi-Fi connection or Bluetooth all of that is governed by some standard of IEC.

So, this is a very big body which encompasses a lot of technologies, then ILAC is International Laboratory Accreditation Corporation. So, two things are there in this one laboratories and accreditation.

So, when we are talking of measurements I made a couple of references to a standard measurement or a standards laboratory. So, that is where this organization comes in what is a laboratory, what is the standards laboratory how about, what are the processes by which we ensure that it is a certain quality or certain level of laboratory that is this organization.

Then we have this organization which is from physics international union of pure and applied physics.

So, all physicists also were on board in this effort all applications related to measurements in physics then we had IFCC which is International Federation of Clinical Chemistry and Laboratory Medicine. In today's time if you are looking at Covid19 test methods instruments, their calibration, their certification, their liability it could be an organization like this which would into play and for them also this whole thing about uncertainty is very very important.

Then of course, we had the major international standards body the ISO International Standardization Organization and this covers practically everything with a very large number of applications on which it makes standards. Then you have IUPAC here International Union of Pure and Applied Chemistry, so the whole professional body of chemists was on board in making this standard and the last one listed here is OIML International Organization of Legal Metrology.

This makes all standards related to measurement of various things, whether it is your petrol being dispensed at a pump or even industrial sites where you say ok how much coal did you give me in a coal wagon, that measurement is governed is the processes for that measurement it is are governed made by OIML and there also uncertainty comes in a very major way. Besides these we have already seen that this standard got homogenized with ASME PTC 19.1.

ASME is American Society of Mechanical Engineers and this standard is now a mother standard for many other applications including Aeronautics and Astronautics and several other things. This standard also got adopted and became an American National Standard which is given a number ANSI American National Standards Institute and this also is practically drawing this and ISO GUM. So, for small differences we leave those aside all these standards are now pretty much the same, if we learn one we would have learnt the method for all.

And the importance comes out from this slide that what we have learnt is not just applicable to the work we are doing today. But may be in the future if we are going to do some work it will be applicable there also and also to various fields including here electrical engineering, electronics, communication, physics laboratory accreditation, laboratory, medicine chemistry, legal metrology everywhere what we are learning is uniformly applicable.

So, that is the document is there on the web freely downloadable please go ahead and have a look at it.

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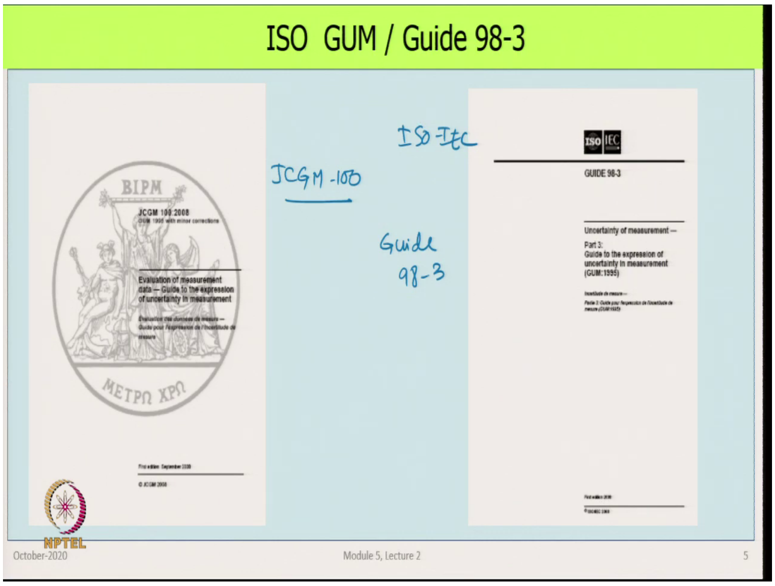

These are the cover pages this is your GUM as JCGM 100 and the same thing is published here by ISO as Guide 98 dash 3, the 2 are exactly the same documents both are available you can read either of those. So, this is Guide 98-3 and it is called ISO IEC Guide 98-3.

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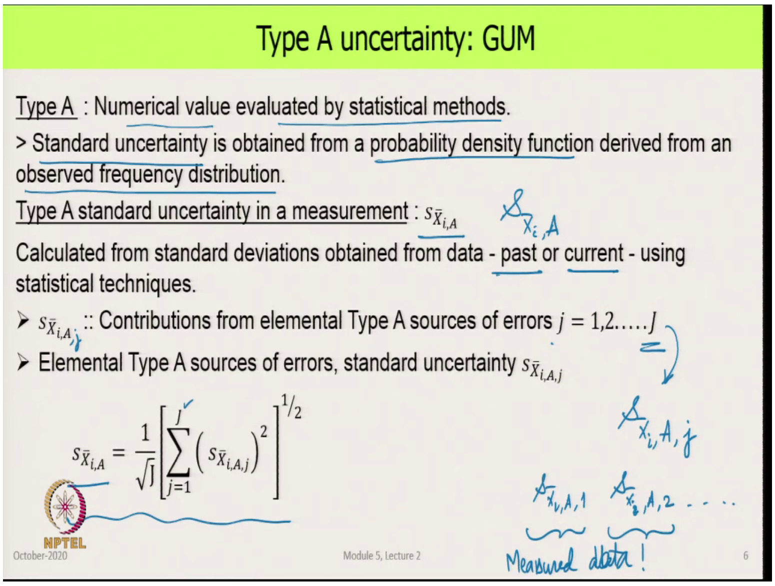

Now, according to this guide sources of error are classified as Type A and Type B. Type A errors are causes of errors result in an uncertainty which is evaluated by statistical methods.

So, Type A errors are those with the numerical value of the uncertainty is evaluated by statistical methods. Further what do you mean by statistical methods is that we have observed data observed frequency distribution. That means data that we collect present it as the frequency distribution from that we derive a probability density function and from there we get the standard uncertainty.

So, the standard uncertainty for Type A errors is obtained from a probability density function derived from an observed frequency distribution. So, this is the real data from which we made a frequency distribution and then calculated all the statistics, this we denote as s subscript X i comma A j X bar i comma A and this is also what the GUM says. The data from which it is calculated could be current or past data or a combination of those. So, there are there will be elemental sources of error on this count.

So, Type A elemental errors they could be j in number and we could qualify this as s X i A with the subscript j. So, that is an elemental uncertainty that is coming in X bar i which is the measure and type of error a j denotes how many sources of error are there and from each one of them what is it that we are getting.

So, you will get X i A 1 first source of error X i A 2 second Type A source of error and so on. Each one of these came from a probability distribution function from measured data that is the important part, this real data from which we are getting these numbers.

And we can combine all of this in this formula here which tells you that the Type A standard uncertainty is 1 upon square root j s X i A j square it is summation over all those elemental sources and its square root. So, that is how we can evaluate the Type A standard uncertainty.

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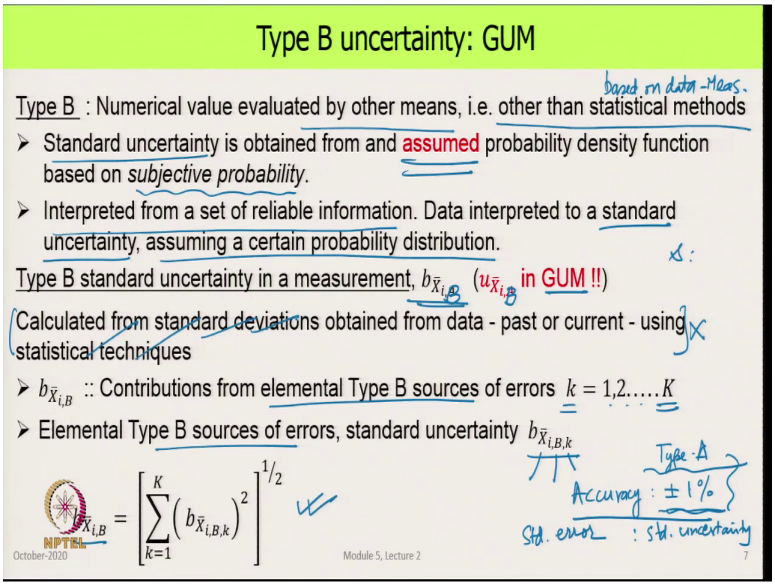

Type B uncertainty is somewhat different, X i in Type B numerical value of the uncertainty is evaluated by what is written as other means. That means, other than statistical methods based on recorded data and data we mean measurements data.

So, in Type A case we actually went got some data by doing an experiment and then interpreted the uncertainty value. In Type B we do not do that and we say I will only look at data that I have or information that I have and then I will proceed and the way you proceed with that is that this standard uncertainty is obtained by assuming a certain probability density function based on subjective probability.

So, what it means is, for example, if I have accuracy and somebody says my accuracy is plus minus 1 percent.

Then we can convert this into a standard error by assuming that the data from which this came comes from a certain distribution could be a normal distribution or a triangle distribution or a rectangular distribution.

Assuming that distribution and we make some more assumptions and say what does this mean and then assign a standard uncertainty to this. So, in terms of a distribution we said that standard uncertainty is nothing but the standard error. So, although the technique could be statistics based the data is not from a hard experiment.

So, what we do is we interpret the values from a set of reliable information or a pool of reliable information like accuracy is 1 linearity is another and there will be so many other things that come in. And assuming a certain probability distribution function we interpret that data in that light and calculate the standard uncertainty. We will see more examples of this in the next lecture and that will give a flavour of how an issue like this is resolved.

It is another case that if in measuring this accuracy we collected data and that data is available to us, then from that data whatever error calculation we do a standard error calculation we do then because it was based on actual data it will end up being a Type A uncertainty and not a Type B uncertainty. But that is generally not the case as in the type of what we do.

The point is then that there are certain things which depending on how one did it could be classified as Type A or as Type B uncertainties and that does not really matter its fine.

So, Type B standard uncertainty we do not get from a in Type B standard uncertainty in a measurement we give the symbol b x i comma this should be a B this is given as u X i B in gum. And I put this in red simply because this causes confusion in the symbols and this is one of the issues with GUM and why I am following PTC 19-1, u we have already reserved this symbol for standard uncertainty in a measurement.

And so for systematic errors we got the letter b for Type B uncertainty calculations we are giving the same symbol b and we are keeping the symbol s for uncertainties that came from data that were random uncertainties or Type A uncertainties.

So, that is the difference here ok. So, then so this is not the case here actually we do not get from past data this is should not have been here b x i B is contributions from elemental Type B sources of errors and we say the number of elemental sources of Type B errors is k. So, 1, 2, 3, 4 like that k and denoted by the variable small k.

And from each elemental Type B error source we can get the standard uncertainty as this for the X i parameter Type B uncertainty and k is the number of Type B uncertainty sources. And using this relation here which is the sum of the squares and the square root of that gives us the Type B the standard error associated with Type B uncertainties in the measurement.

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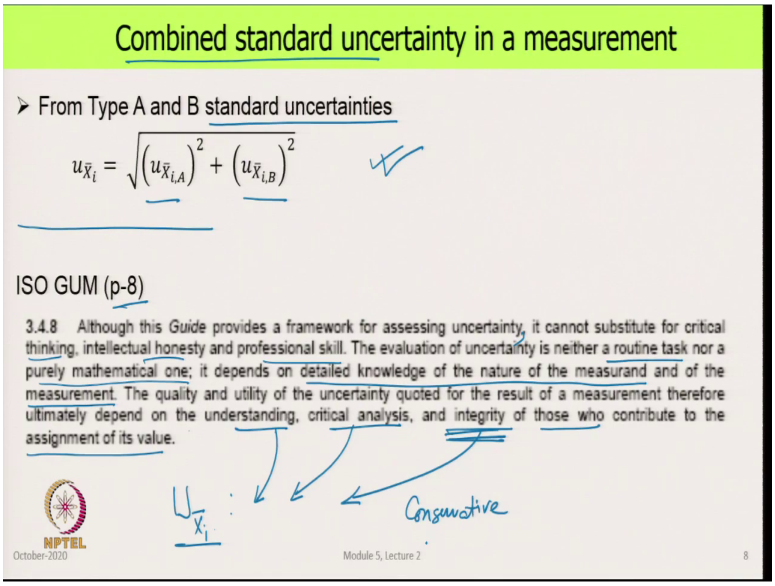

So, we got that and now we are ready to calculate the uncertainty in the measurement. So, this is the combined standard uncertainty or just the standard uncertainty which is the sum of squares and the square root of these two types of uncertainties. So, this gets us to the same point that we got in the first lecture by looking at random and systematic uncertainties.

So, that is the difference between the ISO approach and the PTC approach, the ISO GUM gives comprehensive background to lot of things and relies very heavily on statistics and symbols used in statistics.

And also the standard terms used in statistics now some of those things are fine up to the point, but after that when we start looking at experiments where we have multiple measurements, multiple results, multiple sources of elemental errors then keeping track of the symbols does become an issue. And that is the reason why I have developed a consistent manner of putting symbols and the designations and that is more consistent with PTC 19-1, otherwise ISO GUM in a way the mother standard for everything.

I have reproduced here from page 8 of the standard a few lines which I will read out and it give the prospective on what uncertainty analysis is all about.

And it says although this guide provides a frame work for accessing uncertainty it cannot substitute for critical thinking, intellectual honesty and professional skill. In the end the human being the experimenter is the key. In this it continues the evaluation of uncertainty is neither a routine task nor a purely mathematical one it depends on detailed knowledge of the nature of the measure and of the measurement.

The quality and utility of the uncertainty coated for the result of a measurement, therefore ultimately depends on the understanding critical analysis and integrity of those who contribute to the assignment of its value. So, after all this calculation when we do get u X i although it came out through a very involved mathematical exercise, as it says here this is ultimately coming from an understanding critical analysis and integrity of the people who report it.

So, it is not something which is very very rigidly fixed in some sense, this thing also requires as I mentioned in one of the lectures that we should be always conservative and of course honest in reporting what the uncertainty is.

Conservative means you always report on the higher side it may look a look may it may look see the it may show us in a slightly bad light, but does not matter at least we are being honest. So, with that we summarize what is there in the ISO approach.

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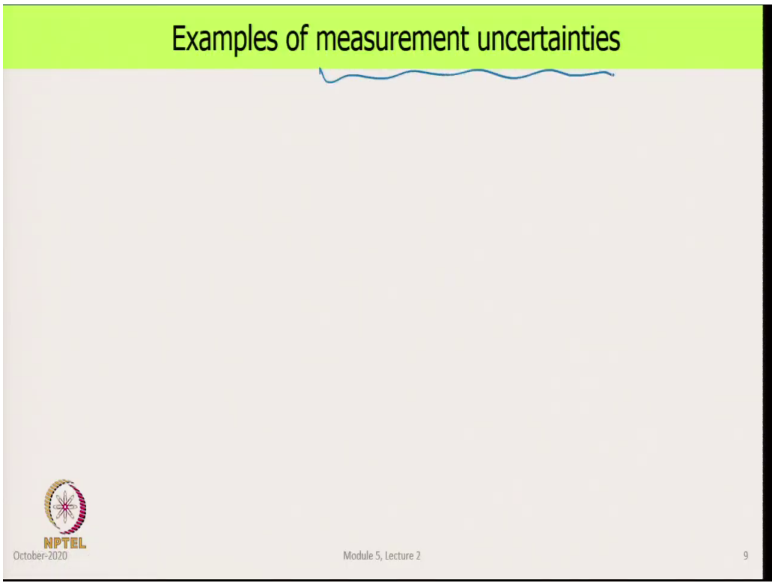

Now we move on to examples and these are examples on measurement uncertainties we have 3 examples we will take them one by one.

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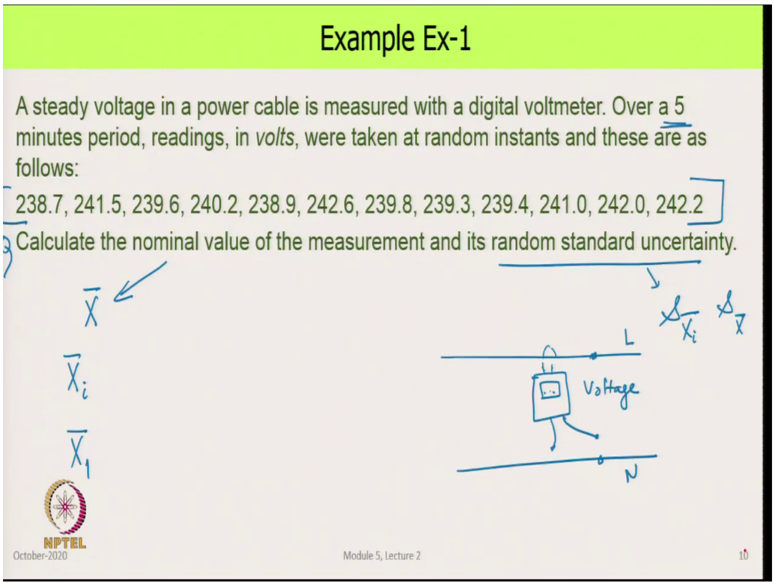

Here is the first one the application is that there is a electric cable around that we have put an instrument which is a voltmeter or a multi meter and we are reading the voltage based on the way it picks it up. This thing the current and there will be another 2 links which are connected between the supply lines this is say the live and neutral or Earth and we are getting the voltage between these two.

We can do that quite routinely in any of our domestic wiring or anywhere else, we just get the right instrument put it and take it. So, this keep showing a digital value and if you actually see the voltage you will find that there is a slight variation all the time, if you just do not get a rock steady constant same number. So, this is the application and I have said that these were measured over a certain time period this is 5 minutes it does not matter 5 minutes or 5 seconds or 15 minutes.

At some random times during that period these readings were noted, when we say readings were noted means you say ok now I will look at it and even if the reading is fluctuating whatever I see I will record it that is what we did. So, these are the numbers that we have got they are all listed here and the question that you have been posed is calculate the nominal value of the measurement and it is random standard uncertainty ok.

So, what we have been asked to do nominal value of the measurement means X bar all the while we have been using X i bar this could be x 1 bar or since there is only one measurement we are working with we can even call this X bar. So, that we tell us the nominal value and the random standard uncertainty this is the symbol as we have got is s X i bar in this case we can just call it s X bar. So, these are two things we need to calculate. So, let us see how the procedure is ok. So, we have all those numbers and our procedure would be like this.

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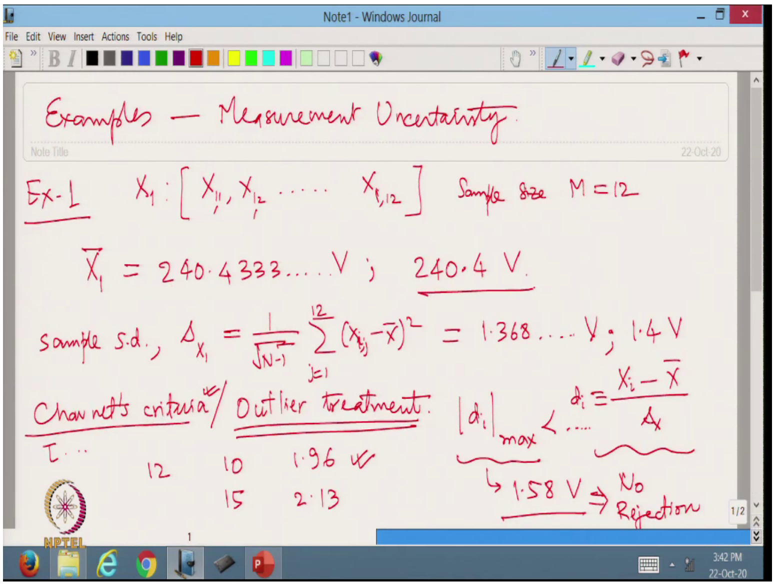

We have the data points and we can call them say we will call it X 1. So, the data points are X 1 1 X 1 2 and like that up to X 1 12. So, our sample this is our reading sample size is the number of readings we have we will call it M or M i this is equal to 12 and to get the nominal value we just need to calculate the mean X 1 bar.

And if you do that just do the arithmetic mean do the calculation and you will get the answer as 240.4333. So, we do an important thing we round off and report it to something that make sense and since our measurement was only up to 1 decimal place we will say that the mean value is 240.4 volts. Whether, the next digit make sense or not it the standard deviation that will tell us.

For doing that we then calculate sample standard deviation s X 1 and this is 1 upon square root n minus 1 summation X i minus X bar square X 1 j. So, this is j equal to 1 to 12 and if you do this calculation you will get 1.368 volts. We could round this off and say this is 1.4 volts. Now before we proceed further when we are dealing with data we need to do one check which is Chauvenet's criteria or in general what this is called outlier treatment.

So, this thing can be done either by using Chauvenets criteria or by a tau based table, I will show it with Chauvenets criteria. So, what this criteria tells us is that if you have these readings then you should look at the absolute value of the normalized deviation which means the d i is defined as X i which is the reading minus X bar over the standard deviation s X. Then according to this criteria the absolute maximum value must be less than something which depends on the sample size, so our sample size is 12.

And if you look up the table on Chauvenets criteria 12 is not listed, it says that for 10 the maximum allowable possible deviation is 1.96 and for 15 sample size this is 2.13. So, what we do we calculate what is the deviation for each reading with this formula and after doing that we see well what is the biggest number that we get and if you do that we get a biggest number which is d i max comes out to be 1.58 volts.

Allowable if we take a more stringent ten criteria10 sample size criteria we are still within that 1.58 less than 1.96 and obviously less than 2.13. So, what it tells us that all our readings are statistically expected. So, nothing needs to be rejected so this is no rejection.

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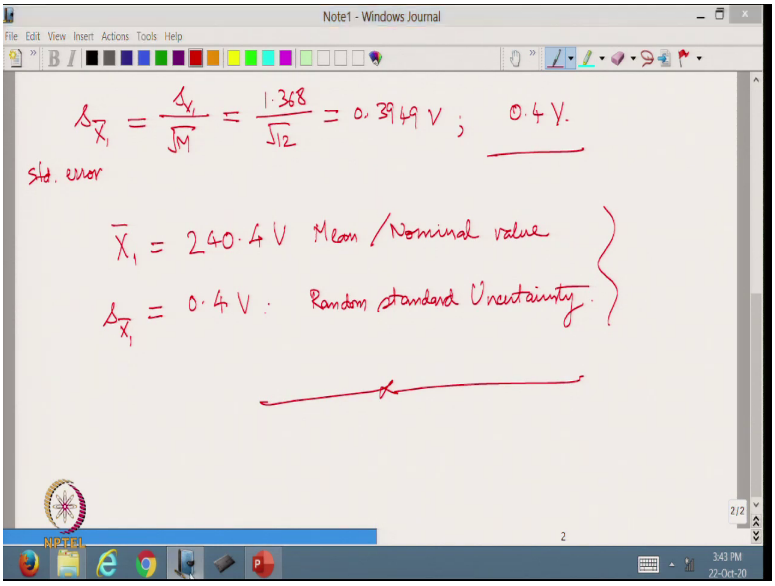

So, we keep all those twelve readings and go ahead and now our job is to calculate X 1 X bar which is the standard error or standard deviation of the mean which is s X 1 1 which is the sample standard deviation divided by the sample size which is 1.386 divided by square root of 12 and this turns out to be 0.3949 volts which we round off to 0.4 volts.

So, we got what we were looking for we got the nominal value X 1 bar as 240.4 volts and the systematic standard uncertainty as 0.4 volts sorry this is not systematic this is random. This is the mean or we may call it nominal value. So, that is the answer.

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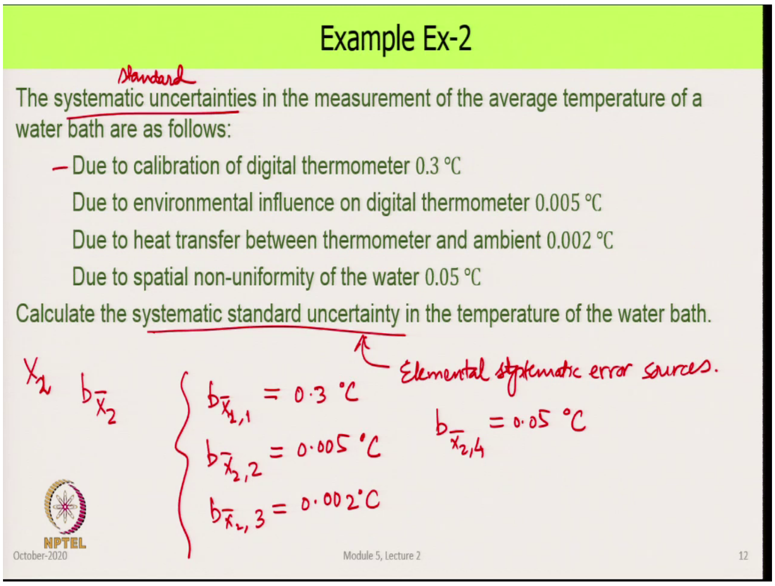

Now, we come to the next example it says the systematic uncertainties in the measurement of the average temperature of a water bath are as follows, due to calibration of digital thermometer 0.3 degree Celsius due to environmental influence on digital thermometer 0.005 degree Celsius, due to heat transfer between thermometer and ambient 0.002 degree Celsius.

Due to special non uniformity of the water 0.05 degree Celsius and it says calculate the systematic standard uncertainty in the temperature of the water bath. So, what this thing tells us is we have to calculate B there is only one measurement we will call it X 1 or let us call it in this case X 2 B X 2 bar this is what we have been asked to calculate. And the systematic standard uncertainty arises as we know from elemental standard uncertainty, from elemental error elemental systematic error sources. In this case we have 4 of them.

So, what we do is we call it the first one this one b X 1 bar X 2, X2 is our parameter and this is the first systematic error source. So, b X 2 bar 1 this is 0.3 degree Celsius b X 2 bar for the second reason is 0.005 degrees Celsius b X 2 bar 3 is equal to 0.002 degree Celsius and b X 2 bar 4 is 0.05 degrees Celsius. We take it that the systematic uncertainty that is being given here is the systematic standard uncertainty and that is why we wrote down these 4 values.

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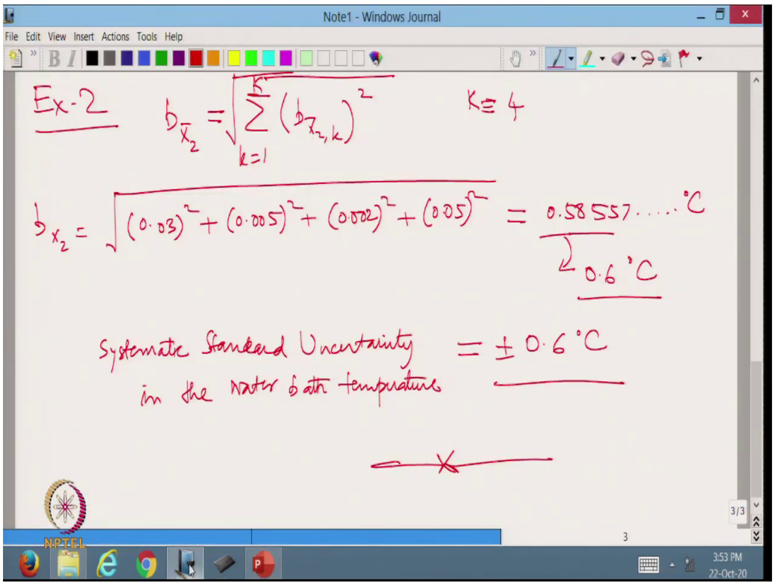

So, now let us see what the solution will look like, the relation that we got to use is that the systematic uncertainty in a result the systematic standard uncertainty in the result is the sum of squares of elemental systematic standard uncertainties from all the sources. In this case we have 4 elemental sources k is equal to 4 the values have been given. So, we can open up this and finally this is all square root of all this.

So, b X 2 is equal to square root of the square of all those terms that we had 0.03 square plus 0.005 square plus 0.002 square plus 0.05 square and square root of all of that and if you do this calculation you will get 0.58557 and you can put more decimal places which we round off and say this is 0.6 degrees Celsius. So, that is our answer.

That the systematic standard uncertainty in the measurement or in the measurement of the water bath temperature this is equal to plus minus 0.6 degrees Celsius, so that completes our solution.

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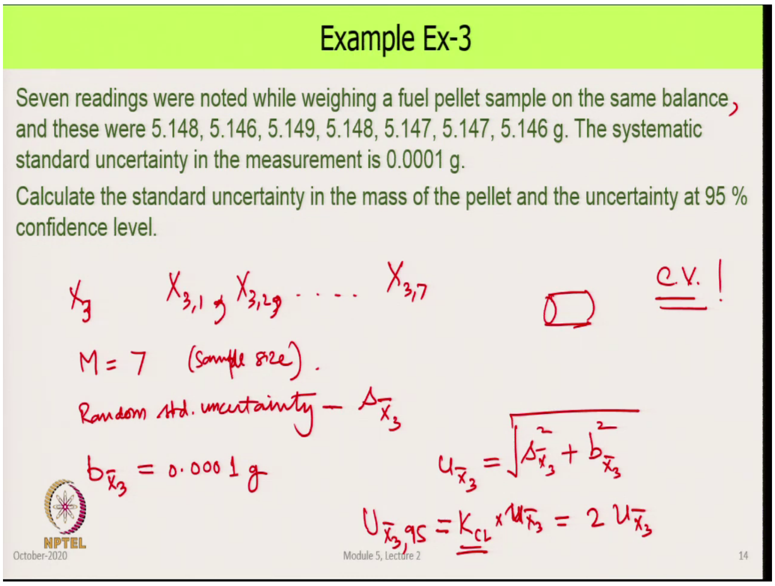

Now, we come to the third example it says 7 readings were noted while weighing a fuel pellet sample on the same balance and these were 5 this this this this this grams. The systematic standard uncertainty in the measurement is 0.0001 grams. Calculate the standard uncertainty in the mass of the pellet and the uncertainty at 95 percent confidence level.

So, to give a background of this measurement what we have is we make a fuel pellet, we say for example, we have some agricultural waste we take that make it into small pieces and compact it and make a pellet. So, this will be like about 10 to 15 millimetres in diameter and length would be like 20, 30 millimetres or even less this is then used as a fuel.

So, we have to do many things with this pellet and one of the things is we are going to measure it and measure it is mass. From a practical applications more than mass the calorific value of this would be of really interest we would we will come to that later on. So, what we have be given some readings that the same pellet was put on the same balance and hopefully by the same person who was operating it that is the that kis how the readings were taken. And so we have been given the seven readings and this becomes that this will get readings will be X.

So, let us call it parameter X3 what we have X 3 comma 1 X 3 comma 2 and so on until we have X 3 comma 7 sample size M is equal to 7. So, what it tells us that we can use this data to estimate the random standard uncertainty, that we saw in the first example we will follow a process very similar to that. What it also tells us that systematic standard uncertainty in the measurement is this much. So, systematic uncertainty means this is b in the measurement b X 3 and this has been given as 0.0001 grams.

So, that procedure would be that from this first set of numbers that we have we will calculate s X 3 bar then we will use the formula u X 3 bar which is the standard uncertainty in the mass of the pellet, this is square root s X 3 bar square plus b X3 bar square root of all of that. And it says what is the uncertainty at 95 percent confidence level uncertainty is u X 3 bar we have been asked at 95 percent confidence level, so it is comma 95.

This will be KCL times u X3 bar which for 95 percent this factor is 2. So, this becomes 2 times u X3 bar that is the second part of the question here. So, let us solve this now.

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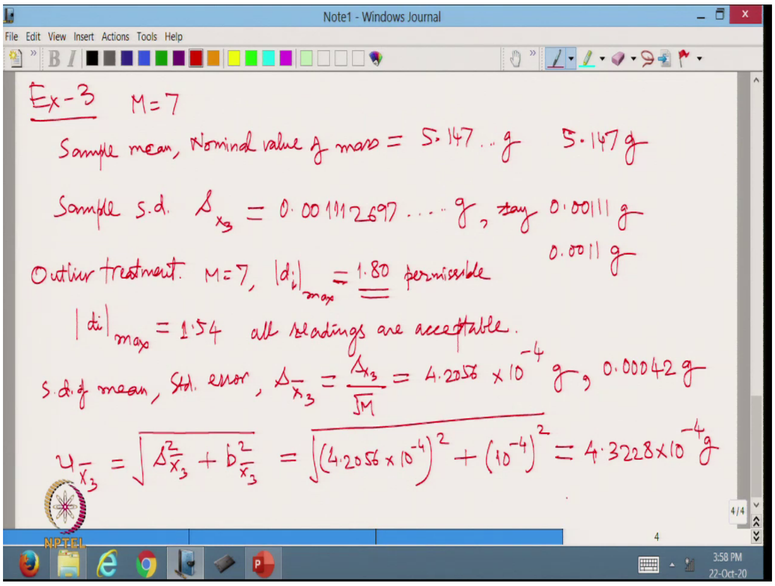

So, we have all those readings and the sample size is 7 we do exactly what we did in the first question, we calculate the sample mean which is our mean value average value or the nominal value of the mass and this we can do the averaging calculation.

And when you do that you will get 5.147 grams which we could round off to as many decimal places as was given to us, which is 5.14 we will keep as this much. Then we calculate the sample standard deviation which is s X 3 sorry it is not X 3 bar it is just X 3 and this same formula is applicable we can put it in the spread sheet or write a code or there are automatic things in the calculator these days we can use that and from there we get this as 0.001112697.

I am deliberately putting that many number of things, so that this is what the calculator showed, but we know we got to round it off and this we will say is 0.00111 grams at the most or we can even delete that and say 0.0011grams. Then we do the same thing outlier treatment for M equal to 7 the maximum possible deviation this is given in the charts as 1.80. So, this comes from the chart you can look up statistics notes books web anywhere else.

And we do the same thing calculate the deviation for every measurement those 7 measurements and pick out the maximum size out of it and we get that in our case this is permit permissible in our case d i max as measured is 1.54 we are well within this limit. So, all readings are acceptable. So, we proceed further we do not need to do any more other calculation, we calculate the standard deviation of the mean which is our standard error or standard uncertainty.

s X 3 bar which is s X 3 by square root M which is 4.2056 into 10 to the power minus 4 grams, we could round that off and say this is 0.00042 grams. So, then we come to the expanded uncertainty now that we got this u X3 bar is 2 times sorry. Now we got this we have already got b X 3 bar which has been given to us.

So, we can calculate u X 3 bar which is square root s X 3 bar plus b X 3 bar square this is square root of this number square 4.2056 into 10 to the power minus 4 square plus b X 3 bar is given to us as 10 to the power minus 4 this square and when you do this calculation we get the answer 4.3228 into 10 to the power minus 4 grams.

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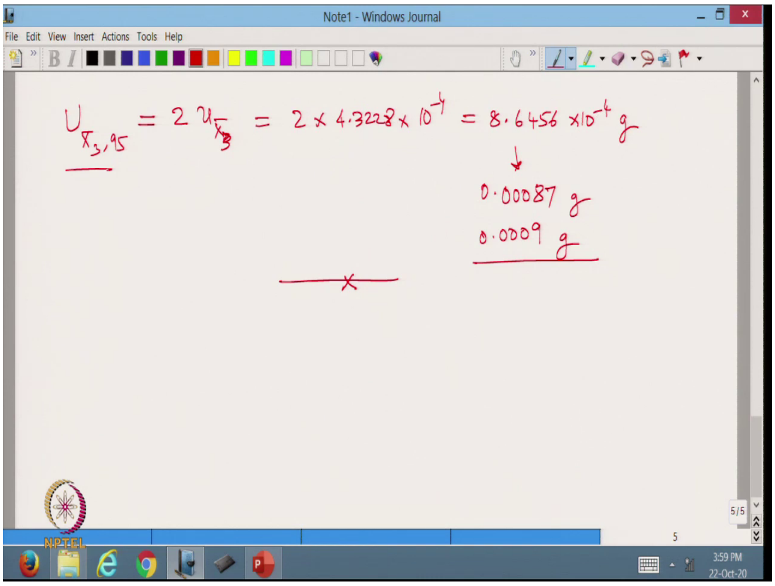

And then in the next step we have to calculate u X 3 bar at 95 percent confidence level this is 2 u X 3 bar which is 2 times that number we got 4.3228 into 10 to the power minus 4 which is 8.6456 into 10 to the power minus 4 grams which we round off to 8.2 0.00086 or 0.00087 we can say or even 0.0009 grams either of those would be ok.

So, that completes our solution and we got the answer that we were looking for. So, with that we conclude our discussion on measurement uncertainty.

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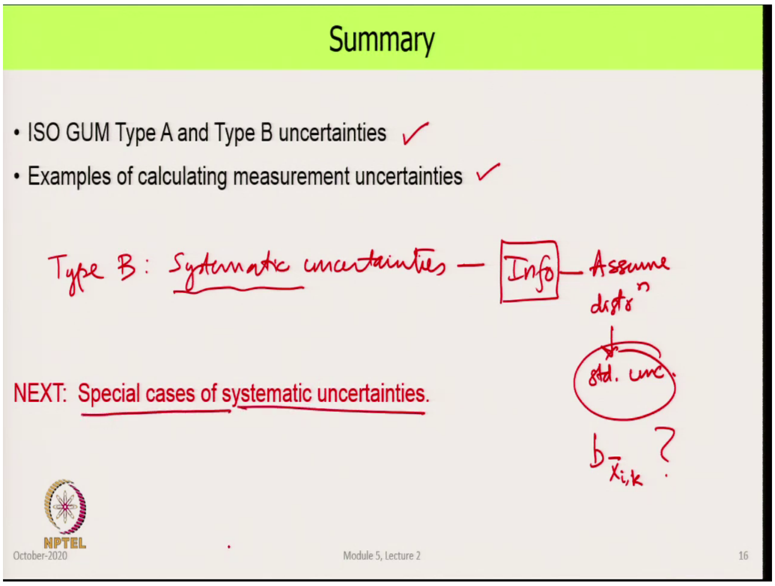

Where we have seen two methods first the PTC 19.1 approach which we shall follow and now we have just learnt about the ISO approach which gives us Type A and Type B uncertainties. And I have also presented a few examples to illustrate what we had learnt so far.

What the discussion of ISO GUM has thrown up is about Type B uncertainties or the systematic uncertainties mostly these would be systematic uncertainties. In fact, ISO GUM says the word systematic uncertainties could be misrepresentated or misinterpreted. So, they do not recommend it, but simply it is easy to use, so I have kept it and this is as per PTC 19.1. So, what we just came across when we looked at the ISO definition of Type B uncertainties is that we have information.

We assume a certain distribution and from there we estimate a standard uncertainty. So, this information as I mentioned there was accuracy there is linearity there is a resolution all these things are coming in. The question is how do we convert that data into a standard uncertainty? So, this would end up being some sort of a b X i bar comma k the elemental systematic standard uncertainty. This is what we will pick up in the next lecture where we look at special cases of systematic uncertainties.

On that note we conclude this lecture which was on the procedure for calculating measurement uncertainties.

Thank you.