Project and Production Management
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Lecture - 37
Aggregate Production Planning: Modeling Approaches

In our last lecture on aggregate production we had looked at some of the basic concepts involved in aggregate production planning and we had looked at heuristic procedures like the graphical procedure to identify the best production plan. We had also looked at the procedure popularly k known as linear decision rules in which costs were assumed to be quadratic and when you differentiate the cost you would land up with linear decision rules. Today we will look at some modeling approaches to the problem of aggregate production planning and see how the best production plan can be determined in various circumstances. Broadly speaking we will look at various solution techniques in today's lecture. For instance the production costs, the costs of holding inventories are linear then you can use linear programming and also the transportation for solving the problem, an ex10sion of the linear cost case is the situation where we have piecewise linear and convex costs. In fact we had a look at Holt, Modigliani, Muth and Simon's linear decision rules last time that was actually a case when costs were convex in nature. However the costs are piecewise linear and convex, then in such a situation the transportation model can be used very effectively as we shall soon see through an example and the case when concave or arbitrary in nature. For this particular situation you can get either non linear programming techniques or there is a very efficient procedure based on determining the shortest path in a network that can be very effective in determining the solution.

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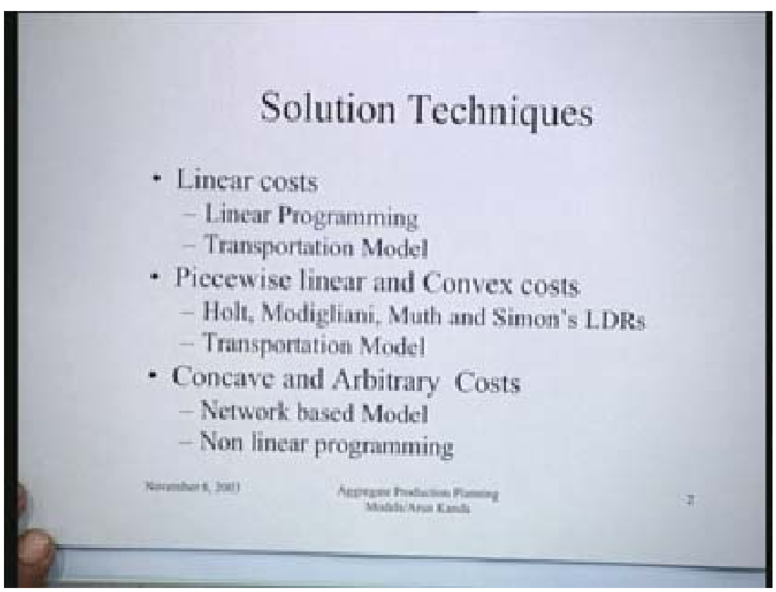

We shall look at the category of solution techniques which look at this particular problem for instance take the first case, if you take a linear programming formulation of the problem then a typical linear programming formulation of the problem will begin with definition of the variables. Typically in an aggregate production planning problem we defined variables like r and v which is nothing but the cost per unit produced during regular time and overtime, so r stands for cost per unit if the item is produced on regular time. v stands for the case if it is produced in overtime, typically what we are assuming is that v would be greater than r, similarly Pt and Ot are nothing but the number of units produced during regular time and over time respectively. So production in regular time Pt production in overtime Ot, then we have the hiring and firing costs, you can lay off people and you can hire new people, so H is the hiring cost and f is the lay off cost per unit respectively. Capital At and Rt are the number of units increased or decreased respectively during consecutive periods which mean that if your production was let us say 200 in the first you would make it either 250, the next period in which case there is an increase or you could cut it down to 180 in which case there is a decrease. So these variables capture the fact whether there is an increase or decrease in production. C is the inventory cost per unit per unit per period, Dt is the sales forecast of demand in period t and capital Mt and Y t are the available regular time and overtime capacities respectively.

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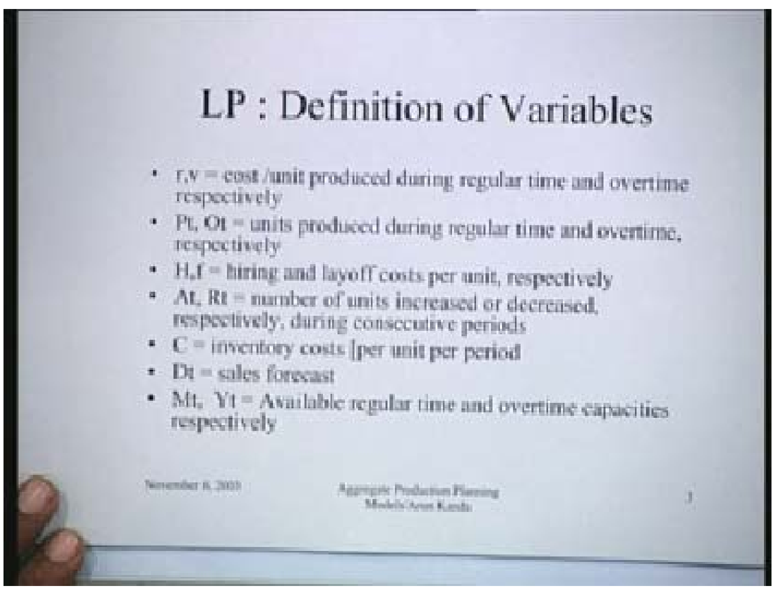

This would be the kind of data that you would need when you are trying to model the problem using linear programming. Let us then develop the linear programming objective function. Basically what we are interested in saying is that we want to determine the production plan such that we minimize the total cost C which is the cost of production, that is summation Pt over t. This is the cost of production multiplied with; this is the unit cost, so quantity produced multiplied with cost per unit on regular time.

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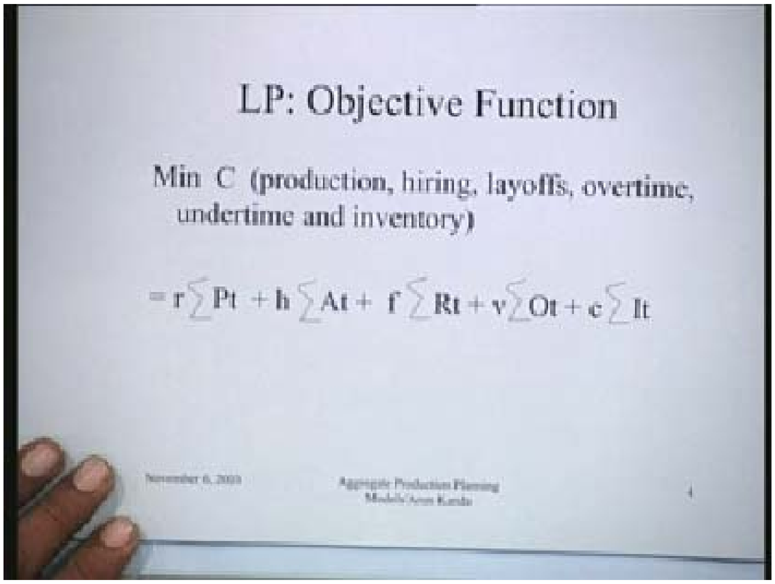

Then we have h into At, so this is the quantity, this is the hiring cost h. Similarly you have the firing cost which is dependent, if there is an increase or a decrease. So if this is the amount of increase, the hiring cost is proportional to the amount of increase of production. Firing cost is similarly proportional to the decrease in the production in the t th period and similarly you have this is the production and over time. V is the unit cost of production and over time and C is the unit cost and It is the inventory. So this is the inventory related cost. Essentially speaking the objective function would look something like this. Now to complete the problem you need to add constraints and for this particular problem typically there will be 5 sets of constraints which are shown here which would show that the production in period t must be less than or equal to the capacity in period t. This would be valid for each time period. Over time production in period t would be less than equal to the over time production capacity in period t. This is a constraint which is like an inventory balance equation, that the inventory at the end of the t th period will be equal to the inventory at the end of the t – 1 th period + the production in period t + the over time minus the demand. This is just an inventory balance equation and here of course we are saying that depending upon whether this is production increase Pt – Pt – 1. This is captured by the variable At and if it is a decrease then this is captured by the variable Rt because all the variables are non negative.

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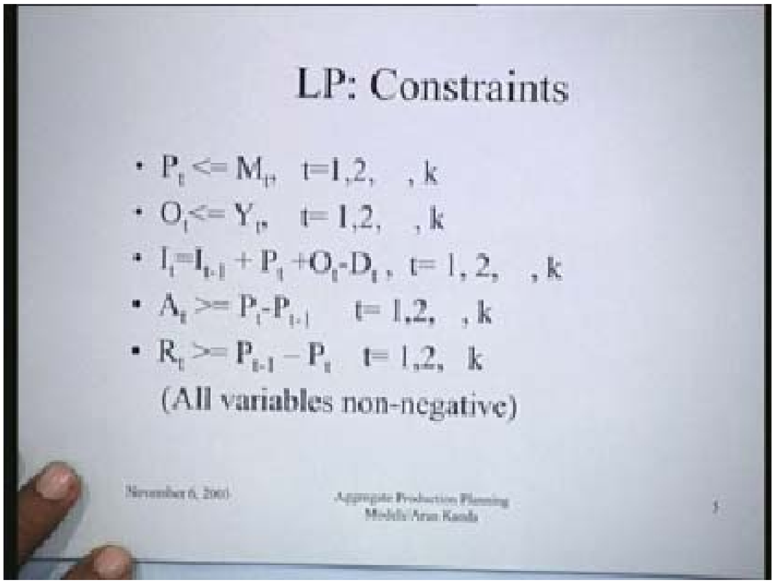

Using this objective function and the constraints we have defined, you can actually set up the problem as an Lp and use any standard Lp code for solving the problem. The Lp formulation is very versatile because depending upon the nature of your cost you can include other types of costs like set up costs and so on and still be able to handle the whole problem through a linear programming formulation. The second formulation that we are going to study is the formulation of problem of aggregate production planning as a transportation model.

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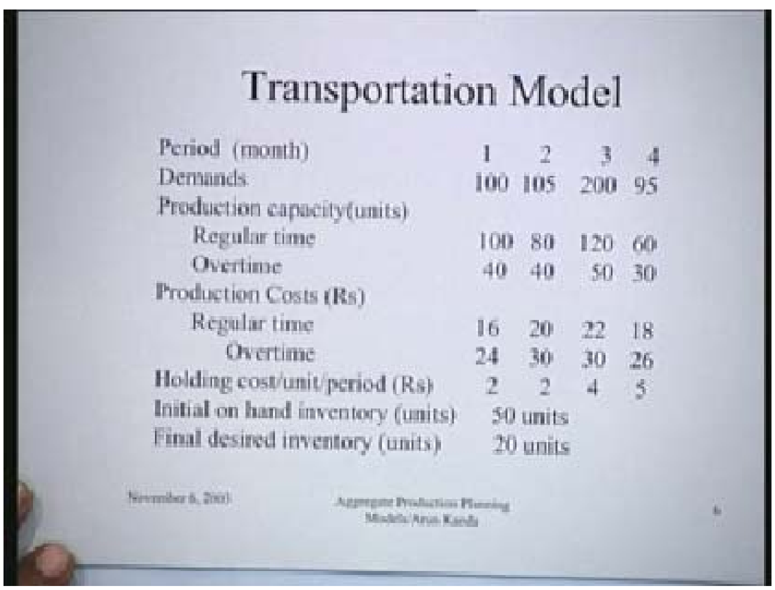

This is a very interesting model because it becomes very easy to solve this problem and then you can get the optimal solution simply by almost by inspection. We will consider the case where there are no shortages to being with. There are no shortages, so we would have the data for the problem in the form of let us take A_4 month horizon. There are 4 months 1, 2, 3 and 4. Let us say the forecast of demand during these 4 months are 100, 105, 200 and 95, so demand is obviously fluctuating and we have these forecasts of demand. The regular time capacity and the over time capacity is given to us, for instance in number of units we can produce 100 items on regular time and 40 on over time, 80 on regular time, 40 on overtime in the next period, 120 on regular time, 50 on over time, 60 regular time, 30 on overtime. Someone might ask as to why the regular capacity keeps changing with periods. It could be changed. It need not change but what could happen is that in a general situation you might have committed your capacities to other products or to other things or to other customers. This is the amount of capacity that is available to you on regular time and similarly for over time, production costs on regular time are 16, 20, 22 and 18. What we are assuming here is that production costs can vary with time. Normally you might expect them to increase but they might crash in a certain period also for various reasons depending upon the economy, depending upon the state of business, similarly in over time you have costs that are higher than this 24, 30, 30, 26 typically higher.

When it is higher what it means is that the regular time and over time cost, the total cost is actually a convex function piecewise linear, convex function because you have a cost slope of 16 followed by 24. It is a typically that is the behavior that you have here. Holding cost per unit per period is 2, 2, 4, 5. This means essentially that it costs you 2 rupees to hold an item in inventory in the first period. Similarly it costs you 2 rupees to hold an item in the second period and then in the fourth and the fifth period in the third and the fourth period respectively the costs are 4 and 5. The initial on hand inventory is 50 because this is a legacy of the past. In the past planning horizon you have 50 units left, that is what it means and the final desired inventory is 20 units. This is what you desire at the end for one to have this. This is the nature of the problem data that you would have before you try to solve the problem using a transportation model. Let us see how this problem can be solved by using the transportation model. The first thing that you do in a transportation model is to set up the supplies and the demands, the suppliers and the demands that you have the various situations. So in setting up this transportation problem what are our sources? Where do we get this supply from? We get supply from the initial inventory. So we have I_{in} as the source, regular time in the first period, over time in the first period, regular time in the second period, over time in the second period, regular time in the third period, over time in the third period, regular time in the fourth period, over time in the fourth period. These become the sources of supply and we note down here as to what is the quantum of supply available in these various sources? What you notice is that initial inventory is 50 units. We write that 50 units here. The production on regular time in the first period is 100. So we put that here. The production in over time in the first period is 40. So we put that down here and similarly regular time production in the second period, over time production in the second period and so on. We have these figures. Now as far as the demands are concerned the demands are nothing but the forecast of demand for the various periods. When you look at the demands, the demands in the first period are 100. Then it is 105. Then it is 200, then it is 95 and you would require that the final inventory should be 20. If you now sum up the column you find that the total is 570.

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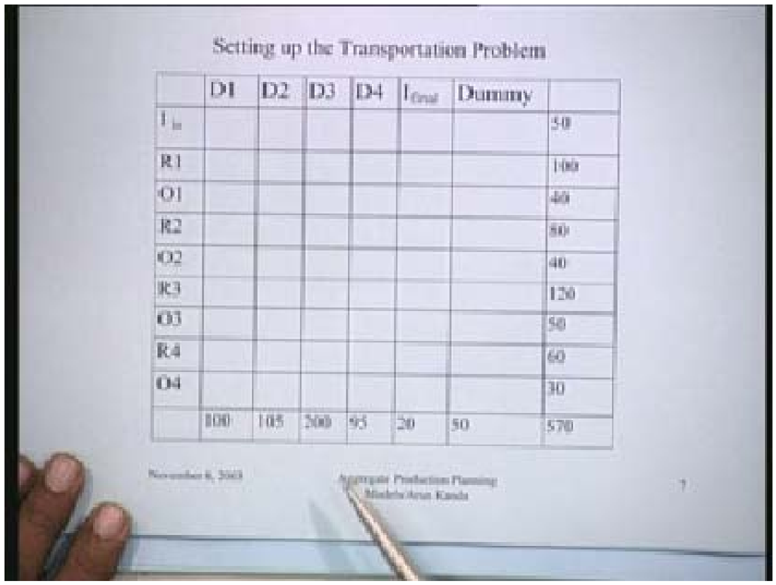

If you sum up the row you find that the total is actually 520. You need a dummy column. So I add a dummy column here and the capacity of this dummy column will be 50, so that now it becomes a balanced transportation graph. This is the first step that you have to do in setting up the transportation model. Set up the sources, set up the demands, note the sources of supply, note the demands, and add a dummy, total the dummy which incidentally will give me the unutilized capacity. You can also interpret the dummy as the unutilized capacity; you have more capacity than what you need. So obviously after satisfying the demand you will always have some capacity which is unutilized and that will be shown in the dummy. So this is the first step that you do when you are trying to set up the transportation problem. Having done this, the next thing that we need to do is to add the costs. How do you add the costs? Typically you will find that if something that you have in inventory is immediately sold in the first period, there is no cost associated so, 0. However if that quantity is being sold in the next period you have incurred the holding cost for the first period which is 2 rupees. So this becomes 2. For this particular period you have incurred the holding cost of this period which is 2 rupees again.

So you get 8, similarly if you have to store it up here in the fifth period then the corresponding costs will have to be added here. You will get this particular figure here in terms of this. If it is produced, if it is consumed in the first period in the second period in the third period in the fourth period and so on you will find that the cost will 10d to increase progressively this way because of the increased holding costs. Now look at this regular time and over time production. What you find is that regular time production is 16, over time production is 24, this is the unit cost. So we put them here. Similarly the regular time production cost and the over time production cost in the second period are

20 and 30. So we put them down here. For the third case this is 22 and 30 and for the fourth case this is 18 and 26. These are the production costs if the demand in a particular period is satisfied by a production either on regular time or on over time in that very period. However what may happen is that you might want to produce the first period but you would consume in the second period. So you carry the holding cost of 2 rupees, and carry the holding cost of 4 rupees, you carry the holding cost of 5 rupees. Progressively the costs are increasing depending upon the nature of the holding cost and similarly here similarly here similarly here similarly here similarly here and so on.

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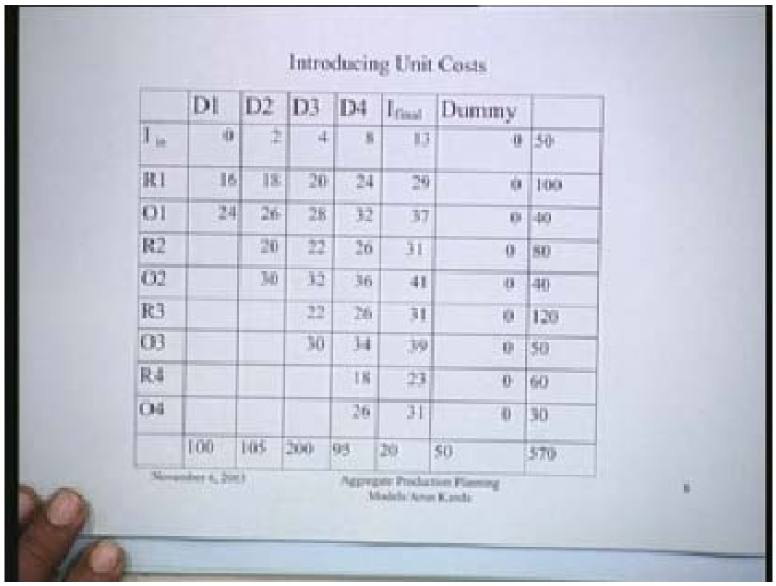
Introducing Unit Costs

	D1	D2	D3	D4	I_{final}	Dummy	
I_{in}	0	2	4	8	13	0	50
R1	16	18	20	24	29	0	100
O1	24	26	28	32	37	0	40
R2		20	22	26	31	0	80
O2		30	32	36	41	0	40
R3			22	26	31	0	120
O3			30	34	39	0	50
R4				18	23	0	60
O4				26	31	0	30
	100	105	200	95	20	50	370

This is the procedure by means of which you can calculate and put down in the transportation tableau, the cost of production and holding inventories as shown here. Since we are not allowing any shortages to take place what does it mean? It means that there will be no entry in this box. There will be no entry in this box. Similarly why will there be no entry in this box? This box will not contain any particular item at all. Why will this box be empty? Because what it implies is that if you produce in the second period you can satisfy the demand for the second period, third period, fourth period but you cannot satisfy the demand of the first period with the production of the second period. That is why this particular block will be empty. Similarly these 2 blocks will be empty here because production in the third period can satisfy the demand in the third period and fourth period but not in the second and the first period and similarly here the production in the fourth period can satisfy demand only in the fourth period and beyond but not in these periods. The structure of this matrix is upper diagonal and this portion below the diagonal will have 0 costs. It will not permit any entries in that particular cost because we are not allowing any shortages to take place.

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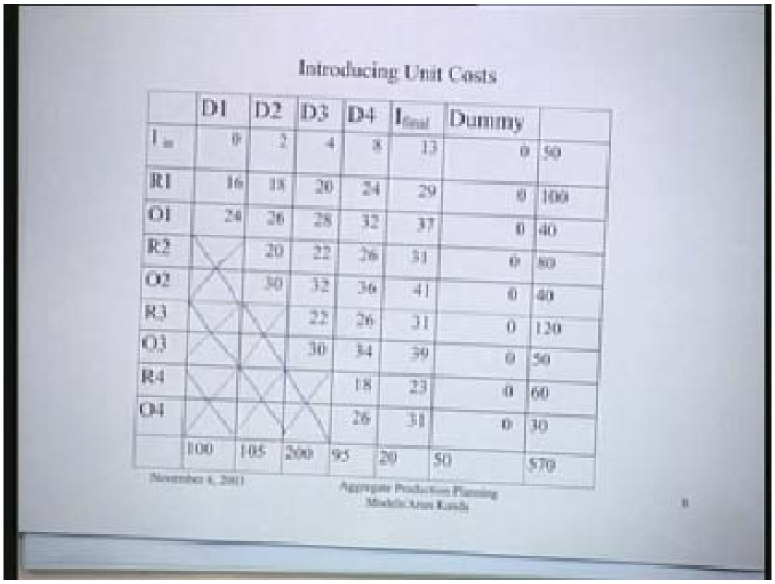

	D1	D2	D3	D4	I_{final}	Dummy	
I_{in}	0	2	4	8	13	0	50
R1	16	18	20	24	29	0	100
O1	24	26	28	32	37	0	40
R2		20	22	26	31	0	80
O2		30	32	36	41	0	40
R3			22	26	31	0	120
O3			30	34	39	0	50
R4				18	23	0	60
O4				26	31	0	30
	100	105	200	95	20	50	570

You can use the net present value also but here we are not taking into consideration the net present value. You see it all depends upon what kind of interpretation you want to give to the cost. If you feel that the periods are not too many, this is after all only a 6 month horizon or A_4 month horizon. In that case you feel that time discounting will not make any significant difference, and then you can take the costs as they are. However if you feel that they would make a significant difference and there is nothing to prevent us from taking the discounted values of these costs and working in exactly the same manner. The choice is yours. Once we have set up this transportation model we want to solve this problem to get the production plan. A solution to this model fortunately can be obtained by a very simple algorithm for want of a better word, can be called the greedy algorithm. Otherwise of course you can solve the transportation problem by using the vowel's approximation method and then using the usual methods of solving a transportation model or of that, but that is not necessary in this particular case because of the special structure of the problem.

We will see how the problem can be solved in this particular case. What we do is in trying to solve this problem we start with the first period demand. For the first period and we satisfy the first period demand entirely. That means this 100 units of demand must be satisfied by taking the least cost sources, for instance the costs are 0, 16 and 24. What you do is, you try to allocate the maximum here. When this is exhausted you go to this value, try to allocate the maximum here till 100 is exhausted. So let us look at the first column, what can we do? Here the maximum that can be allocated is the minimum of 50 and 100. We can allocate 50 here and then this is 0. We have satisfied the demand for the first row in this case but there are 50 units of unsatisfied demand here and those 50 units of unsatisfied demand here comes to the next lowest cost which is 16 rupees and once you do that this figure of 100 just becomes 50. 50 is the unutilized demand. We have satisfied the demand for the first period, that means this demand 100 is fully taken care of and we have updated the availabilities in the other sources.

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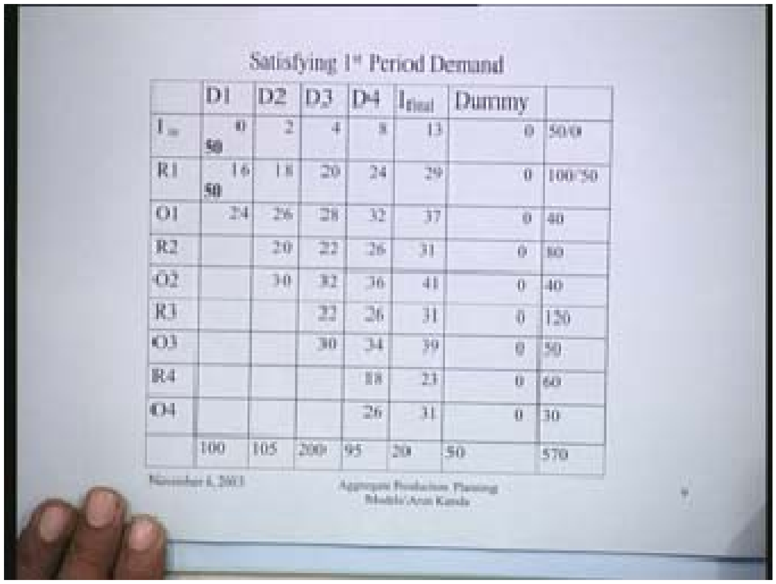

We have done it for the first period. We will have to do something similar for other periods in succession and this is a very simple procedure. You have to look for the cheapest source available and try to allocate the demands for that particular source. What we can do is in the second period. How do we satisfy the demand for the second period now? We again look at this but what happens is 2 is the cheapest but 2 is not available. The next cheapest happens to be 18. 50 units are available here, so we allocate 50 units here. Nothing remains here and then move to the next cheapest source and the next cheapest source after 18 is 20, so in 20 the balance 55 can be allocated to this particular cell and in which case after 55 units are allocated, only 25 units will remain as far as the availability of R2 is concerned. We have been able to now satisfy the demand for the second column or the second period. This 105 is taken care of and we have updated the whole procedure.

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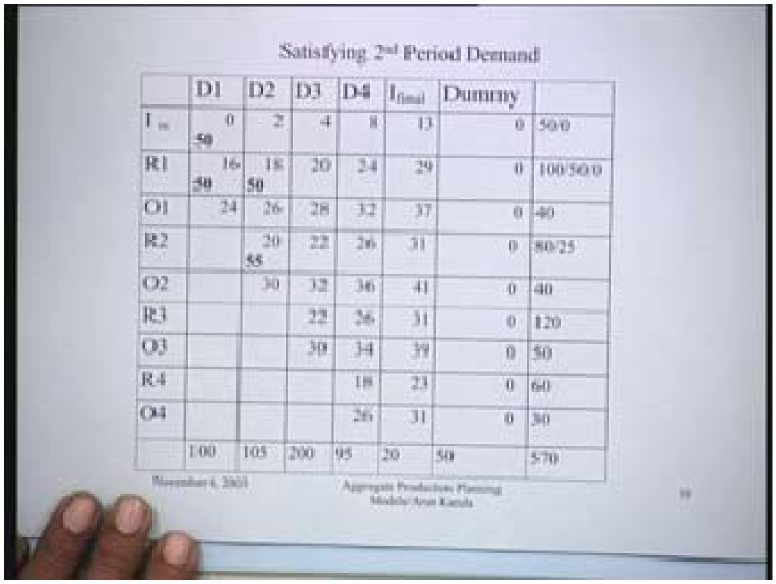

We will continue in this fashion, when you look at the third period, we look at the third period demand and what we find is again you find that 4 is not available. Similarly 20 is not available and after 20 the cheapest value that you have is 22. So 22 is available at 2 locations. 25 units can be taken off from here and similarly 120 units can be taken off from here and making this 0. What we have is actually 145 units have been allocated, 55 more units need to be allocated, so you try to find out which is the next one. After 22 the next cheapest source is 28. In 28, 40 units are available. We put 40 there and bring it down to 0 and then of course after 28, the next cheapest unit available is 30and we need only 15 here to satisfy the entire demand for 200. This now remains at 35. We have been able to satisfy the demand for the third unit in this way by making these 4 allocations.

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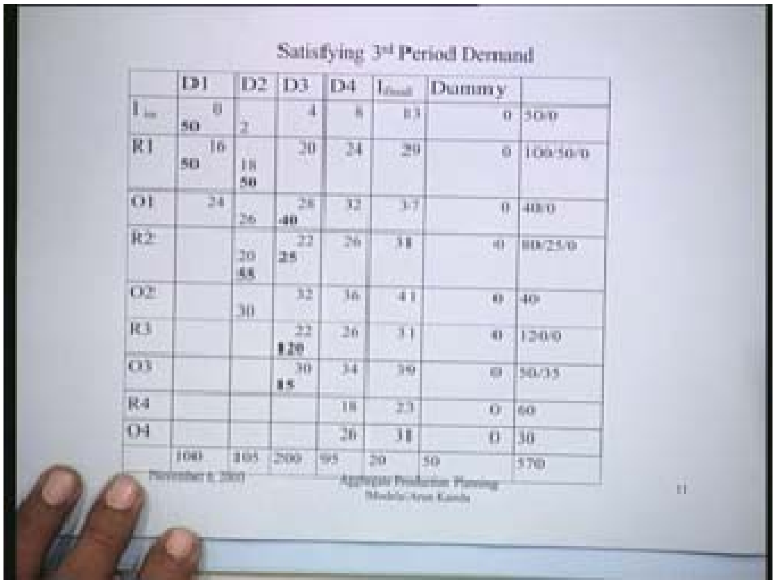

	D1	D2	D3	D4	I_{final}	Dummy	
I_{in}	0 50	2	4	8	13	0	50/0
R1	16 50	18 50	20	24	29	0	100/50/0
O1	24	26	28 40	32	37	0	40/0
R2		20 55	22 25	26	31	0	80/25/0
O2		30	32	36	41	0	40
R3			22 120	26	31	0	120/0
O3			30 15	34	39	0	50/35
R4				18	23	0	60
O4				26	31	0	30
	100	105	200	95	20	50	570

Let us go to the fourth period similarly, when you go to the fourth period you find that you can do the allocations in three cells corresponding to cost of 18, 26, 34 because the other lower cost values are not available and this leads to 95 units in this particular fourth period.

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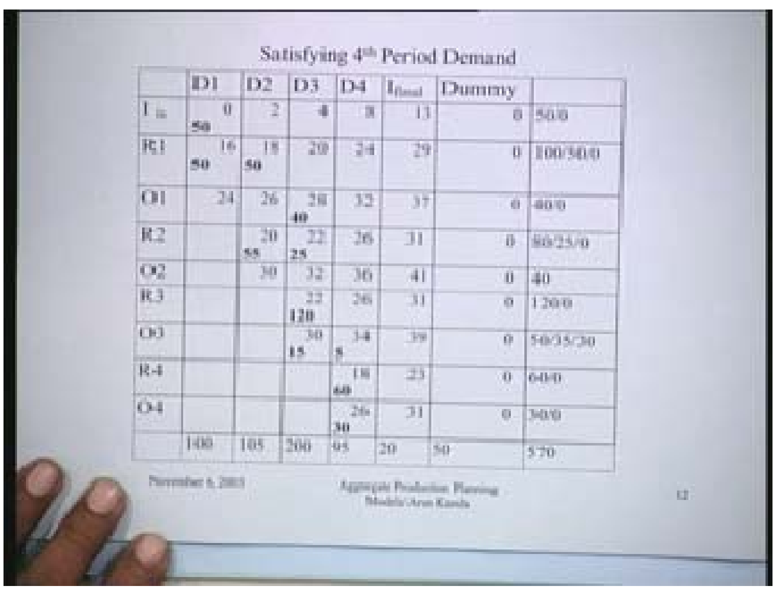

	D1	D2	D3	D4	I_{final}	Dummy	
I_{in}	0 50	2	4	8	13	0	50/0
R1	16 50	18 50	20	24	29	0	100/50/0
O1	24	26	28 40	32	37	0	40/0
R2		20 55	22 25	26	31	0	80/25/0
O2		30	32	36	41	0	40
R3			22 120	26	31	0	120/0
O3			30 15	34 5	39	0	50/35/30
R-4				18 60	23	0	60/0
O4				26 30	31	0	30/0
	100	105	200	95	20	50	570

We have allocations of 5, 60 and 30 in the fourth period. The way we calculated these figures, cost figure that is of course into production + inventory. That is what you have in stock and if you want to utilize that stock then progressively the holding cost will keep on increasing but the same thing will be true of what you produce and then subsequently as you go on the inventory costs will be added on to that. Now the only item that remains to be allocated is the final inventory restrictions I final. Here only 20 units are to be allocated which go into this particular cell and you have this particular solution.

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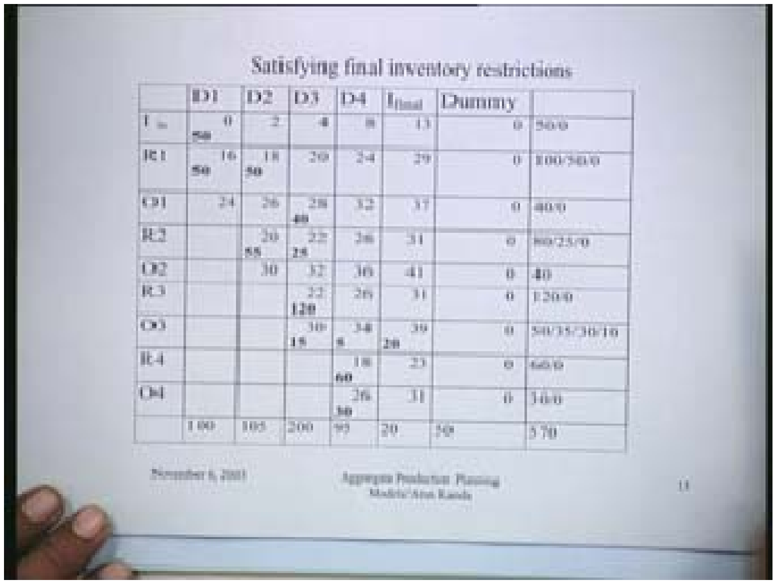

	D1	D2	D3	D4	I_{final}	Dummy	
I_{in}	0 **50**	2	4	8	13	0	50/0
R1	16 **50**	18 **50**	20	24	29	0	100/50/0
O1	24	26	28 **40**	32	37	0	40/0
R2		20 **55**	22 **25**	26	31	0	80/25/0
O2		30	32	36	41	0	40
R3			22 **120**	26	31	0	120/0
O3			30 **15**	34 **5**	39 **20**	0	50/35/30/10
R4				18 **60**	23	0	60/0
O4				26 **30**	31	0	30/0
	100	105	200	95	20	50	370

The next thing is the allocations to the dummy. Those are trivial because what would happen is that whatever is remaining here will come here. For instance you find that this at this particular point of time something is remaining. It comes here and then similarly here also something is remaining. Once this 40 is done, it then becomes 0 and similarly this 10 gets allocated there and once it is allocated nothing remains there. Ultimately nothing remains as far as sources are concerned and the amounts in bold in individual cell defines the complete solution to the production planning problem. Let us see what the solution looks like and what is the total cost. The total cost for this solution will be 50 into 0 + 50 into 16 + 50 into 18 + 40 into 20and so on and summed up over all the values cij Xij summation overall ij. You will have this particular thing, so you find that the total cost of the optimal solution is the sum of the unit cost and the quantity which works out to 10,000 370 for the planning horizon and we have a guarantee that there is no solution better than this available for this particular. It is an optimal solution and this includes the costs of production on regular and over time and the cost of holding inventories overall.

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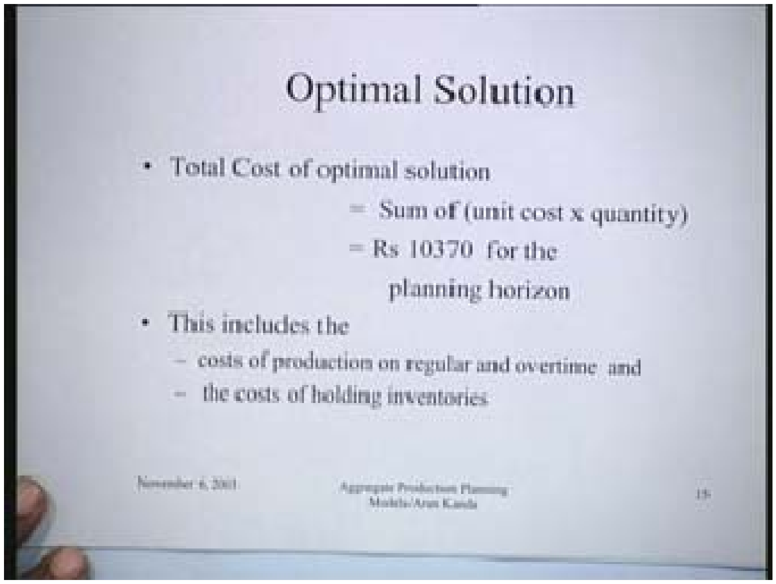

So you are assured of an optimal solution, it is not a heuristic anyway and from the transportation tableau you can work out exactly a production plan. You can see for instance that the regular time production in the first period will be 100. The regular time production in the second period will be 80. In the third period it is 120 and in the fourth period it is 60 and as far as the over time production is concerned it is 40 in this period, there is no over time production. So it is – 40, 40 was the capacity. We are not producing anything on overtime in the second period and similarly the overtime capacity in this particular period was 50 but we are producing only 40. We are not utilizing capacity 10 in the as far as the over time is concerned and finally in the fourth period we are producing 30 units in over time.

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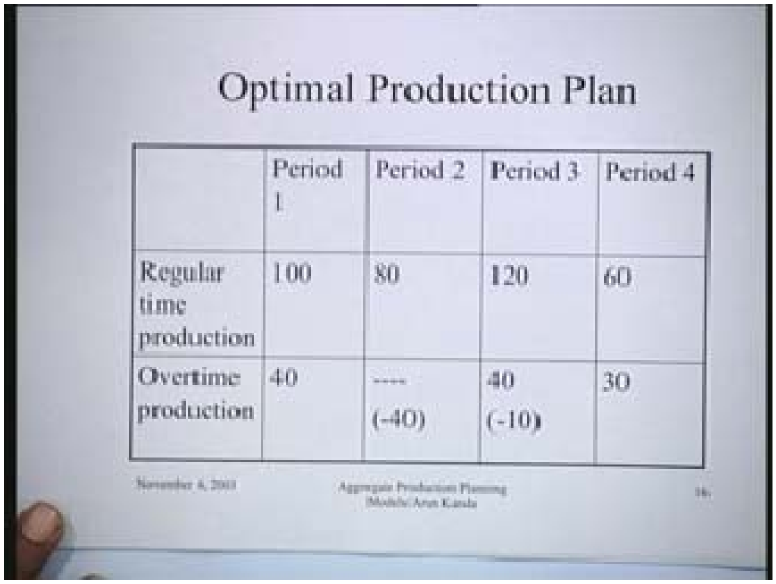

Optimal Production Plan

	Period 1	Period 2	Period 3	Period 4
Regular time production	100	80	120	60
Overtime production	40	---- (-40)	40 (-10)	30

This in fact specifies what is to be produced on regular time and over time in each period. This is the optimal aggregate production plan. In fact there is a theorem which proves this and I will give you the essence of the theorem. The essence of the theorem is that in this particular situation, since there are no entries here at all, the least cost solution will be the smallest possible, cheapest possible way to satisfy this demand. There is no other if you look at the demand for the first period. Once you have satisfied this demand and you update the demands again, then this becomes effectively the first period again and this is then the cheapest way to solve this particular problem and so on.

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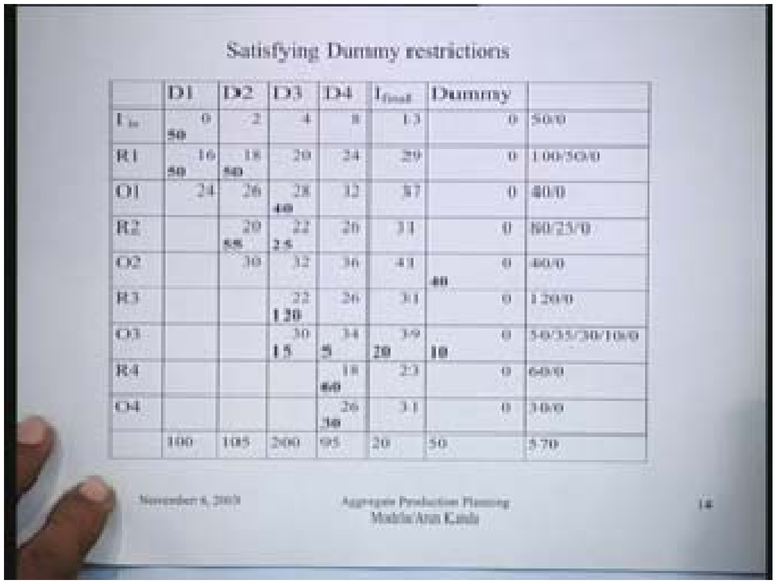

If you go on recursively you prove that in fact you have got an optimal solution, so the optimal production plan in this particular case looks like this. It specifies how much to produce in regular time and how much Pt is produced in overtime and that is incidentally what the objective aggregate production plan is. I must tell you how much I should produce in different periods such that I meet my demands at the minimum cost, so that is exactly what we have achieved through this particular case. Now the case that we considered that is the transportation model was actually valid when the costs piecewise linear and convex because what we are saying is that you will first pick up the regular time source. Only when regular time is exhausted, will you go to the over time source because the cost structure is like that. But yes I think we can also make another comment on the transportation model that we just studied, for instance what may happen is as look at this transportation model and the solution that we have obtained, we have now proved that it is an optimal solution. But what may happen in a problem for instance which has shortages, it means that in this box here you can have an entry of cost.

Why because you can produce in the second period and use the demand or the production at the second period to satisfy the demand in the first period because there is a shortage in the first period and there is a cost of back ordering in the first period. If it costs you 20 rupees now and suppose I take a shortage cost of let us say 5 rupees per period then what it means is that this item here is we utilize this will be 25 cost and this will be 35. Similarly here this will be 27; this will be 32 and so on. What you will have is that if you allow shortages to take place, the transportation tableau will be full and the interesting thing about the tableau is that on the diagonal it is production in the period and consumption in the same period. This side you have production in a previous period and consumption later. These are inventory costs, holding costs added to production and on this side of the diagonal, you will have production costs and shortage costs. So the structure of the cost will be that on the diagonal production costs. On the other hand, you will have production + inventory cost like you have here and on the other side; you have production + shortage costs. So that is the structure of the problem with shortages that is dependent primarily upon how you value your customs. For instance if there is a company which probably values its customers more too each backorder, it will give a cost of 100. If there is a company which does not value its customers more it might give a cost of only 5, that means I do not care for the customers. The cost is much smaller. So it is actually a subjective assessment of how you value your customers which is what the back ordering cost should be. It is in that context you have some values put in here. Once this is full, you cannot use the greedy algorithm anymore because it will not give you an optimum solution. You can still use a greedy algorithm but it will not give you an optimal solution. What you have to is you have to solve this problem as a routine transportation problem, use the transportation tableu or vowel's approximation method or use any package for solving the transportation model and get a solution. Even then you have formulated the problem and it can be solved either regular transportation problem, that is what it is. Now let us look at the case when the cost structure is not piecewise linear, that means we are talking about arbitrary costs.

We have considered situations where for instance the holding cost or any other cost is linear like this. Positive inventory holding cost is linear and this is back ordering cost which is also linear. Here these were arbitrary costs because it is going up and it is coming down. So it is neither concave nor convex. If you have a general cost structure like this or if you have set up costs, the cost of set up in a certain period is so much and thereafter the cost exhibit economies of scale. Economies of scale are that the slope of this line will be the cost per unit. So initial pieces are costly to produce, as you produce in larger quantities cost per piece is small. So the cost function will be concave with setups.

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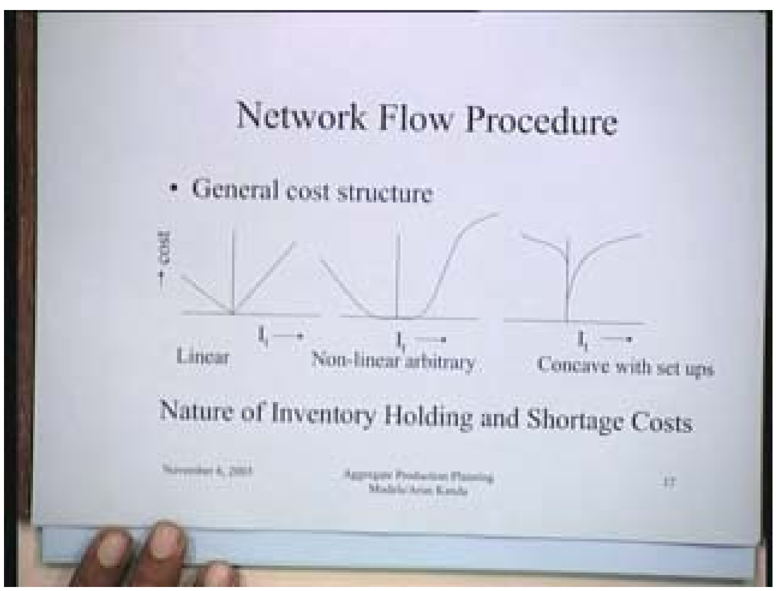

We shall now consider a case as to how we can solve this problem of aggregate production planning with arbitrary costs. They can be concave with setups and non linear or even linear for that matter. In fact if we look at the variation of production cost, if the cost of production is piecewise linear but it is like this is like regular time production, over time production and subcontracting. The costs would be going up but these are now piecewise linear concave with set up. You set a set up cost and then these are of this nature.

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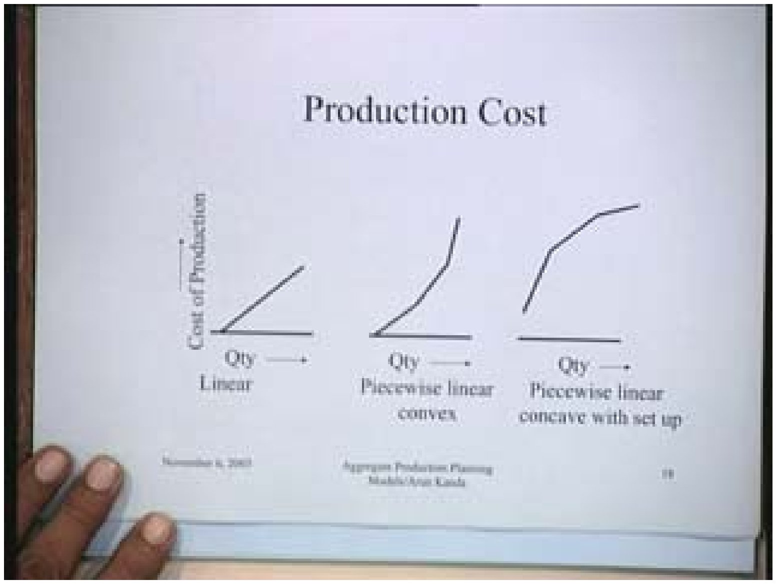

Production costs can also be of this nature and for defining our problem, let us consider a situation where we are dealing with 4 periods as before. t = 1, 2, 3, 4 and the production cost in period t involves a set up cost a t and a unit variable cost Ct which is for raw materials, utilities, labor etc because labor costs, utilities and raw material costs are generally dependent upon the quantity of material produced. The quantity of raw material produced for instance if you are talking about car manufacture, the total amount of steel per car is let us say X, total amount of steel 50 cars will be 50X. That is what we are assuming. So in this case if you have a set up cost we will have Ct Xt which is let us say the cost of production which is equal to At + Ct Xt, if Xt is greater than 0 and is equal to 0 if Xt is equal to 0. This makes the cost function discontinuous.

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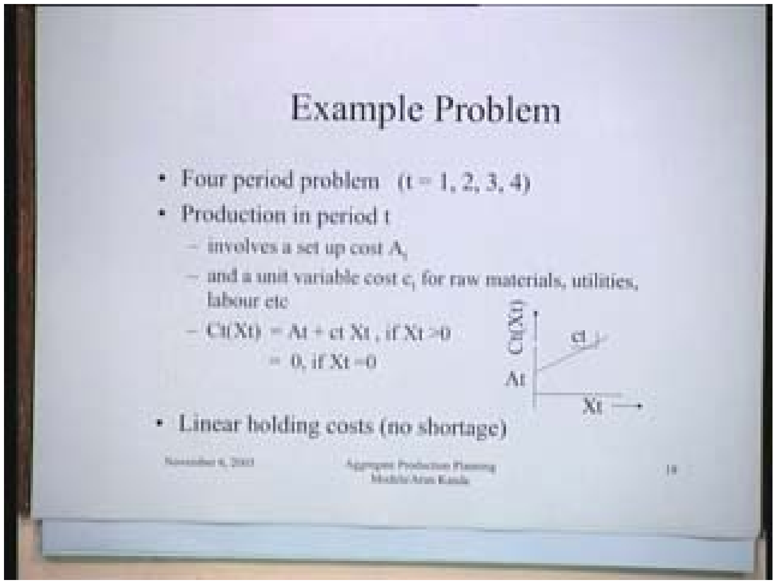

What we have is 0 here if you are not producing anything but if you are producing certain quantity then it suddenly jumps here and you have At + Ct Xt, so this is a set up situation and we assume that let us say the linear holding costs are there and we are assuming to begin with that there are no shortages. We need some data for the problem, so we take some data for the problem. This will be the data that we use for solving the general problem. We are saying for instance that demands in periods 1, 2, 3, 4 are 1025 and 15, so they exhibit the same type of character. The set up cost in different periods could be different because of inflationary factors or other factors that are there 100, 120, 120, and 40. The variable cost per unit can also vary 8 rupees, 9 rupees, 10 rupees, 10 rupees different periods and the holding cost can be 2, 4, 5 and 7 different.

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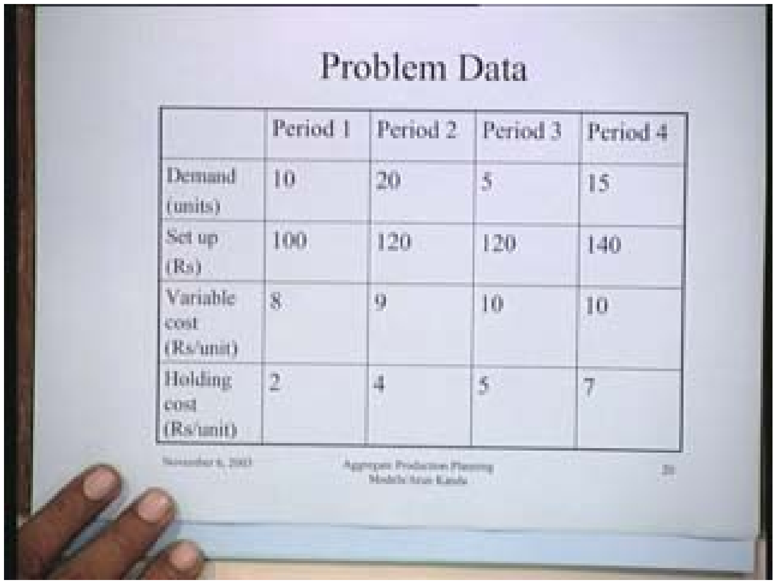

Problem Data

	Period 1	Period 2	Period 3	Period 4
Demand (units)	10	20	5	15
Set up (Rs)	100	120	120	140
Variable cost (Rs/unit)	8	9	10	10
Holding cost (Rs/unit)	2	4	5	7

We will work the problem with this data for purposes of illustrating what we have to do, the first step in the problem is to set up a network and how you set up the network is actually pretty straight forward. That is if you have n periods for which you are planning the problem then there will be n + 1 nodes and exactly nC2 that is n into n + 1/2 arcs with n emanating from node 0 n – 1 emanating from node 1 and ultimately 1 emanating from node n – 1. If you take A_4 period problem, we have node 0, node 1, node 2, and node 4 and from here we take all possible arcs. We have node to 0 arc from 0 to 1, from 0 to 2 from 0 to 3, from 0 to 4, so there are 4 arcs emanating from node 0. That is what we said that in general there will be n arcs emanating from node 0. Similarly there are only three arcs emanating from this node 1, 2, 3 and from 2 there are only 2 arcs. 1, 2 and from this there is only 1 arc emanating from this particular case. In this particular case there are only 10 arcs in the problem, n that is you have 4 into 5/2 and you count the number of arcs, you have only 10 arcs in the problem.

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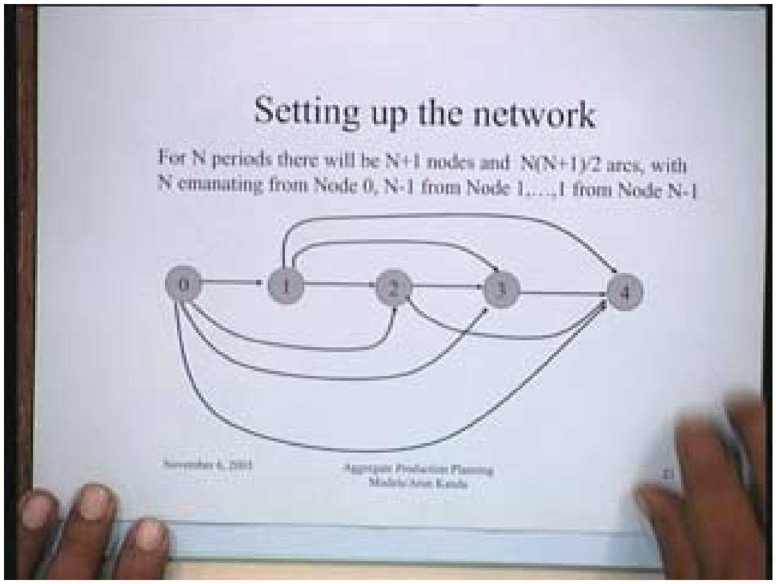

Let us give an interpretation to these arcs. An arc going from node j to node k where obviously j is strictly less than k in the network, in the numbering scheme is defined as Mjk which is the length of the arc. This is the cost of producing in period j + 1 for the requirements of the periods j + 1, j + 2 and so on up to k.

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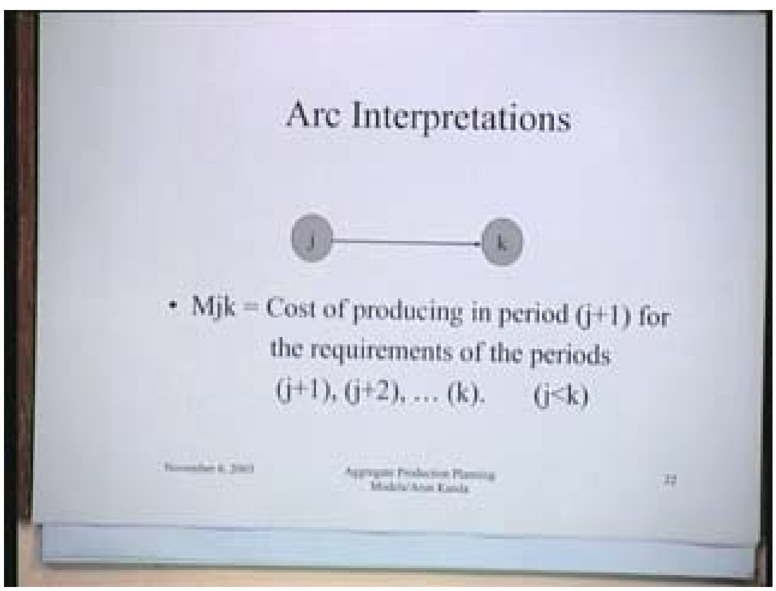

This is a very fundamental definition which you will have to use again and again to actually work out the costs. We are talking about the planning horizon j + 1, j + 2 and so on up to k, for instance if it is going from node 3 to node 5 what we are saying is from between on the periods 4, 5, that is what it is. This planning horizon is from 4 to 5, so

Mjk is the cost of producing in period j + 1for the requirements of the period j + 1, j + 2 and so on up to k. This is the interpretation, so what we do is we can do the cost computations. The cost computations are basically computing the length of each arc, there is an arc from 0 to 1 arc from 0 to 2, 0 to 3, and 0 to 4. There are 10 arcs in all, 4 from 0, 3 from 1. So the 7 arcs that are listed on this page will have a value. What is the value? M01 is the cost of producing in period 1 for the requirements of period 1 alone by that definition Mjk. So this is a 1 which is the cost of producing in period 1+ c1 into the quantity produced is D_1 that is it. So it is produced for the requirements of period 1 and consumed in the period 1 alone. M02 what is that? Cost of producing in period for the requirements of period 1 and 2, so what it means is a 1 + c1, the production quantity is now $D_1 + D_2$ because you are producing this and then D_1 is consumed in the first period. So D_2 will have to be carried over to the next period. So holding cost for the first period into D_2 that is how it is.

There is a cost of production, this is a set up. This is the production cost; this is the holding cost for that period. Similarly M03, MO3 is slightly more complicated. It is the cost of producing in period 1 for the requirements of period 1, 2 and 3. You understand now the significance of the interpretation. This will be A_1 + c1 into production is now $D_1 + D_2 + D_3$. This is taking place in the first period itself + h_1. What was consumed in the first period was D_1. So $D_2 + D_3$ had to be carried to the next period and then D_2 was carried was consumed in the second period. So ultimately h2 in the period, you have to carry D_3. This becomes the total cost in the period in the M03 and M04 which will be the lengthiest expression in the whole problem. Let us see if we can understand that because that would mean total clarity. M04 is the cost of producing in perio$D_1$for the requirements of period 1, 2, 3, 4, it is as simple for the whole 4 periods if you produce in the first period itself for the entire demand that is what it is. $A_1 + c_1$ into $D_1 + D_2 + D_3 + D_4$. You consume D_1 in the first period, so $D_2 + D_3 + D_4$ will have to be carried over and then you will consume D_2 in the next period. You will have to carry over $D_3 + D_4$ in the second period and then you will carry D_3. You will have h_3 into D_4which will have to be carried over in the next period; this becomes the total cost of or the length of this arc M04. This is all production in the first period which is clearly available from $A_{1.}$ $M_{1\,2}$ $M_{1\,3}$and $M_{1\,4}$ is nothing but the cost of production in period 2. This is A_2 now and this is c2 production in period 2, production for second period which is D_2 for the second and third period which is $D_2 + D_3$ and the corresponding holding cost by the same logic and here this is $M_{1\,4}$ will be the cost of producing in period 2 for the requirements of periods 2 3 and 4.

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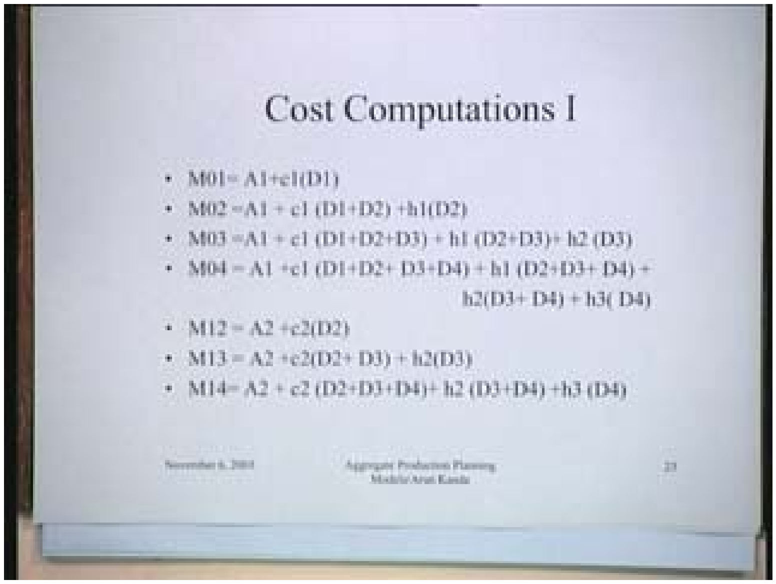

That is what you have here + holding + holding correct. You will know how to write these expressions written each one of them, and then talk the talk about the remaining three arcs. $M_{2\,3}$, $M_{2\,4}$ and $M_{2\,3}$ and $M_{2\,4}$ is the cost of producing in period 3, so this is A_3+ c3 $D_{3,}$ A_3+ c3 into D_3+ D_4 and then h_3 into D_4 which will have to be carried over and what is the last arc $M_{3\,4}$? This is the cost of producing in period 4 for the requirements of period 4. This is A_4 + c4, D_4 that is all.

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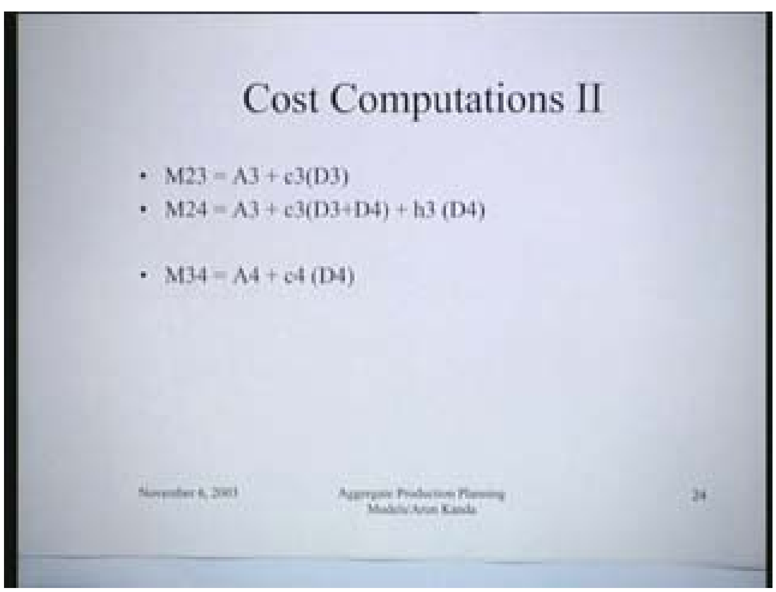

These expressions have been computed for all the arcs in general and now utilizing our data that we have for this particular problem we can actually find out the numerical values of these lengths for each arc. So it is just a question of plugging in the values that you have into the general formulas that they have found out and what you find is that M01 is 180, M02 is 380, M03 is 450, M04 is 735, M12 is 300, M13 is 365, M14 is 635.

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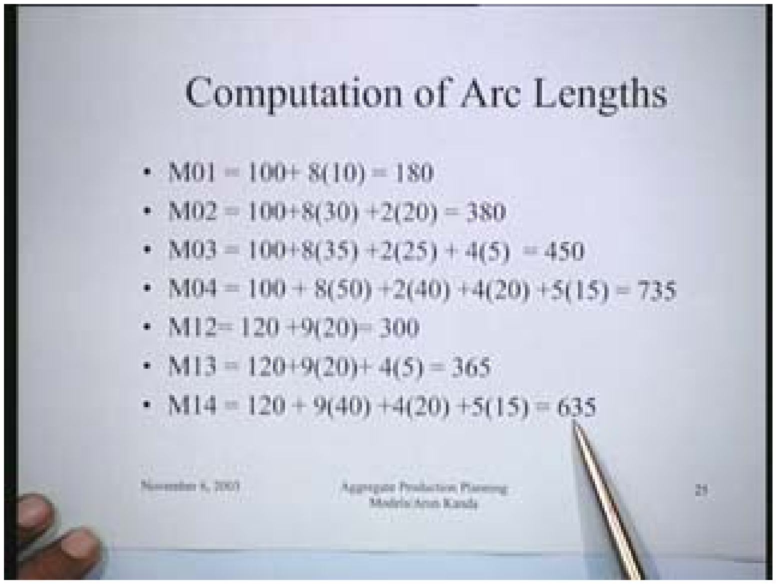

So M23 is now 170, M24 is 395 and M24 is 290.

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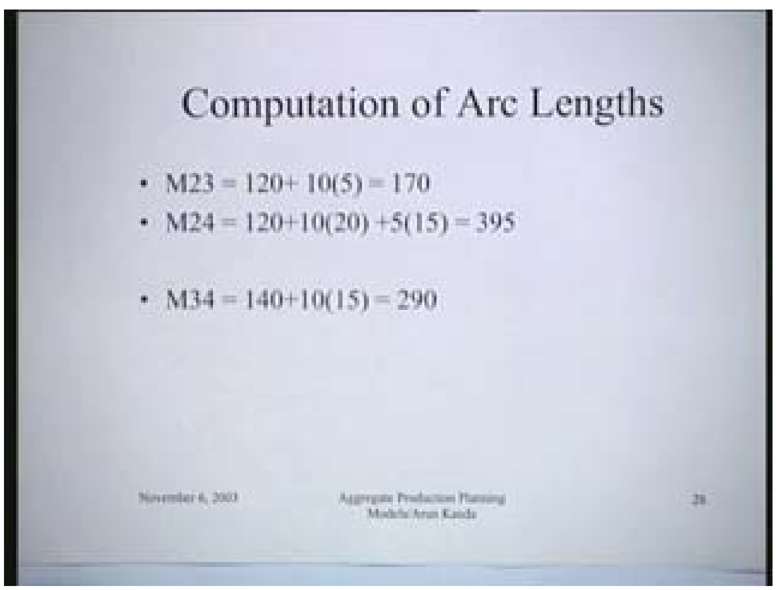

I think up to now the only thing that we have done is for the network that we defined for each arc we have computed the length based on the definition of Mjk and I just illustrated to you how it is to be done. If we do that the next thing would be that on the network which we had set up, we will write these lengths that we have calculated. This is M01, the arc going from 0 to 1. This length is 180 which is what we had computed. M02 is this length which is 380; M03 is this length which is 450.

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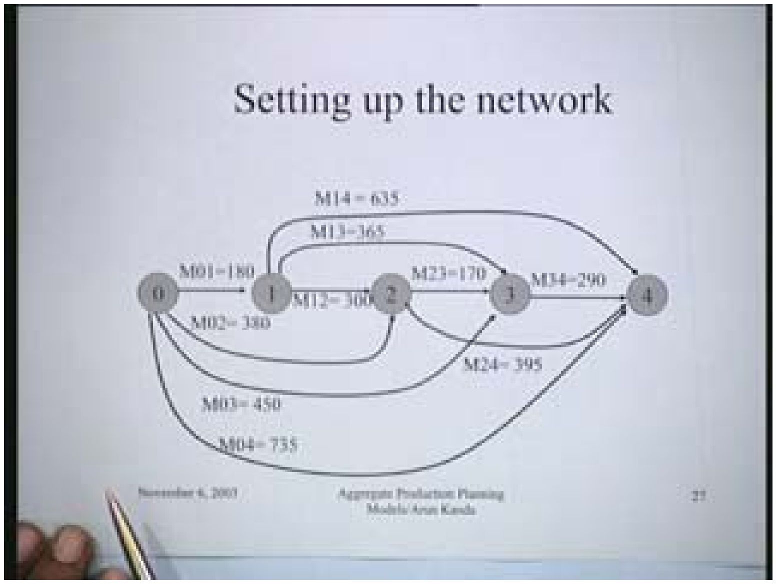

M04 is this length which is 735 and so on for all the arcs, so all the 10 arcs are shown. Now it is a problem of finding out or going from .0 to 4 and you want to go by the shortest path. You want to go from .0 to .4 and go by the shortest path because the shortest path is nothing but the minimum production cost. That is the idea. So you know shortest path algorithms are generally very efficient algorithms. So in this case in fact you don't need to use the general Dikster's algorithm for solving the shortest path because you can solve it much more simply. Let us look at it this way. What you find is you give a value 0 here because this is like starting milestone 0. Now when you come to node 2 there is only one way you can come to node 2 from node 0. That is this one, no other arc comes to node 1. This length is 180. So the shortest path up to node one is of length 180, so we put 180 there and box this value.

Let us go to node 2. At node 2 there will always be 2 values, the number of values that you will have is the number of the node always. So from node 2 you can have arc like this which is M02 and then you have this particular arc here. So what does it mean? If you come from 0 directly it will be 0 + 380, so this length will be 380. On the other hand if you come from node 1 this value will be 180 + 300 which is 480. The smaller of the 2 or the minimand is 380. We will put 380 here and go to the next node and when you go to the next node when you come to node three, there are three possible arcs culminating in node three, which are these arcs. You can have 0 3 which is of length of 450 or you can have up to 1 which is 180. 180 + 365 or from 2 you can have 380+ 170 and the smallest value that you can get from this case, in fact in this case there is a tie between these 2 values. You get 450 and then finally when you come to node 4 you have to take the minimand over 4 values, which are those 4 values. We will have M04 + M1 + M14, M2 which is 380+ M24 and this is 3 which is 450 +290 and the minimum of these 4 values is 735. Ultimately when you compute the boxed value at the terminal node you have found out the optimal solution. The optimal solution to this problem says that the minimum cost will be 735 units and if you identify the path which led to the optimum in this case; this was the path which is shown in bold line which led to the optimum.

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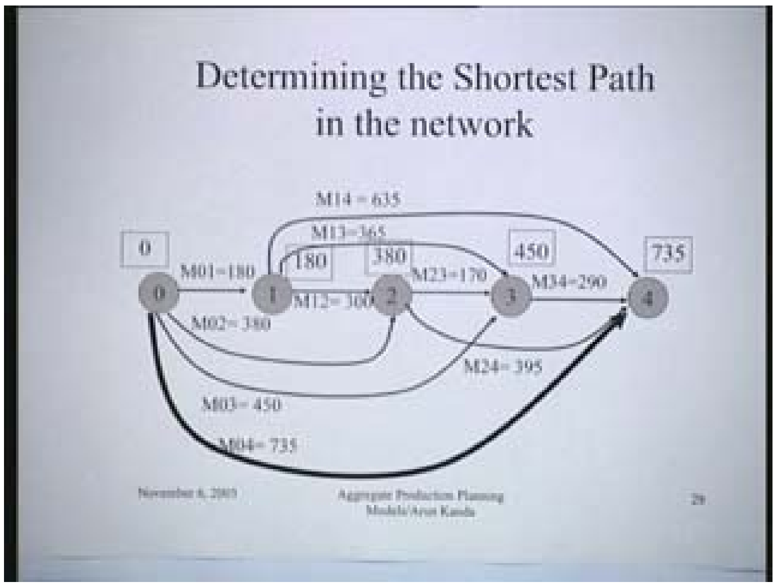

This is the length of 735, it is there. So it shows clearly that in this particular network, the shortest path to go from node 0 to node 4 is this one which has a length of 735. This is the solution to the aggregate production planning problem for this particular situation. Let us try to identify what the plan would be because after all in this case the shortest path involves the cost M04. So again invoking the meaning of M04 which is like Mjk, M04 is the cost of producing in period 1for the requirements of periods 1, 2, 3 and 4. It says that in this particular situation the optimal production plan is this.

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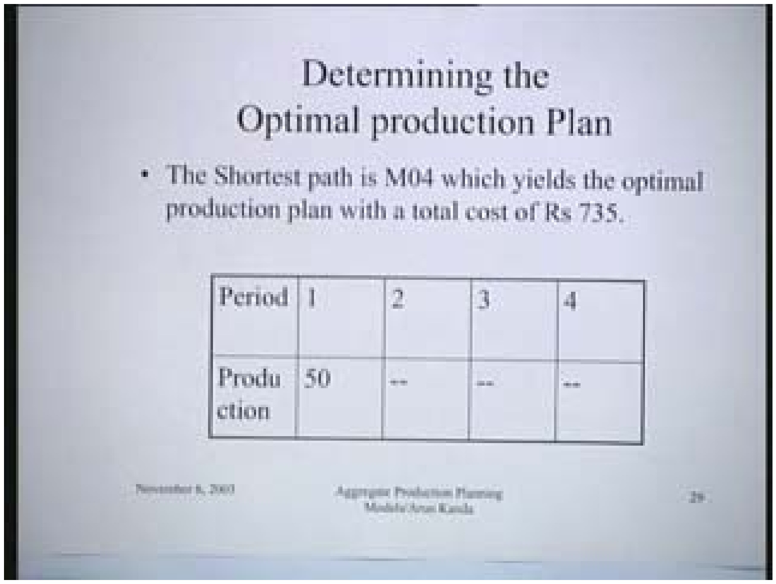

That means the entire demand of all the 4 periods which is 50 be produced in the first period itself and nothing be produced in the second third and the fourth period and this particular production plan, the shortest path M04 will actually yield the optimal production plan with the total cost of seven 135. Now this procedure is general. It can handle a concave cost and in fact something like the variation of a dynamic programming solution procedure. Basically what we are doing is you are trying to enumerate systematically the various policies that you may follow in producing the production plan. You have this particular solution here. What would happen in a situation when there are shortages? So far in this problem we said that there are no shortages involved. The only thing that would happen is the word minimum is added to Mjk. This is the only change which takes place and adding the word minimum creates lot of problems for computation. It is not at all that simple, just this word minimum what it means? So the arc interpretations for the problem with shortages are that Mjk is now to be interpreted as the minimum cost of producing for the requirements of the period j + 1j +2 and so on up to k, that is exactly what we were doing earlier.

You see what this means is something very interesting because what you can do is we have said that earlier if you have period j + 1, j + 2 and so on up to k in the no shortage case, we had assumed that production will take place only in this period. This is the only time when production will take place and this production will satisfy the demand of the entire planning horizon because we were not allowing any shortages. What is going to happen now is notice that production may take place at any period between j + 1, j +2 and k which means you can be producing here, then you can be producing here also because if you produce here there will be inventory holding cost here and shortage in this period. Then here also you have this. So what you have to possibly do is calculate the total cost Mjk for each one of these possibilities of producing here or here or here and then take the minimand over these values and that is actually the minimum cost. Mjk in this particular situation might work out something like producing in the fourth period for the requirements of period 1, 2, 3, 4, 5, 6, 7 somewhere in the middle.

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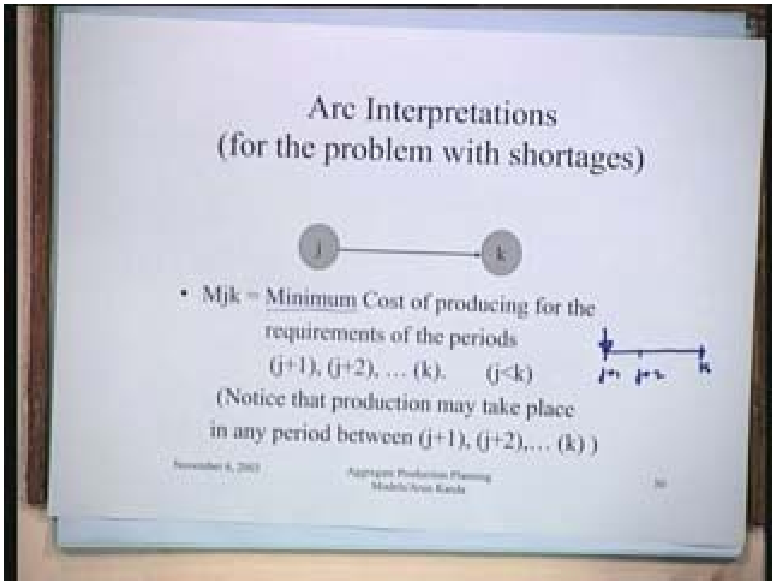

This is actually known as Zanguil’s extension to Wagner and Whitin’s formula. What we did earlier was Wagner and Whitin’s formula essentially. This is Zanguil’s extension which will lead to this. So the no shortage case was relatively easier. The case with shortages will involve additional search and you have this problem.

So let us see what we have tried to do in this particular lecture. Basically we have shown that linear programming is a versatile tool to handle aggregate production planning problems with a variety of linear costs and constraints. In fact we took up a LP formulation of one particular situation where we set up the LP objective function and also the LP constraints. Something like that can be set up for a variety of problems and you can vary the type of production costs and so on and set up. There is it is a very versatile tool in that sense, then after the linear programming formulation we looked at the transportation model and the transportation model we have seen is capable of dealing with problems with piecewise linear convex costs, that means for instance it could deal with regular time overtime and subcontracting together. Costs are progressively increasing. So the no shortage case can be handled by the simple greedy procedure. However the case with shortages has to be treated by the regular transportation problem, the usual procedure.

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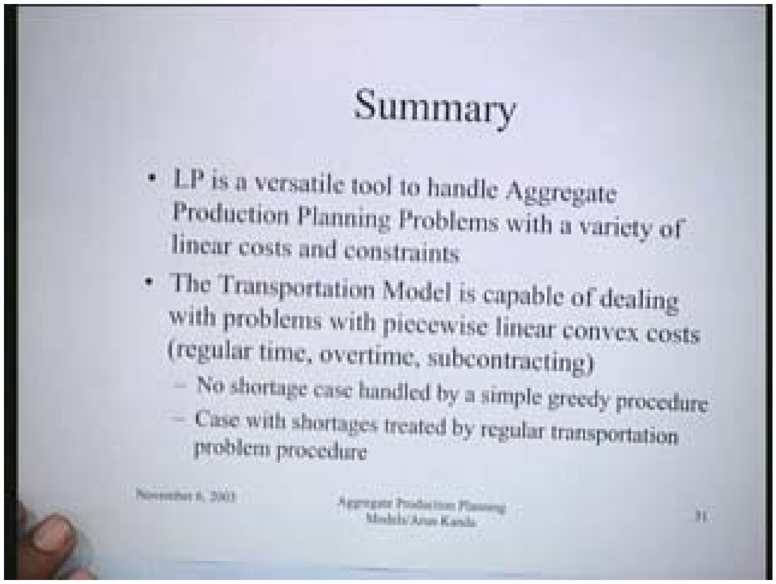

You have solutions for either of these 2 cases and then finally we took up the case of general concave and arbitrary costs in which we considered a problem in which there was a set up cost and there was a production cost and a holding cost in that sense. So the case of concave and arbitrary costs can be solved using a shortest path procedure on a network and finding out the shortest path procedure in case there were no shortages was relatively simple.

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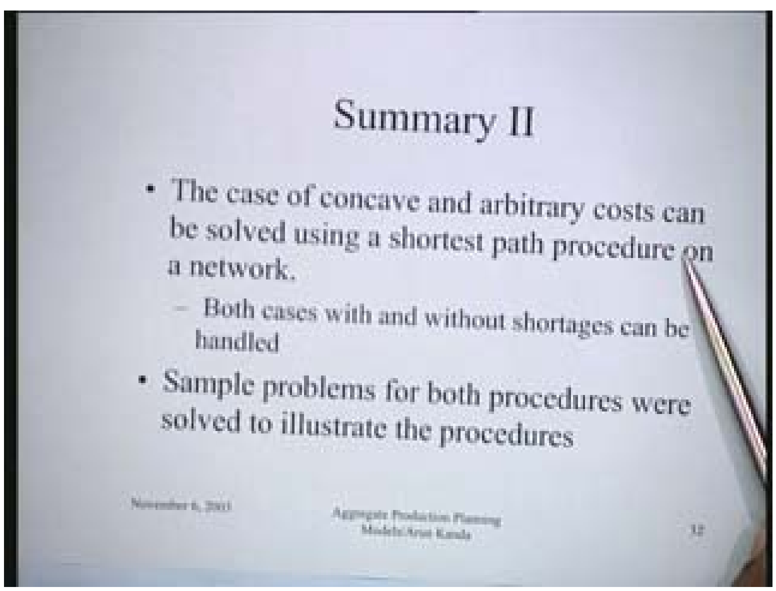

All it involved was calculating the lengths of each arc and then determining the shortest path and both cases with and without shortages can be handled in this particular situation and sample procedures for both these sample problems, for both these procedures was solved to actually illustrate the procedures. That is what we essentially do.

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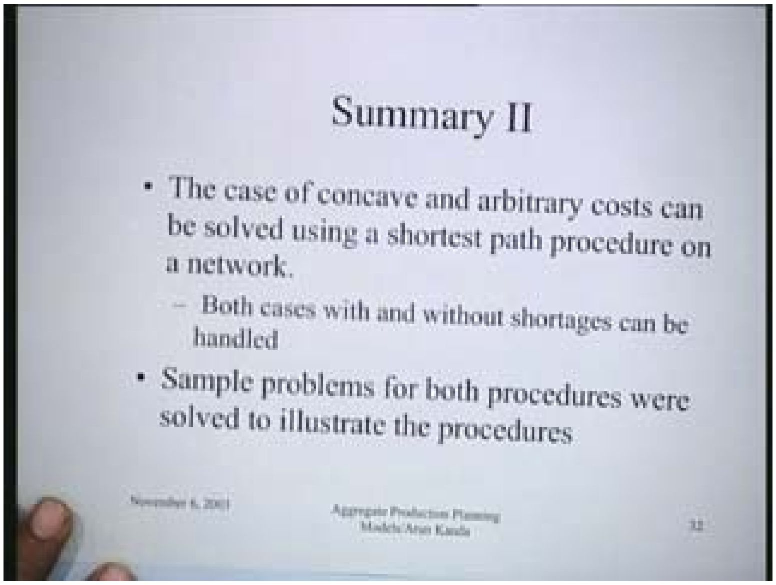

So I think with this, you have some idea of modeling procedures for aggregate production planning and in the next class we are going to be talking more about inventory models and how they are used for taking decisions or when and how much to manufacture for a particular situation.

Thank you!