Introduction to Uncertainty Analysis and Experimentation
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Module - 04
Methodology for Uncertainty Analysis
Lecture - 10
Uncertainty Analysis Processes

Welcome to this lecture, in the course Introduction to Uncertainty Analysis and Experimentation. We are now at module 4, where we look at the Methodology for Uncertainty Analysis. In this lecture, we will look at the various processes that will take us through the Uncertainty Analysis Procedure. Our broad approach is like this. Our objective is to establish uncertainty in every measurement.

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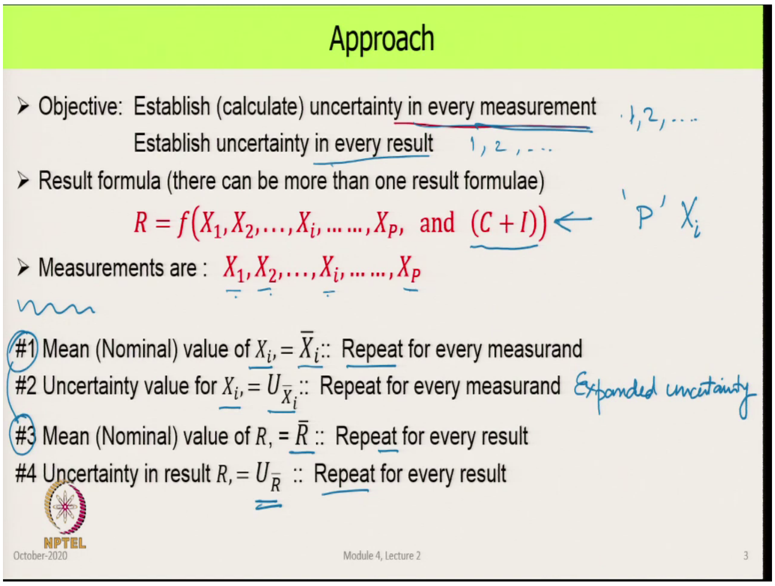

And then, with that information, we want to establish uncertainty in every result. There can be one or more measurements in an experiment and there could be one or more results in an experiment. A typical result, the formula that we shall denote is here R is a function of X 1, X 2, X P.

So, these are P number of parameters and a typical one, we have been denoting as X subscript i and along with these parameters, there are a set of constants; universal constants, physical constants, numbers whose value has no uncertainty associated with them and, the data that we have to work with is measurements of every parameter.

So, broadly what we will do? We first calculate the mean value of every measurand or the parameter X i bar, then we can calculate the uncertainty for each of the measurands which is capital U X i bar. This is also the expanded uncertainty as we have seen earlier. Then, we will calculate the mean or nominal value of the result which is R bar and then, uncertainty in the result or the expanded uncertainty in the result U R bar.

So, this we repeat for every measurand, every parameter, every variable in the experiment which is these X i's and this we will repeat for every result. If there is only one result, its only one; if there are more than one results, we have to do it as many times. So, in a typical experiments that we have gone through in school and colleges, we have usually just done step 1 and then, step 3 and then, finished off; steps 2 and 4 is the new addition that takes us through the complete story of an experiment.

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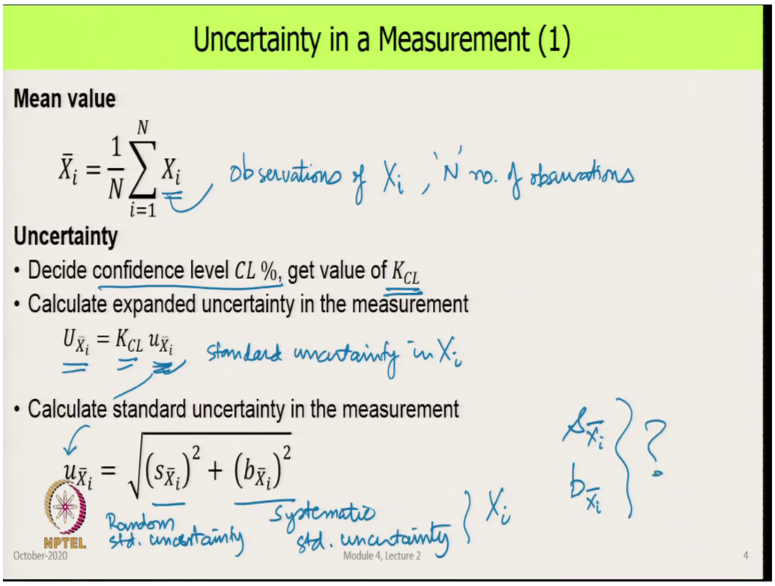

How do we calculate the mean value? This is nothing but the arithmetic mean of all the observation. So, this X i is all observations of the variable X i and like this we have so many more observation for each variable. N is the number of observation. For calculating the uncertainty, we have to beforehand decide at what confidence level, we want to tell the uncertainty and from tables, we get the value of the constant K CL. Then, we go backwards and we say that the expanded uncertainty in each measurand is K CL times the standard uncertainty in X i.

So, now our objective is how do I calculate U X i. So, for that, we come here, which is the square root of the sum of two parameters. These are the random standard uncertainty and this is the systematic standard uncertainty; both of these are for X i. Our job now, how do I get the values of s X i bar and b X i bar that is a big challenge.

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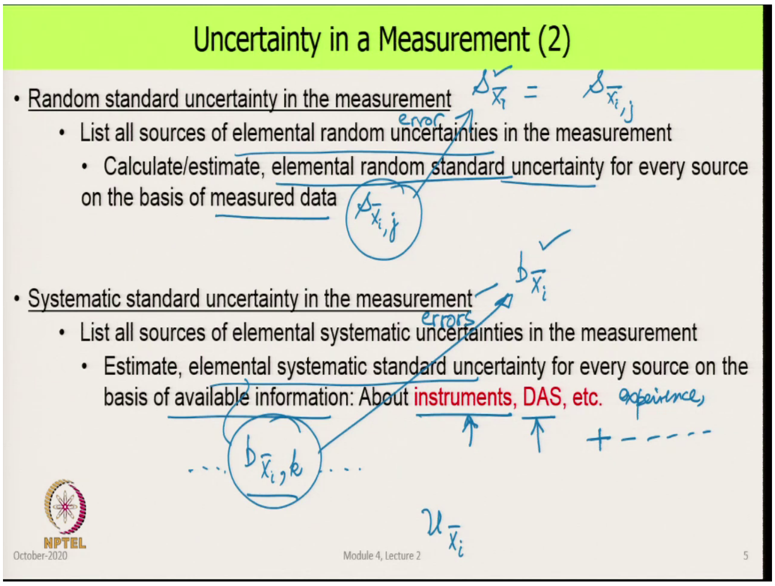

So, what we do is, for the random standard uncertainty in the measurement which was your s X i bar, we list all sources of elemental random uncertainties, elemental sources of uncertainties or we can even call sources of error in the measurement and this as we saw is the decision of the experimenter. And then, we calculate from that data which will be measured data.

We calculate the value of each elemental standard uncertainty. So, s X i is the function of s X i bar for j; j is the jth source of elemental random uncertainty. So, with this calculation, this is what we will get X bar comma j and then, we will see using a formula, from this, we can calculate this.

The same thing we do for the systematic standard uncertainty in the measurement. We list all sources of elemental systematic uncertainties or elemental systematic errors. Estimate the elemental systematic standard uncertainty for every source and this is on the basis of available information and this will give us b X i comma k; k denotes which one of the elemental systematic standard uncertainties of the measurement, we are looking at. And from all of these, we get this thing that we wanted, which is b X i bar. This comes out.

So, our objective now ok. So, where does this come from? This is not coming from measured data, it is coming from information about the process, our own experience, information about instruments, about data acquisition systems, our experience and anything else that we have a reason to believe in.

So, in an another lecture, we will look at these things and say what are the reasons, why these are contribute to systematic uncertainty. But the point is that if we can get all these done, then this can give us this; this can give us this and then, from the previous slide formula, we can get u X i bar, this standard uncertainty in the measurement.

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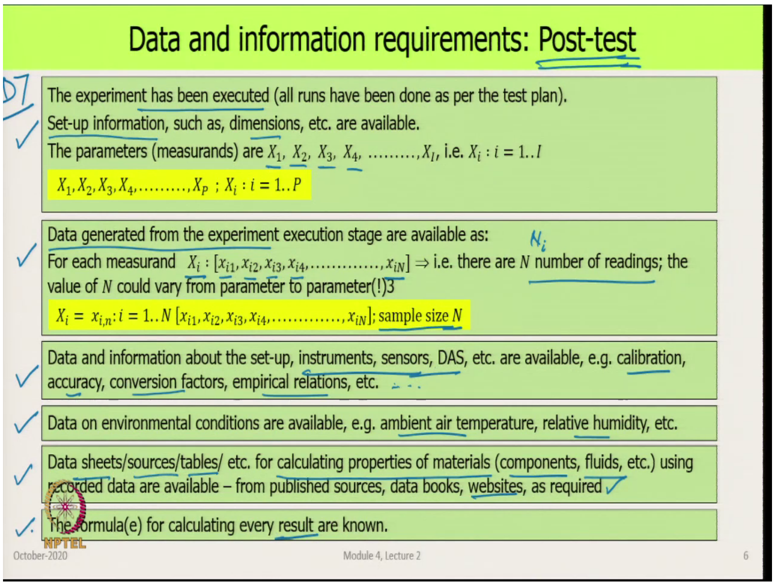

So, here is a presentation of all the data that we require to do uncertainty analysis and for the most general case, I have listed that we are in the situation, where we have done the experiments, executed it.

And it is the data analysis stage, where we are now going to calculate the uncertainties, we are at that stage D 7 from that slide and we say let me know what is all data I have, on the basis of which everything I have to calculate. So, experiment has been executed. So, we are doing the posttest analysis. We have all the information about the set up.

For example, its dimensions and other things and we have all the measurements for each one of these measurands. So, each one was mentioned measured many times and that we said that we have individual data in that. So, the data generated from the experiment itself at the execution stage.

This is for each measurand X i, we have these readings X i 1, X i 2, X i 3, X i 4, X i N; N being the number of readings that have been taken for that particular measurand or we could differentiate it and say different readings may have different number of readings taken. So, this is N sub i.

So, this is what we have. So, X i is x i n, where i is from 1 is to N. So, we have this set which is the sample; sample of measurement of size N. So, this is where we are now saying that this is the sample which is the same as what we learnt earlier in statistics. So, this is the data, we got from running the experiment. This is the data about the setup of everything that was there in it. Then, we have information about the set up and instruments, sensors and data acquisition system.

So, for each one of the instruments, we will have to collect certain information's like calibration, accuracy, conversion factors, empirical relations, all of that and that is another input for our uncertainty analysis. If need be, we also need to have data on environmental conditions which we have measured in the experiment.

So, (Refer Time: 11:13) the air and temperature, relative humidity and any other parameters. Then, we also have information such as data sheets, sources or tables from books, hand books or websites, where we can take our measured parameters and on the basis of that, we can get calculates being calculate the properties of a material.

For example, if we measure the temperature, we can then go and measure from the tables, we can get the viscosity, its density all of that. So, for solids, liquids, gases and for all other types of systems, we whatever data sheets we require, whatever standard property data we require that is over here.

And finally, we know right from the beginning, when we started planning for the experiment, we need we have the formula for each result that we have to calculate. The result formulae are known. So, this is all the information that is there with us and we now proceed with this, to do to the calculations.

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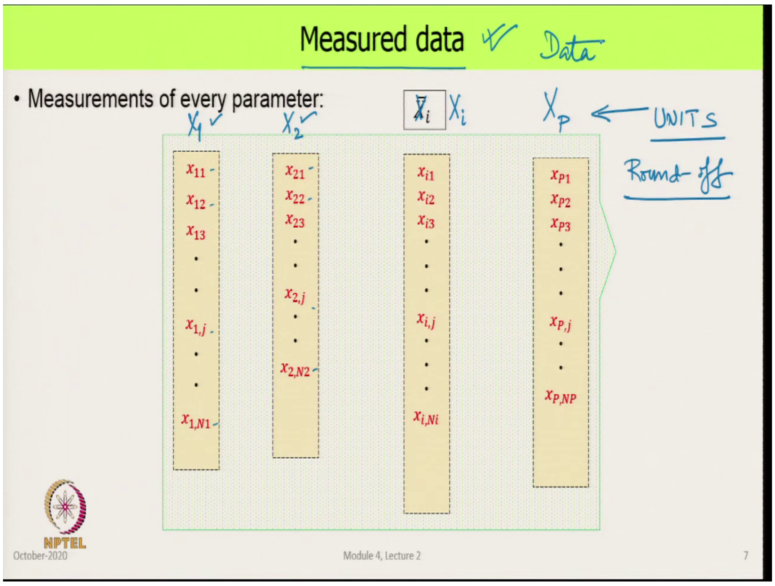

So, to represent the data in a pictorial way, this slide shows what is all the measured data that we have collected in the experiment. So, in this first column, it is x 1 1, x 1 2, x 1 j, x 1 N1. So, for X 1 parameter may be a pressure somewhere or a flow rate, we took so many readings under the same conditions and this is one set of data that we have and these are the measured values for X 1. Similarly, for every other parameter in the experiment, we took that number of measurements. They may be same, they may not be the same.

For the ith parameter, so this should be only X i. We have x i 1, x i 2, x 1 j, x 1 N i and if there are P number of parameters in the experiment, this is X P and these are all the readings that we have. So, each one of these (Refer Time: 13:44) would represent a column in our data file and we should be able to know that this column pertains to this variable, this is for this variable and in every case, we have to be clear and careful about what are the units in which we have got this data.

The other thing also to be careful about is that, you have the right number of significant places. So, we express the result by rounding off to the value which is realistic about what the instrument was capable of doing. So, this is the data that we have obtained. This is what we call the measured data or we just call it data. This is one set of information.

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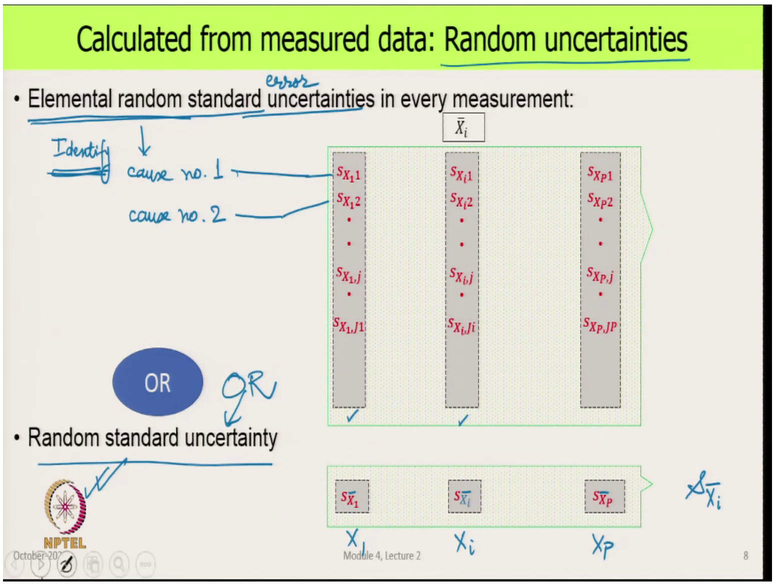

The second set of information from that basis, we now calculate the random uncertainties in every measurand. So, for that, we identify various elemental random sources of error and if we have data on every one of them which is listed here, from that we can calculate s X 1.

This is elemental random uncertainty; elemental random standard uncertainty due to re-cause number 1. This is elemental random standard uncertainty due to cause number 2. These causes we have to identify separately. If there is only one thing one set of data that we have, then the number of causes is only one; we do not need to do this that many times.

So, in every parameter we estimate every element elemental error source that contributes to the random error. Alternately, if we do not have information about every elemental random error source, then we can do one more thing and this is simple and direct. We calculate the random standard uncertainty based on the data that we have collected, which I showed on the earlier slide and from there, we can calculate S X 1 bar, S X 2 bar S X i bar, S X P bar. So, that was our data set, we calculate its standard error and that is the answer.

So, in each measurand X 1, X i, X P, we have calculated the random standard uncertainty or the random standard error. So, this gets us to how the story, where we have calculated all the X i bars. In most of the problems that we will look at, we are unlikely to be able to identify all the causes of elemental random errors because that would require us also to get the data about each one of them, which we in normal experimentation, we do not have that. We have only one set of data coming out of an instrument under the same conditions. So, it will be just the random standard uncertainty which is based on the readings that what we will calculate.

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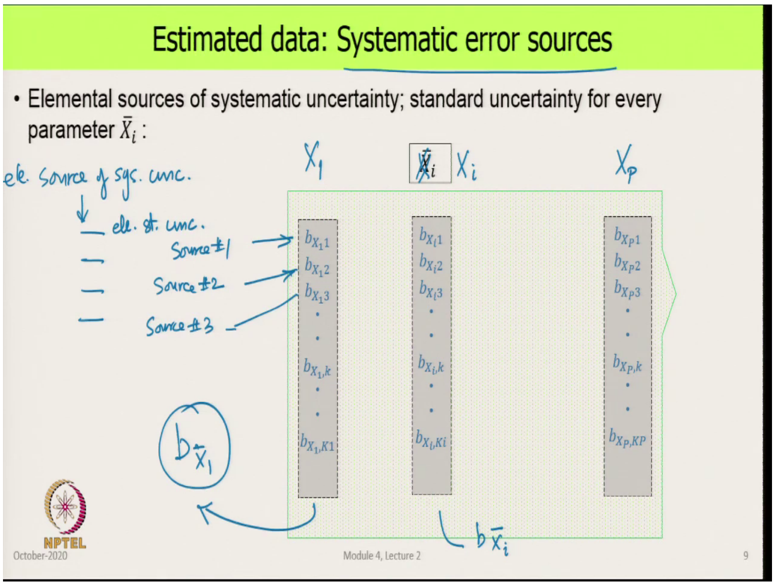

Now, we come to the other part, which is the systematic error sources and in this what we do is, for each measurand say in this column this is X 1, we have already listed every source elemental source of systematic uncertainty.

And now, we go ahead and with all the data that we do not have that much data, we have some information. For example, we have the accuracy, we have the linearity and on the basis of that, for each one of these sources, we estimate its elemental standard uncertainty; the elemental standard error and this we list out here as b X 1 from cause number 1.

So, this is error source number 1; this is from source number 2 in X 1; this is from source number 3 again in X 1 and so, we write these values. And from this, all this information we can use and get b X 1 bar and from this, we will get b X i bar. So, this is for the ith variable. This is for the last variable X P. So, we now have this. From the earlier slide, we had s X i bar, we know the relation; we can get U X i bar and from knowing the confidence level, we can get the expanded uncertainty in the measurement, that completes our calculation. Now, what is the overall methodology for getting the uncertainty in the result?

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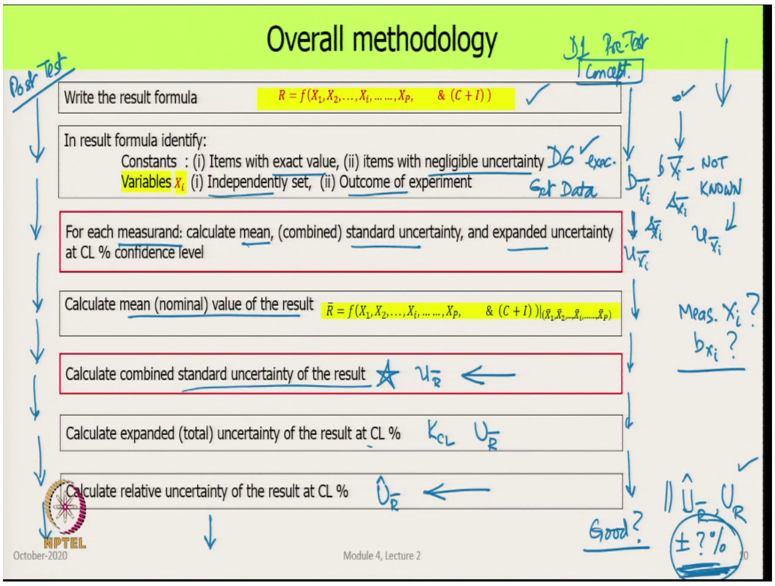

So, if we have done, the we are doing the post-test uncertainty analysis, this is what we do. We write the result formula. Identify which are the constants which are those with negligible uncertainty. Identify the independent and dependent variables.

We do the experiment and collect all the data. So, D 6 stage has been done; execution has been done, we got all the data. So, we wrote the formula; did the experiment and then, as we have seen in the earlier result, we calculate the mean, this standard uncertainty and the expanded uncertainty of every measurand.

Then, we will develop a method to calculate first the mean nominal value of the result which is straightforward. Then, we will develop the formula for calculating the combined standard uncertainty of the result which will be U R bar. Then, we get the value of K CL and from there, we get U R bar and then, relative value from U hat R bar. And then, we use this data for doing whatever conclusions or any other analysis, we want to do with this. So, this is broadly the posttest scenario.

Now, we look at the pretest scenario. Our concept design is done. So, we know the result formula. In the pretest, we are looking at it as a fresh case, where we have to make a setup from scratch.

So, we did brainstorming, came up with many concepts and we converged on one method and we wrote down the result formula for that, may be one or two or three or many formulas were there, we wrote those down. Within that we identified which are the exact values, which have values whose nth uncertainty is can be neglected, which are the independent variables, which are the outcome variables. So, this also gets done. So, this is also done.

Now, we do not execute the experiment. So, there is no data. So, we use all the information that we have to calculate only b X i's we can estimate s X i bar or at least for this level of analysis under analysis, we could even set it at 0. So, this becomes equal to u X i bar. If s X i bar were known from previous results or from experience or from a gut feeling, we could use that value also. And then, we can continue with all of this process and finally, we ask the question is this good enough? When I set up my experiment the mandate was that I want a result within plus minus something percent.

How does this compare with that? If this overall uncertainty is less than what we want in the first place in that initial starting stage, then we are in good shape. We could fine tune it by doing the exercise again and by saying, if I make a compromise in putting a lower quality instrument, I have save one cost, but do I get a penalty on uncertainty? If it works, fine; if it does not, abandon it. So, that is what we do in this part. There could also be a case where we have to go the other way around, where we are say that our final result uncertainty is specified.

So, U hat R bar or U R, these are specified and we are asked that this is the setup, what instrument should I use and how many times I should do the experiment so that I get a result within plus minus so much percent, which is what this is. So, now we have to go and do the backward calculation. We have the result formula.

So, this point we are there and we have some of the X i, b X i bars known; most X i bars are not known, some of these not known. Now, we have the issue. The result uncertainty is known, but one or two of the U X i bars, these are not known. So, now, what we do is we get the calculation done and say that if I do this strategy, what uncertainty will I get.

So, you say what is the measurement strategy for X i bar or X i and with that, what b X i can I expect; is that good enough to achieve this? If yes, we are in good shape; otherwise, we go and change this instrument and there could even be a case that if this was specified, we had a result formula and we had all the instruments that we have.

So, we knew all the b X i's. We could calculate beforehand and check whether this mandated uncertainty in the result is achievable or not achievable. So, there have been instances, where somebody said I want this thing to be tested with so much percentage uncertainty; do you have a set up and we could say look I have a set up, let me check what uncertainty I can get with my set up.

We do that check calculation in this way and then, compare it with what this is needed and we can even say to them that look yes, my uncertainty is less than yours; we could do it or we could say that this my uncertainty is more than this value that you want, my set up is not good enough for your purpose, I cannot do the experiment. So, that is an example of the use of uncertainty analysis, where even before taking any data, we can decide whether an experiment is doable or not doable.

So, this is the overall strategy we have and the way we will proceed further in the course now is that we will learn the methods of calculating the combined standard uncertainty of each result. This is the next big module. After that, we will look at another two modules on how to calculate the uncertainty of the result.

So, when we get into that there will be a lot of nitty gritty mathematics coming in; but we should keep in mind that this is the overall scheme of things under which we are working. So, we do not lose sight of what the big picture was.

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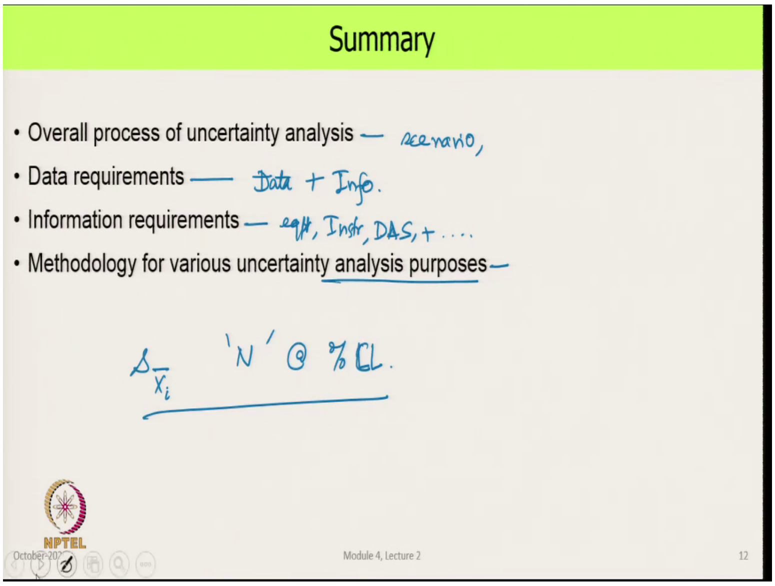

So, that what we do. After this, we will look at how to calculate uncertainty in a measurement and then, how to calculate uncertainty in a result; those are our next steps. So, that concludes this lecture on methodology, where we have learned what is the overall process of uncertainty analysis for different scenarios; pretest, posttest, checking goodness of test all of that. Then, we saw we got charts, then what is the data that we should get in an experiment. It is not just the measured data that we got, that is one of the parts (Refer Time: 28:56) much more information over and above that.

So, experimental data plus lot of information about instruments, electronics all of that. Information came about equipment, about bought out instruments, data acquisition system and what not and finally, for various purposes, we saw what is the scheme of things by which we will do the uncertainty analysis. So, this will tell us the overall picture of uncertainty analysis; but there is one thing which you will find in statistics or in an experimental methods course, which we will not touch here.

And that is that for an individual measurement, how many time should I make a measurement to get a certain uncertainty, which is to say that how does s X i bar depend on the number of readings that I take. This analysis and that too at a particular confidence level. So, this goes in to the experimental methods part of the course. We will not look at it over here. We will look at the overall picture and establish uncertainties in measurements and then, in results.

So, with that we conclude this lecture on Methodology for Uncertainty Analysis. After that, we will see how to get information about instruments and data acquisition system and then, we proceed to do uncertainty analysis calculations for a measurement. With that we conclude this lecture.

Thank you.