Project and Production Management
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Lecture - 36
Aggregate Production Planning: Basic Concepts

In the last two lectures we have been talking about forecasting and various methods of forecasting. As you all know the forecast of demand is a very important input to the planning exercise and in today's lecture we are going to be talking about utilizing demand forecasting for purposes of aggregate production planning. Aggregate production planning is the exercise of developing an overall plan for a company which specifies how resources of the company are going to be committed overall the next six months to one year. So the typical planning horizon that we are talking about is a planning horizon up to about one year and in this particular lecture we are going to be talking about some of the basic concepts of aggregate production planning and we will see how aggregate production planning is actually useful for a company.

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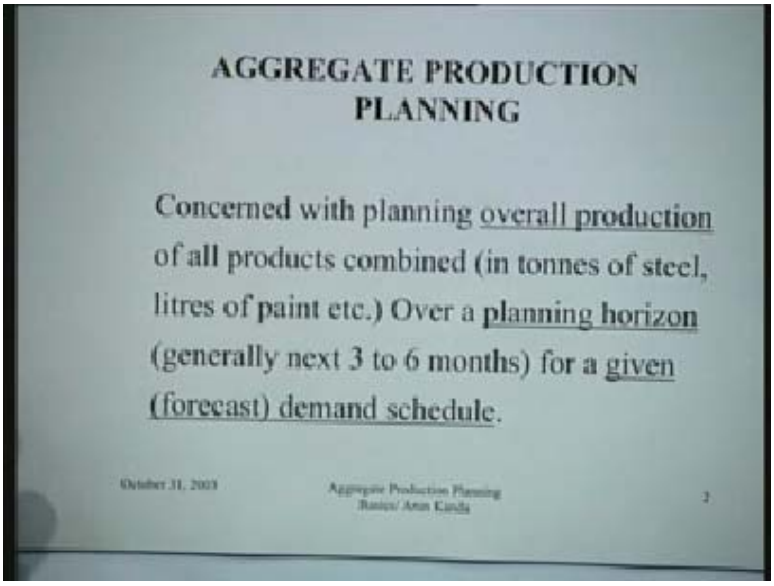

This is the basic definition of aggregate production planning. So if you see aggregate production planning is basically concerned with planning overall production. The emphasis is on overall production of all products combined and this overall production could be in tones of steel liters of paint etc and we are talking about a planning horizon for generally next 3 to 6 months or even extending up to a year for a given forecast demand schedule. So the basic idea here is that the demand for the various products of a firm could be fluctuated and if the demands are fluctuating how do we commit our resources to meet this fluctuation in the demand? That is the basic intention. I think a word about how we call it aggregate production planning and how is aggregate

production planning different from production planning within the context of a factory. You see for a planning horizon for the next six months or one year, the company is basically interested in knowing how the overall performance of the company shall be as the demand keeps on fluctuating. So a company might be manufacturing let us say hundreds of products. The demand for one product may be going up.

The demand for the other product might be coming down and as far as the hundreds of products are concerned each demand would exhibit its own kind of behavior. So if the company tried to track the performance of these individual products it could probably go mad in tracking the demands of hundreds of products which are there and since the company is interested only in the overall growth and the overall commitment of resources for the next one year, it is therefore interested in aggregating the demands of the individual products and talking about the aggregate production plan and rather than talking about individual production plans. So if you are talking about a steel plant, for instance a steel plant could be manufacturing sheets scalp, various types of ingots of different sizes, rail sections strips in a variety of sections. So you would have a large variety but when the managing director of the steel plant wants to know how the steel plant would be performing over the next year he would probably be concerned about the total production in terms of tones of steel of all types of products put together. So tones of steel would be a convenient aggregate measure of the performance of the company and you could then aggregate the demand of all the products in terms of the total tones of steel and use this as an aggregate measure of the requirements of the company. Similarly if you are talking about a paint manufacturing company, a paint manufacturing company produces paints in different colors, different sizes and for different applications and for each of those paints. The demand would be different but if you want an overall idea of how the paint manufacturing company is performing, you could probably aggregate the total requirements of paints and the total liters of paint requirement for the whole company as a whole.

This concept could be extended to even an automobile manufacturer. For instance if you talk about an automobile manufacturer, let us take the typical example of Maruti udyog, the company produces different types of cars. It produces a Maruti 800. It produces a zen, it produces a van it produces a gypsy it produces a versa and so on. It produces different types of cars and the demands of these cars will be different over the planning horizon of the next one year but if the company is trying to find out what would be its growth level for the next one year, it could probably try to mix up the demand of all these cars. How would it mix up the demand? It could probably have to think in terms of a unit like standard passenger cars. So may be they might say that a Maruti 800 is like a 0.8 of a standard car, a Maruti zen is like one standard car. Esteem is like 1.2 passenger units and develops some units like this and then ultimately measures the performance of the company in terms of standard car units something like this and then it would have an aggregated variable which would be able to deal with this. So the point that we are trying to say is apart from this aspect of focusing on the overall growth, the second major advantage of using aggregating or aggregating the demand is that normally if you have 50 products and you make their forecast and then you try to work with individual forecast each forecast will be subject to some errors and if you try to combine these forecasts the

aggregate demand figure would be subject to much less error because the positive and negative errors will tend to randomly cross each other out and therefore there is greater accuracy in obtaining the aggregate demand forecast then the individual demand forecast. So this is another reason for dealing with aggregate production planning. So the whole process in aggregate production planning is essentially to determine the aggregate forecast. First you do a process of aggregation, i.e., do aggregate production planning.

Once you have done it, you will have to disaggregate to obtain the individual pattern of demands or rather production levels for individual products in that sense. So I think this perception of aggregate production planning must be kept in mind. Basically what are we trying to do in aggregate production planning is we are working with a forecast of demand and we are trying to work out the capacities cost and commitments that you are making. So you know that this is the capacity of plant so much in January, so much in February, so much in March because you might have committed capacity to other products. So the capacity availability in different periods is known. So you know the various types of costs that are incurred and you are therefore trying to find out how these capacities should be committed to the aggregate demand over a certain period of time and this is what we call an aggregate production plan. From the aggregate production plan we are then able to work out the work force requirement over the planning period and we are also able to determine the machine time, allocation over the planning horizon. So basically this is the purpose of aggregate production planning.

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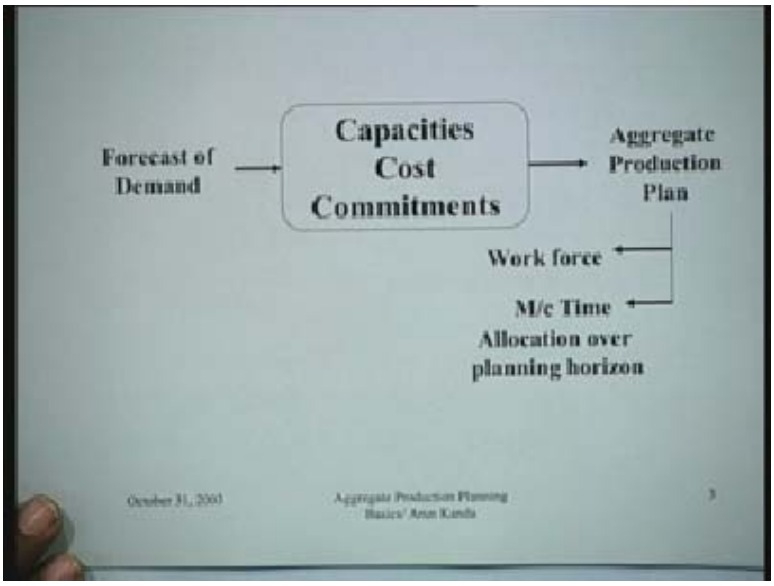

n trying to determine the commitments of the capacity to various products, we have already indicated that the basic problem in aggregate production planning that we are trying to solve is the problem of meting a fluctuating demand. If you were sure for instance that the demand in each month is going to be hundred units and this will continue add infinitum then production planning will be relatively simple affair but that is not what happens in practice. In practice demands keep on fluctuating. You have to

forecast these demands and therefore let us be clear about the options that management has to meet. Fluctuating demand in any business, how can it meet a fluctuating demand? Probably the first method is one that is used quite commonly used in industry is to build inventories in slack periods in anticipation of higher demands later in the planning horizon. So you can keep on building inventories when there are no sales in the hope that in some future you will be able to sell off the inventories that you have made. This could be a good strategy provided the demands actually show up in the future but if you have been able to if you have not been able to forecast the demands properly then this could be a risky strategy.

You probably are aware that in the Maruti factory in Gurgaon at one stage, they had such a large stock of cars that they had to procure additional land to just store these finished cars outside and so much so that the cars were actually getting damaged in the adverse weather conditions in which they were being stored. So this strategy of using inventories in slack periods in anticipation of higher demands may work in practice but it is not without its problems. The other option could be that you could carry back orders or tolerate lost sales during peak periods. Do you know the difference between a back order and lost sales? A back order is something when the customer comes to a shop and you don't have the item in stock and he places an order with you and he is willing to wait. So whenever you are able to supply you give in the order. So that is called back ordering. A lost sale would be when a customer comes to your shop and he finds that you do not have the brand of shampoo that you need and therefore he just walks away to some other shop and that is a lost sale. So both lost sales and back orders are actually situations which occur when there is a shortage. When you do not have enough stock available of whatever item is demanded so whenever the stock level is positive you will inventories. Whenever the stock level is negative you either carry back order or tolerate lost sales in that sense. The third option that management may have to meet a fluctuating demand is to use over time in peak periods under time in slack periods to vary output while holding work force and facilities constant.

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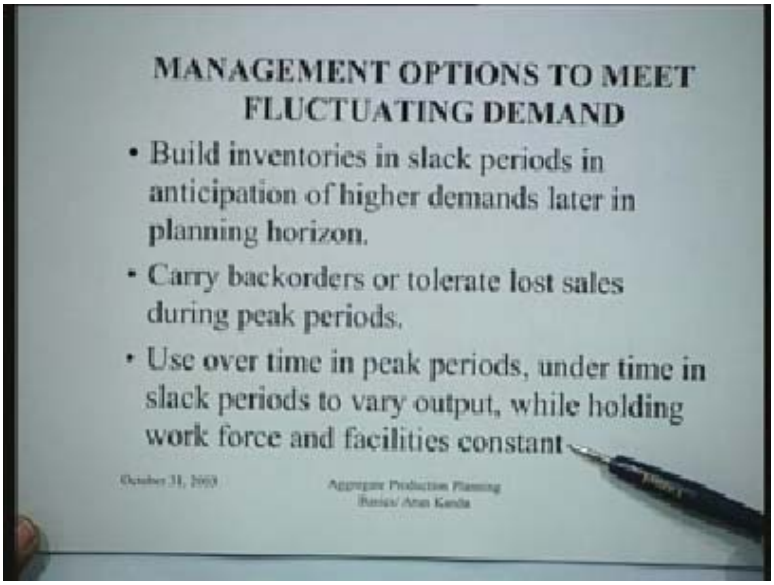

This is a very common strategy. This is the kind of strategy that students typically tend to use. When there is a peak period, that is five assignments had to be submitted, they work over time at night and get these assignments ready and otherwise when there is a slack period, they don't. I mean they use under time in slack period. They don't work and that is the way they vary their output and while holding the work force and facilities constant. I think another point that needs to be mentioned in the context of aggregate production planning is that normally we are not talking about the option of increasing or machinery or other kind of infrastructure within your plant. For instance if there is an increased demand for something, one option might be to set up another factory. We are not talking about those capital intensive options in this particular situation because our planning horizon is in typically one year ahead. So we are talking about those options in which generally the facilities are constant though of course work force can be varied in general. The other option available to the manager for dealing with fluctuating demand is you can vary the capacity by changing the size of work force through hiring and firing. It is a very common strategy these days.

You probably heard or read a couple of days to go about Cadbury's laying off something like 20,000 workers all over the world in their various plants. So they are adopting this strategy of changing the size of the work force through either hiring or firing. So most of the companies fire their workforce and when there is a need you can always hire the work force. So changing the capacity by hiring and firing is a very common strategy. It will be contract labor. For instance most farmers would engage additional labor when they require them for purposes of reaping the harvest and then when they do not need them they would fire them. This is exactly that but hiring and firing as a typical term as a normal in normal parlance is the legacy of the American culture. The American culture people are very fond of hiring and firing. You find that an executive is doing very well. He is given a promotion today but when he comes to work, the next Monday he finds a pink envelope on his desk which says thank you Mr. Ralph you are fired today because

we do not need you and here is our cheque for whatever number of days you have served us. This kind of culture prevails more in the united states not in India as much though of course, with the advent of multinationals and so on this kind of culture is also coming into being the hire and fire culture. But normally when you talk about employment in the government of India then if you are hired once, you are fired only when you retire. That kind of a situation but then this is an option available to management for changing capacity by changing the size of the work force through hiring and firing. I would like to mention here that hiring and firing is not without its costs. When you hire people you have costs of hiring. That is the process of selecting people, conducting an interview, training people before they actually become productive. So these costs are the costs of hiring and similarly when you fire people, there is a cost associated with firing as well.

There is loss of customer good will and the loss of customer good will would mean that the person whom you are fired will probably spread the message in other organizations. Do not ever join this company because they hardly keep you for two weeks and then fire you. So that reputation travels. So there are costs associated with hiring and firing and one of the major purposes in aggregate production planning is actually considering these various costs which are associated with these various options and try to select a strategy which is the optimum strategy in terms of meeting the fluctuating demand. You can vary the capacity through changes in plant and equipment but generally as I said this is the long term option and is not considered when you are talking about aggregate production planning. So the point really is that each option that we have considered here involves cost. The cost may be tangible or intangible cost and the aim in aggregate production planning is to choose the best option considering the various costs.

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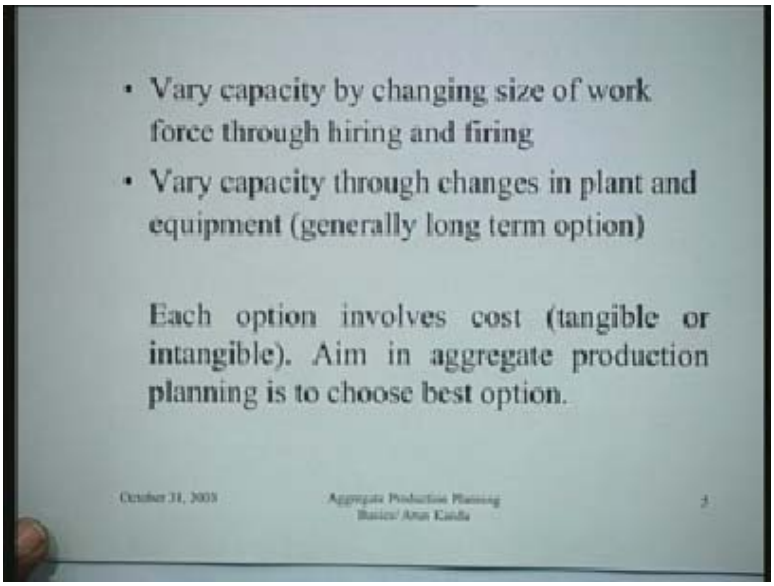

It is essentially an optimization problem with these types of costs, fluctuating costs and the kinds of constraints that we will just try to mention. So before we talk about the problem involved let us try to identify the costs which are there in typical aggregate

production planning. One of the major costs involved is procurement cost. So whenever you are producing you have a typical option of either buying the material or producing the material in either cases procurement is to be made. You can either buy the material for hundred rupees or then process it within your factory or you can procure the item may be for 500 rupees whatever it is. So costs of procurement have to be taken into consideration and in fact all these costs could be fluctuating or varying with the time period over the planning horizon. Then you have costs of production. Cost of producing a piece in the first period might be 100 rupees. In the next period it might be 110 rupees and then it might be constant at 110 rupees a piece and then it might go up again in the sixth period to something else. Inventory holding costs the inventory holding cost is that cost by which if you produce something in a certain period whatever is not consumed will have to be held for that particular period and that is the inventory. There are costs of storage costs of spoilage cost of pilfering etc.

These are basically all aspects of inventory holding costs and typically one of the most common components of inventory holding cost is the loss of interest on the capital that you have invested. For instance if I stock something worth one thousand rupees, if i had kept that one thousand rupees in the bank or given it to somebody else, I would have been able to earn some interest on it by holding it in inventory in the form of a car or a whatever product I have. I have actually foregone any returns on that particular product. So what we mean by saying that there are always inventory holding costs, Shortage losses associated with back orders and lost sales. So depending upon the situation that is if you are not able to supply an order because you don't have the quantity ready with you then you incur a shortage and the shortage losses could be in the nature of back orders. So in a back order, the cost would be in terms of rupees per unit time. Because you are satisfying the demand at a later point of time but for a loss sale it is a one time. So it will be like so many rupees per units lost. The element of time would not be there in lost sales. So that would be the unit of a back ordering cost and lost sales. Then we talk about costs of increasing and decreasing the work force. These are typically the hiring and firing costs that we were talking about. If you increase the work force the cost of employing additional people training them and bringing them up to the mark so that they can contribute effectively to your process. That is the cost of increasing the workforce. Cost of decreasing the work force could be the cost of ideal capacity or it could also be the cost of loss of good will. So these types of costs are involved. Then we have the cost of overtime and under time. Cost of changing production rates.

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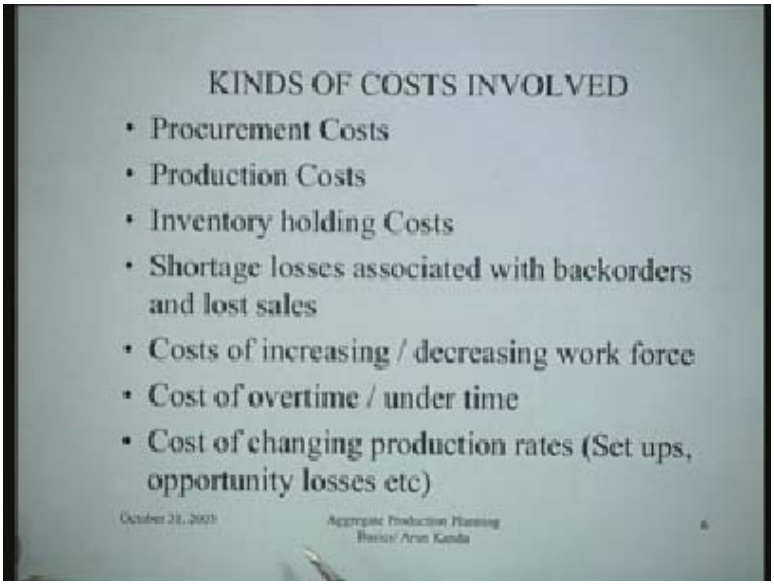

What can happen is that there could be set ups. There could be opportunity; losses etc and these are the costs of changing the production rate. For instance if you are producing at a certain level in the month of January and you decide to change the production rate in february it would either mean that you have to change some set ups, make some new adjustments in your plant and that is the kind of set up cost that you have here and or there could be some kind of opportunity losses as a consequence of these kinds decisions. Let us now take a simple example to show how aggregate production planning could be done. Suppose that we have data on demand for the next let us say one year. So during the first period the expected demand is 100 and I also have a cumulative demand figure which tells me what is the cumulative demand up to the end of that period. So, in the second period the demand is180. The cumulative demand is now 280 and so on. In the third period the demand is 220. So this added to 280 will give me 500. Now the significance of this cumulative demand is that it tells us that, up to the end of the third period the total demand for all the periods is 500 units. Similarly the total demand up to the end of the eighth period is 1500 units and this is how the demand has been fluctuating over the various periods.

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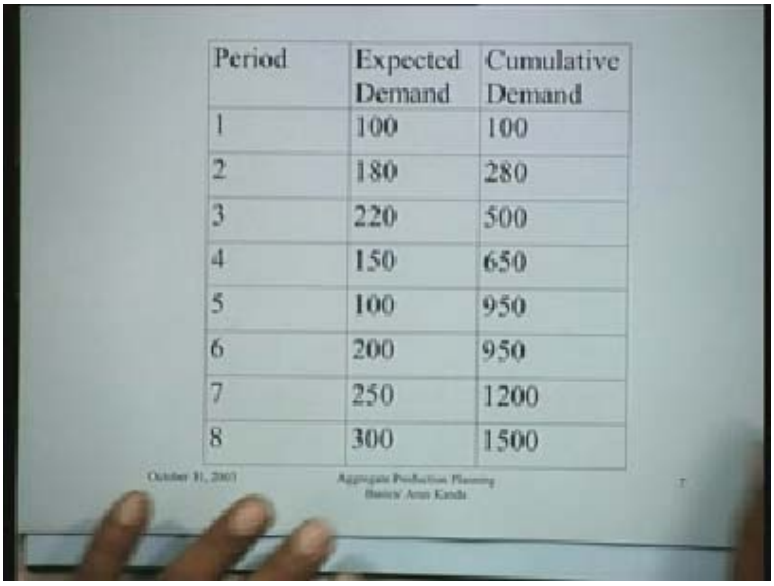

So this information would be available to us from a demand forecast. We have been talking about demand forecasting in the previous two lectures. So basically the output of an exercise of this kind would be that we would have access to information like what is the expand demand over the next period. What we are seeing here is that there are these are four week periods, so there will be thirteen four week periods in a year 52 weeks. So we have the demand figures available for all the four week periods. So these are like four week periods in that sense.

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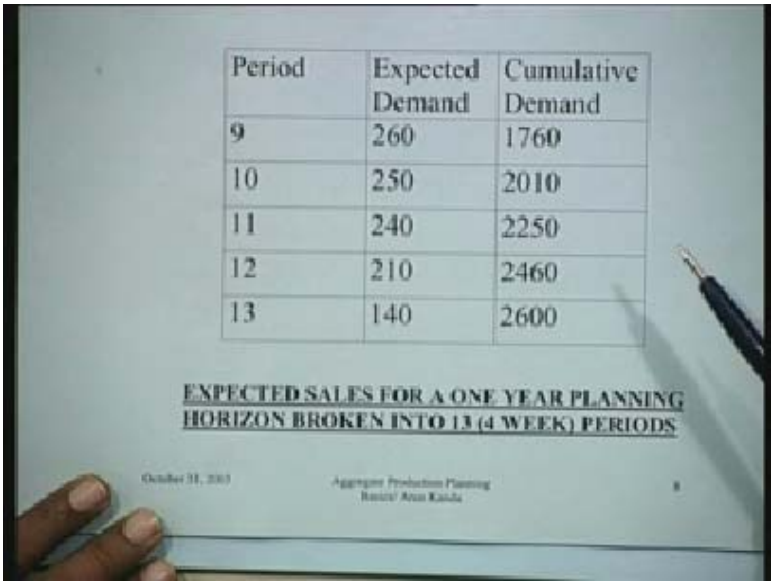

The cumulative demand is 2600 for all these thirteen four week periods and that means for the entire year. Now what we would like to do is to find out that in the face of this uncertain demand what would it be what should be the policy for the company to produce so that you can meet this kind of demand and we will approach this problem from an intuitive perspective and we will see how we can solve this particular question. We will use a graphical procedure and incidentally this graphical procedure is the most common one with practical managers because they find it simple and it does not involve any complicated mathematics and obviously the prize is in terms of the fact that this leads to only a heuristic solution. It does not guarantee an optimal solution but let us say it has certain advantages to offer. The first thing that you do in the procedure is that you plot the cumulative demand. So the solid line which you see here is a plot of the actual cumulative demand that we had in the table. So you plot the cumulative demand which is shown here and you go up to the thirteenth period and you have this demand.

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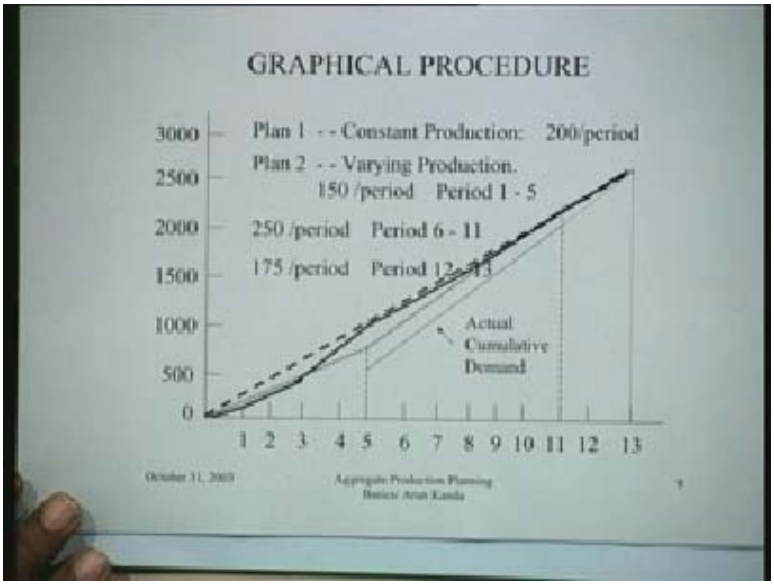

Now let us consider two possibilities. We call them plan one and plan two. After all what we need is, what is the production plan? A production plan is nothing but a path going from this point to this particular point here and any line which goes from here to here is actually a production plan. Since infinite number of production lines can be drawn from zero to this terminal point therefore there are infinite numbers of production plans. But normally what is the ideal thing for a production manager? He would like to vary the quantity that he produces every year. So the total quantity that is to be produced in thirteen period's is2600. So if we assume that the constant production level is to be maintained at 2600 divided by thirteen, we should maintain a constant production level of 200 per period and we would have a constant level of production.

If you plot this plan this plan is actually shown by this top dotted line, it is a straight line from here to here and that is what it is. Do you notice something about this plan? One is the constant production. If you plan at a constant production rate, you compare the actual demand forecast with this line. You find that the actual demand forecast for most of the time except somewhere here is below this line. What does it show? It shows that the cumulative production is greater than the cumulative demand which means if this particular line, that is the production line is above the cumulative demand line we would have costs of carrying inventories. So one thing that you can immediately see from this plot is that if we follow this policy of having a constant production level of 200 per units, in most periods we will be carrying inventories except in some periods where there could be a shortage. This amount could be calculated as we will do subsequently. So if the cost of carrying inventory is not too high this should be a good plan because you are most of the times not incurring any back orders or shortages. When you are carrying inventories and this is what it is. Of course one might say that another possible plan may be that you follow this demand line closely which means you produce exactly as per demand. That could be a production policy. So if you produce exactly as per plan what will happen is that your cumulative production will exactly follow this and you will neither be carrying

inventories nor there shortages. But the kinds of cost that you will involve are cost of changing production rates every period. So there would be a situation where you would not have to carry any inventories. You would not have to incur any shortages but you will have to change production every period because you are following the demand. You are chasing the demand. It is actually a chase model and when you are chasing the demand every time you are changing the period there is a cost involved. This cost could be through other hiring and firing or it could be by varying production rates. So whatever it is in various ways, you would have a situation like this. So this policy although good in terms of minimizing the costs of carrying and back orders it would not be so good from the point of view of changing the production rates because you will have cost of changing production rates will be pretty high.

Practically you will be changing the production rate every month and this would mean havoc for the production manager. You would not like to do this kind of thing. So you might argue that can we not have an intermediate plan and the intermediate plan could be that rather than changing this production rate every period, we have minimum number of changes and then see. For instance by looking at this curve you might find for instance that it is going up and there here. You could have a policy that let us say up to the fifth month I maintain a constant production rate. So in these months I am having inventories and in this period I am carrying shortages here. So the total production here the cumulative demand up to this level is750. So 750 divided by 5 is 150. So the policy is that you carry on the varying production level at a rate of 150 per period during the periods from one to five. So you get this lower dotted line up to here. Then you can look at this and say up to here from this point from the fifth period to the eleventh period. You again calculate what the cumulative demand here is and then see, divided this by the total number of periods and you get 250 per period is what you produce during the period from 6 to 11. So you have production plan is something like this and in the subsequent periods twelve and thirteen which are the last periods, you again know that this is what you have to produce. So you are producing 175 per period during the period from twelve to thirteen. So what is the advantage? The advantage is this is like a compromised plan. It shows that there will be some holding inventory some shortages but the number of changes in production are limited because during the entire planning horizon first five periods we are changing, keeping the production level at 150. Then we are changing it to 250 during this period and then we are changing it to175.

So one slope next slope next slope. Only these three slopes are involved and it involves starting with this production period and then only two changes. One changes here another change here. In the earlier plan that we were talking about where you are changing the production level each time, that would be more expensive. So the basic advantage of the graphical procedure is that it helps us to in fact by looking at the demand profile suggest what could be the possible alternative plans and this is the feature that practical managers love most. What they do is they can put down their plan on it and then basically it can be evaluated. So if you put down a couple of plans on this particular line you can then evaluate them and pick up the best one. Evaluation is generally done in a convenient tabular form like this. We are talking about plan one which is constant production. So production is constant in all the periods and because of the varying we have the demand.

So if you have produced hundred the demand was hundred, so the inventory at the end of this period is going to be hundred and the back order is zero and the capacity changes plus 20. Assuming that the capacity was set for something else the capacity changes so much, the overtime is zero and the subcontracting is also zero and you compile this table.

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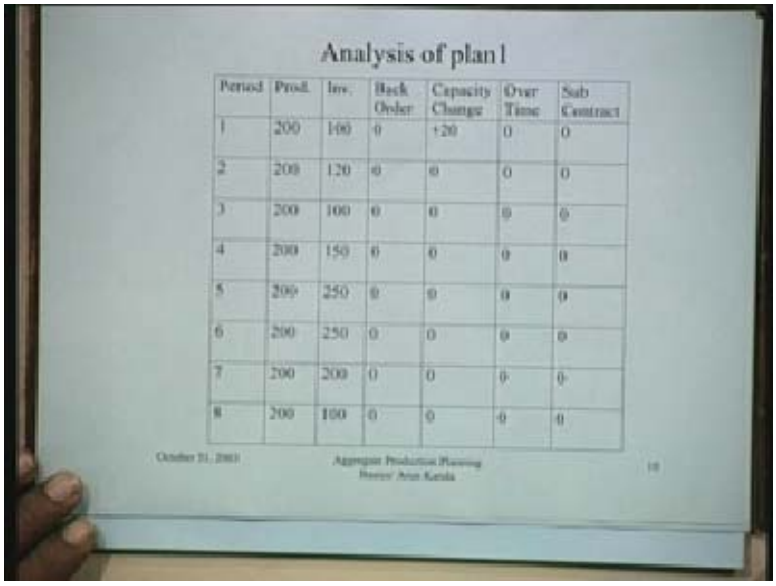

Obviously what you notice from the plan is the first plan is that there are back order costs generally. There is only one capacity change cost. There is no overtime cost, no subcontracting cost at least in the first eight months but there are mostly costs of holding inventories. So depending upon the relative cost of this you can determine what the total cost is. Capacity change means that if I am operating at hundred units today and if I change this capacity to 120 in the next period there is going to be some cost associated with that capacity change. That is what it is. So this plan for instance it does incur some back order costs as I said in the tenth, eleventh and twelfth periods. So what you have to basically do is you know these various cost parameters? You have to talk about the cost of production, the cost of inventory, the cost of back order, the cost of capacity change, the cost of overtime, the cost of subcontract and we now know each of these multiply with the appropriate figures, you will get a here. So you will know the total rupee value in terms of the various types of costs involved for this particular plan.

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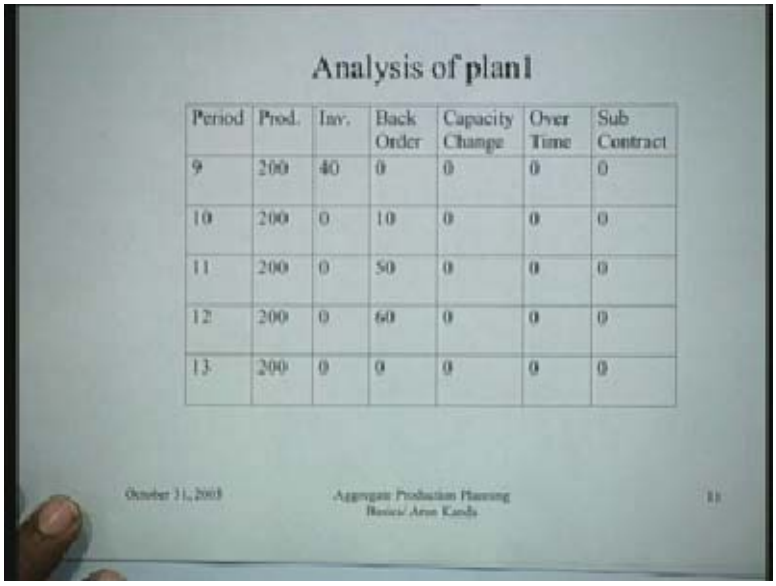

We can do an exactly similar exercise for the second plan. What was the second plan? We were keeping the production constant up to the fifth period up to150. Then it was 250 and thereafter it was 175 in the last two periods. So you can calculate similarly the inventory, the back orders, the capacity changes, the over times and the subcontracting which is necessary. Of course this is all based on a certain set of assumptions.

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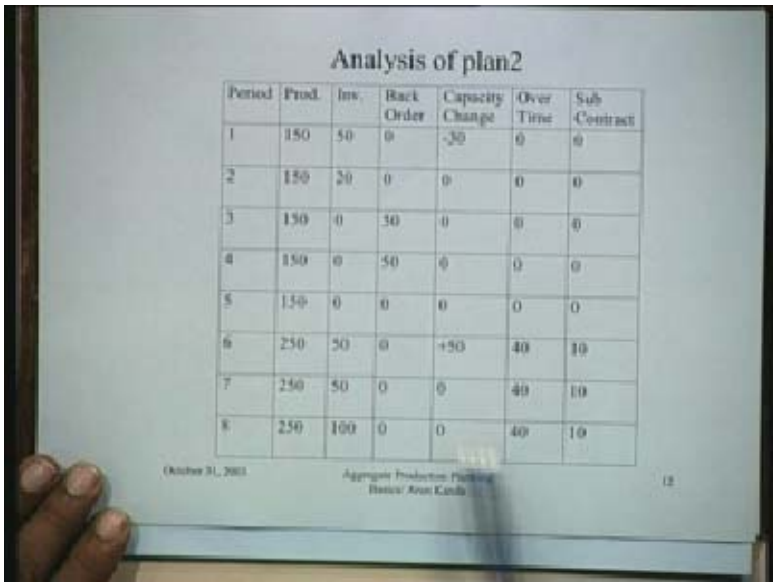

We will talk about them in a short while and ultimately even for this plan you will have this. So the inventory costs here are zero and back order is there.

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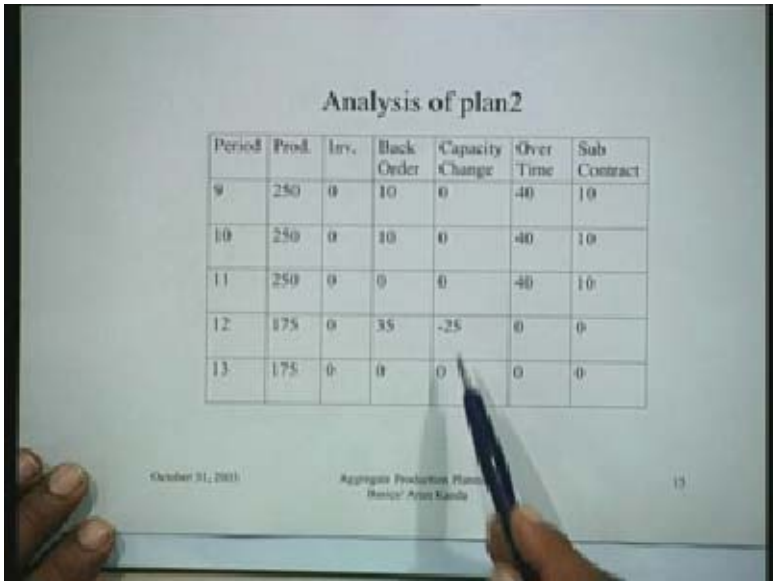

Capacity change means reduction in capacity over time costs, subcontracting all these costs are there. So if you know the relative values of these particular costs, you can find out this particular component. This is the basic frame work that you can follow for analyzing a plan. So you can compare plan one with plan two with plan three, find out the overall costs and then take a plan which is the best for your use. So this is how the graphical method will actually be a way of choosing the best plan out of the ones that you have been trying to that you have listed. So if you have not listed the optimal plan obviously you will not get the optimal plan. It is a heuristic procedure but it is good in the sense that it tries to consider all the costs involved. Now in the carrying out those computations some assumptions were made which are quite commonly followed. All shortages were assumed to be backlogged which means that we did not have any loss sales. If there was a shortage of 20, we said that it is backlogged. Regular time capacity was taken to be 200 units per period. The maximum overtime capacity was taken to be 20 percent of the regular capacity. So, 20 percent would mean that the maximum overtime was 40 units per period. Overtime is generally preferable to subcontracting and that was an assumption. Normally you would tend to produce something on regular time followed by something on over time and if you cannot make it on overtime then you will subcontract. That is the idea. So the relative preference was regular time overtime and subcontract. Assumed initial inventory was zero. The initial regular time production capacity was 180.

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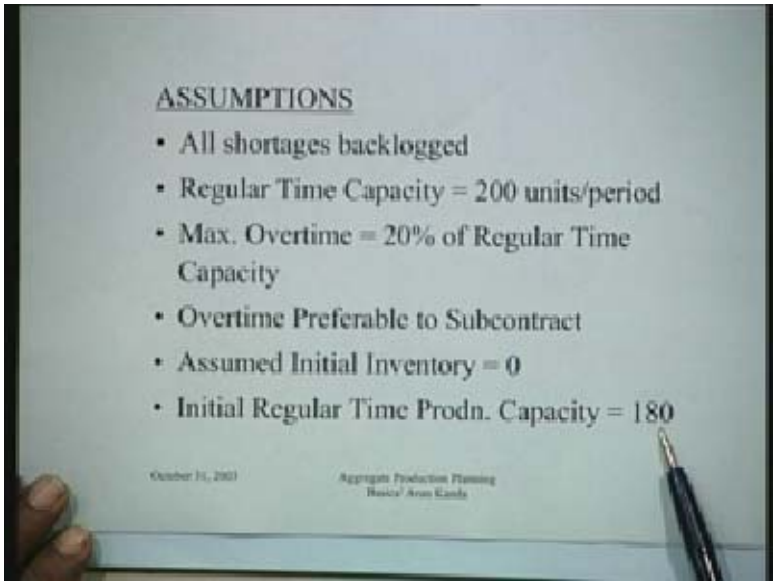

So these were the figures that were assumed that last year the company was operating at an initial capacity of 180. So that this here if you want to produce 200, we have to change the production capacity to 200, so the production capacity change costs are + 20 and that is what you found in the table. So you will always have a set of situations something like this and you can use the graphical procedure which is the simplest method of aggregate production planning to actually identify the best plan of action for your situation. However the biggest limitation with the graphical procedure is that it does not give us an optimal solution. It does not give us the solution which has the minimal cost and therefore we have procedures which in fact are dependent to a very large extent on the nature of costs and depending upon the nature of costs we can choose an appropriate procedure. For instance if you have a situation where the production cost in a particular period say X_t is the production and period and C_t X_t is the production cost. If this is linear that means it passes through the origin. It is like saying that then slope of this line is the cost of per piece.

So if it is whatever is the cost like this it is a linear production cost, similarly the inventory cost. The inventory cost could be of two kinds, it could be a situation where the inventory is positive which is the holding cost. The situation when the inventory is negative is the shortage cost. What is positive inventory? It is your holding stocks. What is negative inventory? You have items on back order. So it will be one of the two situations. Either I can supply immediately 20 cars, if I have more, stocked with me. So it is a situation where the inventory level is high and if there is any demand less than this I can supply it off hand but in this case there is a shortage. So I back order. If I somebody places an order for 20 cars and I do not have 20 cars in stock I will say okay, the next two months I will manufacture these 20 cars and supply them to you. That is back ordering cost but back order is also at a cost. It is in terms of loss of goodwill whatever it is. So there are these costs which are determined by the slopes.

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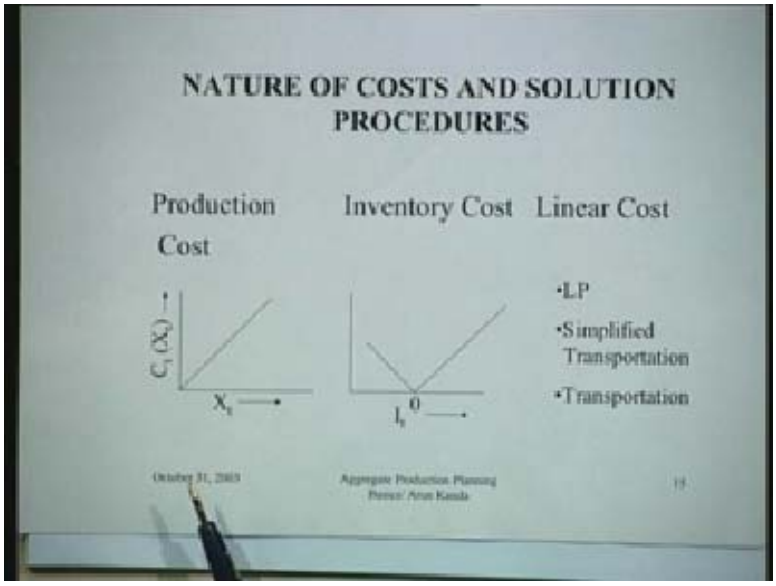

You can look at it this way. If it is allocated some additional land, if he is not stocking cars on it, he can probably give it on rent and on revenue on that particular land which it is forfeiting by putting some cars on stock. It is a potential loss of an opportunity which is a cost. That is what it is. So it would always be a situation of this nature. So if you have linear cost of this kind that is production costs are linear and the inventory costs are also linear, then you can use linear programming for solving this problem. You can use a simplified transportation model or you can use a transportation model for solving this particular problem. I am just trying to give you an overview of the various solution procedures. In the next lecture we shall go into the details of how these models can be applied for determining the optimal solution but the point here is that these solution procedures that we have identified here are valid only if the cost structure is linear for both production costs and for inventory holing and shortest path. Then you can use any of these procedures. On the other hand if costs are convex this is now a case when all the costs are convex. What do you mean by convex a cost, that is the production cost is something like this. What does it mean? If you produce the first unit the cost is less, as you keep on producing more units the cost keeps on increasing. When do you think this would be a relevant thing to happen? This would happen for instance when initially you have lower costs and subsequently beyond a certain production level the costs are lower. So if you are talking about regular time production over time production and subcontracting the cost slope would be progressively increasing piecewise linear in that sense right

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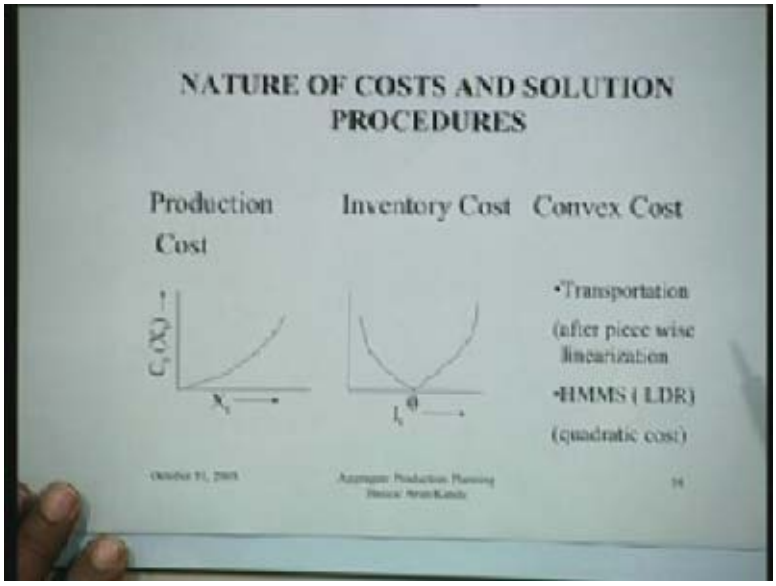

So those costs are there for convex. In a convex cost if you join any two points on this cost function, the line segment lies wholly above this particular cost. That is how you define a convex cost function. Similarly you will find that the inventory costs are also convex. This is convex and in a situation like this where the costs of production cost of inventory are convex again you can use a transportation model but after piece wise linearization, piece wise linearization is quite common. Regular time, over time subcontracting the cost will increase per unit or another very classic example of convex cost is a model due to holt Modigliani muth and simon for people and they have developed what are known as linear decisions rules or LDR's and they are typically valid for convex costs of this kind and in fact what has what they have assumed is that this is based on a case study in a paint manufacturing company and for that paint manufacturing company they have approximated all costs as quadratic costs of this type. This is a quadratic function and when you differentiate quadratic function you get a linear function. The solution that you get is a linear decision rule. That is how the term comes about. We will have a quick glance at this particular method. If the costs are concave and piece wise concave this is the most complicated category of costs. Mathematically you are saying that the cost is first increasing and then flattening out. What happens is this is like economies of scale the more you produce the lesser is the cost per unit. Similarly for inventory costs you have this behavior and for concave and piecewise concave costs we can use dynamic programming for determining a solution or we can use a network flow solution which is based on the shortest path in a network developed by Wangner and Whitin. In fact it is Wagner and Whitin. It should be Wagner and Whitin.

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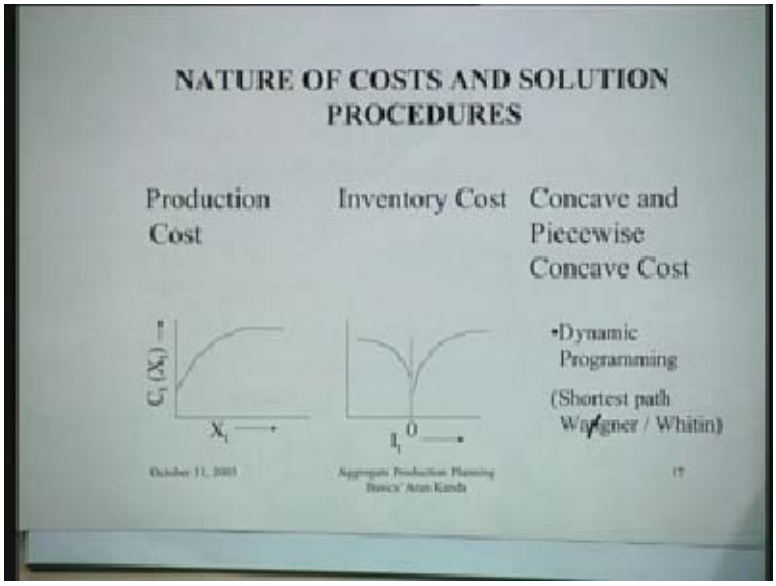

To give you an idea of these linear decision rules let's quickly run through this example. This was a study done by Holt Modigliani, Muth and Simon in the in 1955 on a large paint manufacturing unit. What they did was they defined these variables. W_t is the work force level in period t; t going from one to capital T, X_t is the aggregate production level in period and then we have I_t is the actual production net inventory at the end of period and I_t star is the ideal aggregate net inventory at the end of period t.

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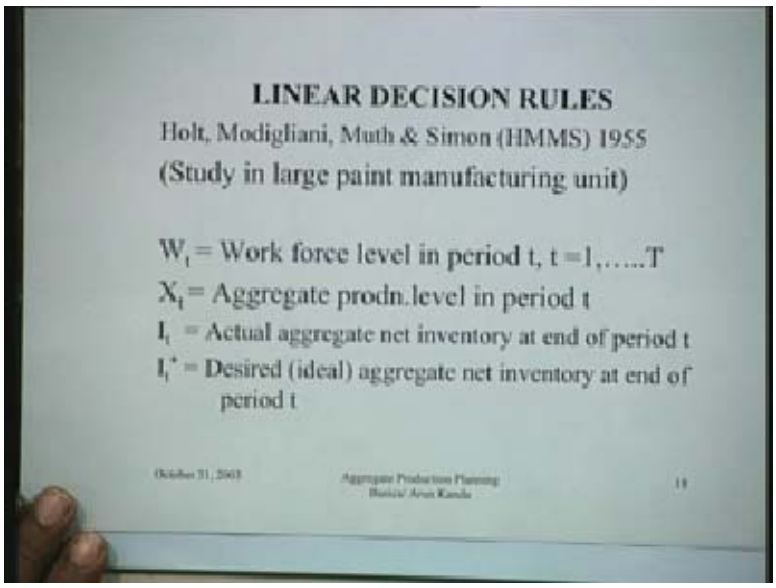

What is done in this particular model is that various types of costs are modeled. How are they are modeled? What is the regular time payroll cost? It will have a fixed component

and a variable component like this. This is the workforce W_t number of workers in period t. So this cost is modeled by $C_1 W_t + C_{13}$ which is a constant. So C_1 and C_{13} is constants. Similarly workforce change cost. Workforce change cost is something like this. This is for shortage and this is for holding inventory. So you have $W_t - W_t - 1$ is the increase or decrease in workforce. So this is the hiring cost. This is the firing cost. They approximate this function by this quadratic component and this quadratic component is written as $C_2 W_t - W_{t\ 1} - C_{11}$ whole square

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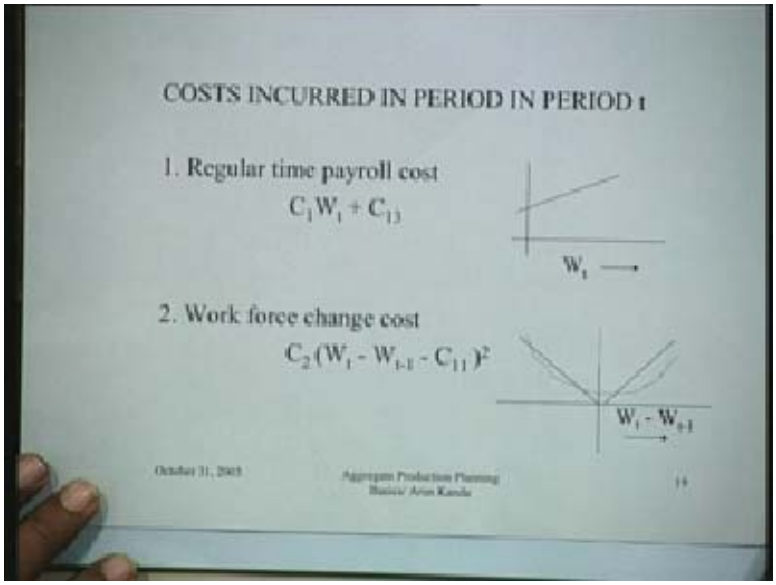

This is how the workforce change cost convex costs. Then coming to the over time cost, over time cost will also be that, beyond this level the cost goes up. This approximated by this particular function and it is $X_t - C_4 W_t$ whole square multiplied with $C_3 + C_5 X_t - C_6$ X into $W_t + C_{12}$ into $X_t W_t$. It is nothing but a function of all X_t and W_t. The $X_t - W_t$ whole square, X_t term W_t term and the product of the two terms, all the components are there with various parameters. Now these parameters will have to be determined from the actual cost of the company. Similarly invented in related cost which of this nature are approximated by function which is quadratic of this type and then you have C_1 $I_t - I_t$ star whole square which is equal to C_7 into $I_t - C_8 - C_9$ into D_t whereas D_t is the demand for the T th period whole square and these are all the costs and this is for just to give you an idea that once you sum up all these costs and you take partial derivatives with respect to W_1 star and X_1star which are your variables. You will get lineary functions of D_t W_0 I_0. What really happens is that W_1star W X_1star are now becoming out linear functions of D_t W_0 I_0 in the previous period. Similarly W_2 X_2 will be linear functions of W_1 $X_1$and so on.

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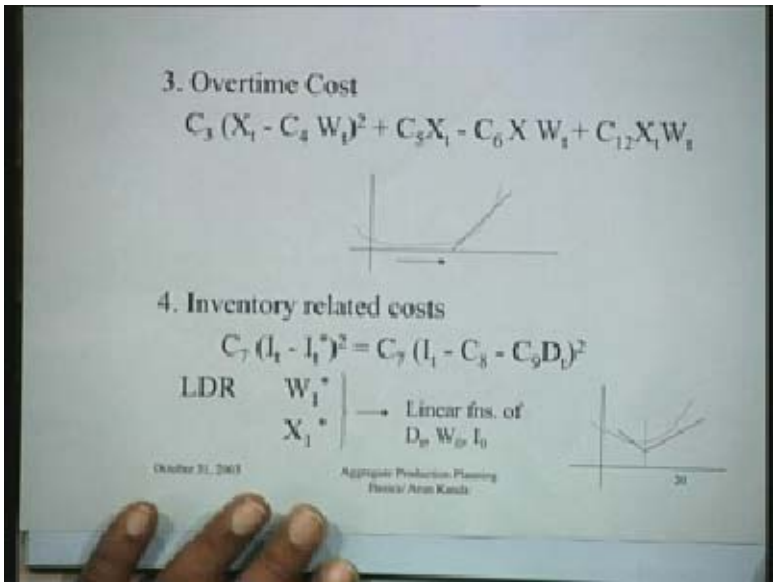

Those are the linear decision rules that this particular model is actually trying to use. There are a number of other methods to handle aggregate planning, aggregate production planning. These are some of the models which are available in the literature for solving aggregate production problems this type. So a very common formulation is the linear programming formulation. So if the costs are linear, you can easily write down a LP to represent the costs and the various constraints. What would be the constraints? The constraints would simply be inventory balance equations. I have so much, I produce so much. I sell so much and therefore at the end of the period, I have so much. This kind of a constraint will have to be written for each period. That is the inventory balance equation conservation of mass equations. Then any other constraints that you may have and the costs that we have discussed will constitute your objective function. This is the typical LP model formulation. Search decision rules was one category of procedures which were actually determined by Taubert in the sense that you talked about one type of rule, another type of rule that you are using and you can then talk about a combination of these rules. It is like rule one into alpha + rule two into one – alpha and you are trying to basically search with what particular value of alpha you would have a rule which would give you an appropriate kind of solution. These have not been produced to not come out very popular in aggregate production planning. Goal programming formulation is an extension of linear programming and it would be useful when you are dealing with multiple goals. Parametric production planning is a procedure developed by Jones in which basically some parameters are introduced which can change over certain periods and through the process of optimization you are actually determining the values of those parameters.

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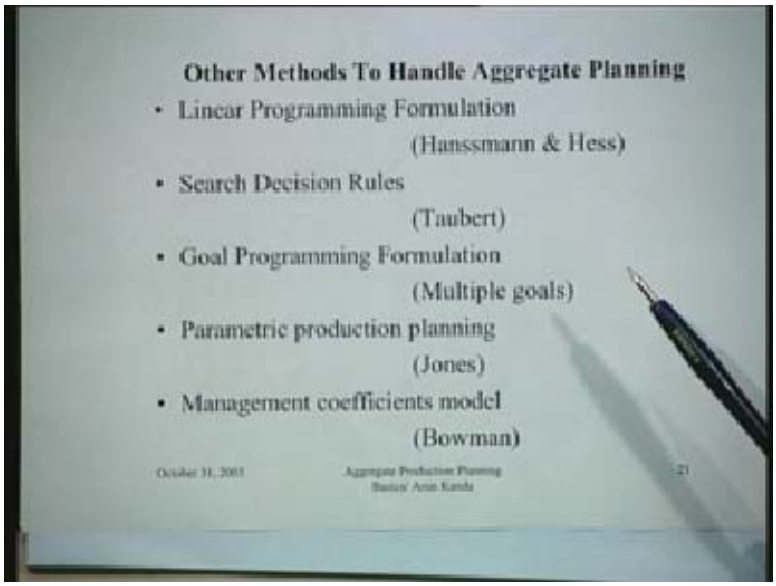

That is the basic idea and a management coefficients model is an approach in which you basically try to talk to different managers and find out what is the optimum strategy. You then try to find out something like a weighted average of the methods that are suggested or the strategies that are suggested by different managers which might happen. For instance if you gave the example of the graphical procedure we had two plans. So one could be the possibility of product one manager one. Second one could be the idea of manager two. So what you can do is if you thought of may be 75 percent confidence in this manager and 25 percent confidence in the other manager. You could take the two plans and find out what would be the production quantity in individual period something like that.

That would be the approach that you would follow to determine this. There are a variety of methods for solving the aggregate production planning problem. To conclude, let us summarize what we have tried to see in this particular lecture. We have seen aggregate production planning is relevant for fluctuating demands for medium term horizons typically in the range of 6 months to 1 year. I think that is one key factor that you have to understand is that basically it is the horizon of 6 months to 1 year that we are trying to see and we are talking about the fluctuating demand of the company and we are trying to see how the resources should be allocated to meet that fluctuating demand. This is the problem. This is the problem that we have been trying to see. A simple graphical procedure that generates good solutions by examining the demand pattern was present. It was simple in the sense that you could pose a procedure and then just find out what is the cost of that procedure. That was the idea and there could be various ways to meet a fluctuating demand. It could be through hiring firing through storing inventories through shortages through back orders etc. We saw that there were a variety of ways. In fact the very fact that there are a variety of ways that is why the optimization problem is so intense.

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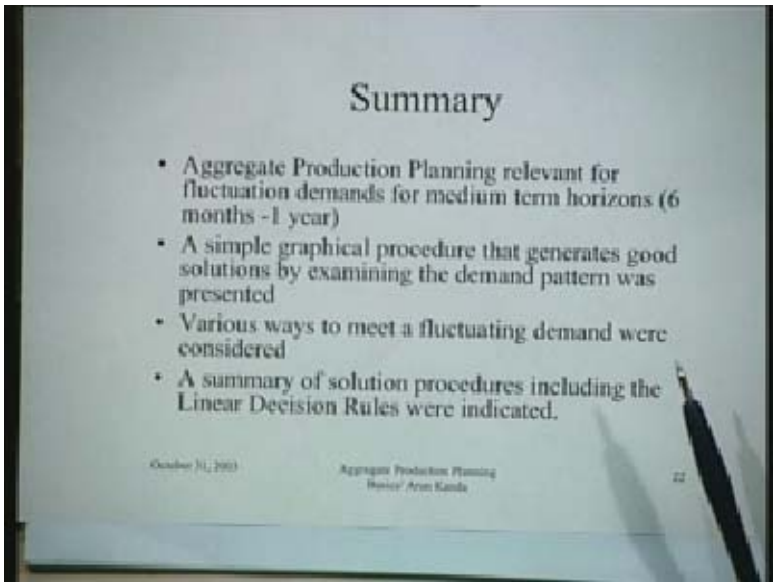

You have to find out what is the best way and finally we had taken an example of solution procedures including the linear decision rules where basically the costs were convex and because they were modeled by a quadratic function, when you differentiated the cost function you actually landed up with linear decision rules and this was one approach. Now in the next lecture we shall be talking about some specific procedures that I will use for optimizing the production plan. That is finding out what is the best way to determine the production plan.

Thank you!