Project and Production Management
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Lecture - 39
Inventory Modeling

In the last lecture we were talking mainly about selective inventory management and we had actually talked about aspects like ABC analysis, VED analysis, FSN analysis which were actually ways to conveniently classify different kinds of production inventories and storage inventories so that some meaningful work could be done with greater ease.

Today's lecture is going to be concerned with modeling of inventory systems that means we are going to talk about developing mathematical models where the fluctuations in inventory depending upon the ordering policy are taken into consideration and we try to find out what is the best way to conduct the inventory in a particular organization. The essential problem in it in very simple terms could be described as a problem of lot sizing where we are essentially concerned with when to order and how much to order of any particular item which might be there. The first case that we are considering is that of a purchase situation, so in a purchase situation assuming that there is no back logging or there are no shortages taking place what would happen is the inventory would normally show a gradual reduction with usage till of course a point that the inventory becomes zero. It is at this particular point of time that an order is raised and for a certain quantity q. So the stock level suddenly rises at this particular point of time and thereafter the inventory level tends to drop gradually and it keeps on dropping till at this particular point of time. You again have zero inventories and again the next order is realized and so on. Basically it is a repetitive process which resembles a saw tooth curve, this fluctuation of inventory and what are our decisions? Our decisions in the purchase situation are to identify what should be the order quantity Q such that the total quantity, total cost is actually minimized. That is what it is. In a general sense the decisions variables are the order quantity Q per lot and also the maximum back order level b which you will consider in classes three and four. This is the first case that we are considering when we are not allowing nay back orders to take place.

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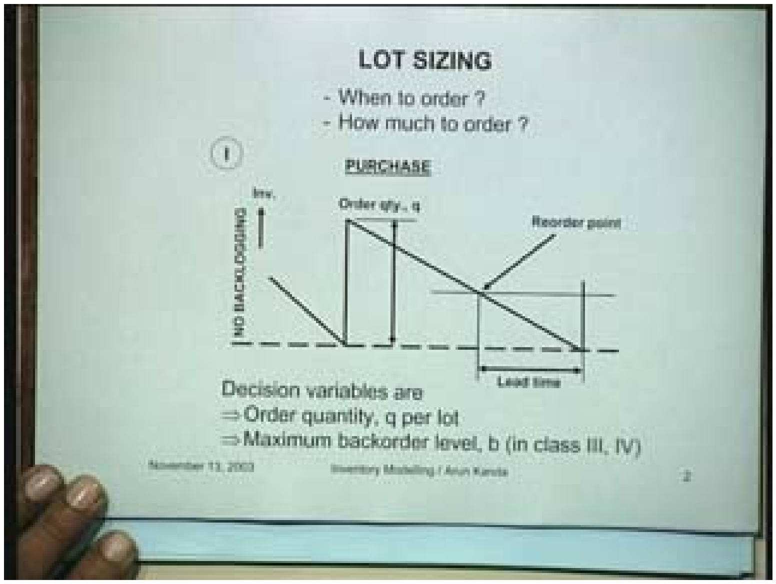

This is the simplest pattern of lot sizing that we consider where the inventory fluctuates in this manner and it rises suddenly when a stock is received and then because of the gradual consumption, there is a consumption of inventory in that sense. We can now compare the situation that we just considered, which was a purchase situation to what would happen if it was a manufacturing situation. That means we are now considering a production situation. In a production situation again our decisions are when to order and how much to order. But what do you notice here? You notice that for instance this dotted line shows the rate of increase of inventory would be if there was no consumption. The slope of this line is the production rate p for that particular item and this rate should be reduced by the consumption during this particular period. If the demand rate is d, this value, this particular line will be P – d. What is going to happen is that we keep producing from this point to this point. The inventory would have risen to this level but because we are consuming constantly, therefore it does not rise to this level. It comes to this level and at this point of time we stop producing and then again there is a gradual depletion in the inventory stock and till at this point you are again encountering a zero inventory level and then again so this is actually the production of a batch. In batch production you have this much, this is the period of production of a batch and then you are allowing the items to be consumed and then again there is batch production and so on. The decision variables here are again the order quantity Q per lot. How much to produce and of course we are not specifically concerned with backorders because we have not allowed the inventory to become negative, that means we are not allowing any shortages to take place in this particular situation.

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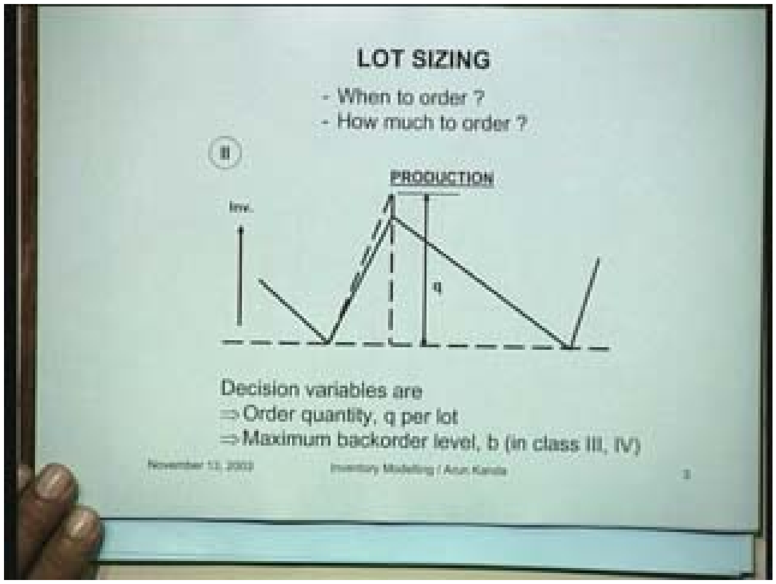

This is a second model which is a model of a production situation rather a purchase situation in which the saw tooth curve is now inclined as you see here. Now you take a situation with backordering that means you are allowing shortages to take place. This is basically a situation of purchase because this is a vertical line and we are now allowing the inventory to come below the zero level. This is backordering. In this particular situation there are two variables that we want to solve for. You are trying to find the optimum order quantity Q per lot and the maximum backorder level b. So Q and b are the two variables that we would like to solve for so as to minimize the overall costs and this is a situation of purchase with backlogging.

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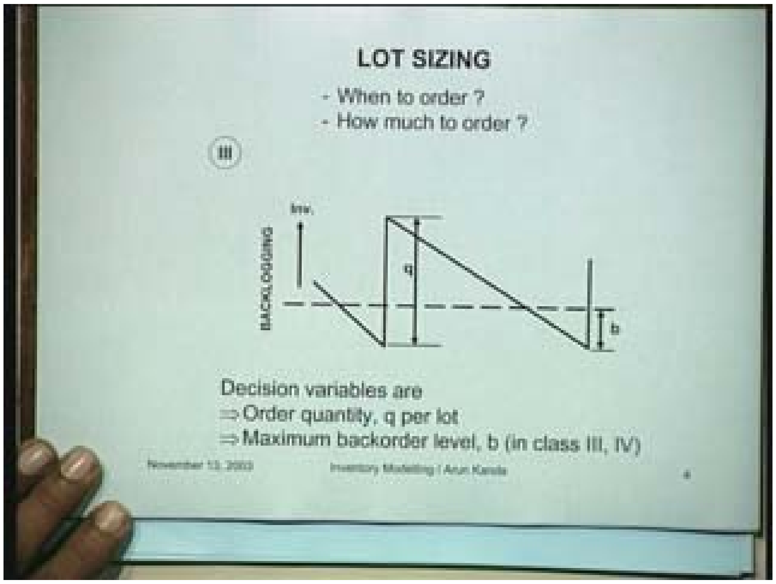

Negative inventory means that you keep on consuming and booking orders in this particular period. We have finished the stock and when we are talking about back ordering, what it means is we normally consume let us say ten per day. If we go on for five more days we would have accumulated fifty orders. These we would satisfy when we have the stock. You are back ordering, that means your requirement is that say ten per piece, ten per day. So you do not have it for the first day, for the second day, third day, so the total back order is fifty. You do not have that fifty. That fifty would actually become available when you have the stock and therefore out of the stock you will first clear the back orders and then satisfy the remaining items. This is what it means, so back ordering is a situation where you are allowing shortages to take place. It is very much like if you are looking at a car dealer for instance, he has the cars he supplies and he books the orders and says I will supply you one week later or two weeks later, that is a shortage, that is a back ordering situation. Let us now consider the fourth case which is basically a production situation again but with back ordering. In fact this is the most general case which embodies all the four cases that we considered earlier. The decision variables here are the order quantity Q per lot. How much should you produce and what should be your maximum back order level which is this quantity b? In these classes with backordering you have to find out the optimum back order level as well as the quantity so that the total cost is actually minimized.

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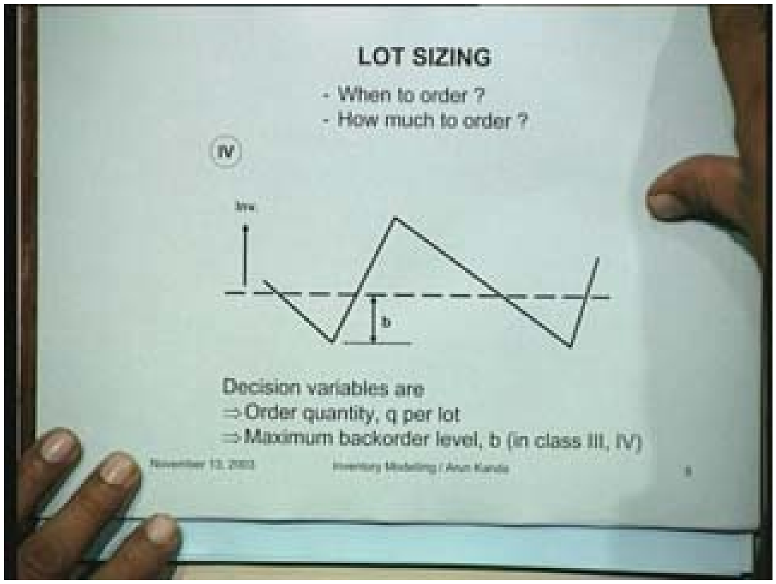

The broad approach to the analysis is summarized in these steps and what you find we have to solve the lot sizing problem in cases one two three and four that we have considered so far. The first step is generally to identify the cost components in each cycle of length t. It is a cyclic cost behavior. So you look at_1particular cycle and during that cycle t you identify what is the carrying cost, what is the shortage cost, what is the set up cost for that particular order. Having identified these costs then you express the costs in terms of the decision variables which are order quantity Q and the back order level b, that is what you do. First know the costs then express the costs in terms of Q and b by using simple geometrical relationships and then develop an expression for the annual cost by multiplying one by the number of cycles per year. So this is the cost per cycle, multiply this with the number of cycles per year you will get the annual cost which is this and then from this particular expression, the total annual cost have to be optimize to find Q star and b star which is the optimum lot size and the optimum back order quantity.

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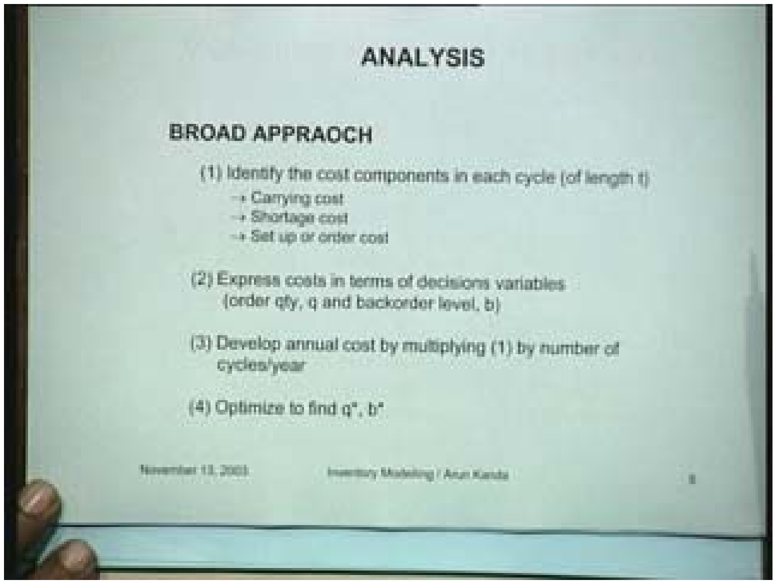

The basic philosophy is examining the cycle each cycle and the cycle will repeat itself. If you identify the cost in that cycle, the same type of behavior will be repeating for each cycle and then expressing cost in terms of the decision variables developing the annual cost and then optimizing to find out Q star and b star. If for instance you apply this approach and the final solution that you would get is in a simple lot size formula which is the purchase situation without back logging. You have Q star = 2 C_3d divided by C_1 where C_1 is the carrying cost in rupees per unit per unit time and normally the carrying cost is expressed as the product of inventory carrying cost rate i and the unit cost, so the unit cost is C. C_2 is the shortage or the back logging cost which does not appear here because this is a case without backlogging and C_3 is the order cost which is in terms of rupees per order, that means each time you make a decision to replenish your stock, you incur a certain fixed cost which is the order cost or the set up cost and that particular cost is denoted by C_3. The so called economic order quantity is being determined by 2 times C_3 into d divided by C_1 under the root and of course the back ordering cost or the back ordering value in this case is zero.

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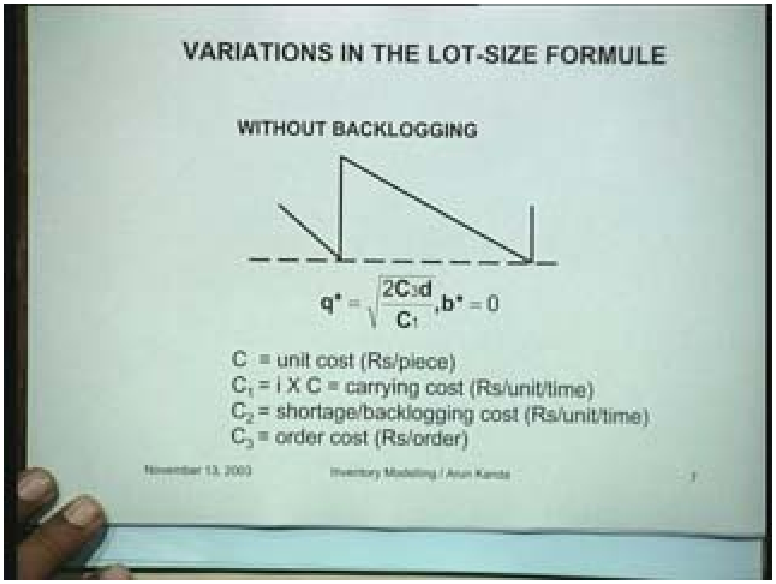

That is what it is. However if we consider a situation like this where the production is taking place or this is a batch production situation. From here to this particular point, production is taking place and thereafter only consumption is taking place. Then for this situation the optimum lot size formula works out to $2C_3d$ divided by C_1. Only thing is this is the additional factor which comes in the denominator. That is 1 – d/p. Notice that if the production rate is infinity, that means this is a vertical line as before. Then this particular factor d /p. This would become simply zero and so this reduces to the earlier formula that we had for the classical lot size formula.

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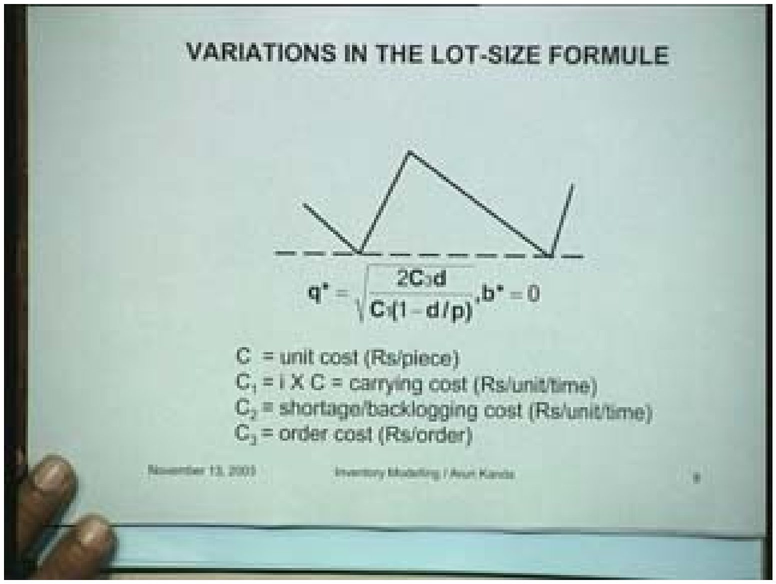

This was a production situation without backlogging. If you now consider the situation of production with back logging what happens really is that this inventory level is now allowed to become negative. This is let us say some value b. When this happens, the optimum Q star value that formula is $2C_3d/ C_1$ under the root which is the standard formula for the EOQ and this is now multiplied with $C_1+ C_2 /C_2$ under the root and the value of b star is equal to C_1 into Q star divided by $C_1+ C_2$. So, depending upon the situation you are using your formula for the optimum lot size and the optimum back order quantity would vary.

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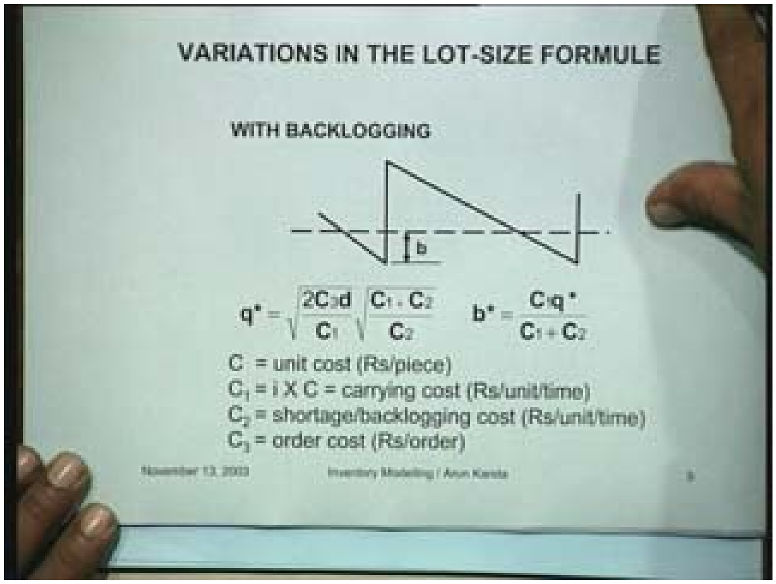

Of course this is the most general case where you have back logging. You also have production and therefore the cycle goes from here like this. So during this cycle you find that portion of the cycle and there is back ordering. Then there is inventory holding and then again there is shortage during this particular period and the formula for Q star which is the optimum production quantity is given by $2C_3d$ divided by C_1 into 1–D/P and multiplied with the square root of C_1+ C_2 divided by C_2 and b star which is the optimum quantity to which you should allow the back orders to accumulate is governed by C_1q star into 1–D/P divided by C_1+ C_2. This would be the kind of formula that you would have for a situation like this.

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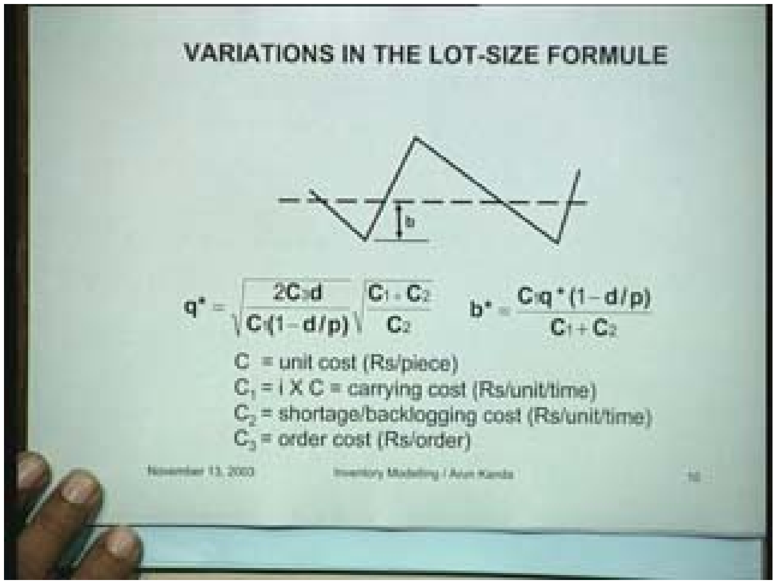

I will give you some examples of the basic assumptions in the classical lot size formula which we have just considered, starting with the simplest one. We have basically assumed that demand is constant is fixed at a constant rate of d units per unit time that means we are essentially assuming that demand to be deterministic and we are saying that it is constant and the value of that constant is known to us. The second thing that we are saying in the classical lot size formula is that replenishments are made when inventory reaches zero level so that no shortages occur. So we are permitting any shortages to take place in the classical lot size formula. We are also assuming that the lot size Q is fixed, that means the quantity in which you place the order every time is Q which could be whatever it is a truck load whatever it is a fixed quantity. There is infinite replenishment rate which means the moment the order comes, the stock level rises suddenly. Lead time is known, which means you know that your stock is going to be depleted in another two days and you know it takes two days to get the order.

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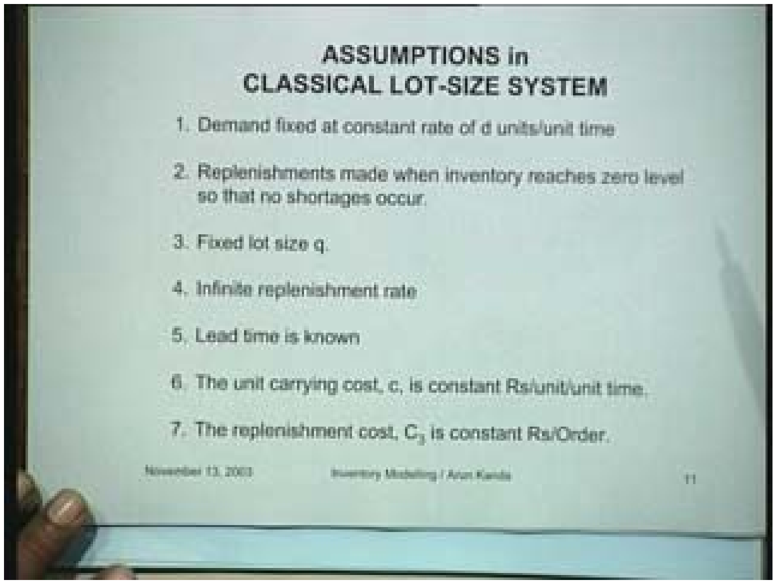

You can always place the order two days in advance so that the stock arrives exactly when the demand is zero. That is the kind of assumption that we are making here. The unit carrying cost c is a constant in terms of rupees per unit time and the replenishment cost C_3 is constant in terms of rupees per order which is the order cost. Now here are some interesting sensitivity studies in the classical lot size formula. If you see the classical lot size formula basically there are two types of costs involved. One is the inventory carrying cost and the inventory carrying cost is C_1into Q/2because for a pattern of fluctuation inventory of this kind, the average inventory is q/2. C_1into Q/2or iC Q/2will be the carrying cost and this is a straight line that means as you keep on increasing the lot size, this cost would tend to increase. However the ordering cost is determined by this quantity C_3d /q because after all each in each year C_3 is the cost of placing the order. So d/q represents the number of orders you place each year. So C3 D/Q would then be the total ordering cost per annum and if you happen to see behavior of this q, it is occurring in the denominator. This should be a hyperbolic function of this nature. The two opposing types of costs are the carrying cost and the ordering cost and the sum of these two costs is this one.

The total cost and the order for a certain value of Q star, you have the total cost coming out to be the optimum lot size formula. So k Q which is the cost function is the carrying cost and this is the ordering cost and if you take partial derivative with respect to q, you get Q star as 2 D C_3 /C_1under the root and you can also work out the total cost which is the total minimum cost. Just plug your values, take this particular value of Q star, put it into this expression into this particular cost function and you will get TC min which is the minimum cost as 2D C_1into C_3. You can now define a quantity b which is b Q star, which is measuring the amount of deviation you have from the optimum lot size here. So you have b Q star and then the total cost for that particular value divided by TC min. If you calculate this, this is simply 1/2 into b + 1/b and this value is 1 + b square /2b. The interesting thing to note here is that t c divided by t c min is not dependent upon the costs at all. It is not dependent upon C_1 C_3 etc. It is just dependent on the parameter b which is how far you are from the optimum lot size and then if we look at this ratio t c over t c min that is 1 + b square /2b. For different values of b you find an interesting thing. For instance if I am at the order quantity, my total cost is 1 but suppose I increase my cost by 10 percent, I increase that is I depart from the optimum lot size by a factor of 10 percent. Make it 1.1 q. Then you find that the cost has increased to 1.005 whereas if I decrease it here, the cost has increased to 1.006 which is higher. Now you can do it for two intervals. If you decrease it by 20 percent, the cost has increased by 2.5 percent and here the cost has increased only by 1.7 percent. The point is therefore from this, and you can easily see if you make it half. The cost has increased by 25 percent but here if you increase it to 1.5 times a value, the cost has increased only by 8.3 percent. The issue is really that this curve is generally much steeper here and more flat in this region. So it is much easier or rather the cost penalty that you will pay by increasing the lot size is generally much smaller than the penalty you would pay by decreasing the lot size because this curve is steeper here and generally less steep or rather more flat in this region. So an increase of 20 percent will lead to little cost increase.

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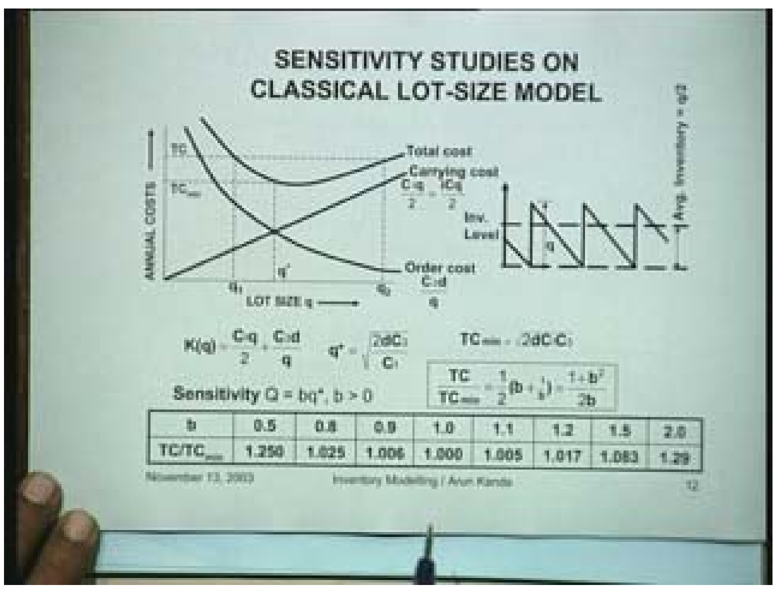

A decrease of 20 percent would lead to a significant cost increase. This gives you a very good rule of thumb that when you are calculating lot sizes, it is always better to increase or round off values to the higher side rather than to the lower side because the costs are much higher on the left hand side than on the right hand side. This is an insight that you get by doing a sensitivity analysis on the classical lot size formula. Let us now consider the determination of the economic order quantity with what we call quantity discounts. What we are trying to say is our total cost will be C_1Q/2which is the holding cost d/Qinto C_3which is the ordering cost and this is of course the annual usage which is the cost of purchasing the item. I purchase d item at the cost of C. This is the amount that I have to pay and these are costs of holding and placing the order ordering cost. If you take a small example and let us say that the annual demand is 5000 parts for a certain item and let us say that the order cost is rupees 49 and the inventory carrying cost is rupee 1 per part per year. To store each part per year we are paying a cost of one rupee. So this is the holding cost, obviously the EOQ for this situation will be two into the demand into C_3which is the ordering cost divided by C_1which is 1. This value is simply 700 units. So, 700 units is your economic order quantity. Suppose this particular item was available to you under a discount scheme which is shown like this, it means if you buy in the range of zero to 999 you can buy the piece for 5 rupees a part. If you buy in greater quantities up to about 2500 units, you get it for 4 rupees 85 paisa per unit and for quantities above 2500 units, if you order then your costs are you rupees 75 paisa per unit.

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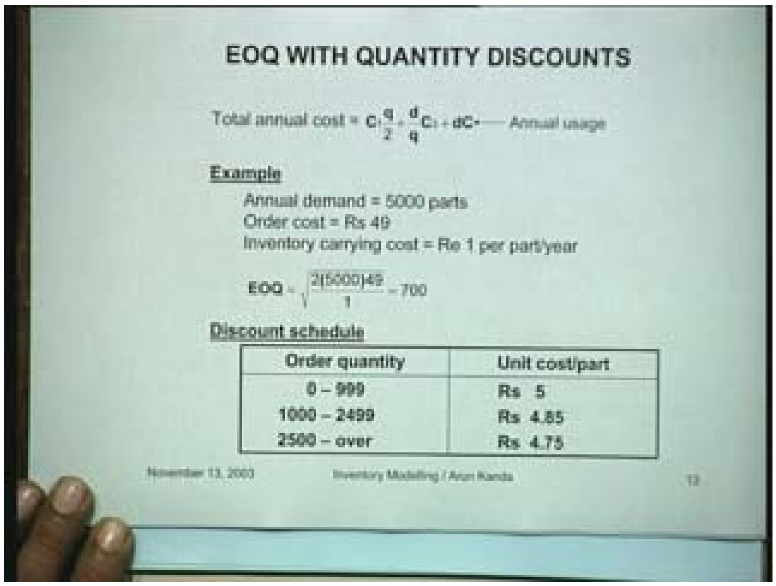

Now what is going to happen here is we can find out here that if we look at different costs you order 700 or 1000 or 2500, the annual holding cost is half of whatever we ordered. So 350, 500, 1250 and the annual ordering cost will obviously be much less if you order in larger lots because you will be required to place lesser number of orders. Here you place 350 orders. Here you place 245 orders. Here you place 98 orders and of course we can calculate the purchase cost, which is the total cost of purchasing the item which is 25000 here, 24250 here, 23750 here, this is because of the different costs of the item from 5 rupees to 485 to 475 and if you then consider the total cost of this particular case, you find that the total cost for an ordered quantity for 700 works out to 25,700, for 1000 it works out 24995, for 2500 it works out to 25098. If you see a graphical plot of this information, you find that for all these three situations, three situations meaning that the purchase costs are different for 5 rupees, for other 1000 and for 2500,where the costs are in one case you have 250700. In the other case you have 24995 and in this case you have 25098. The minimum cost in every case is here but what is happening is that this particular cost function 5 rupees is valid only till a cost of 1000. So the first cost function is valid from here up to 1000 only. Then this cost function is valid for this region up to here and then this cost function is valid up to here. If you are now looking for the global optimum for this particular situation, the global optimum in this case will be only this particular value. What is happening now is that because of these quantity discounts operating at a lot size of 1000 rather than the EOQ value of 700, it is warranted here.

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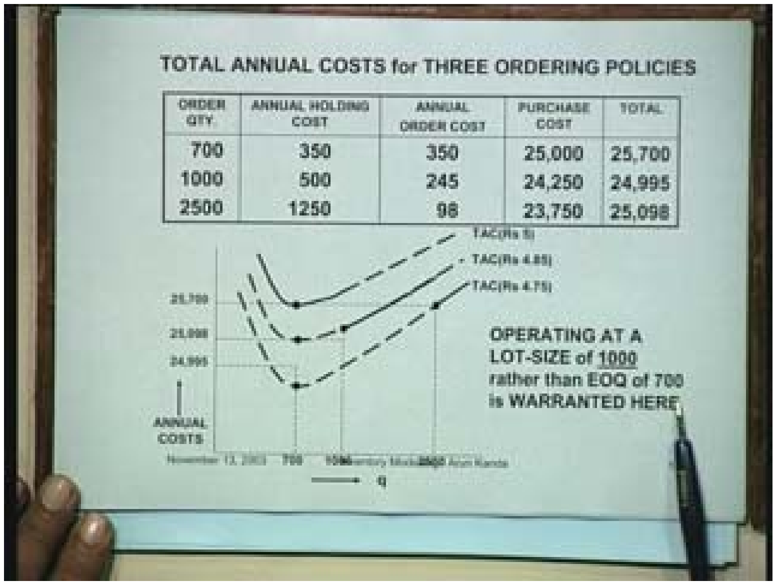

This now becomes a discontinuous cost function from here and then from here to the next one and the next to this one. So whatever is the dotted line actually these cost functions are not valid during that particular period. This is how you can consider the various types of quantity discounts in determining what the optimum lot size should be. This was actually a case where the EOQ value for all of them remained the same. Why the EOQ remained the same was that EOQ is determined by $2dC_3/ C_1$. If the carrying cost and the ordering cost and the demands are the same then obviously the EOQ would be the same. This was the situation here but what may happen is the EOQ itself might change. The second example shows how the EOQ might change. Suppose for instance that you can buy an item up to 100 units at rupees 2 and from 100 and above at rupees one per piece, so at the rate of rupees 2, your EOQ is 79 and your demand is 250 per year. C_3 is rupees 5 per order and the value of i that is the inventory carrying cost rate is 20 percent. So the EOQ for rupee 1 would be now 112. If you look at the cost figures you find that the cost of 79 is the EOQ for this particular case.

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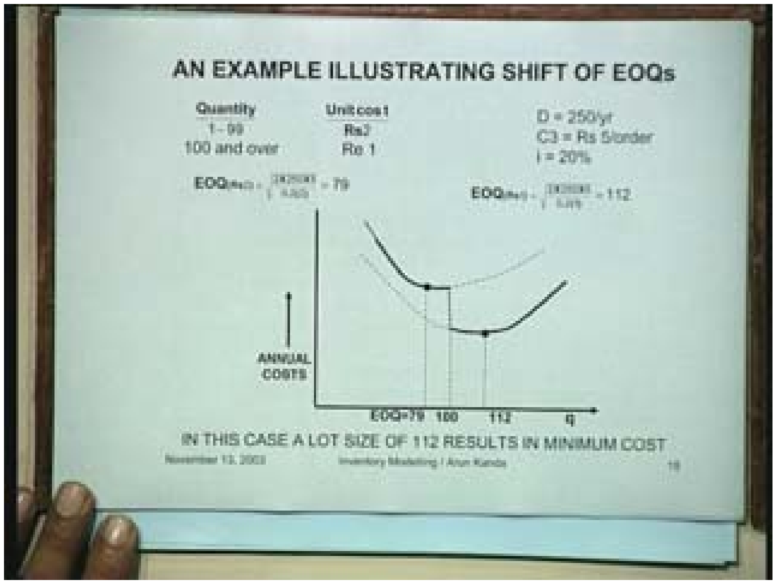

In this particular case since the inventory carrying cost is directly taken as iC, the C value is the cost of the item is changing. The inventory carrying cost is changing here. Here it is 0.4 and here it is 0.2. As a consequence the EOQ has changed from 79 to 112 and therefore you can see that if this is valid for let us say 1 to 100, we will have this cost function which is valid up to 100. Then of course we gradually come down to this function here. So in this particular case the overall minimum will be at this point and this particular minimum shows that a lot size of 112 actually results in the minimum cost. Depending upon the nature of the discounts and the breaks you have to actually look at the total cost curves and see how the optimum would be shifting from one place to the other. Just to give you an idea of how the derivation for the formula is concerned we take the most general where inventory is fluctuating in an arbitrary manner. That means this is a production situation. This is then the consumption part of it and we will try to evolve a general formula for this and a procedure very similar to this would then be applicable to find out any general lot size for a given situation. What we are saying is we look at the cycle. What do you find in the cycle? You find that the cycle has four essential times. We call them t_1 t_2 t_3 and t_4 . So t_1 and t_2 is actually the time of production TP and this capital t is the time of the entire cycle which is taking place. If we take this notation as d for the demand rate, p for the production rate, Q for the order per unit production quantity t for the cycle length, b for the maximum backorder level permitted. Imax for the inventory on hand inventory level maximum inventory on hand inventory level. C is the usual unit cost, C_1 C_2 C_3 in fact what you can do is we have in this particular situation considered 2 types of shortage costs.

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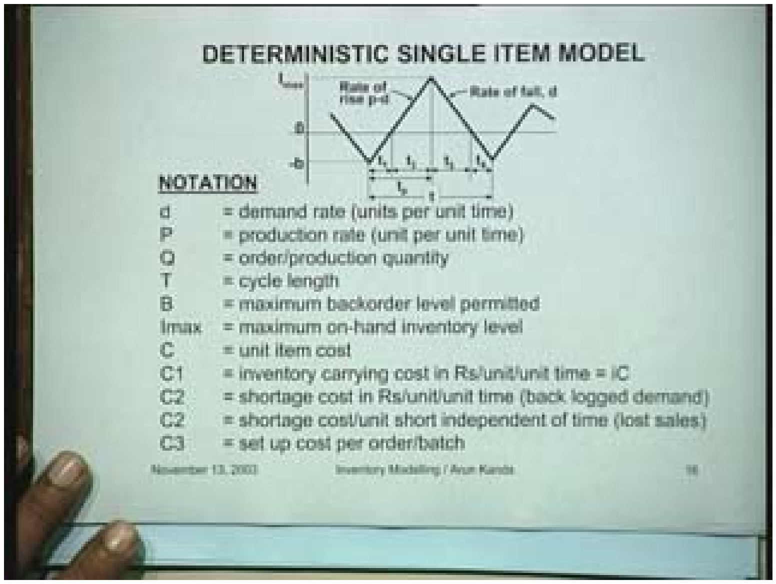

We have considered C_1 is the inventory carrying cost which is equal to ic as usual but what we also do is C_2 is the shortage cost in rupees per unit per unit time. This is the back logging cost as you have before. This you can call as C_2 prime. C_2 prime is the shortage cost per unit independent of time. So this is lost sales. We are now modeling both the backlog demand as well as the lost sales together. C_1, C_2 and C_2 prime and C_3 is the set up cost per order per order per batch. So if you look at this formula what we try to basically do is we express all these times in terms of the production quantities that we have, for instance what is the cycle time QT? The total cycle time is, you have a lot size of q, how much time will it take you to consume Q, Q/d where d is the demand. This is an expression for the cycle time which in this expression is the total time for the cycle and time to produce a lot TP will be by the same logic will be q/P where P is the production rate.

Maximum inventory Imax will be q/P into P – d – B will therefore be Q into 1 – d/P – B, so that becomes the maximum inventory and then using this Imax TP, T we also express the various components of the cycle time t_1 t_2 t_3 t_4 in terms of Q and B. That is what it is, so t_1 is the time required for back order b to be cleared once production starts. This will be simply time for back order b to be cleared. So just it is, what is t_1? Look at this cycle. You have accumulated a back order of so much, t_1 is the time that the back order is cleared, that is what it is, B/P – d. t_2 similarly will be i max over P – d. So this is the time required to build up inventory from zero to Imax. t_3 is the time for inventory level to drop to zero from i max at a constant demand d. This is nothing but Imax/d. It is like saying if I am here how much time does it take for the inventory to fall down to zero, that is the time t_3. t_4 is the time for back log b to build up at a demand rate which is simply B/d.

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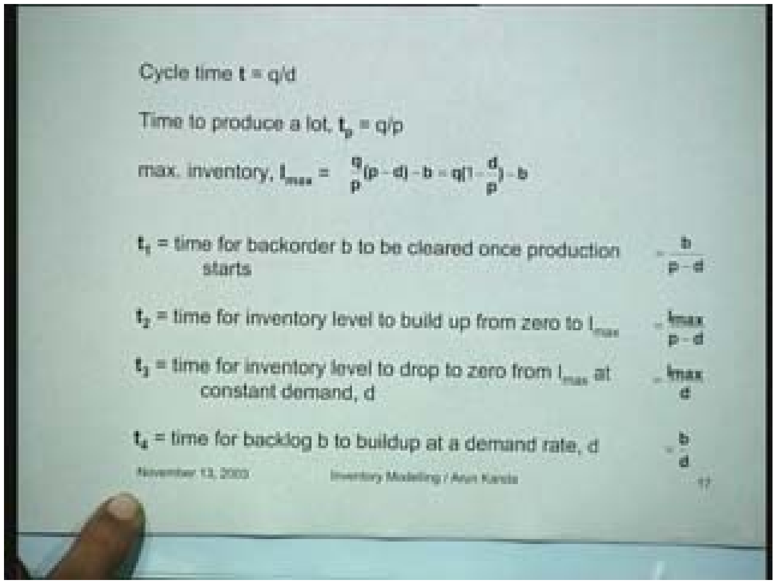

Having got these expressions you can develop the expression for the total cost per cycle that is our intention. The total cost per cycle you notice an interesting thing. During the period of the cycle from t_2 to t_3 this period, you are actually having inventory and during the periods, $t_1$and t_4, you are having shortages. During this period what we have is the carrying cost during the period t_2+ t_3 is half of i max into t_2 + t_3 into C_1which is the cost of carrying inventory and during t_1+ $t_4$you have a shortage cost and what we have here is we have both a back ordering cost which is C_2 and we have a lost sales cost which is C_2 prime. So C_2 prime into b because b is the number of lost sales. For each order lost you lose certain goodwill, so what we are saying is since you have lost or backordered b units you have lost so much in terms of good will or a loss sale and this is of course your cost of back ordering. The ordering and the replenishment cost per cycle is C_3. Then you express t_1+ t_4 which is the period during which you incur shortages. t_2+ t_3 is the period which incur carrying inventory and Imax is this particular quantity.

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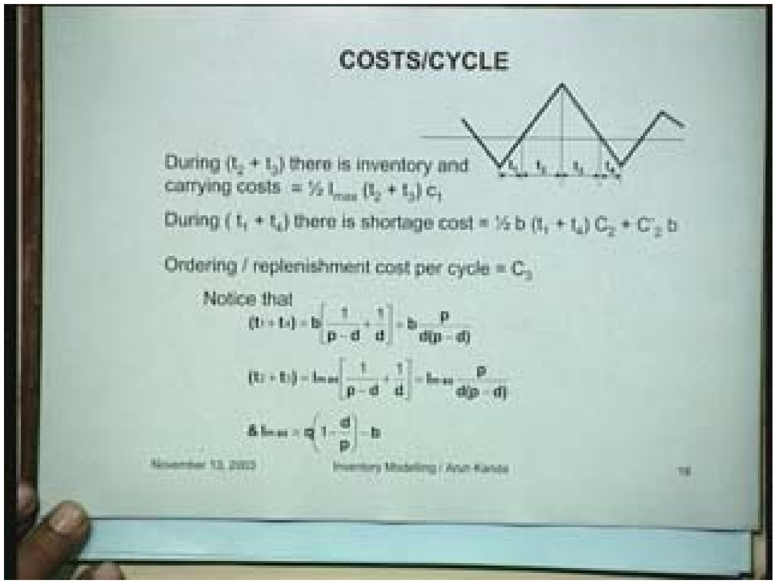

Doing a bit of mathematical jugglery you arrive at the expression for the total average annual cost kQb which is of this nature. This is the cost of holding inventory, C_1 into Imax/2 into the period of time, the fraction of the time that you are holding inventory. This is the period of back ordering. This is the period of lost sales and this is the ordering cost. You are placing an order only once in a cycle of t. That is why it is C_3 /t and all these values, you can substitute for T, t_1+ t_2 etc which we have just now calculated and you can write this particular expression Q for the total cost.

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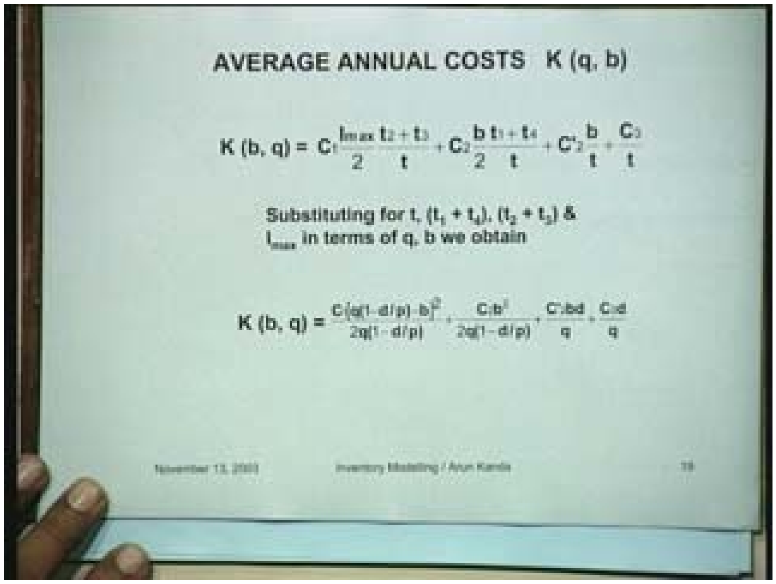

Important thing to note here is that the total cost is expressed as a function of B and Q primarily, B is the back order quantity and Q is the order size that you have. That is what we are interested in expressing and if you then express the take partial derivatives of k with respect to Q and B and solve these equations simultaneously the general equation for Q star and B star therefore for this situation is governed by this formula. This is a more general formula because it now that occurs that both the lost sales as well as the back ordering costs. This is a very general formula which says that Q star is determined by this formula and b star, the optimum quantity is the determined by these where we have both C_1, C_2, C_2 prime and C_3 which come into this particular equation.

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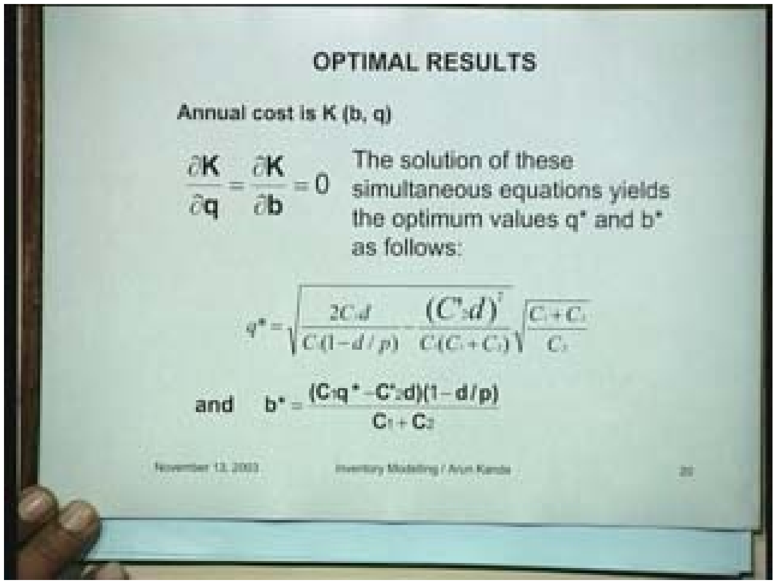

This in fact summarizes the general approach that you have to take for finding out the lot size in any arbitrary kind of situation depending upon how the inventories fluctuate. Let us now try to make a distinction between items which are fast moving and slow moving items. Last time you would recall in the previous lecture we had made this distinction between FS n, you know fast moving, slow moving and non moving items but fast and slow moving are generally important because the fast moving are the ones which are generally subject to inventory control. Those items which are slow moving or non moving they are actually candidates for stock disposal. Slow moving items have these two things. They have a low level demand and they have frequent periods of no usage. If you find that in your stores, there is a low demand for some item and then for one year you don't need an item, suddenly you require an item after one year, that is an item which can be classified as a slow moving item and thresholds are difficult to define. What should be the threshold? These are dependent on the item nature and normally in the literature it is said that the threshold is likely to be between 550 to 100 units per year. That means if your demand for an item is in the range that is likely to be less than this, then it is a slow moving item. Otherwise you can call it a fast moving item. Peterson and silver recommend classifying items according demand over the replenishment lead time, so these people have given a different criterion. An expected lead time demand of 10 units or larger puts an item in the fast mover's class whereas an expected demand time less than ten units defines a slow run up. So these people have defined their own way of looking at fast and slow moving items.

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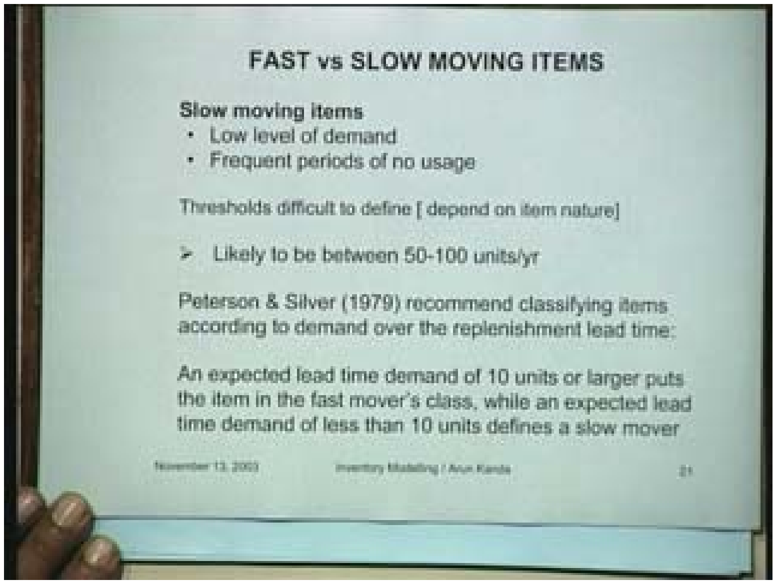

Let us now talk about inventory control. So far we have been talking about determining the quantities that you should actually have in an ideal situation so, that were inventory planning. Once you establish a plan go ahead with reviewing your orders. What are the inventory control policies that you should have? There are essentially two types of policies that we can deal with. One is called the continuous or the perpetual review system with the replenishment quantity Q and reorder level r. What this simply means is that suppose, this is the stock level, now in an actual situation this stock level will be fluctuating and will be coming down. So what happens really is that when it comes down to a certain level called the reorder level r which is what you have to determine in an inventory control policy, then you place an order and when you place an order then you have placed an order at this point of time. Your order actually materializes after this much point of time.

You placed an order here and in the mean time your inventory stock is coming down and then ultimately you have received the order. The amount that you have stocked rises to this level and the next cycle repeats itself. What will happen is both there could be uncertainties. The uncertainties typically are that you are now governed by the reorder level and what may also happen is that the lead time that you have could be varying. For instance this you took three weeks. Next time it may take only two weeks.
Next time it may take three weeks, or whatever it is, less time. So, one has to actually go in practice with the uncertainties in the lead time and also about the fluctuations in the rate of demand which could be different from period to period. Essentially this is how it operates. This system is called the perpetual review system. You are keeping a constant eye on your stock. The moment the stock level reaches r, you place an order. This is more like a two bin system. You remember you mother's stock let us say sugar in two containers, the container in the top and the big container in the bottom. Once the first container is exhausted, she comes to the second container. That is the time she places an order for the sugar. So this is exactly that kind of a system. It is a two bin policy for

controlling the stocks. You have a reorder level r. You place an order when your stock level reaches r and then of course your lead times can changes. Your order quantities can also change. I mean depends upon how much you order. For instance in this case, the order quantity that you change could be the same. It could be the economic order quantity but it need not be the same. In many cases you might try to bring the order. You bring the stock up to a maximum level and thereby restrict the maximum inventory.

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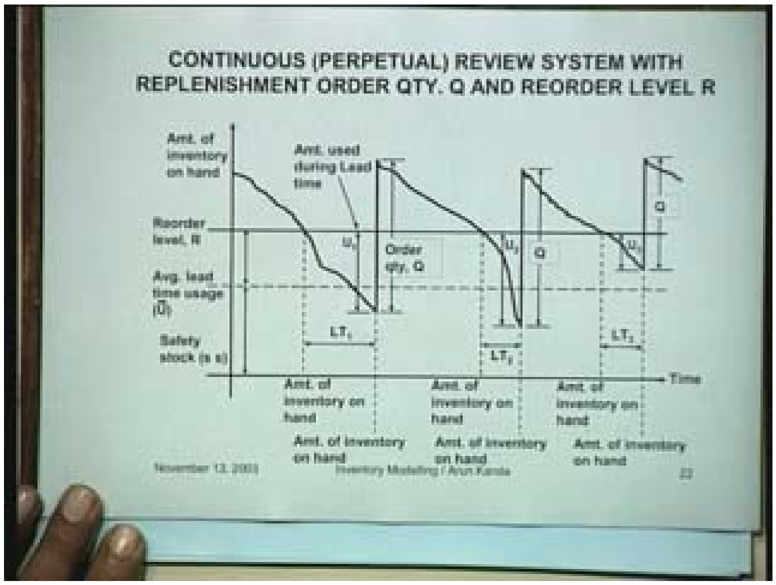

We just saw that the lead times could be uncertain. So how do you consider those kinds of uncertainties? If U is the lead time, demand in fact the u value here, you just saw that the value from here to here is the total amount consumed during the lead time. During this time from here to here, you have consumed this much quantity. So we are very much concerned about this lead time demand. So we are trying to estimate this. LT is the lead time which is in working days and d is the daily demand that you have some value. The total lead time demand will be the product of the daily demand multiplied with the lead times, so d into LT average, because these are random numbers. The variance of u will be equal to the variance of LT+ variance of variance; it will be variance of D. This is the variance of d multiplied with LT bar + the variance of LT into d bar whole square and therefore sigma u which is the standard deviation for the lead time demand will then be the square root of this particular value which we have just computed. This is now known and therefore ultimately what should be your reorder level? This is what we are trying to determine.

Reorder level will be equal to u bar which is the average demand during lead time + z into sigma u. z is like one sigma, two sigma, three sigma in that sense, so depending upon the value that you have, you have something like this. So if the demand is normally distributed and you fix your reorder point here then this area represents the probability of stock out during the lead time. So basically we are using this z which is obtained from normal tables based on either a pre specified stock out probability P. You can say that we do not want the stock out to be a probability of more than 1 percent or 0.05 percent or whatever it is. You can have the optimal stockage frequency, based on costs of shortage and carrying inventory.

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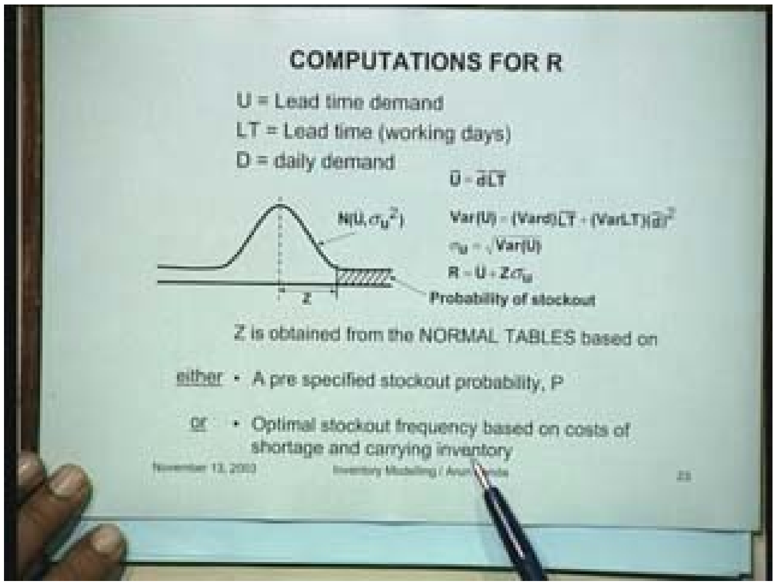

Depending upon the situation we are trying to balance the shortage and the carrying inventory. So you have this kind of situation here. This would be a way to estimate the reorder point because then inventory control policy is governed by the reorder point, so the reorder point depends upon the demand during the lead time. The lead time is a variable. The demand is a variable. So you can consider their variances and work it out in this fashion. Let us take an example. Let us say that the, per unit holding cost is one rupee per year. It is a small item which requires one rupee per year to hold. The ordering policy is that you order it 4 times a year, not more than that and the pre specified service level is one stock out per three years. The number of stock outs per year will be 0.31/3. That is what we are trying to talk about this stock out coefficient per year.

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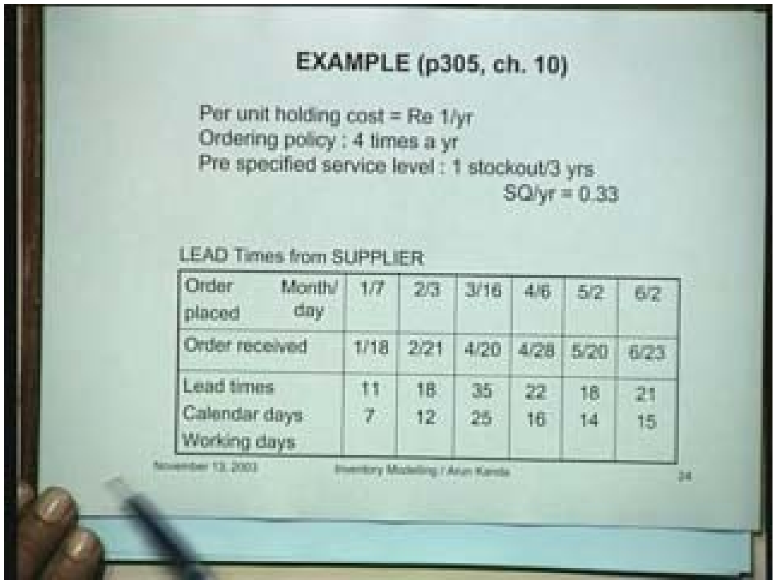

If you talk about the lead time what kind of data would you have? You would typically have data of this nature which would say when the order was placed. So he says it was ordered for instance in the seventh of January. Then the third of February, the sixteenth of march, the sixth of april, the second of may and the second of June, suppose the orders were placed in this order and the orders that were received were actually on 18/1, 21/2 etc. So you have this record available, so the difference of these two will give you an idea of the lead time. The lead time in this situation was 11. On this time it was 18, 35, 22, 18, 21. However if you want the calendar days, you want to remove the so called holidays from here then the number of calendar days for the lead time are 7, 12, 25, 16, 14 and 15. In terms of the number of working days, you have this. This defines you the actual distribution of the lead time. So what we can do is we have the average lead time. We have these values of the lead time. So the average lead time is 14.83. The variance of the lead time can be calculated as x – mu whole square and summed up for all the values. So this is 34.97 days square and similar data on demands for last 6 months can be used and this yields for instance the average demand is 40 units per day and the variance of demand is 30 units per day whole square. This is how you can extract data on lead times and demands from the actual facts that you have available with you.

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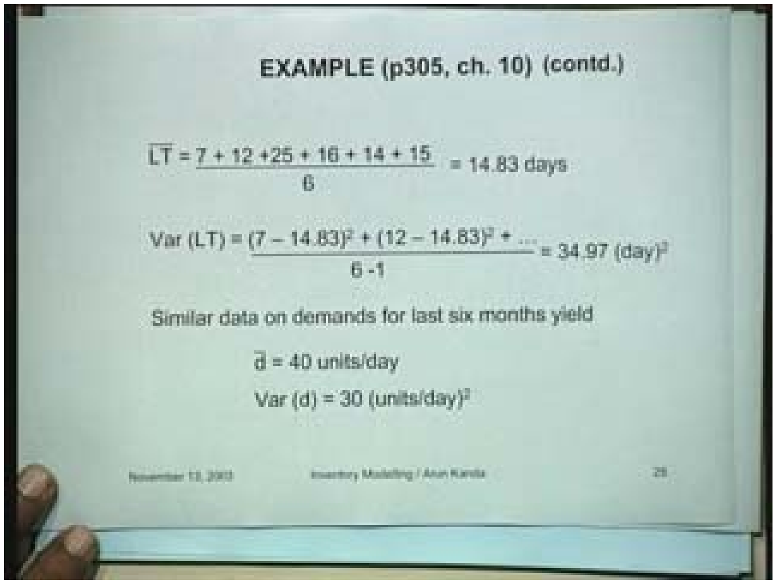

We are using this data, what is the demand? U bar is nothing but the average consumption during lead time. So this will be 40 into 14.83 which is what we have calculated here. Demand multiplied with lead time approximately and then 593 is the units demanded per lead time. The variants of u by utilizing the formula that we had just derived can be worked out to 56397. So sigma u can be obtained as 237.5. So the desired stock out per year is only 0.33 as stated earlier. Therefore order cycles per year 4 is given. P the probability. Desired probability of stock out per order cycle is actually determined by stock outs per year divided by n which is 0.33/4 which is so much and from the tables the corresponding value of z that you have for this probability. This is the desired stock out probability which we have computed for the situation. This is now 0.083, so you have now specified this area.

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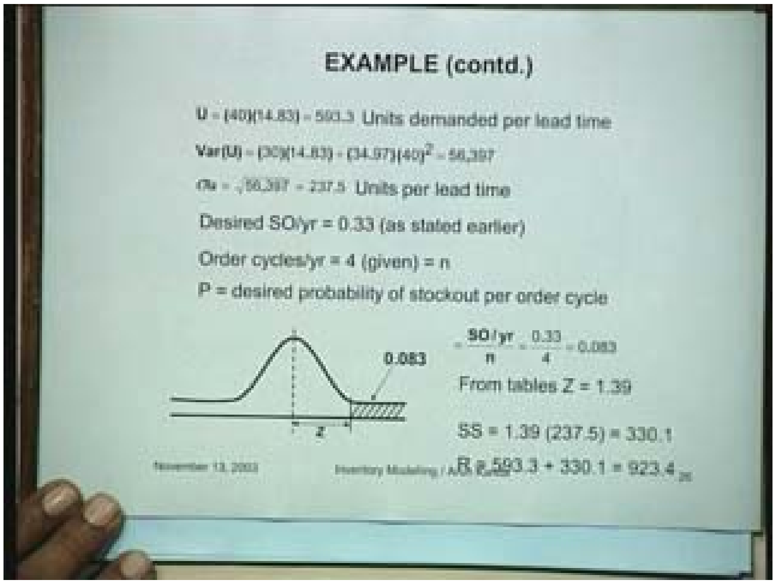

From standard tables you can say z is 1.39 for this and therefore the safety stock and r is the safety stock. It will simply be 1.39 times 237.5 which is the sigma u value here and this value comes out to 330.1 and r is nothing but 593 which is the average consumption during the lead time plus the safety stock and this value is now 923.4 which means that if you want to get the required amount of protection for this particular product, you should have a value of r or a reorder level of 923.4. This is how you can compute the values of the value of the reorder point. That is when you should place the order. That is the size of a second bin in a two bin policy. That is what we are trying to talk here about. We can also use this information to improve the reliability of the lead time. For instance, if the variance of lead time is 0, then variance of the u which is demand of the lead time demand during the lead time will be 444.9. Sigma u will be then square root of this. Safety stock is worked out to 29 as compared to the 330 value that we had earlier. We have actually reduced the variance of the lead time to zero and r would be equal to 622 which is compared to the 923 value earlier and as a consequence of this change, that means if we make the assumption, this gives you an idea of the amount of savings you can make if your information is correct. If the variance lead time is 0, that means you are sure about your lead time there is no uncertainty. Then you can have a safety r value that can go down to 622. Inventory is lowered by 301 units. Annual savings are rupees 301, thus it is worthwhile to improve the reliability of the lead time.

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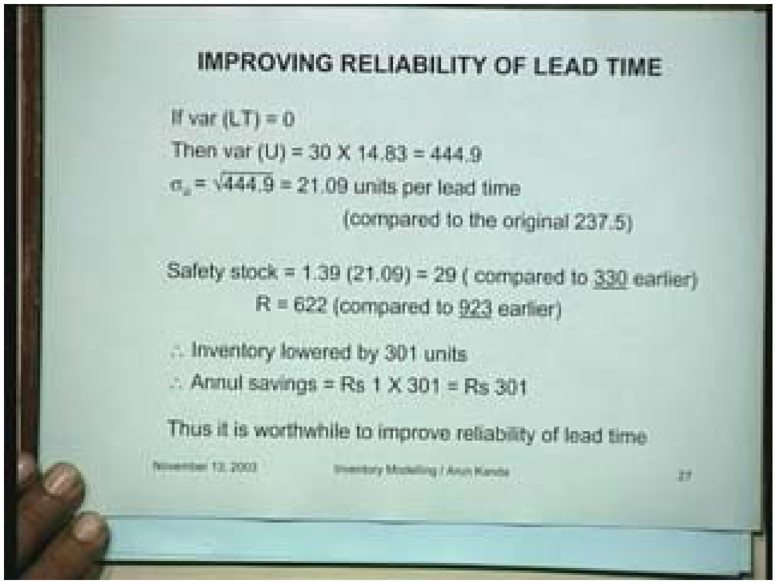

That is what this example actually shows. If you were uncertain about lead time you have to keep a higher level of reorder point. If you are sure about the lead time you can cut down the amount of value of the reorder level and thereby have costs. In quantitative terms you get this estimate. Let us quickly look at some of these inventory control policies. As I indicated to you one kind of policy is a continuous review system where you are constantly monitoring the stock and then whenever an inventory reaches a certain level S, you are placing an order Q. The Q is a fixed quantity. In the SS policy what is done is that whenever the inventory becomes equal to or less than s, you then order up to a level S. That means the order quantity in the s s policy will be varying. It will not be a fixed order. You will try to bring the stock up to the maximum level S. That is the difference between the SQ and the SS policy. Essentially both of them are continuous review policies. Generally a more convenient policy is a periodic review policy. A periodic review policy would mean that you have a policy of the type SR or a policy of the type SSR and you are not reviewing the system continuously.

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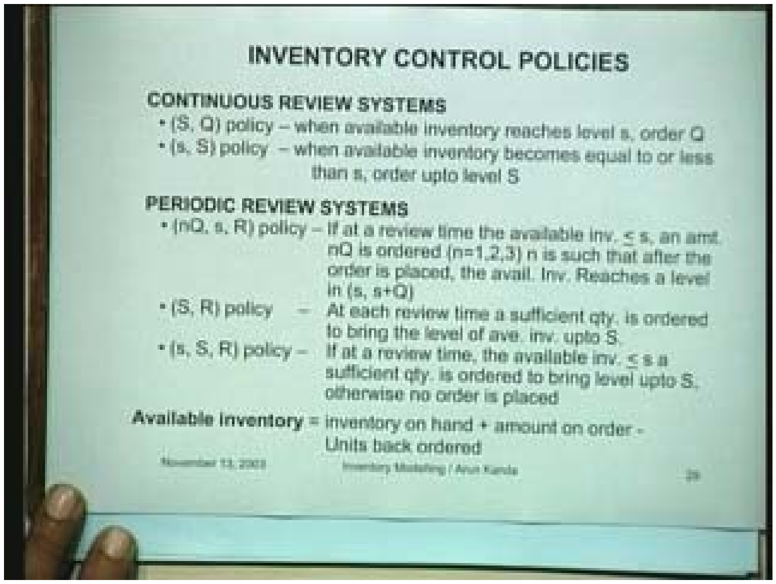

You are trying to review the system periodically every period. Finally let us conclude and see what we have tried to do in this particular lecture. We have seen the EOQ formula with sensitivity analysis which was discussed, various variants of the lot size formula for the different situations and their examples with back logging, without back logging for both the purchase and the production situations were considered, and quantity discounts were explored.

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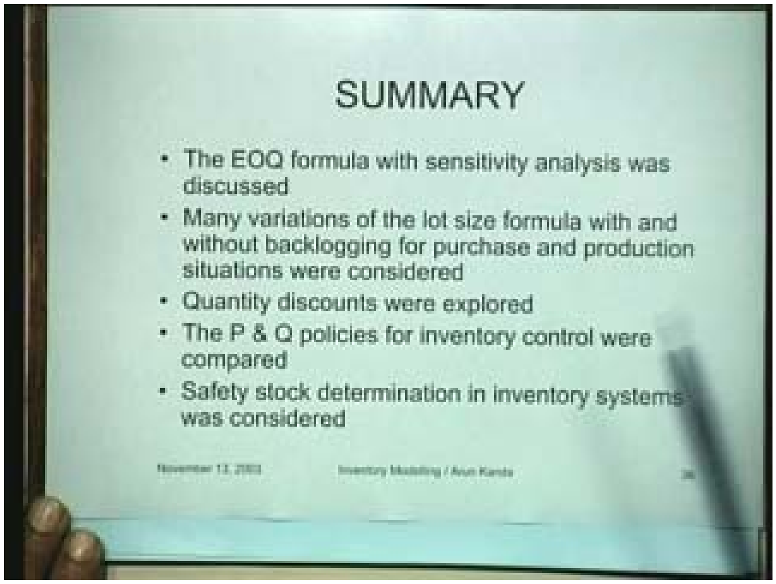

How to find out the optimum EOQ levels in this case was explored. The P and Q policies for inventory control were compared, continuous review and periodic review policies, what they are were discussed. Finally we had a look at methods of determining the safety stock in inventory systems. I think we will conclude here.

Thank you very much!