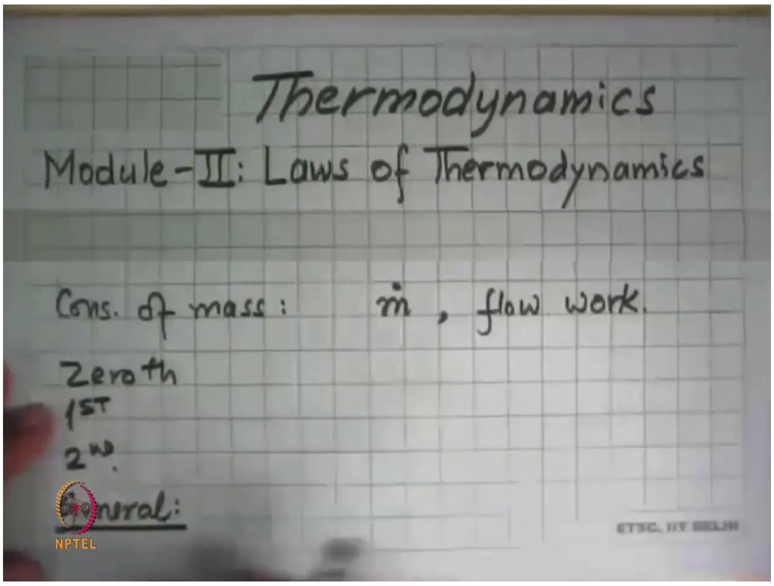
Engineering Thermodynamics
Prof. S. R. Kale
Department of Mechanical Engineering
Indian Institute of Technology, Delhi

Lecture - 11
Laws of Thermodynamics: Mass flow rate. Conservation of mass. Flow work.

This is the second module of our lecture from Thermodynamics.

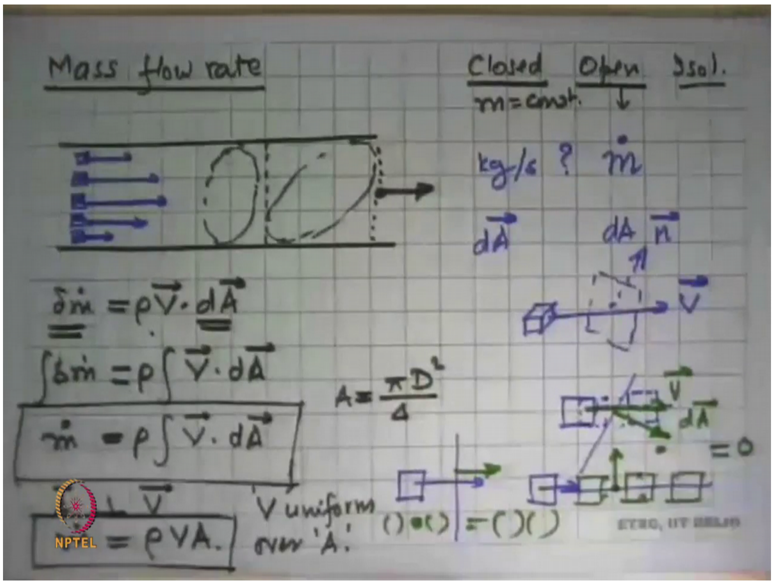
(Refer Slide Time: 00:21)



In the first module we looked at several concepts and definitions, and the ways of using them. We did that because that the stepping stone to what we do next which is to look at the laws of thermodynamics. So, before we look at the laws of thermodynamics itself what we will do is we look at conservation of mass which is a fundamental law that every system has to follow. And in deep, so we will define something called mass flow rate and flow work. With then move on to 0th law of thermodynamics, and then the first law and then second law.

So, that the plan for today, tomorrow and day after. We shall look at laws the way they are in the most general sense, which means that these are applicable to all systems, all types of applications and are very very general. So, it is this that one always has to remember applications may add on, applications may change issues may change but the laws do not change they remain the same. So, we begin with conservation of mass and we first look at the concept of mass flow rate.

(Refer Slide Time: 02:03)



We have seen that there are two types of systems, closed open and we can put the third one isolated. And in closed systems we said that the mass of the system is constant. So, even though the system may change its shape and size, the same molecules are always there in the system. It is the open system where mass can cross the system boundary and that is where mass flow rate comes in. So, very generally if it say that there is a duct or a pipe in which there is a fluid that is flowing and need not be that every element has the same velocity. These are all fluid elements, and like that we can build a complete thing across the space of this. So, this is the two-dimensional area and we are looking at fluid elements that cross that area.

And we ask the question how can I get across this area, what is the flow rate that is how many kg per second of the mass of this substance is flowing to. And for that we will use the symbol m with a dot on top. So, this symbol will keep coming on later on also when we look at the laws of thermodynamics.

Now, we will see how to connected with this. The basic concept in that you take any surface and I am showing an elemental surface and there is an elemental fluid element which is crossing this, then how much mass cross this surface that is the question. So, we have two issues here, this is the velocity vector of the fluid and the surface is there and there would be a normal to that element. And now, wherever we tick this surface the total flow rate is not affected because the fluid does not care which where we choose our surface. So, in this problem if the surface where like this or like this or like this, the mass flow rate to this pipe does not change; only our mathematics changes.

So, what is happening here is that we say that this normal to this elemental area this is dA times V normal. So, that is a vector now, so this we put as dA star. Now, if you look at the two-dimensional counter part of this is the surface this is the element and it is crossing this. So, at the some point it begins to cross then somewhere here it is crossing it and after sometime it has completely cross this. So, after this has crossed we say how much mass went through this. Now, before we write the formula let us look at another case that this surface could be perpendicular to this slope or this surface is here and this is going this way.

If you look at this second case we ask how much mass is crossing the surface it must necessarily be that the mass goes from this side to this side or this side to this side. But if the fluid element is just going across it like this then there is no mass flow rate across this surface. So, if what this happened because the normal area vector here is here and this velocity vector is here and we get the dot product of these two which is 0. So, basic idea is that if all such surface is such that the velocity vector is along the surface or always the area vector is normal to the velocity vector then there is no mass flow rate taking place across the (Refer Time: 06:58).

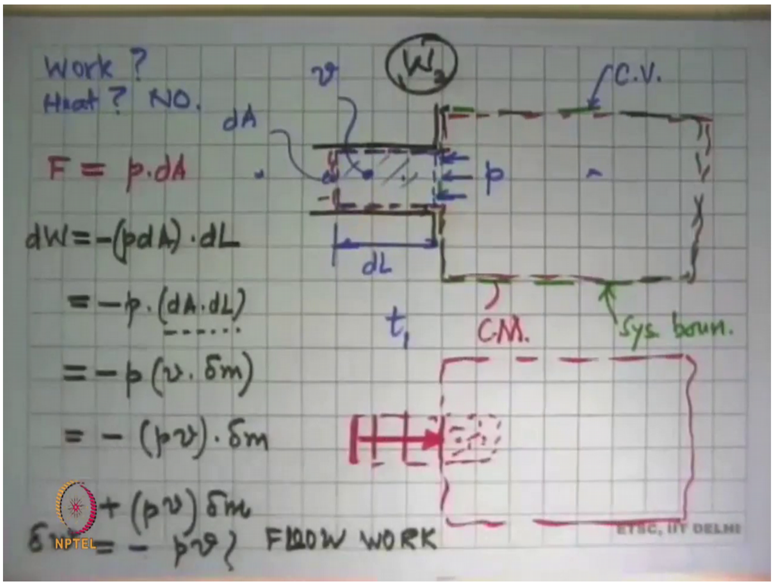
In this case the normal vector is here, velocity vector is here, both are co-linear both are in the same direction. So, the mass flow rate in this case if you take the dot product again will be just equal to the MAC product of the magnitudes of these two. And in between here it will be a mixture this is the dA vector, this is the V vector and we get the dot product of that. So, the fluid did the same thing. It did not care where the surface was.

So, if you do that and just we are doing for one little element over here, then we said that that delta m dot this is equal to V multiplied by dA and then we say that this is what is happening over a small element. I want to know what is happening across the entire cross section of the pipe which say in this case we could have taken a cut it could have been like this or this could have been like that or it could have been any other shape. There is no restriction on the type of surface that we are taking here. And then we say we do this integration over this entire surface so that the surface is intersecting the boundary of this (Refer Time: 08:31) or the pipe.

So, we then said that this is integrated. And we get this is integral of V dot dA and so m dot is integral over the surface V dot dA. So, that is our starting equation. And in case A is perpendicular to V then m dot this has to be density here this rho V into A, rho is the density, V is the velocity, A is the area and in doing this integration and getting one V here we made one assumption that V is uniform over the entire area A. In reality that may not be the case but that does not matter, at least in this course for the purpose of calculation we will say that area is uniform everywhere and when we say velocity is so many meters per second in duct or in a pipe, we will implicitly assume that this velocity is applicable over every point in this area. And that is what gives us the flexibility to write m dot as rho V A.

A now, is the area of this and it is not this area, but the area normal to V so then we have to look up a cross section of this which in the case of pipes A becomes pi into D square by 4, where D is the inside diameter of the pipe. (Refer Time: 10:49) inside which the duct it is the inside dimensions of the duct somewhere we get this area. So, this is the first definition that we looked at. Let us now look at the concept of what happens when a fluid element is pushed into a control volume.

(Refer Slide Time: 11:19)



So, we take a system and I denote the system here. This could be any shape does not matter, that is a system boundary. And it say that this system at some point is such that it is connected to a pipe or a duct, across whose face we have taken this hypothetical surface. And we say that in this case what about fluid is here would be here, and an element of this fluid at some time t 1 is just outside this system.

So, we have we have two systems, one is this big one which is our control volume and the second is this little element of fluid. And what we do for this is first we say that, the area of this is dA, this length dL, the pressure acting on this is p, and its specific volume is V. And what this could be in the real case? Would be an application, like say air been blown into a tank or this could even be like air been put into a tyre of a vehicle, air is going in. The system boundary is the inside space of the tyre in that case.

And we ask that if this fluid element were to cross the system boundary is there any work associated with this. So, the question is what is the work is there any heat, the answer is no. Because there is we are saying that the temperature difference is not there, and to push this element we do not need heat but we need to do work which is of course, it is pushing this element inside the control volume.

So, what is that force? In a (Refer Time: 14:15) way that one can say is that we are looking at this whole thing as a system, that now our system is what we are drawn outside plus this little element. If this whole thing is looked at as our system now we are fixed that mass. What is going to happen to this system if at every, this fluid element goes in this system boundary begins to disappear it comes here, then it comes here, then it comes here, and finally, it comes here. The mass of the system remain the same but that little fluid element pushed in which means that this system boundary was pushed in.

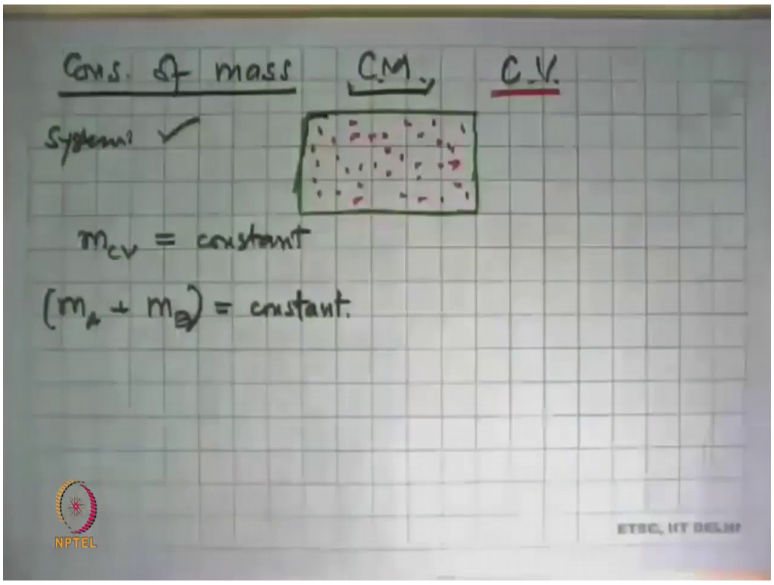
So, the final system will then look like this only. And whatever that the mass was it went somewhere over there. So, effectively what we did that this system boundary which was here got displaced until it came over there. So, this was the amount of displacement of the system boundary. And what is the force that cause this displacement? It was this pressure acting over this area, we said the pressure is uniform over the area. So, the force that push this was p times dA, and the displacement it did was dL and you go back and say what is our convention for work that we learnt in the previous module. We said that work is done on the system that is negative, work done by the system is positive.

So, with that we get that the work done in this case and we say this will be dW, elemental work done this is this force with a minus sign because it is work done on the system force, multiplied by the distance through which it went dL, we assuming that dL and a are normal are in the same direction with are normal to each other. If the area had been like this and it was been pushed like this the relation would have to modify itself. So, that is the work required to be done by the surroundings on the system to push an elementary mass into the control volume. And we rearrange this becomes minus t times dA into dL, this is the volume of the element and that we can write in terms of the specific volume. So, this is minus p into small V into the mass of this element delta m. So, this becomes minus pv delta m.

So, although this work does not come in that work symbol that we used in the first will use now W 1 2 type of a thing, there to accept that whenever there is a flow in this much work is associated with the flow. And similarly, if this fluid might element what will pushed out the work associated with that would have been plus pv times delta m. And the work for unit mass that would be dW by d m this would be delta small w this would be minus or plus as the case may be pv and this is called flow work. So, to get any fluid into a system there is work associated with that and to get a fluid out of the system there is some work associated with that also.

This is an important thing that we have to keep in mind and this particular type of a relation this p into v combination which is pressure into specific volume. Now, remember pressure and specific volume are both properties, so this will also be a property of the system. It will come back a little later when we look at the full statement of the first law. But there is an important thing that first law will be defined mass flow rate that is one thing (Refer Time: 19:10). Now, in the second thing we have just done is defined the case of flow work which is work associated with the flow.

(Refer Slide Time: 19:27)



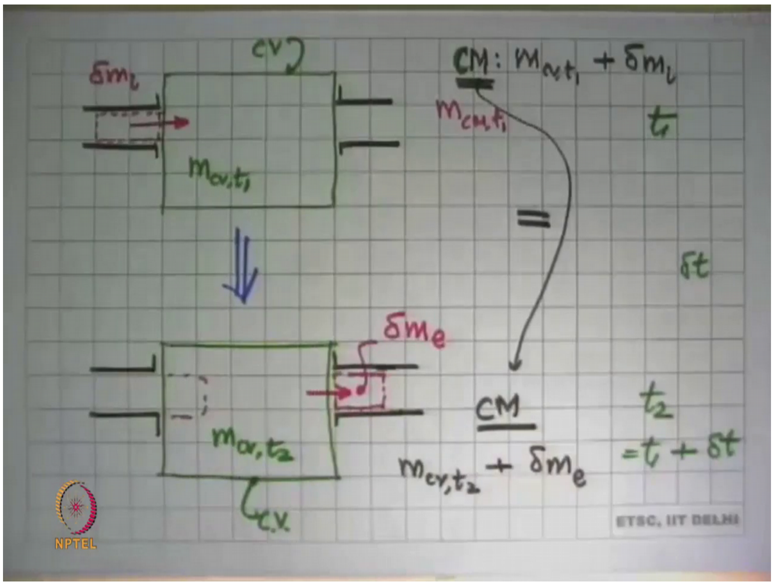
So, with that we do the next step which is right the conservation of mass, first for a control mass and then for a control volume there are two things we will do. Now, why conservation of mass? This is the most fundamental conservation equation that every system has to obey. There is no question of violating this it is always true and so this is the first condition that the system has to specify, before we can even talk about applying the first law of thermodynamics, the second law of thermodynamics or second law of motion or any other law. If this law is not satisfied if just put the problem at that point and make sure that we have conservation of mass without that there is no point talking about energy and interval, that is how important this is, ok.

So, the control mass a very definition was that we have a system boundary and a control mass is something that the system boundary is such that every molecule which is inside this system boundary always remains inside this system boundary. This system may change its shape, its size, that does not matter, but every molecule which was inside this has to be inside this. It does not have to be a single type of a molecule it could be a mixture of two different types of molecules, would be different phases of the substance, that is all ok.

So, once we say that every molecule inside the system has to be inside the system the conservation of mass becomes very simple that the mass in the control volume is constant. And this could be just a single substance or it could be there are two substances there it could be the individual sum of masses and then this has to be constant. There could be change of other properties, but the mass has to be constant that is the thing.

So, is nothing very unique or very tricky about the control mass issue. We all do that more happens to a control volume. And let us first see how we are going to frame the problem and then derive an equation. We will do what we did a minute back, but with the slight difference.

(Refer Slide Time: 22:27)



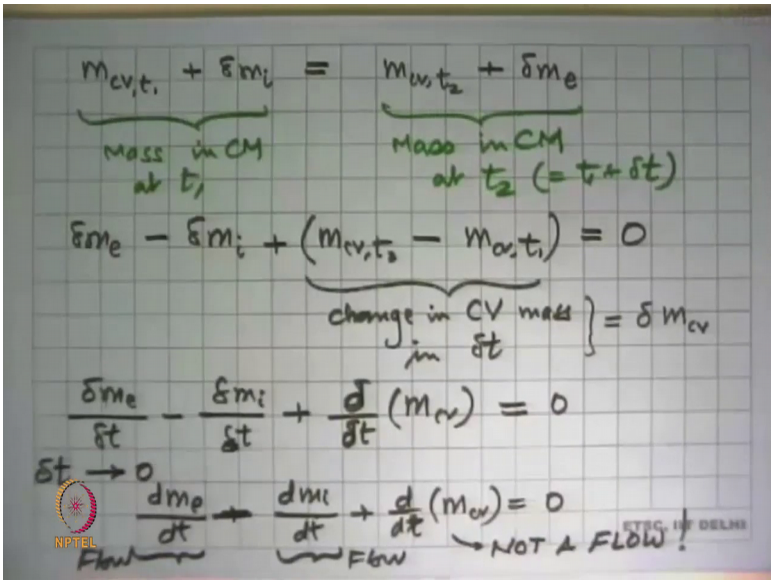
We say that here the system, this is the at time t 1 same system at time t 2 with a difference being a very small time delta t. So, t 2 is t 1 plus delta t, and it say this is a system boundary which is a control volume which is fixed in space and its shape and size does not change. So, this is our control volume, same control volume in both cases it does not change.

And now, we say that this has got a connection over there and connection over there, so same thing will happen here. And what we say is that at time t 1 we have this much mass in the control volume and this little element just adjacent to the control this system boundary. This is about to go in, so will say that the mass of this is delta m i it is wanting to go into the control volume. And now, we say that this mass plus this mass say the mass of the control volume which was at time t this is m cv at time t 1, we define a control mass which is m cv t 1 plus delta m i. So, mass in this control volume plus this element that was going to go inside we have take all of this together say that this is the total mass and this is a control mass that we will track.

Now, if we track this control mass what it would mean that after little while t, then this system has gone down over there this little elementary mass somewhere went and got inside this system. But they also give this flexibility that some other mass got out of the system. So, tracking the control mass and we said when I push this some little thing also came out over here, that what we are basically saying that the mass of this is delta m exiting delta m e. So, a control mass change its shape from being this, it became this the total mass here and the total mass here is till the same. And now, the mass at time t 1 control mass, so mass in the control mass at time t 1 was that much and now we say that (Refer Time: 25:44) time t 2 mass in the control volume this will become mass in the control volume at time t 2 plus what was this going out delta m e.

Now, this is the same control mass that means, this and this are equal. It is not necessary that m cv t 1 is equal to m cv t 2. This could be more than this or this could be less than, this both are ok. And now, we say that seems these two have to be equal, what you we put that as an equation and the terms m cv t 1 plus delta m I which is the mass at time t 1 is m cv t 2 plus delta m e.

(Refer Slide Time: 26:37)



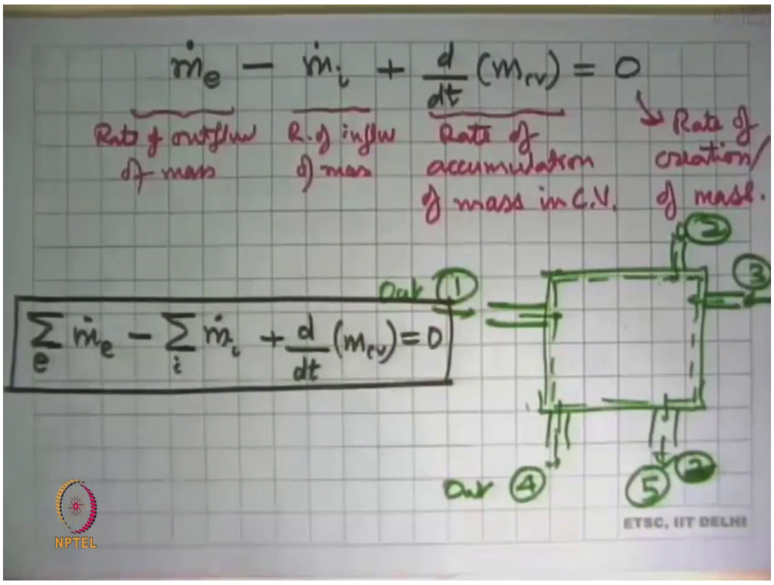
So, what this equation is telling us is that for that this is mass in the control mass at t 1 this is mass in the control mass at t 2 which is t 1 plus delta t.

Now, we do a small amount of arithmetic on this. We keep the delta m e term on one side this comes that becomes minus delta m i plus m cv t 2 minus m cv t 1. This is equal to 0. And the next step. So, this is the change in the mass in the control volume in time interval delta t.

So, what we do next is divide everything by delta t and we get that delta m e by delta t minus delta m i by delta t plus this whole thing become d of delta t m cv change in mass in delta time t this become designate as delta m cv. And we say look at the limit then delta t tends to 0 then this equation will become d m e by d t minus d m i by d t plus d d t m cv is equal to 0.

Now, we look back what we just derived this term and this term they represent mass flow rates going into the system and going out of the system. This is the rate at which the control volume is changing its mass. This is a flow, this is a flow, this is not a flow. And that tells us how to write the next step of this equation then it get m dot e minus m dot i plus d d t mass in the control volume is equal to 0.

(Refer Slide Time: 29:45)



So, what it is telling us is that rate of out flow of mass this is rate of inflow of mass, and this rate of accumulation of mass in control volume is equal to 0. And the 0 comes because you can think of this as rate of creation or destruction of mass which in classical thermodynamics is 0 that will write the 0 once here. And so if we generalize this equation, we just have to put a summation that if the system had more than one inflows and outflows. Then, we have a tank into which from one pipe there is water coming in, there is a second pipe from which there is an inflow there is a another inflow there and there is an outflow there and an outflow there and this is our control volume.

Then this would be inflow number 1, inflow number 2, inflow number 3, outflow number 1, outflow number 2. And to just make it unit numbers you just call it 1 2 3 4 5. And that is what we learnt in the first part of the course that when you get a problem first define both the system is, make the system boundary, identify all mass inflows and outflows, in this case we got all these things coming in, give the stake points these are the stake points that you define.

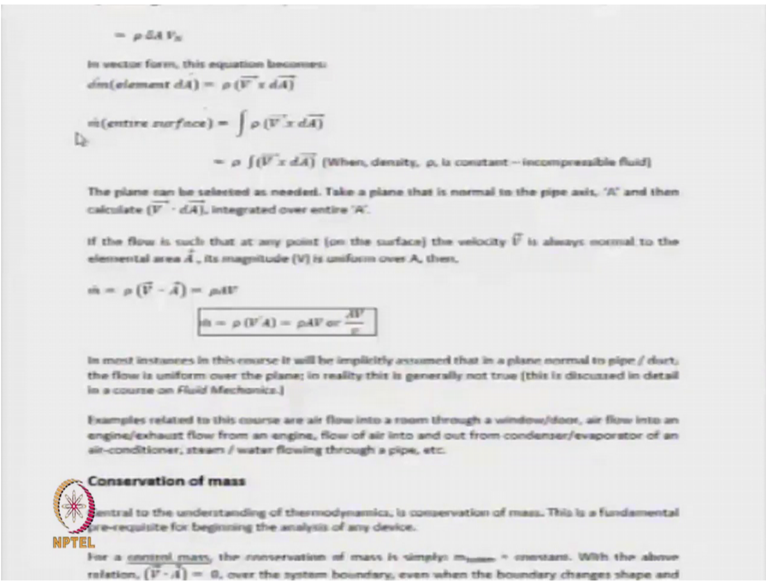
And then now we have learnt the first in do is apply the conservation of mass which tells you, but now we have more than one inflow. So, all will do is modify this equation little bit and say that summation over all outflows m dot e minus summation over all inflows, over all inflows plus d d t of m cv is equal to 0 and that is our conservation of mass, most general type of a problem.

And its form that we have seen here outflow rate minus inflow rate plus rate of accumulation in the control volume is 0. This type also relation is true for everything. The second law of motion for a control volume will become instead of mass it will become momentum, for the first law of thermodynamics instead of mass it will become energy. So, this becomes a very robust definition of a conservation equation for a control volume and it is not just that we are looking at these types of problems, but if we are looking at say reservoir of a dam, then same thing applies there that the rate at which you have inflow into the reservoir (Refer Time: 33:43) rate at which there is outflow is the rate at which that level of the reservoir both up and down how much water it accumulates or its depletory.

This all some incidentally will applied to your bank account where whatever money you had outflow rate which is what you withdraw minus inflow whatever comes in is plus the rate of accumulation of money in your account will be there rate of creation of money if you want interest or something like that. So, this is a very very fundamental equation. And there have been many industrial cases where the industry was having lot of problems, and instead of worrying about energy and momentum and all those details the problem got solved the moment we looked at conservation of mass, and realize that something was not been accounted for the accounted for that things can sell back in line.

So, this is a very very basic equation that will always be there as we have first starting point in analyzing any problem especially when it is a control volume. So, this is what is there in the part a, of the notes that I have given out. There are couple of changes you need to make there because on page 2, you can see this here you need to make a important change, there is the mistake here.

(Refer Slide Time: 34:57)



$= \rho\, \delta A\, V_x$

In vector form, this equation becomes:

$dm(element\ dA) = \rho(\overline{V} \times \overline{dA})$

$\dot{m}(entire\ surface) = \int \rho(\overline{V} \times \overline{dA})$

$= \rho \int(\overline{V} \times \overline{dA})$ (When, density, ρ, is constant – incompressible fluid)

The plane can be selected as needed. Take a plane that is normal to the pipe axis, 'A' and then calculate $(\overline{V} \cdot \overline{dA})$, integrated over entire 'A'.

If the flow is such that at any point (on the surface) the velocity $\vec{V}$ is always normal to the elemental area $\vec{A}$, its magnitude (V) is uniform over A, then,

$\dot{m} = \rho(\vec{V} \cdot \vec{A}) = \rho A V$

$\dot{m} = \rho(V A) = \rho A V$ or $\frac{AV}{v}$

In most instances in this course it will be implicitly assumed that in a plane normal to pipe / duct, the flow is uniform over the plane; in reality this is generally not true (this is discussed in detail in a course on *Fluid Mechanics*.)

Examples related to this course are air flow into a room through a window/door, air flow into an engine/exhaust flow from an engine, flow of air into and out from condenser/evaporator of an air-conditioner, steam / water flowing through a pipe, etc.

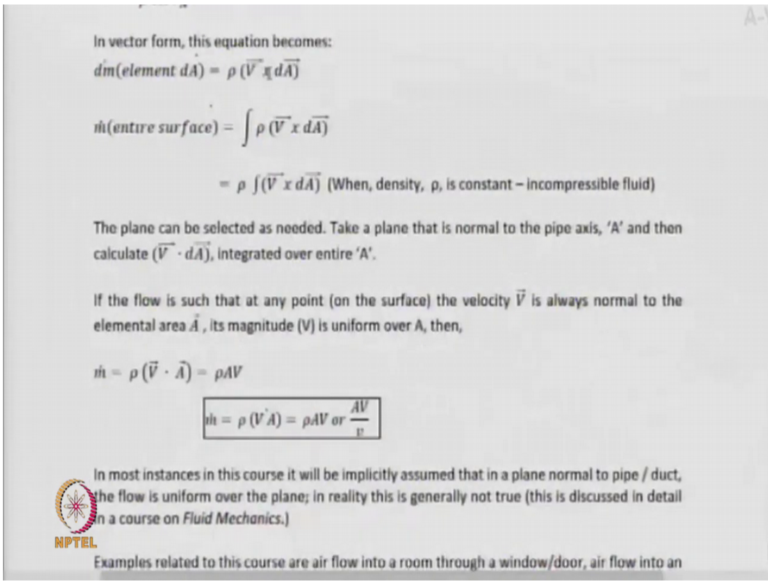
Conservation of mass

Central to the understanding of thermodynamics, is conservation of mass. This is a fundamental pre-requisite for beginning the analysis of any device.

For a control mass, the conservation of mass is simply: m_{system} = constant. With the above relation, $(\vec{V} \cdot \vec{A}) = 0$, over the system boundary, even when the boundary changes shape and

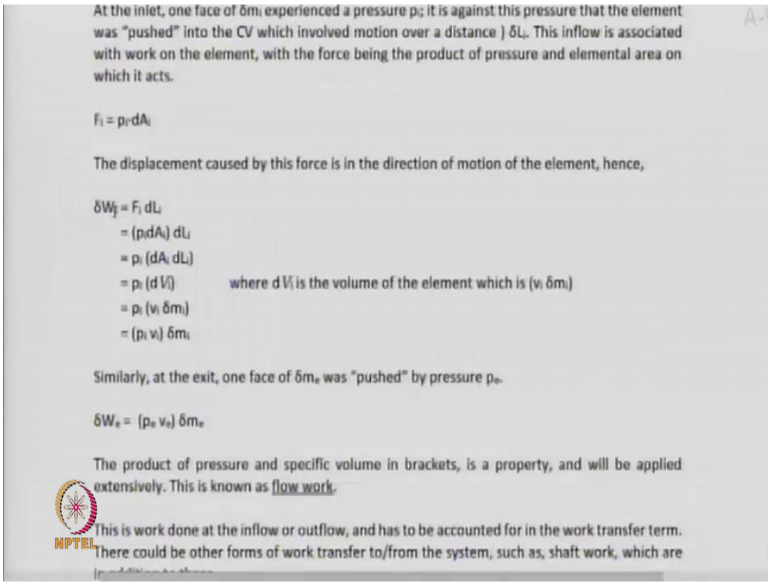
It is V cross it is not actually across, but multiplied by dA.

(Refer Slide Time: 35:07)



This should be replaced by the dot; it is the vector dot product and need not to be taken as the vector cross. So, this equation and the next two equations both are dot products. So, please make that change. I will update the notes and put up a revised version on the forum. And then everything else has been derived the way before and in the flow work this is the inflow this has to have a negative sign there.

(Refer Slide Time: 35:35)



So, in all this expressions we have to put a minus sign that I have been just now. So, that is that that what we discussed just now. So, this part was the. So, with that we completed our formulation of the conservation of mass.