Introduction to Uncertainty Analysis and Experimentation
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Module - 05
Uncertainty in a Measurement
Lecture - 12
Basic procedure - I

Welcome to this course Introduction to Uncertainty Analysis and Experimentation. Today we look at the module number 5, which is Uncertainty in a Measurement. So, this entire module we will look at the techniques of calculating the standard uncertainty in a measurement. Today we start with the Basic procedure.

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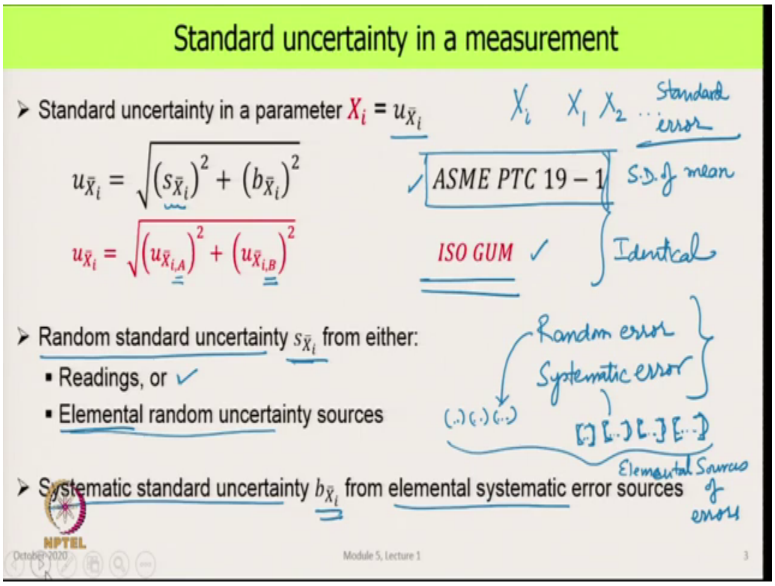

We get the standard uncertainty in a parameter or a measurand which is X i that means, this could be different measurements X 1, X 2 and so on. And that we denote by u subscript X bar i. So, this is the standard uncertainty in the parameter which from statistics is the same as standard error or the standard deviation of the mean.

To calculate this we have two options; one by PTC 19 1; which says that u X bar is the root of the sum of squares of s X i bar and b X i bar. The first term are the contributions from random sources of error and the second term comes from the systematic error sources.

The other way of calculating u X i bar is from ISO GUM; guide to the expression of uncertainty in measurement. And this is that this is the square root of the sum of the squares of type A uncertainties. The standard uncertainty in type A errors; type A causes of error and from type B errors. No matter which method we choose they have sufficiently been harmonized, the terms have been made relatively standard that for all practical purposes these two are identical.

In a sense ISO GUM is now the major standard across the world and PTC 19 dash 1 adopted itself to become like ISO GUM. For our purpose the method that I will explain is based on the ASME code. And the only reason is that in a starting course it is a little more easier to grasp what is random source of error and what is a systematic source. So, that is the reason we are looking at this.

So, now, to calculate it we have the strategy goes as follows. We calculate the random standard uncertainty s X i bar, which is this term here, in one of two ways. Either we collect lot of measurements from the instrument those are our readings and then do a calculation on that.

So, that is a very straight forward thing that if I want to get the standard uncertainty in say a pressure measurement and make multiple measurements of the pressure note down the readings assuming that everything else is in steady state. And then we do a bunch of statistics based calculations and establish s X i bar with a one step calculation.

In practice this is usually the case and later on we will also see examples and in the assignment you will see problems on this. The second technique is little more involved; and it says that the random standard uncertainty is up combination of elemental random uncertainties. That means, random errors were coming from several sources which we call the elemental sources of random error we get calculate or estimate the magnitude of these and from there we can calculate this.

So, in the lecture we will also look at the formula for doing this. So, that takes care of s X i bar; now b X i bar. This is systematic standard uncertainty, they has only one method which is a calculation based on the elemental systematic error sources.

So, like the random error the systematic error also has lots of n sources or systematic error. So, we can denote it slightly differently; that means, these are not based on data, but on other information including the experimenters experience past information and all of that.

Each one of these elemental sources so, all these are elemental sources of error. In the systematic errors we take the magnitude of all of these we will see a procedure for doing that; then we combine this in the mathematical formula and get b X i bar. So, that is the big picture.

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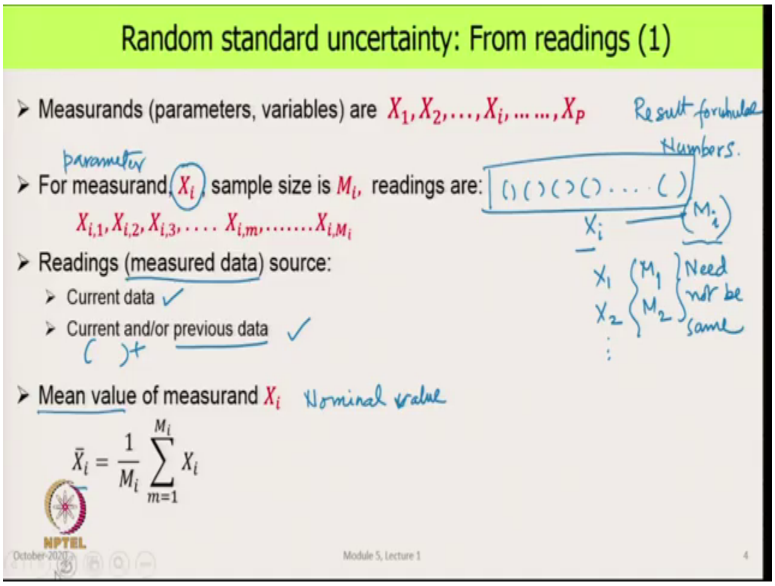

Now we will start by looking at the first thing which is s X i bar. So, how do we get the random standard uncertainty from readings? First we do what we have already talked of many times either when you are designing the experiment or you have done the experiment we have all the measurand listed and of course, we also have all the results formulae.

Then we look at one measurand or one parameter or variable at a time and we call that X i. And we say that we made so many measurements of this particular parameter, this could be pressure, temperature, flow rate, PH, humidity whatever. And that set of all those observations this forms a sample in the context of statistics, and these are M i in number.

That means, for X i variable I have this many number of measurements individual independent numbers. This M i could be M 1 for X 1, could be M 2 for X 2 and so on. And these M’s need not be equal or be the same. Now where do we get these readings from? And we have two possibilities. Readings which means measured data this could be a current measurement.

For instance, if I am using an instrument say a thermocouple based temperature indicator, before doing the experiment I collect lot of that data and then establish the standard error in that. That is the idea of current data; that means, the experimenter is doing it in themselves and using that as a basis for doing the calculation.

Another option is that we may not do all that data collection ourselves. We could rely either on some data collected in the past by somebody else or we use that data plus some from our own data with the same instrument, same setup, same everything. So, this is the second way in which we can get or the data that we are talking of.

And based on this data we then start up processing and what is this data for each measurand X i we have these numbers which are there. These are (Refer Time: 10:38) numbers values if you want to call it. And then we perform the few statistical operations on this; the first one is you to calculate the mean value X i bar which is the same standard formula that we come across in school and many other courses is the arithmetic mean. We also call this the nominal value.

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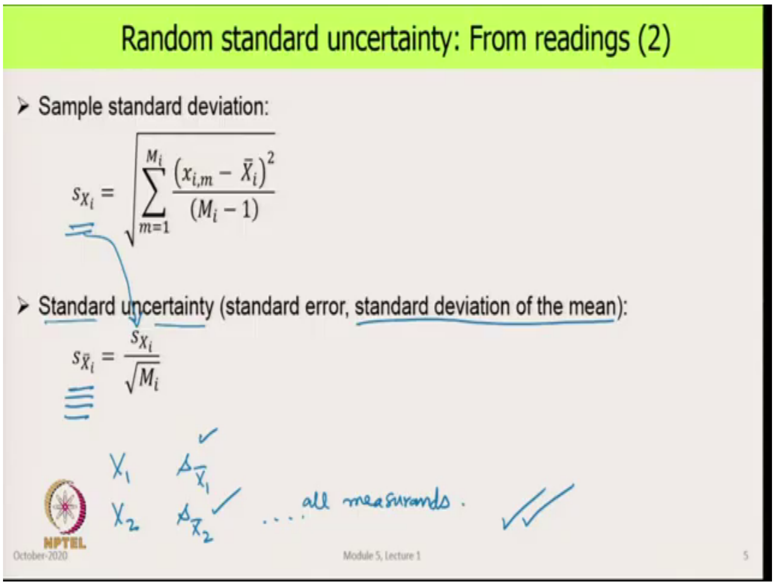

Then we calculate the sample standard deviation s X i bar again the same formula that we saw in school and in earlier courses basic courses. So, this is a sample standard deviation. And then we establish the standard uncertainty or what statistics is known as standard deviation of the mean.

So, this is now s X i bar which is s X i divided by square root of M i. And this is our standard uncertainty that we have been wanting to calculate, that completes our calculation. So, for each X i so X 1 we calculated X bar, s X 1 for X 2 variable we got X s X bar 2 variable calculation. So, these values came up and like this we do for all the measurands. So, you are repeating this calculation that many number of times and we are done with what we wanted to do.

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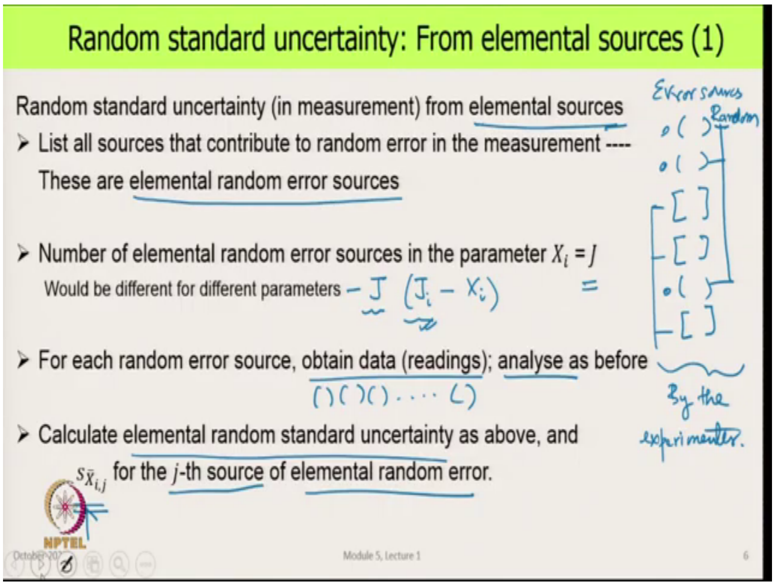

Now, we come to the second method from elemental sources of error. So, these are sources of error which are not calculated from data. So, they are which are calculated from data sorry and so they are based on statistical operations. So, what we do first? We list all sources of error possible sources of error in a measurement and they will do that for every measurement. And from all those possible sources of error we identified those which can be estimated on the basis of data and they are random in nature.

So, these are the elemental random error sources. So, you have error sources and we say that these are various error sources and we say that this, this and this they are based on data and they are random in nature. So, each one of these together they form a family which is the elemental random error source or source of elemental random errors. The others these will be the systematic errors.

So, we say that the number of elemental random error sources in the parameter is J; J could be 1, 2, 3, 4, 5 whatever. And this is all of this remember is done by the experimenter; that is you. So, for different parameters we could have different number of random error sources which means that J would technically be J i which is for variable X i and like that. For simplicity instead of J i, I have just kept it as J rigorously this is what should have been done.

For each random error source, we obtain the data which means that when we take data for that random error source we are holding everything else constant. So, we can say that this variability that we are seeing or these errors that are being caused are being caused only because of this error source and no other error source. So, we get a bunch of numbers and then we analyze as before and get all the data that we want.

Then we calculate the elemental random standard uncertainty in the same way as we have done before. We calculate the mean the standard deviation and standard deviation of the mean and this now is the elemental random standard uncertainty for the Jth source of elemental random error this is what we got. S X bar i J tells you which is the Jth source of elemental random error.

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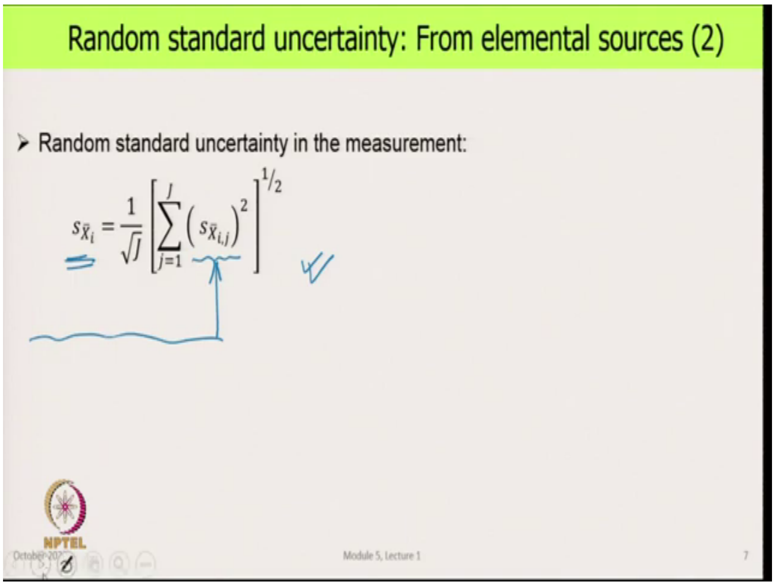

So, this is how we can calculate s X i J bar and then we combine all of these in this relation to calculate the random standard uncertainty in the measurement. That s X i bar is equal to 1 upon square root J sum of the squares of the elemental random standard errors. So, that is the formula we have, and this gives us the value of what you were looking for.

Now clearly this is a more involved method we can even say this is in some way more rigorous and quite often we would like to use it. But remember estimating sources of random errors is a little tricky and sometimes we will not be able to even do it. We will see how to handle that.

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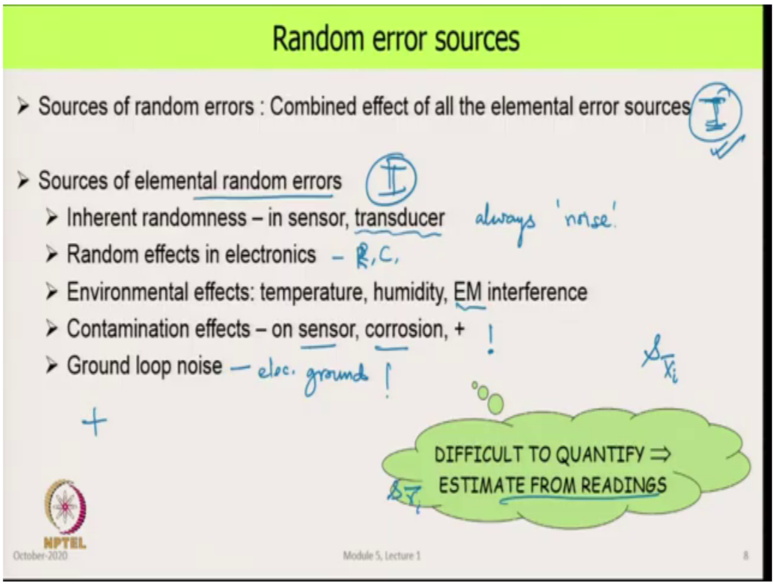

So, we have two methods both of us both of those methods gave us what we were looking for which was s X bar i we got that. Half the story of uncertainty in the measurement has been done. So, in the first technique that we saw technique number I we only took readings and based on the readings which was the combined effect of all elemental error sources.

We estimated X i X s X i bar. In method II that we just saw we calculated based on different error sources and the reasons why we have elemental random errors are that there could be inherent randomness in the sensor or transducer this is always there. At some level this is also referred to as noise.

So, whether it is an infrared sensor or a thermocouple bead or a strain gauge this will always be there; we cannot escape it. Then there would be random effects in electronics. So, you have elements like resistance capacitors inductors all of them nothing is exact, and they always have some randomness in their behavior so, this contributes to the errors.

Then you have environmental effects like temperature humidity and I put here electromagnetic interference. So, we cannot fully control these we think their impact is very minimal, but they are contributing some small error in the measurement. Then you have contamination effects for example, after using something the property of the sensor changes or the sensor itself gets contaminated it gets a deposit of say dirt.

And every time we clean it we damage it a little bit it again introduces a different amount of random errors. So, this is a little tricky issue in the practical world where there are many instruments where cleaning and keeping it clean is a major challenge. The errors are not insignificant and if you do not regularly remove the contamination, the measurement would be quite seriously off.

Yet another source of error is the ground loop noise this is something which is inherent in all the places where we are and this refers to static and electric currents in the ground; ground means literally the ground or the earth or the floor of the laboratory. Because that is connected to the ground of the electric power system and depending on how well the grounding is done.

A machine operating somewhere could influence a measurement done in its neighborhood. This is always there. We can minimize it and of course, if we are designing our system thoroughly we can even eliminate this. And you can add more to this and we see that that forms a sort of a list of elemental random error sources.

And as you can see from this it is very difficult to quantify what is the effect of each. To say what is the effect of the randomness in the resistor or randomness that is inherent say in a piezoelectric crystal this very difficult to quantify. And so more often than not we take recourse to the first method and we estimate s X i bar from readings of the instrument.

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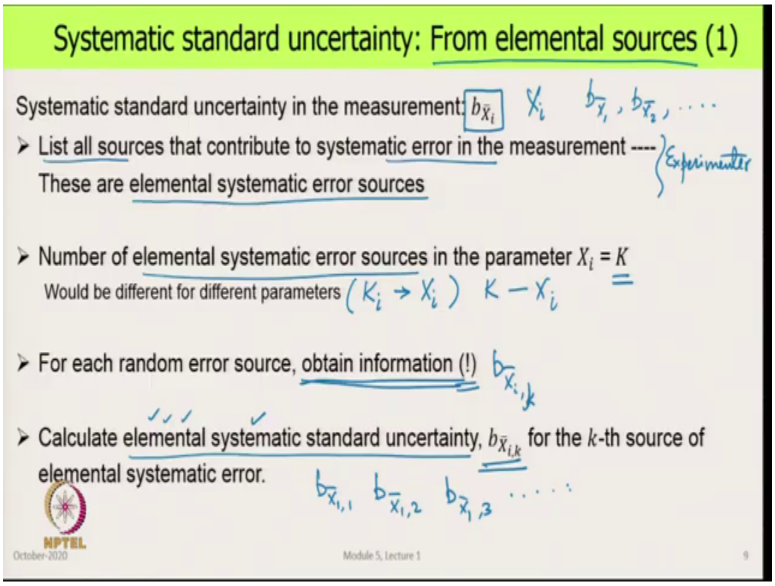

So, that was about random uncertainty now we look at systematic standard uncertainty. And as I mentioned systematic standard uncertainty there is only one way to estimate it; that is from the elemental error sources. So, in a while I list some of the elemental sources of error, but the procedure goes like this.

That we want to calculate b X i bar that is our objective; thus the systematic standard uncertainty in the measurement; in the measurement X i. So, again there will be b X 1 bar for measurement number 1 b X 2 bar for measurement number 2 and so on as many measurements are there.

So, first we list all sources that contribute to systematic error in a measurement. These are the elemental systematic error sources that is what we have. So, we have made a list 1, 2, 3, 4 as many as we can think of; Again who makes it the experimenter. Then let us say that the number of systematic errors that we have identified, we call this by the symbol K.

And as we J which was the random uncertainty counterpart of this number here also K, would strictly have been K i which is the number of element of systematic random sources in measurement X i. But we have to interpret that K does depend on which parameter we are looking at and it could have different values for different parameters.

Then we have to pick up each one of these sources of elemental systematic errors, and obtain information about them. Now this is being a systematic error; it is not based on data is based on information. So, how to get this we will come back to it in a little while and in the next lecture we will see how to identify it and based on data that we get how do we converge and calculate b X i bar; b X i J bar.

So, we have to do what we saw in an earlier lecture on instruments that we need to get all the details about the entire measurement process for each parameter. The sensor, the transducer, the wiring, the electronics, amplification, analog to digital converter, all of that; plus anything else that we know about the measurement.

Then our next step is that for each elemental source of systematic error we calculate the elemental systematic standard uncertainty. So, we are looking at this systematic uncertainty within that there are many elements we are looking at each one of them and so we get these numbers.

So, this is the K'th source of elemental systematic error; there could be a b X i bar say b X 1 comma 1, first measurement first elemental source of error. The order in which we number them that is completely immaterial then we could have 2, then you could have b X 1 bar 3 and so on.

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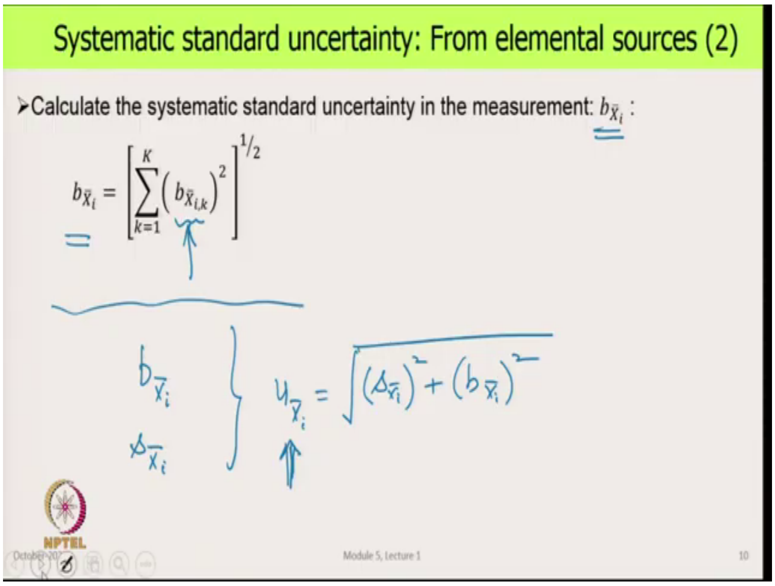

Once we get all these numbers we then have to combine them and the formula for that is given here; that the systematic standard error in the measurement X i which is the symbol is b X i bar this is the square root of the sum of the squares, of elemental systematic standard errors from each one of the error sources.

So, with this operation we also get b X i bar from an earlier operation we had got s X i bar; and we then combine these to get u X i bar which is nothing but, the square root of the square sum of the squares s X i bar square plus b X i bar square. And that is what we were wanting.

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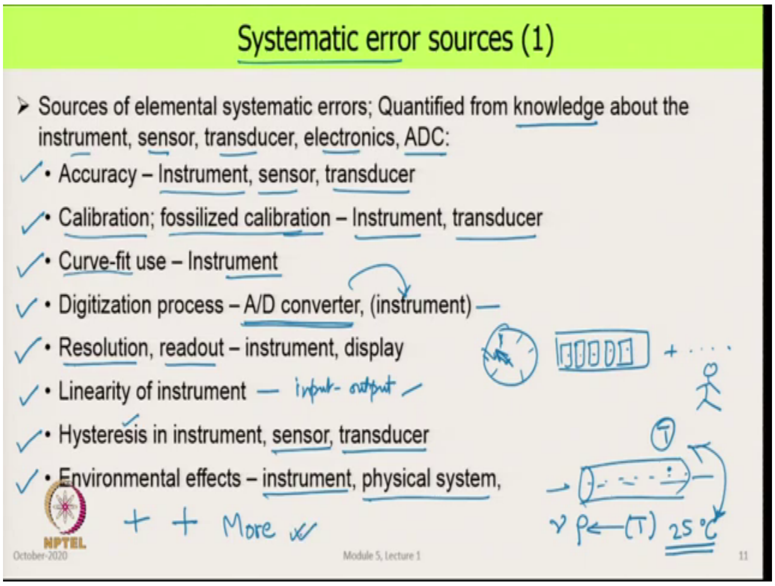

Now, what was the systematic error sources? And on this we later on so we will come back and spend more time on it. Right now I will just list them and just explain what is it that this is. How to get information on this and then convert it into a standard error, this we will look at in a later lecture.

So, we have to have knowledge about the instrument sensor transducer electronics A to D converter everything else. And here are the various sources of errors systematic error sources accuracy. This is something we come across very often something is accurate and that is of course, define how well we are able to predict the correct exact true value.

And as we said earlier when we were looking at methodology, the true value is never known the best estimate of true value is the one that we get from a standard laboratory using a standard instrument which has very very very small error in it. And that is the most accurate instrument that we have on the planet Against that instrument we calibrate our own instruments.

So, accuracy is issues come up with the instrument itself if you are using a sensor and doing something else we have to look at the sensor accuracy, transducer you will have the accuracy of the transducer. So, depending on how the instrumentation system is set up, we could have one or more of these most of the cases we will just have the instrument in which case we only talk about the accuracy of the instrument.

Then we have issues of calibration; that means we see that for the same signal what our instrument has given and for that same input what does a calibration instrument give us or a standard instrument give us, and then we make a correlation between those two. So, this we have to do for instruments and for transducers.

And then we have the issue of fossilized calibration where we are using some old calibration or calibration based on some other technique, that has gone out because of time and that could also be a cause of systematic error. The third source of systematic error is curve fit use.

So, what we do here is that; we do not go to a standard instrument we just take our own instrument and make a calibration of the dependent and independent variables and get a curve that is what is called the curve fit. You can call it a calibration curve, but it is not strictly a calibration at all; because you did not go to a standard instrument.

And with this curve fit we go back and when we make the measurement, we interpret the value of the dependent variable based on this curve not on the calibration curve. So, this causes a systematic error and this is usually associated with the instrument and not just the instrument as we have, but the fact that we use it get data and then make some information out of that.

The fourth source of error is in the digitization process. And this is associated with the analog to digital converter if it is a separate entity or in many cases it could be integral to the instrument itself. And an example of that is these days everywhere we are seeing non-contact temperature measurement for COVID-19 detection. What it does is that it has a sensor which picks up a signal and inside that it does all the filtering amplification and the A to D conversion and finally, displays a digital number.

So, that is an example of an instrument where the A to D converter is an integral part of the instrument itself. Then we have systematic errors associated with resolution or readout of the instrument or the display. So, this is simply what it means we have a dial on which there are certain markings and there is a pointer and we read from there. Alternately we have indicators digital indicators which put various numbers and we read those. And there could be some other techniques also.

So, here what is happening is we have cause one thing which is that? Inherently what the instrument displays it could already have built in an error into it, that is one thing. But it could also be an interaction with the experimenter who reads something particularly when the instrument value is fluctuating very quickly. So, there is issues with resolution and readout. We will look at examples later on that if the needle of a pointer in between two readings; what is the reading this is a question that comes up quite often.

Then we have questions of linearity of the instrument. By linearity we mean that the relation between the input and the output typically of the sensor or the transducer this is linear function. You double the input you get the double output or more most sensors and transducers would be like that, but when they are put in an instrument or when we are using them there will be regions especially at the extremities where the instrument is not linear.

It could also be the case that the transducer sensor of the instrument is inherently not 100 percent linear it has got some non-linearity in it. So, we how do we correct for it we expect that it is the linear behavior, but it is not behaving like that we estimate the error and that is the error due to the linearity of the instrument.

The next source of error is hysteresis. Hysteresis is what you have learned in school, magnetic hysteresis it also happens in when materials are stressed and strained. Same thing is that is the same phenomena on which many sensors are based sensors and transducers.

A strain gauge is a good example of that, where when you slightly strain it; that means, you have deformed it a little bit its resistance changes which is what you detect and interpret this value of the strain and then from there you get other numbers. So, these types of devices are subject to hysteresis and we need to do a lot of experiments and establish how much is the error that is caused by hysteresis in the sensor.

Then there are systematic errors coming because of environmental effects which affects the performance of the instrument. For example all the electronics or resistor would have a certain value of resistance at a particular temperature if it becomes warm or if it is operated in a hot environment or a very cold environment if resistance will be different it will start giving different numbers.

Environmental effects could also influence what I have written here as the physical system. And what I mean is; that if you are studying something say flow of water or in a pipe or air in a duct. So, this is a pipe say and something is flowing through this. We want to do a pressure drop calculation and connected with the viscosity velocity or we need the value of viscosity we may need the value of density. Both of these a temperature dependent.

So, the best thing to do which would to actually measure the temperature even though in this case we are not doing anything else with the temperature, and use that temperature to get these values. Alternately we could say no temperature is about 25 degrees Celsius and we will get properties from the table or the book or anywhere else and use that in our calculations.

The real temperature was slightly different from 25 degrees C at which the properties were taken, this variation ends up producing a systematic error.

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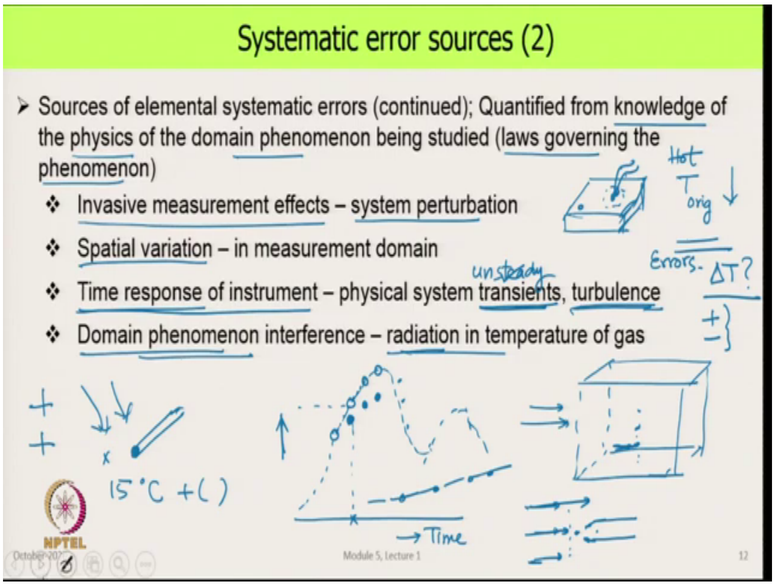

Then we have another set we can add even more errors to this class here depending on what the instrument is what the conditions are. So, these are in general something that I have written more will come from the knowledge and the use of which experiment of which instruments you are using. So, this is very somewhat quite experimental specific.

Now, we come to another set of systematic errors whose quantity can be established from knowledge of the physics of the domain. What you are studying, we understand the laws governing that phenomena that what I have put here.

And using that knowledge we can estimate that if there is some extraneous thing coming in how much would be the error because of that. I have listed here 4 items the first one is invasive measurement effects.

Invasive simply means that you have disturbed the system that you wanted to measure by inserting the instrument or the device. The moment we do that, we have perturb the system and that has caused a systematic error. A quick example on this is the say this is a surface or a say a plate and we want to measure the temperature of this plate.

So, we put a sensor there say a thermocouple there are two wires coming out of the thermocouple. And what happens is if the thermocouple were not there this whole surface would have had some temperature let us say the original temperature T original. The moment we put a thermocouple on it if this was hotter than the surrounding air this will conduct heat and locally it will suppress the temperature and you would find that some space around this, the temperature is less than the temperature far away.

And what this instrument is going to tell us is the temperature at this point which is now slightly less than this point in this temperature. So, if it is a hot surface compared to the ambient you will get a lower reading, if it is a cold surface compared to the ambient you will get a higher reading both are systematic errors.

But we do know the science and the engineering behind heat transfer we can write those equations get data about this, this, this and the air around it what it is doing solve some equations and establish how much is the magnitude of the dip in the temperature, that such a thing causes. And that can say that we introduced this much error because of this invasive effect and as we see here it could be either positive or negative, but not both it is not a random error it causes error in one direction only. So, that is one example.

Anytime there is an invasive measurement this is an issue, we have to worry about it. The second point I will put here is spatial variation in the measuring domain. For instance we have a duct. So, these are the two walls of a duct that which says something like this, and say air is flowing through this. In the practical world if you take any plane and take a bunch of points over there the velocity here, here will all be slightly different.

So, if you take a measurement over here that putting an instrument, you would get a slide. So, this is the instrument say a pitot probe that you have put, but the air coming on it instead of being exactly uniform everywhere here the velocity could be less here it is slightly more here could be even more.

So, what this sensor will pick up is some sort of a mixed average of these velocities. So, we cannot say exactly that what you got as a calculation from this is the velocity at this point it is affected by what was happening around it. That means in space the parameter being measured is varying and because of the finite size of our sensor or the instrument.

We are sampling a little a bunch of variable parameters and the instrument is making a signal out of it which we interpret as a measurement that is spatial variation. We may or may not be able to quantify this. The third systematic error source and it is a very issue big issue when it comes to studying transient phenomena and turbulence.

Is when the phenomena that you are studying is changing with time that is what we mean by a transient phenomenon or an unsteady phenomena. And we have put an instrument there and we want to measure that parameter value at different times. So, what happens here?

Say the system was going like that. Say it is going like this and then picking up again and this is time and this is the signal value or the parameter value sorry. So, at this time our sensor should have been reading this value, but the value itself was changing like this and a sensor was not a made was not able to keep track of that.

So, as the value went there it change the signal from say here to there, but only to some extent that is there; when the signal went there it got only up to there, when the signal went there the parameter went there are our instrument showed this. So, only two things can happen that either this phenomenon is so slow that our instrument is able to track it. The say phenomena is like this going very slowly then our instrument has enough time to respond to it and capture the value at its true value.

But when it is a rapid change while increasing we will have a lag and while decreasing we will end up having a over prediction. That is because our instrument is not able to respond as fast as the temperature or the pressure or the parameter is changing. A good example of that is a clinical thermometer that we put in the mouth and we have to wait a minute two minutes before we take a reading.

And that is simply because it has been introduced into our physical domain whose temperature is something its original temperature was something else. And it takes two minutes say for the entire instrument to come into thermal equilibrium with what the body and only then the temperature measurement makes sense.

So, this is a quite an important thing in many measurements where we are making unsteady measurements or transient measurements. This is also the case when you are studying turbulence in fluid flow or turbulent heat transfer turbulent mass transfer or even atmospheric phenomena, which are usually turbulent and varying with time.

The fourth cause, I put here is what is called as domain phenomena which interfere with the measurement. And an example of that is the radiation effect while measuring a temperature. The very simple example of this is; say we have an instrument say a thermometer or a thermocouple and we want to measure the air temperature. So, if you just hold it in the room and say this is the air temperature, but that is a measurement, but we do not know what are the errors that have come into it.

If the room where air were cool say 15 degrees Celsius and the walls were 30 degrees Celsius or 25 degrees Celsius, then this instrument will not show 15, but 15 plus something. So, whose temperature are we reading? Well it is not the air temperature, but air temperature of the sensor which is slightly different from the air whose temperature we were trying to measure.

The same thing will happen if we take this instrument and say I want to measure the temperature of air and we go stand outside and there is sunlight coming on it this instrument will show something else. Ambient temperature may be 30 degrees Celsius this may end up showing 38, 39, 40 degrees Celsius.

So, that is what I call as a phenomena and there is an example where radiation is influencing this. And what if has to be done is that in this particular case we have to go back to the physics of what happens in this situation and see how you can eliminate or minimize the radiation effect on this.

Like this there will be many more things which can add which are again very user specific phenomena specific and then using knowledge of that particular topic you can go back solve some equations and analytically estimate the quantum of error that we are getting. And as you can see here these errors usually are on the plus sign or minus sign they are not completely random errors.

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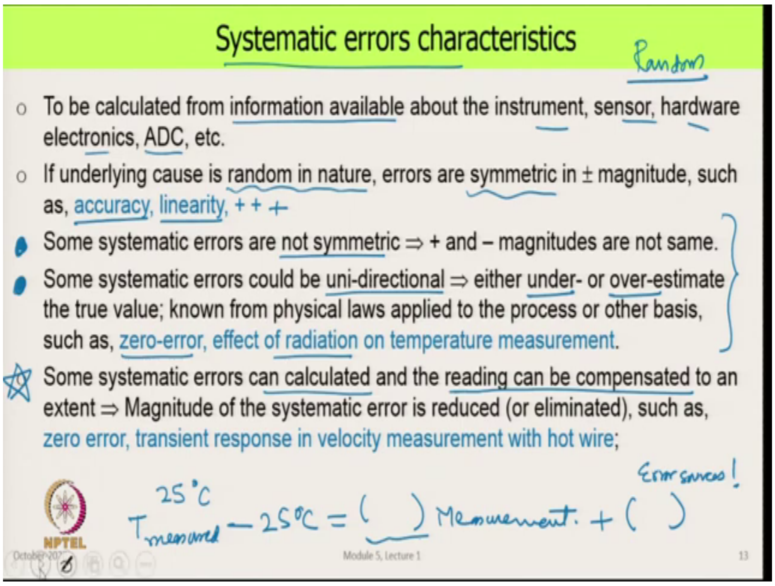

So, what are some of the characteristic of systematic errors? And we are talking of the systematic error characteristics, but we did not talk of characteristics of random errors. For the simple reason that when we say the random, the probability that it will be overestimated or underestimated is equal and so there is no issue of any difference coming in. Systematic errors as we have seen in some examples just now need not be like that, some could be like that some need not be like that.

So, the first thing is that we do not calculate systematic errors from data unlike random errors and we calculate them from information that we have to collect, about the instrument sensor hardware electronics analog to digital converter and everything else. If the underlying cause of that systematic error is random by nature, the errors are likely to be symmetric; that means they are equal in plus and minus magnitude. And examples of these are errors due to accuracy and linearity and others you can add.

Some systematic errors are not symmetric; that means a plus and minus magnitudes are not the same. We have seen some examples just now we also saw that some of these are errors that could be unidirectional; that means we know for sure that we are either over estimating the value of the parameter or under estimating the value of the parameter.

A simple example of that is the zero error say in a vein balance or sprain or whatever it is zero error you will always be over estimating the value that we are getting and the effect of radiation we just saw. Now, to these are how to treat this in uncertainty analysis is a bit tricky in the more involved.

And it is good to know that they are there, but we are not going to talk about these errors in this course. These are topics for an advanced course. So, we are only saying that we know that there is an issue we are not going to tackle problems which are these issues in this course, this is for a advanced study.

And then we have another interesting thing here as we seen in the earlier slide, that some systematic errors can be calculated from knowledge of the phenomena and the equations in an analytical way. And what we often do is we compensate the reading to that extent from that calculation.

For example, if we say that from my calculations the radiation error is over estimated by 25 degrees Celsius then we can say I will take my measured value subtract 25 degrees Celsius and this I will treat as my measurement. So, one error we have taken care of we have a much better estimate now, we are fairly confident about it but all the other error sources are still there; they have not been eliminated.

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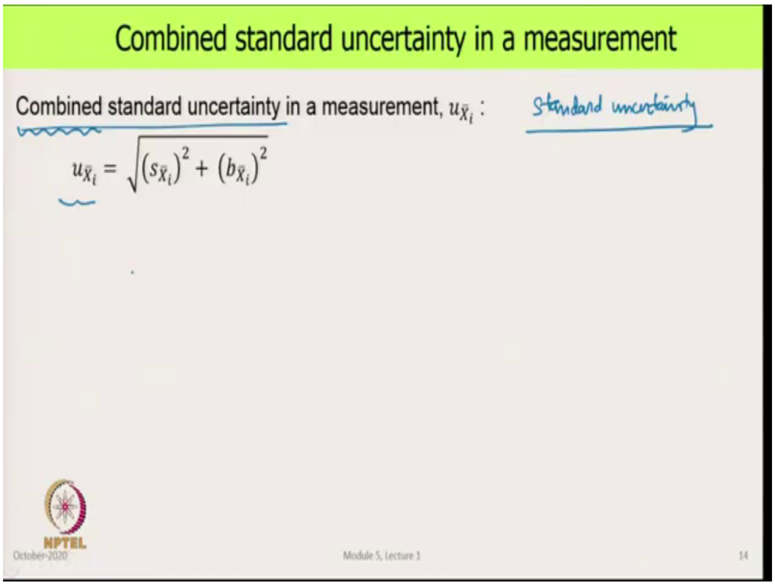

So, these are some of the issues that come up with systematic errors. And then having got s X i bar and b X i bar the random and systematic standard errors we can now calculate the standard error in the measurement which we call the combined standard uncertainty in a measurement. Or in simply in many cases this is just referred to as standard uncertainty. So, what combined is often dropped, but this is not the same as uncertainty this is very specific it is standard uncertainty u X i bar.

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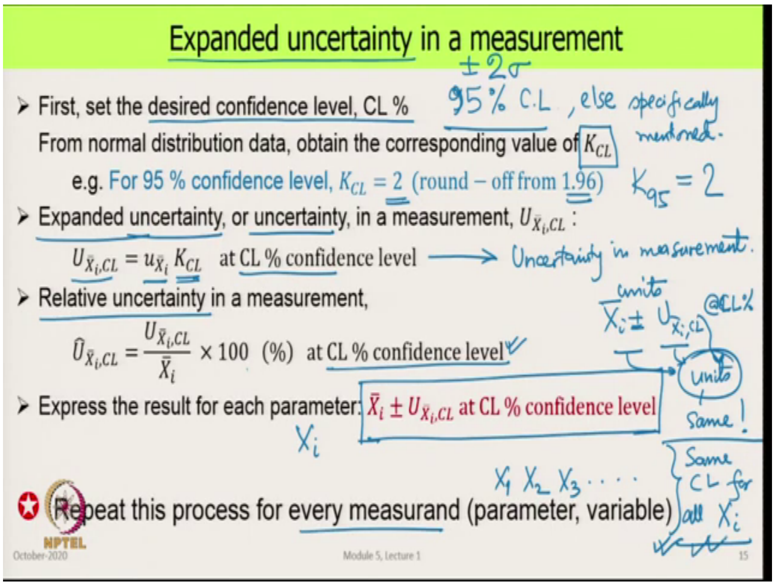

And this will now we will convert into the uncertainty by calling it the expanded uncertainty. And for that our first step is we have to decide what is the confidence level at which I want to report my uncertainty. So, as we have said earlier in most of engineering uncertainty analysis most other applications also 95 percent confidence level is the norm.

Anything else can be done is done, but has to be specifically mentioned. If there is no mention of what the confidence level is by default we assume this is 95 percent confidence level or in statistical terms this is plus minus 2 sigma. So, now what do? We do we decide our confidence level, go to the tables charts whatever else we want to go to and for that value we get the value of the constant K for that value of CL.

For instance for 95 percent confidence level we would get K 95. And if you go strictly by what the table says it would be 1.96, in uncertainty analysis everywhere we round it off and take it as 2. So, K 95 is 2 and then we take the standard uncertainty in the measurement multiply it by this factor, and report the expanded uncertainty in the measurement at a particular confidence level.

So, this is what it is when we say what is the uncertainty in a measurement. This has the same units as the measurand. So, the result that which is X i bar plus minus u X i bar at some confidence level. This always has the units and preferably both should have the same unit.

It is quite troublesome if you say that the pressure, I am reporting in kilo Pascal and the uncertainty in Pascal. It is best to report both at with the same units we can also report one more value and often you will see this in many applications is the relative uncertainty.

So, we got the uncertainty or the expanded uncertainty which is u X i bar CL. And now we call the relative uncertainty in a measurement which is the expanded uncertainty divided by the mean value of the parameter multiplied by 100. So, this is so much percentage at that particular confidence level.

Somewhere in the beginning we should have always specified what this is and later on in solving the problem or doing the analysis we should be clear that we are always referring to every measurement uncertainty at the same confidence level. So, this is our final answer that is what this whole exercise was all about. We will take up some examples so, it becomes clearer how we are applying it, but that is the process that we follow.

We have got this for parameter X i we repeat this for every measurement or parameter that we should do it first for X 1 then X 2, X 3 as many parameters are being measured in the experiment; you have to repeat it that many times. And the most important thing is that all these parameters, the uncertainty has to be reported at some confidence level which should be the same.

So, we should have the same CL for all X i, this is absolutely essential even more so in the next step when we have to calculate uncertainty in the result. If our objective of the experiment was only to get the measurement and report that even if we had different uncertainty levels that is ok, but we want to go to the next step and report the uncertainty in the result using these values then this is absolutely essential.

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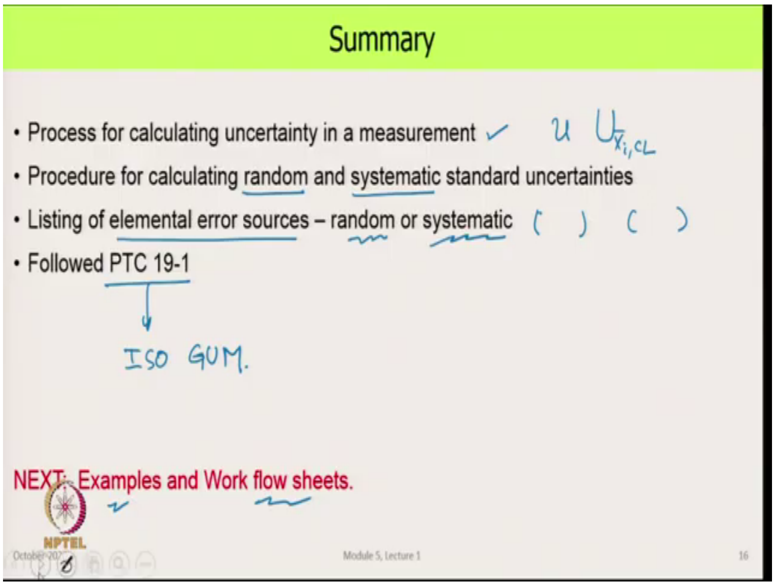

So, we have done what we set out to do and we conclude the lecture with a summary that we have looked at the process of calculating uncertainty in a measurement. We said that there are two steps to it, first we calculate the random standard uncertainty and separately the systematic standard uncertainty.

We saw how we could get those uncertainties from elemental error sources for both random and systematic error sources. And finally, we got the relation by which we can combine these for both of them random and systematic and get what we were looking for which is U X i bar comma CL.

And what we have done we have largely followed the procedures of PTC 19 dash 1, which for all practical purposes is identical to ISO GUM and about that we will see a little more in the next lecture. So, we will then also look at examples and finally, workflow sheets as to how to go about performing uncertainty analysis in a measurement in a step by step procedures.

On that count we conclude this lecture, which is on the basic procedure for calculating uncertainty in a measurement. We will continue on some more topics in the next lecture.

Thank you.