Solid Mechanics

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Lecture - 10

Mohr's Circle (contd.), Stress Invariants, Decomposition of the Stress Tensor

Hello Everyone! Welcome to Lecture 10! In this lecture, we will continue with Mohr's circle.

1 Mohr's circle Recap (start time: 00:25)

Let us have a quick recap of what we discussed about the Mohr's circle. Figure 1 shows the square on which we had drawn the components of the stress matrix. Here $\underline{e}_3$ direction is coming out of the plane. We wanted to find out σ and τ on a plane which is at an angle α in the counterclockwise direction from $\underline{e}_1$ plane (see Figure 1).

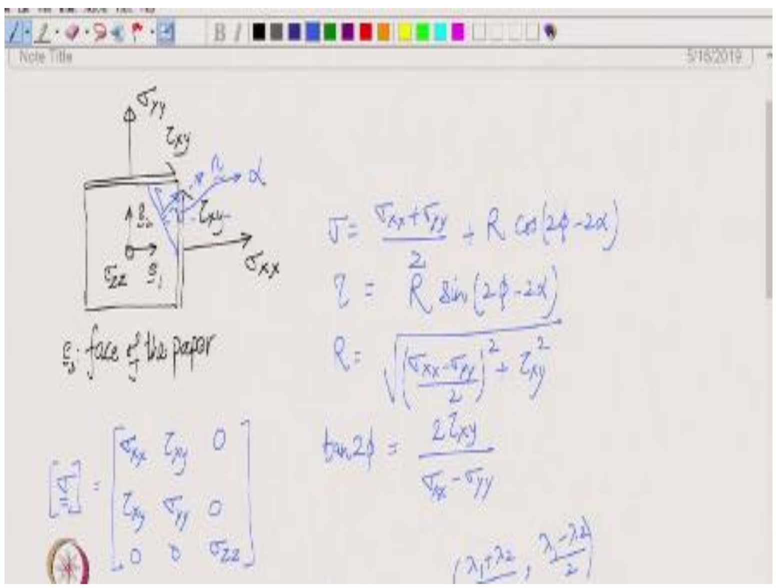

Figure 1: representing stress components on a square.

We had figured out that

$$
\begin{aligned}
\sigma &= \frac{\sigma_{xx}+\sigma_{yy}}{2} + R\, cos(2\phi - 2\alpha) \\
\tau &= R\, sin(2\phi - 2\alpha) \\
R &= \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \tau_{xy}^2} \\
tan(2\phi) &= \frac{2\tau_{xy}}{\sigma_{xx}-\sigma_{yy}}
\end{aligned} \tag{1}
$$

This set of equations is for the stress matrix which looks like the following:

$$[\underline{\sigma}] = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & 0 \\ \tau_{xy} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix} \tag{2}$$

We also saw that the locus of σ and τ for all α-planes is a circle which is called the Mohr's circle. Figure 2 shows this Mohr's circle. The steps involved in drawing this circle were shown in the previous lecture. We had also seen that we can extract the information of principal stress components and maximum shear component of traction (τ_{max}) from the Mohr's circle. We had verified the values obtained from the Mohr's circle with our mathematical derivations done earlier. The expressions obtained are:

$$\lambda_1 = \frac{\sigma_{xx} + \sigma_{yy}}{2} + R, \quad \lambda_2 = \frac{\sigma_{xx} + \sigma_{yy}}{2} - R \tag{3}$$

From the above equations, we had also got the expression of Radius R. Also, as $R = \tau_{max}$, we get:

$$R = \frac{\lambda_1 - \lambda_2}{2}, \quad \tau_{max} = \frac{\lambda_1 - \lambda_2}{2}$$

Also, σ on the plane having maximum shear (topmost and bottom-most points of the circle) was the σ corresponding to the center of the circle, i.e., $\dfrac{\sigma_{xx} + \sigma_{yy}}{2}$ or $\dfrac{\lambda_1 + \lambda_2}{2}$. So, the coordinates of the topmost point of the circle become $\left(\dfrac{\lambda_1 + \lambda_2}{2}, \dfrac{\lambda_1 - \lambda_2}{2}\right)$.

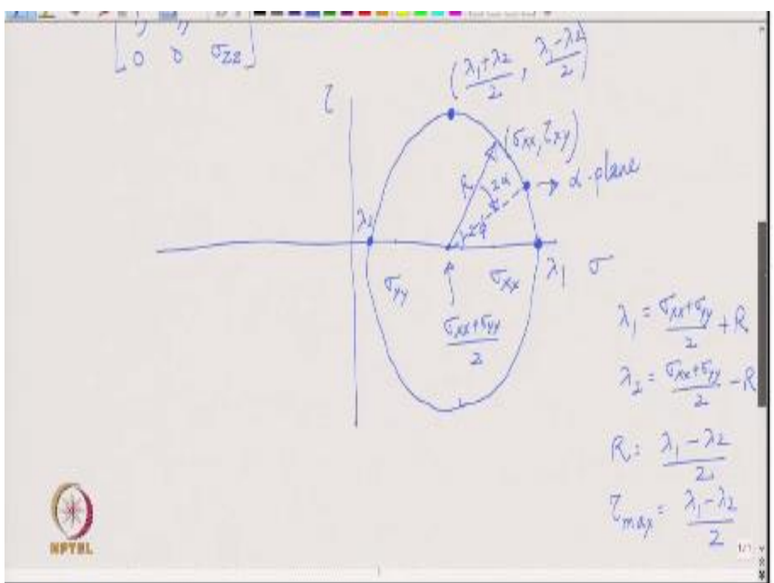

Figure 2: Mohr's circle

1.1 Planes of principal stresses in Mohr's circle (start time: 08:40)

We now want to find the angle of the principal planes with the x plane. We can see in Figure 2 that we need to go by an angle of (2φ) clockwise from the x plane in the Mohr's circle to reach the first principal plane, i.e. where λ_1 is acting. That means that in the original coordinate system (Figure 1), we need to go by an angle of φ in the anticlockwise direction from the x plane to get to the first principal plane. Similarly, for the second principal plane, we have to go $(2\varphi+180^\circ)$ clockwise in the Mohr's circle from the x plane. Thus, in the actual coordinate system we need to go by an angle of $(\varphi + 90^\circ)$ in the anticlockwise direction from the $\underline{e}_1$ (or x) plane. Earlier, when we were finding principal planes and principal stress components by algebraic techniques, we had to solve for the eigenvalues and eigenvectors of the stress matrix. The stress matrix being a 3 × 3 matrix, to get its eigenvalues, we have to find the roots of its characteristic equation which would be cubic. This mathematical process turns out to be complex and time consuming when compared to the graphical method, i.e., by using the Mohr's circle. But, Mohr's circle procedure has a limitation that one of the principal axes must be aligned with any of the coordinate axis. Only then the stress matrix will have the particular form as in equation (2) and only then will our above analysis work. If we have a general stress matrix, Mohr's circle does not work but the method to obtain eigenvalues from characteristic equation works for every case.

1.2 An Example (start time: 13:23)

Let us consider an example to understand it better. The stress matrix for a given coordinate system is given:

$$[\underline{\sigma}] = \begin{bmatrix} 4\sqrt{2} & 2\sqrt{2} & 0 \\ 2\sqrt{2} & 8\sqrt{2} & 0 \\ 0 & 0 & 10 \end{bmatrix} \tag{4}$$

We want to find the principal planes, the plane corresponding to maximum shear stress, values of principal stress components and the maximum shear stress. We can see from the stress matrix that $\underline{e}_3$ axis is aligned with the principal axis and hence we can use Mohr's circle analysis here. On $\sigma-\tau$ plane, we first plot σ_{xx} and σ_{yy}. The center of the circle will be at the mid point of these two, i.e., at $6\sqrt{2}$ on the σ axis. Then we plot the point corresponding to $\underline{e}_1$ plane which is $(4\sqrt{2}, 2\sqrt{2})$. Now we can join the center and the point corresponding to $\underline{e}_1$ plane to get radius. Using Pythagoras theorem:

$$R = \sqrt{((2\sqrt{2})^2 + (2\sqrt{2})^2)} = 4 \tag{5}$$

With this radius and the center known, we draw our Mohr's circle as shown in Figure 3. We can see that the principal stress components and τ_{max} are:

$$\begin{aligned} \lambda_{1,2} &= (\sigma \text{ at center}) \pm R = 6\sqrt{2} \pm 4 \\ \tau_{max} &= R = 4. \end{aligned} \tag{6}$$

We now want to find the principal plane having principal component as (6√2 + 4). In the Mohr's circle, we need to find the angle of the point corresponding to this plane with the point representing √ $\underline{e}_1$ plane. Both sides of the right angled triangle shown are 2√2, thus making this an isosceles triangle. Thus, angle inside the triangle is 45° while the required angle is (180−45)° = 135°. As we have to go 135° clockwise on the Mohr's circle to get to first principal plane, in the actual coordinate system, we have to go $\left(\frac{135}{2}\right)^{\circ}$ counterclockwise. Similarly, we can find the angle for the second principal plane. For the plane having maximum shear, we need to rotate by 45° clockwise on the Mohr's circle, so in the actual coordinate system, we need to go $\left(\frac{45}{2}\right)^{\circ}$ counterclockwise.

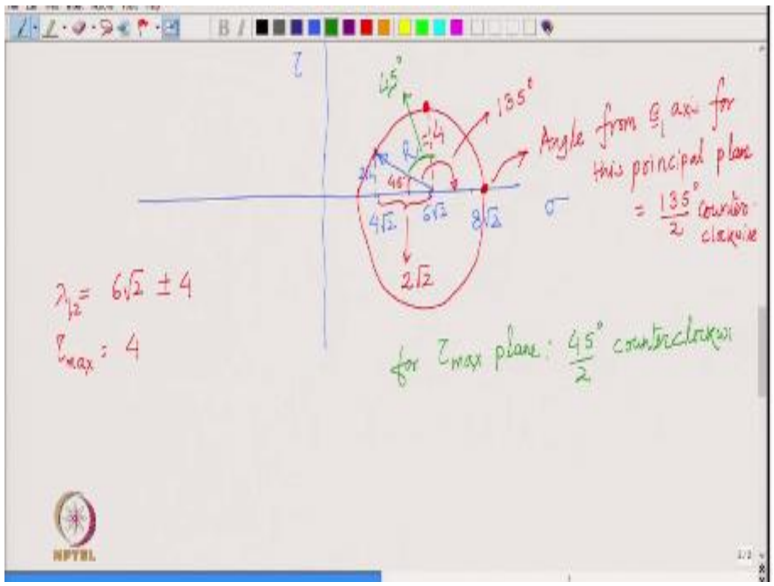

Figure 3: Mohr's circle for the given stress matrix in (4)

2 Mohr's stress plane (start time: 21:42)

We now move to the next topic: Mohr's stress plane. Suppose that the stress tensor at a point is such that the three principal stress components are λ_1, λ_2, λ_3 in decreasing order. The stress matrix corresponding to the coordinate system formed by principal directions is diagonal and given by

$$[\underline{\sigma}] = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \tag{7}$$

With this stress matrix, suppose we take arbitrary planes and plot ($\sigma - \tau$) on those planes: we are not confining to planes whose normal is perpendicular to one of the principal axes. However, first consider the planes whose normal is perpendicular to $\underline{e}_3$ axis. If we start plotting ($\sigma - \tau$) on such planes, we will get the Mohr's circle bounded by λ_1 and λ_2 as shown in Figure 4. Similarly, we can draw the Mohr's circle corresponding to planes whose normals are perpendicular to first principal axis: this circle is bounded by λ_2 and λ_3. Then we take the planes whose normals are perpendicular to second principal axis. The Mohr's circle corresponding to such planes is bounded by λ_1 and λ_3. This will be the biggest of the three circles.

These three circles correspond to very specific normal directions, i.e., they have one of their components as zero. These directions are also shown in Figure 4, e.g., The circle passing through λ_1 and λ_3 has second component of normal vector as zero. If we plot (σ,τ) for all planes with arbitrary normal directions, we will get the region shown as the shaded region in Figure 4. The region within the two inner circles is not realized by any plane. This $\sigma - \tau$ plot is called the Mohr's stress plane.

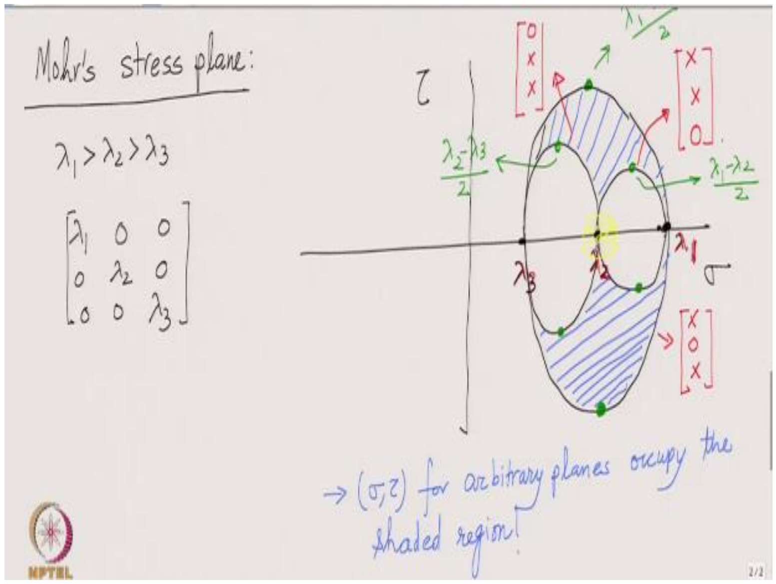

Figure 4: Mohr's stress plane with normals of the planes corresponding to the three Mohr's circles also shown

Recall that we had derived the principal planes by maximizing/minimizing the normal component of traction (σ). In the process of maximization/minimization, we had just set the first derivative to zero and had not checked the second derivative to identify whether it's a minima or maxima. Hence, they could be a maximum, a minimum or even a saddle point. We have in Figure 4, a graph where (σ,τ) is plotted for all the planes. From this graph, we can comment on which principal stress components are points of maxima and which of them are points of minima. λ_1 is a global maximum because the normal component of traction (σ) on this plane is the highest among all the planes. Similarly, λ_3 is the global minimum. But, λ_2 is neither a global maximum nor a global minimum. To find if it is a point of local maximum or minimum, we make a small circle around λ_2 as shown in yellow in Figure 5. However, small the size of this yellow circle be, it contains points both with higher and lower σ compared to λ_2. Thus, λ_2 is neither a local maximum nor a local minimum. It is a saddle point. Similarly, we can comment on the nature of τ_{max}. We have three τ_{max}, one for each Mohr's circle with their magnitudes as $\frac{\lambda_1 - \lambda_2}{2}$, $\frac{\lambda_1 - \lambda_3}{2}$ and $\frac{\lambda_2 - \lambda_3}{2}$, shown by green dots in Figure 5. We can immediately see $\frac{\lambda_1 - \lambda_3}{2}$ that is a global maximum as no other plane has τ greater than this. The global minimum value of $|\tau|$ is zero, so the other two points cannot be global minima. If we visualize a small neighbourhood area around the other two points shown by yellow circles in Figure 5, we can conclude that they are neither local maxima or minima. They are again saddle points.

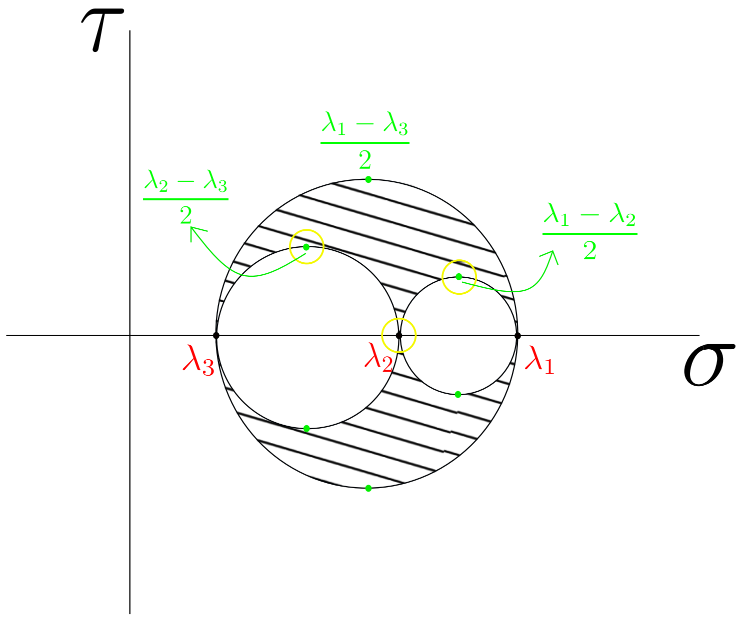

Figure 5: Mohr's stress plane with τ_{max} plotted for the three Mohr's circles. Yellow circles are drawn for the analysis of local region around corresponding points.

2.1 Special Case I (start time: 34:46)

Consider the special case when two eigenvalues of the stress matrix become same. Let's say $\lambda_2 = \lambda_3$. So, we have $\lambda_1 > \lambda_2 = \lambda_3$. In the Mohr's stress plane, one of the circles shrink to a point as shown in Figure 6. We can visualize this by thinking of λ_3 going towards λ_2. So, we have just one circle corresponding to λ_1 and λ_2 and the stress plane reduces to a circle. Let us also visualize what happens to the shaded region from the general case: when one of the circles becomes smaller and smaller, the shaded region also vanishes.

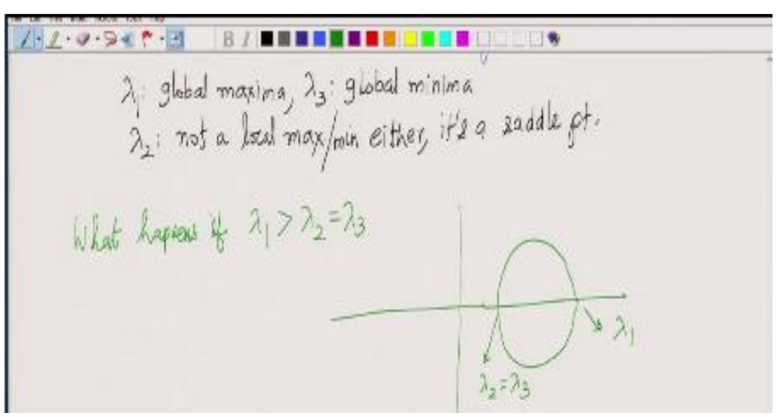

Figure 6: Special case of Mohr's stress plane when $\lambda_1 > \lambda_2 = \lambda_3$

2.2 Special Case II (start time: 36:44)

We can even have a situation where $\lambda_1 = \lambda_2 = \lambda_3$, i.e. all the principal stress components are the equal. In this case, the whole region shrinks to a point as shown in Figure 7. So, for any arbitrary plane, we can only have $\sigma = \lambda_1$ and $\tau = 0$. In one of the past lectures, we had discussed the state of stress in a fluid in static equilibrium. In such fluids, all planes have only pressure acting on them and it acts in the normal direction. That was called hydrostatic state of stress ('static' because the fluid was assumed to be in static state/equilibrium). So, this case of $\lambda_1 = \lambda_2 = \lambda_3$ is also called hydrostatic state of stress. This ends our discussion about Mohr's circle. Let us discuss few more concepts about stress.

Figure 7: Special case of Mohr's stress plane when $\lambda_1 = \lambda_2 = \lambda_3$

3 Stress invariants (start time: 38:53)

After a body is deformed, stress tensor at a point gets fixed and does not change. But the corresponding stress matrix changes when the coordinate system is changed. We had also derived the transformation that takes the stress matrix from one coordinate system to another. It is given as

$$[\hat{\underline{\underline{\sigma}}}] = [\underline{\underline{R}}]^T\,[\underline{\underline{\sigma}}]\,[\underline{\underline{R}}] \tag{8}$$

Here, $[\underline{\underline{R}}]$ represents the rotation matrix that transforms the initial coordinate axes to the new ones while $[\hat{\underline{\underline{\sigma}}}]$ and $[\underline{\underline{\sigma}}]$ denote stress matrices in new and old coordinate systems respectively. The above relation implies that the components of stress matrix change with coordinate system but there are some quantities related to the stress matrix that do not change. For example, we learnt about principal stress components. They are obtained as the eigenvalues of the stress matrix. We will show now that principal planes and principal stress components are independent of the coordinate system. We have the following eigenvalue-eigenvector equation for the stress matrix:

$$[\underline{\underline{\sigma}}]\,[\underline{x}] = \lambda[\underline{x}] \Rightarrow \left[\underline{\underline{\sigma}} - \lambda\,\underline{\underline{I}}\right][\underline{x}] = [\underline{0}] \tag{9}$$

The determinant of the matrix in parentheses is then equated to zero to get eigenvalues, i.e.,

$$det\left[\underline{\underline{\sigma}} - \lambda\,\underline{\underline{I}}\right] = 0 \tag{10}$$

Setting the determinant to zero, we get a cubic equation in λ (also called characteristic equation) solving which we obtain all the eigenvalues. Let us obtain this equation for the transformed matrix. We thus have

$$\begin{aligned} det\left[\hat{\underline{\underline{\sigma}}} - \hat{\lambda}\,\underline{\underline{I}}\right] &= 0 \\ \Rightarrow det\left(\,[\underline{\underline{R}}]^T\,[\underline{\underline{\sigma}}]\,[\underline{\underline{R}}] - \hat{\lambda}\,[\underline{\underline{R}}]^T\,[\underline{\underline{R}}]\,\right) &= 0 \\ \Rightarrow det\left(\,[\underline{\underline{R}}]^T\left(\,[\underline{\underline{\sigma}}]\,\hat{\lambda}\,[\underline{\underline{I}}]\,\right)[\underline{\underline{R}}]\,\right) &= 0 \end{aligned} \tag{11}$$

As the determinant of product of matrices is the same as the product of the determinant of matrices. Also, rotation matrix being orthogonal, its determinant equals 1. Thus, we get:

$$\begin{aligned} \Rightarrow det\left(\,[\underline{\underline{R}}]^T\right)det\left(\left[\,[\underline{\underline{\sigma}}] - \hat{\lambda}\,[\underline{\underline{I}}]\,\right]\right)det\left(\,[\underline{\underline{R}}]\,\right) &= 0 \\ \Rightarrow det\left(\left[\,[\underline{\underline{\sigma}}] - \hat{\lambda}\,[\underline{\underline{I}}]\,\right]\right) &= 0 \end{aligned} \tag{12}$$

Thus, we get the same characteristic equation as in (11). Thus, we have proved that the eigenvalues and hence principal stress components are invariant. Indeed, that is also expected since principal stress components are also the properties of the stress tensor. If we write equation (10) in component form, we get

$$det\begin{bmatrix} \sigma_{11}-\lambda & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22}-\lambda & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33}-\lambda \end{bmatrix} = 0 \tag{13}$$

Upon further expanding it, we get

$$-\lambda^3 + \lambda^2(\sigma_{11}+\sigma_{22}+\sigma_{33}) - \lambda(\sigma_{22}\sigma_{33}+\sigma_{11}\sigma_{33}+\sigma_{11}\sigma_{22}-\sigma_{12}^2-\sigma_{13}^2-\sigma_{23}^2) + det(\underline{\underline{\sigma}}) = 0 \tag{14}$$

This is the characteristic equation (cubic in eigenvalue) in terms of matrix components. A cubic equation, in general, can also have imaginary roots. But here, we know that this equation corresponds to the symmetric stress matrix which always has 3 principal stress components. The characteristic equation in the hat coordinate system will be the following:

$$-\lambda^3 + \lambda^2(\hat{\sigma}_{11}+\hat{\sigma}_{22}+\hat{\sigma}_{33}) - \lambda(\hat{\sigma}_{22}\hat{\sigma}_{33}+\hat{\sigma}_{11}\hat{\sigma}_{33}+\hat{\sigma}_{11}\hat{\sigma}_{22}-\hat{\sigma}_{12}^2-\hat{\sigma}_{13}^2-\hat{\sigma}_{23}^2) + det(\hat{\underline{\underline{\sigma}}}) = 0 \tag{15}$$

However, as the three roots have to be the same, the coefficients of this equation must be the same in all coordinate system. Thus, the coefficients of the characteristic equation are also the invariants and are called I_1, I_2 and I_3 as shown below:

$$-\lambda^3 + \lambda^2 \underbrace{(\sigma_{11} + \sigma_{22} + \sigma_{33})}_{I_1} - \lambda \underbrace{(\sigma_{22}\sigma_{33} + \sigma_{11}\sigma_{33} + \sigma_{11}\sigma_{22} - \sigma_{12}^2 - \sigma_{13}^2 - \sigma_{23}^2)}_{I_2} + \underbrace{det(\underline{\sigma})}_{I_3} = 0 \tag{16}$$

The expression of these three coefficients become similar if they are obtained from the stress matrix in the coordinate system of principal directions, i.e.,

$$\begin{aligned} I_1 &= \sigma_{11} + \sigma_{22} + \sigma_{33} = \hat{\sigma}_{11} + \hat{\sigma}_{22} + \hat{\sigma}_{33} = \lambda_1 + \lambda_2 + \lambda_3 \\ I_2 &= \sigma_{22}\sigma_{33} + \sigma_{11}\sigma_{33} + \sigma_{11}\sigma_{22} - \sigma_{12}^2 - \sigma_{13}^2 - \sigma_{23}^2 \\ &= \hat{\sigma}_{22}\hat{\sigma}_{33} + \hat{\sigma}_{11}\hat{\sigma}_{33} + \hat{\sigma}_{11}\hat{\sigma}_{22} - \hat{\sigma}_{12}^2 - \hat{\sigma}_{13}^2 - \hat{\sigma}_{23}^2 \\ &= \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1, \\ I_3 &= det(\underline{\sigma}) = det(\underline{\hat{\sigma}}) = \lambda_1\lambda_2\lambda_3. \end{aligned} \tag{17}$$

It could happen because the matrix has no shear component in the coordinate system of principal directions. One can also note that I_1 is the sum of the roots, I_2 is the sum of the three combinations of pair products of roots and I_3 is the sum of the combination of all triplet products of roots.

4 Octahedral stress components (start time: 55:25)

To define octahedral stress components, we need to know what an octahedral plane is: they are the faces of an octahedron having 8 faces whose normals have the following form

$$[\underline{n}] = \begin{bmatrix} \frac{\pm 1}{\sqrt{3}} \\ \frac{\pm 1}{\sqrt{3}} \\ \frac{\pm 1}{\sqrt{3}} \end{bmatrix} \tag{18}$$

(see Figure 8). Let us consider the octahedron whose face normals are equally inclined from the three principal directions. Accordingly, the coordinate directions in Figure 8 become the principal stress directions. The normal and shear component of traction on these octahedral faces are called octahedral stress components. The normal component of traction on octahedral planes (denoted by σ_{oct}) will be

$$\begin{aligned} \sigma_{oct} &= \left([\underline{\sigma}]\,[\underline{n}] \right) \cdot [\underline{n}] \\ &= \left(\begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \frac{\pm 1}{\sqrt{3}} \\ \frac{\pm 1}{\sqrt{3}} \\ \frac{\pm 1}{\sqrt{3}} \end{bmatrix} \right) \cdot \begin{bmatrix} \frac{\pm 1}{\sqrt{3}} \\ \frac{\pm 1}{\sqrt{3}} \\ \frac{\pm 1}{\sqrt{3}} \end{bmatrix} \\ &= \frac{\lambda_1 + \lambda_2 + \lambda_3}{3} \\ &= \frac{I_1}{3} \end{aligned} \tag{19}$$

which also turns out to be an invariant. Hence, its value in terms of components of a general stress matrix becomes

$$\sigma_{oct} = \frac{\sigma_{11} + \sigma_{22} + \sigma_{33}}{3} \tag{20}$$

We should remember that octahedral planes are always defined relative to the principal coordinate system and not relative to any general ($\underline{e}_1$,$\underline{e}_2$,$\underline{e}_3$) coordinate system. Similarly, we can find τ_{oct} as

$$\begin{aligned}
\tau_{oct}^2 &= ||\underline{\underline{\sigma}}\,\underline{n}||^2 - \sigma_{oct}^2 \\
&= \left|\left| \begin{bmatrix} \frac{\pm\lambda_1}{\sqrt{3}} \\ \frac{\pm\lambda_2}{\sqrt{3}} \\ \frac{\pm\lambda_3}{\sqrt{3}} \end{bmatrix} \right|\right|^2 - \frac{(\lambda_1 + \lambda_2 + \lambda_3)^2}{9} \\
&= \frac{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}{3} - \frac{(\lambda_1 + \lambda_2 + \lambda_3)^2}{9} \\
&= \frac{2(\lambda_1 + \lambda_2 + \lambda_3)^2 - 6(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_1\lambda_3)}{9} \\
\Rightarrow \tau_{oct} &= \frac{\sqrt{2}}{3}\sqrt{\left[I_1^2 - 3I_2\right]}
\end{aligned} \tag{21}$$

which again turns out to be an invariant. Also, note that irrespective of what face out of the 8 octahedral planes we take, we have the same σ_{oct} and τ_{oct}. We will see their significance later on when we discuss various failure theories.

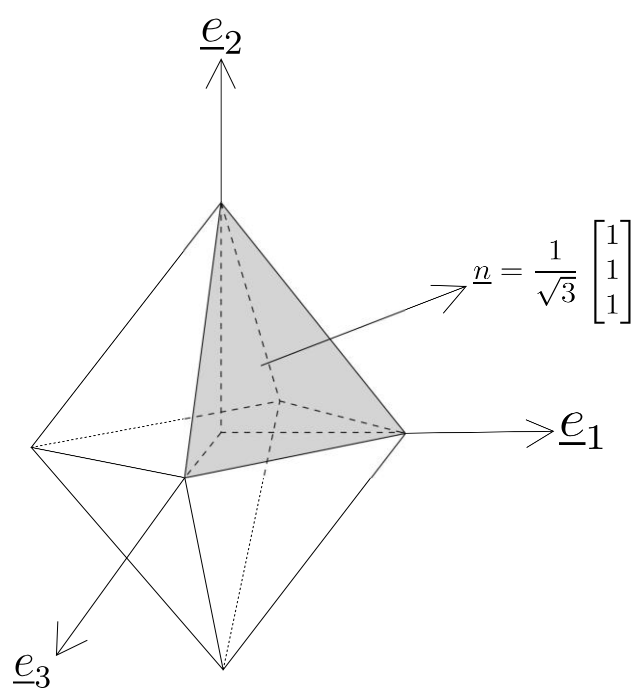

Figure 8: An octahedron along with normal to its shaded face shown

5 Hydrostatic and Deviatoric parts in stress tensor (start time: 1:05:48)

We can additively decompose the stress tensor into hydrostatic and deviatoric parts. Consider adding and subtracting $\frac{1}{3}I_1(\underline{\underline{\sigma}})$ $[\underline{\underline{I}}]$ to a general stress tensor $\underline{\underline{\sigma}}$, i.e.,

$$\underline{\underline{\sigma}} = \frac{1}{3}I_1(\underline{\underline{\sigma}})\underline{\underline{I}} + \left(\underline{\underline{\sigma}} - \frac{1}{3}I_1(\underline{\underline{\sigma}})\underline{\underline{I}}\right) \tag{22}$$

Let us denote the part within the parantheses as $\hat{\underline{\underline{\sigma}}}$. This is a special kind of decomposition because the first term is proportional to the identity tensor whereas the second term is such that its first invariant is 0 which can be proved as follows:

$$I_1\left(\hat{\underline{\underline{\sigma}}}\right) = I_1\left(\underline{\underline{\sigma}}\right) - I_1\left(\frac{1}{3}I_1(\underline{\underline{\sigma}})\underline{\underline{I}}\right) = I_1\left(\underline{\underline{\sigma}}\right) - \frac{1}{3}I_1(\underline{\underline{\sigma}})\ I_1(\underline{\underline{I}}) = 0 \tag{23}$$

Here, we used the fact that the first invariant of an identity tensor equals three. Due to this, $\hat{\underline{\underline{\sigma}}}$ is also called the deviatoric part of the stress tensor whereas the first term that is proportional to the identity tensor is called the hydrostatic part of the stress tensor. There is a physical reason for calling the second term the deviatoric part. There always exists a coordinate system such that the matrix form of the deviatoric part has all its diagonal entries zero, i.e., $\hat{\underline{\underline{\sigma}}}$ has the following matrix form:

$$\left[\hat{\underline{\underline{\sigma}}}\right] = \begin{bmatrix} 0 & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{bmatrix} \tag{24}$$

As diagonal elements of the stress matrix are zero; there will be no normal traction on all the three planes whose normals are aligned with the coordinate axes. These planes only have shear traction. If we visualize the cuboid corresponding to this coordinate system, the faces of the cuboid will be under pure shear which will just try to change/distort the shape of the cuboid keeping its volume fixed (see Figure 9). This is why we call it deviatoric part of the stress tensor. A stress matrix of such a kind as in equation (24) is also called the state of pure shear. On the other hand, the hydrostatic part has only got normal traction acting on the faces of the cuboid. This normal traction is same on all the faces and so, it just tries to change the size of the cuboid without distorting its shape.

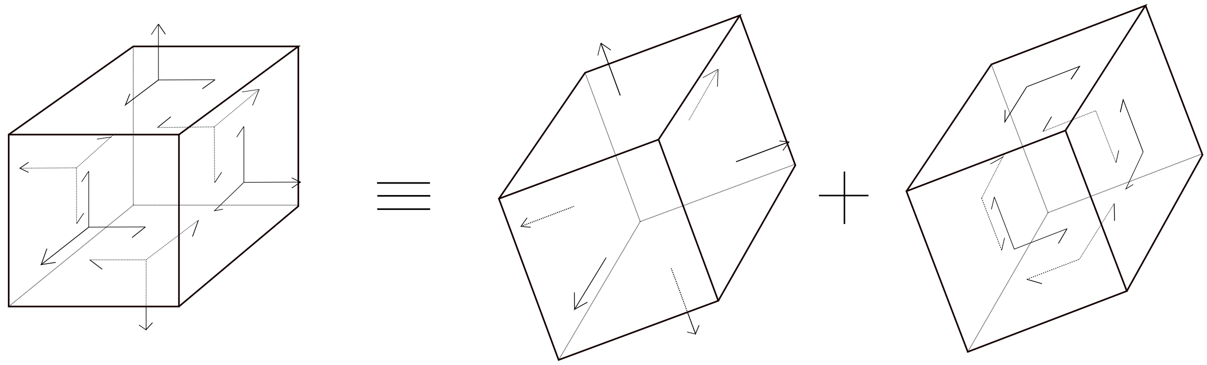

Figure 9: Equivalence of a stress matrix as the sum of hydrostatic and deviatoric parts: the leftmost cuboid represents stress components in an arbitrary coordinate system while the remaining two cuboids have their faces aligned along a special coordinate system in which the deviatoric part is in pure shear state