Project and Production Management
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Lecture - 30
Mathematical models for facility location

In the last lecture we had looked at some of the issues in the location of facilities and we had got a broad overview of the problem of location of different types of facilities. In this particular lecture we are going to be talking about some mathematical models for facility location. In fact before we take up any mathematical models we initiate the discussion.

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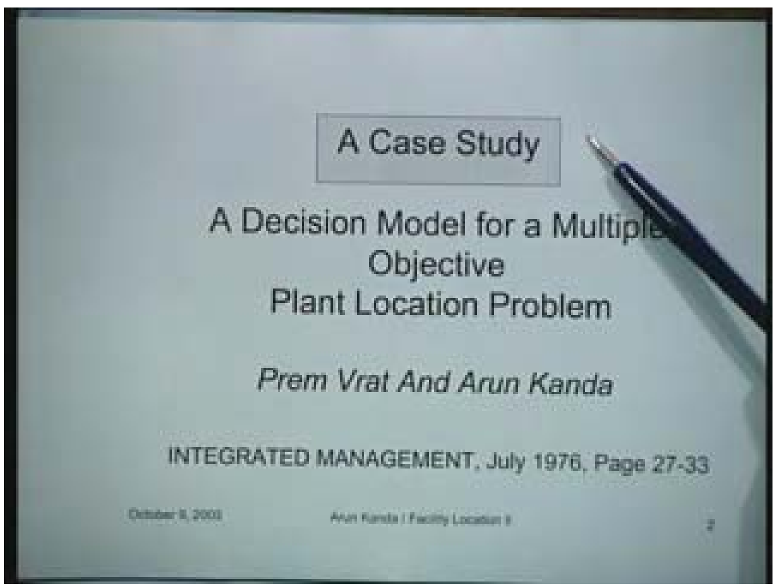

I will discuss the case study. This case study is actually based on paper that Prof. Prem Vrat and I wrote and this is the reference. It is called a decision model for a multi objective plant location problem and the basic intention in this particular case study is to look at various factors which are relevant for the plant location problem and try to then determine after conducting a feasibility study as to what should be the best location for the manufacturing plant. So this would be the kind of study that any new entrepreneur or any new company contemplating a new location for a manufacturing plant would go through.

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In this particular case study the objective of the location was actually to setup a straw board plant. Straw board as you know is a packaging material which is made from industrial waste and basically the idea was that this would be setup in an industrial setting so that there would be availability of industrial waste automatically and that particular industrial waste would serve as the input for the plant and this would then be processed to supply various industries needing packaging material in the form of cottons and so on and various things. This was the basic idea behind the location of this particular plant. Now what was actually done was that in order to conduct this study systematically, interviews were carried out with the plant managers and various other people in the organization to identify what were the various factors that they consider important for this particular decision.

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After series of discussion with the management it was found that the factors that needed to be considered are A nearness to raw material source. Obviously the nearer you are to the raw material source, it would cut down the cost of transportation and it is also easy to get your raw materials. Second factor that was considered important was the availability and dependability of power. Then the manager identifies transport facility as an important factor. Labor supply was another important factor, employee facilities, the kinds of facility that employees will have in terms of shopping recreation schools etc. This was considered to be a factor in the beginning. Competition for the market was considered to be another major factor because if they were already a large number of manufactures of straw board in that particular area it would be difficult to sell your product. So the lesser the competition for the market, the better it was. Proximity to market was considered another important factor. Government incentives, as we know, typically the government might want to give certain kinds of incentives for certain locations may be to encourage development of certain backward areas. These are the kinds of incentives that are available and then finally the cost of land in that particular site which we are considering. In fact you see these factors are almost generic in any plant location problem. You would have to consider various factors. The only thing that would happen is the relative importance you might give in these factors. They could vary depending up on the nature of the plant. Our approach in the problem is essentially something very simple and it is summarized in the form of this table.

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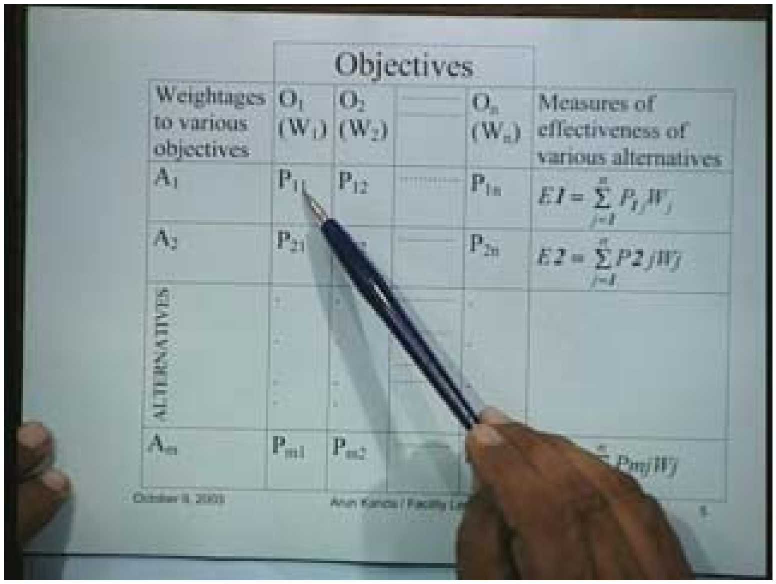

What you would like to do is we have identified the objectives that we have. You would like to prioritize these objectives and give certain weights and then we identify alternative locations A_1, A_2 and up to A_m and each alternative location is evaluated on the corresponding objective. For instance P_{11} is the score given to the first alternative on the objective, P_{12} is the score given to the first alternative on the second objective and so on. So if we can then get these scores, somehow then what we simply have to do is we can calculate the effectiveness of alternative A_1, alternative A_2, alternative Em if there are m alternatives we and get a kind of a score. This is very similar to the manner in which we evaluate students in a class. These are the candidate location of the candidates (Refer Slide Time: 07:19), and these are the performances in various subjects as you go through. Each subject has different weightage or different number of credits and what we assign basically to you is some kind of marks based on some scheme and ultimately you calculate a CGPA or a grade point average. So what we are trying to say here is that, we use this kind of a scheme to evaluate different alternative locations. Now let us see how it can be done.

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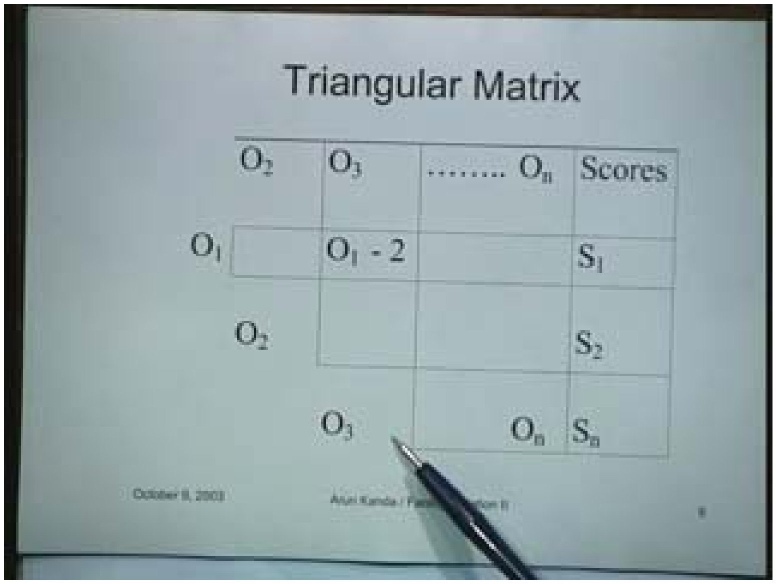

Earlier however we do what would be necessary are the various objectives that we have, the objective that we just listed out nearest to the raw material source availability and dependability of powers and so on. You have to score these objectives to determine the weights, so what is actually being suggested here is that if you construct a triangular matrix, the purpose of triangular matrix is because there are n objectives. There will be nC_2 possible comparisons or pair wise comparisons. All pair wise comparisons. So we look for pair wise comparisons, for instance in comparing objective 1 with objective 3. You might want to give o_1, two marks whatever the interpretation is; to these marks. Then ultimately you can determine the scores by counting the number of scores or votes polled by different objectives. So you will able to get scores. The basic advantage of scheme like this is when you have a large number of objectives; it is very difficult to give weights to the individual objectives. But if you are comparing only 2 at a time, it is much easy for a person or a manager to say between o_1 and o_3; I can compare and say o_1 is more important. The difference is medium. So it is much easier to adopt a pair wise comparison approach and evolve a set of scores.

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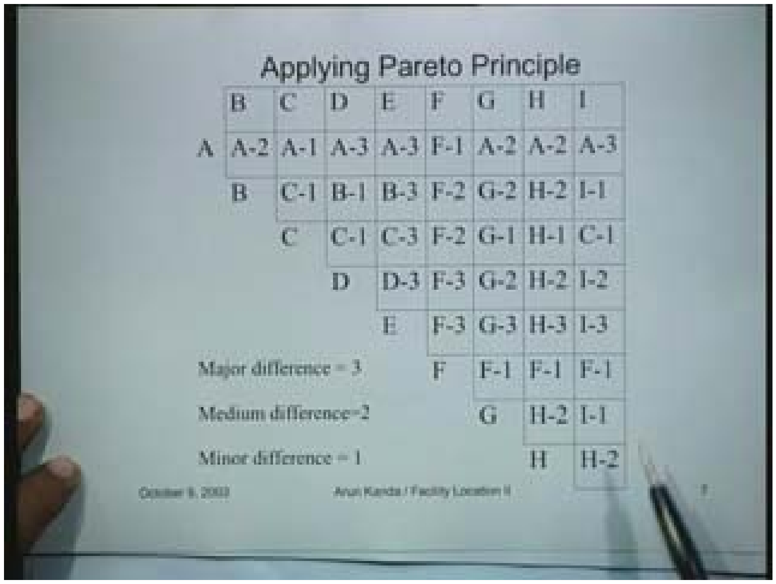

For instance for this particular problem what was done was this triangular matrix was constructed and these were the objectives A, B, C, D, E, F, G, H, I. The management was asked to compare between A and B and the management felt that A was more important and the difference was medium. So you wrote A_2 here. Similarly in comparing for instance C and G, the management felt that G was more important, but the difference was minor. So you put minor. This is because you are basically distinguishing between (Refer Slide Time: 10:17) and then you say how much better, so this is a simple way of doing it. So having got this information we can simply compute, for instance total A, you can count the rows in this particular row. For B it can be in this column and this row for C, you can count the votes here and so on. So you would know exactly what is the score or the weightage for each of these various objectives.

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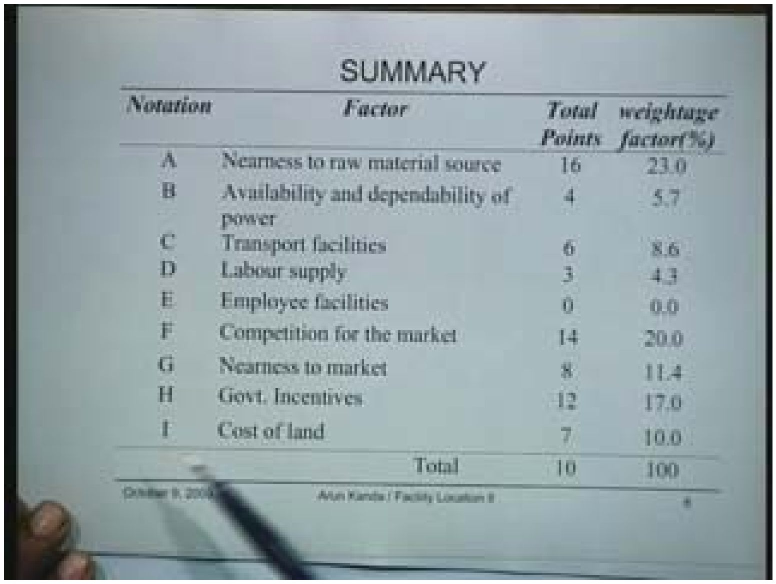

SUMMARY

Notation	Factor	Total Points	weightage factor(%)
A	Nearness to raw material source	16	23.0
B	Availability and dependability of power	4	5.7
C	Transport facilities	6	8.6
D	Labour supply	3	4.3
E	Employee facilities	0	0.0
F	Competition for the market	14	20.0
G	Nearness to market	8	11.4
H	Govt. Incentives	12	17.0
I	Cost of land	7	10.0
	Total	10	100

So in this particular example we have these objectives A B C D E F G H I which were nearness to raw materials source availability and dependability of power, transport, facilities etc and the total weightage given to these particular factors came out from their as 16, 4, 6, 3, 0, 14, 8, 12, 8, 7, and this was what was basically involved here and the weightage for all these the percentage weightage you sum it up this seems to be an error here sum it up and then divide this by the total you get 23.0 percent weightage given to this particular factor here and so on similarly so hundred percent weightage this is how it is finally distributed to these individual objectives. So this was the rational way of arriving at these weightages, basically using pair wise comparisons. There are a number of techniques available these days which you can employ if you so desire. AHP is one example. Analytic hierarchy process is also pair wise comparison technique which essentially gives you these weights and so on. But you have to go through a similar kind of an exercise.

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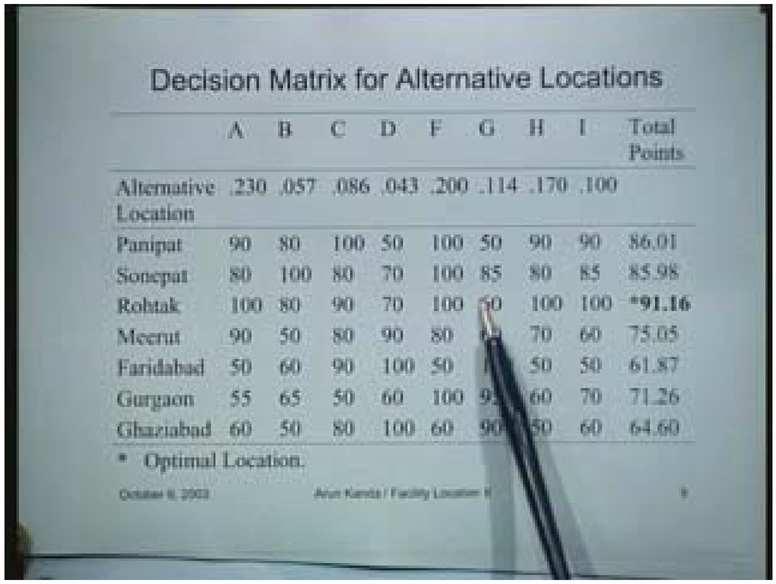

So ultimately the results for this problem could be classified as the above. A survey was conducted as to where the location should be. Now the management in this case was interested that the location of the plant should be anywhere in north India. We did not want to go to South or West so the major industrial districts in the north were considered and these districts Panipat, Sonepat, Rohtak, Meerut, Faridabad, Gurgaon, and Ghaziabad. These were the tentative locations that were possible for setting up the plant and now we have these objectives and these are the weightages that we have discovered and just determined from the pair wise comparisons. Finally what we are required to do is to evaluate each of these locations on these individual factors. If you recall the analogy this is not very difficult, it is like saying if this subject is Physics I have to give quizzes in Physics to determine the marks. I have to determine this factor which I will find out and so on. What we are trying to say is that the first factor which was essentially near to raw materials sources was found in this particular classification, nearness to raw material source, Rohtak was the best.

Hence, Rohtak got 100 points. The scheme was the worst location which will be given 50 points and the best location will be given 100 points. It is very much like trying to say that we normally do not fail candidates in the IIT and therefore the worst fellow will get 50 marks. The best fellow will get 100 marks in the particular. So in comparing these, in terms of distance in the raw material source, Rohtak was relatively raw at that time and plenty of raw materials were available there and so on, so it was 100 which were the worst. Faridabad, Gurgaon and Ghaziabad were all low in the hierarchy and Faridabad was the worst. Faridabad was the worst although it is an industrial town. Whatever, there were already large number of industrial units set up for the availability of raw materials. It was difficult, more difficult as compared to other factors. So this was the scheme, this was on a normalized scale of 50 to 100. Similarly all these evaluations were done on a normalized scale from 50 to 100. For instance in the second objective it was felt that

Sonepat was the best and Meerut and Gahziabad were the worst. For instance the third factor was transport facilities. As far as transport facilities were concerned, it was Panipat that was the best out of these options. It got 100 points, Rohtak was petty good, Faridabad was petty good Gurgaon was bad and Ghaziabad was okay. So this was basically an evaluation of these locations on all the factors.

Once all these scores are available what we can do is 90 into 0.23, 80 into 0.057 so on. The final points that you get here are 86.1 and similarly 85.98, 91.16, 75, 61, 71, 64 etc. So from this analysis it emerged that Rohtak was the best option out of these. Now however you find that this was a very good option, no doubt but the second best option was petty close. This is 91 marks, this is 86 marks. So it is like trying to say that a company comes to recruit, you need not only recruit the best candidate, but the second best, and the third best will also get jobs. So in the same sense you find that here 91 and then 86 will be petty close and then 85, 0.8 which is almost 86, so these are petty close. So you could do a more detailed comparison between the first three. What this means is that Rohtak was best on one, two, three and four. Out of these various objectives, Panipet was the best one on only two of these objectives.

Let us see where Rohtak was worst. It was worst on this particular factor i.e., 60 marks. But it was even worst on this factor and this was likely to be better but then it was best. So on the whole it seems that Rohtak would be the optimal location. So the optimal location was in fact suggested as Rohtak and the company adopted this. After many years the feed back received from the company is that they are very happy in Rohtak which means the decision that was taken more than 15 years ago was probably the right decision for them in terms of whatever we did. Now having used the decision matrix for alternative location, you could use this methodology for any real life situation. You see the tricky part here is how do you assign these marks to these candidates?

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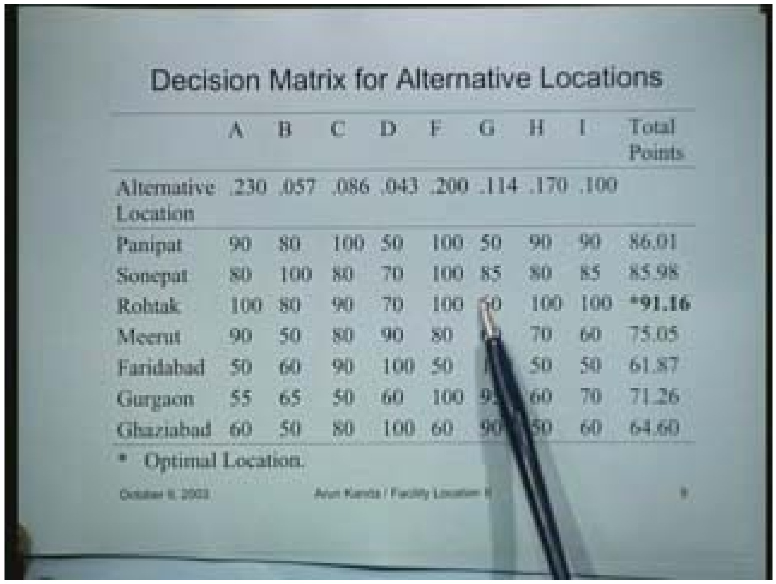

	A	B	C	D	F	G	H	I	Total Points
Alternative Location	.230	.057	.086	.043	.200	.114	.170	.100	
Panipat	90	80	100	50	100	50	90	90	86.01
Sonepat	80	100	80	70	100	85	80	85	85.98
Rohtak	100	80	90	70	100	[illegible]	100	100	***91.16**
Meerut	90	50	80	90	80	[illegible]	70	60	75.05
Faridabad	50	60	90	100	50	[illegible]	50	50	61.87
Gurgaon	55	65	50	60	100	[illegible]	60	70	71.26
Ghaziabad	60	50	80	100	60	90	50	60	64.60

* Optimal Location.

If a particular factor is let us say capital cost, then the highest capital cost will be the most undesirable, least capital cost will be the most desirable and so on. So the manner in which you assign these marks is basically the process of normalization.

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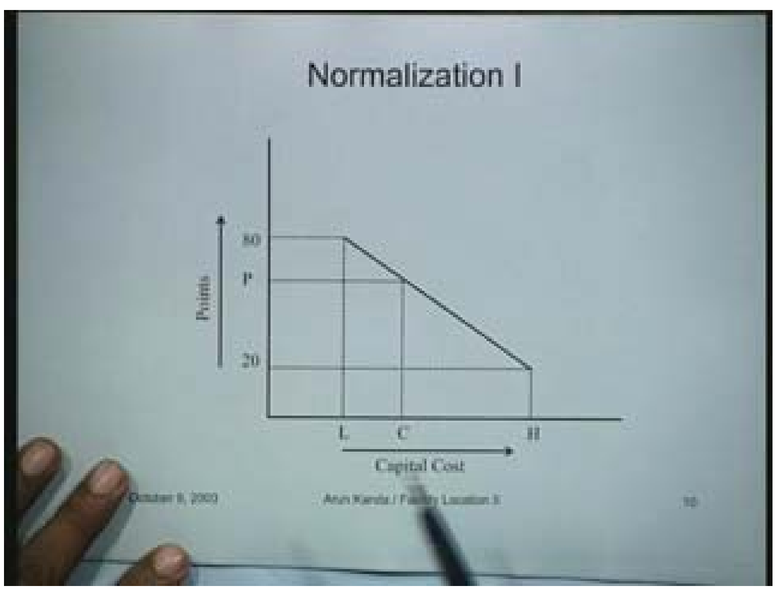

Now what about normalization? What we mean in normalization is that depending up on the objectives suppose our objective is capital cost and we have investigated a number of different alternatives then in the alternatives the one which is the highest in terms of capital cost should be given the least marks, i.e., is 20 the one which is the lowest should get the highest marks which is 80 and the most convenient way to represent this relationship is a linear straight line or a normal straight line. You might use a normalization curve of this kind or a teacher who is giving a normalization curve of this kind. So this is exactly what normalization would mean. It is not necessarily true that normalization would be linear. Take the same example of the capital cost for instance we have something like this, the lowest capital cost is L and the highest capital cost is H. The company might be indifferent for instance, lowest cost might be 2 lakh or whatever and say upto 3 lakh. The company says it does not make much of a difference. You are indifferent between 2 and 3 lakhs. So you give 80 marks to all of them who have done this particular factor and then from here it could be a straight line, it could be a concave function or a convex function as the case may be. The difference between the concave and convex function would be that in the convex function which you have here, the drop is very steep in the beginning. Slope is highest here and it tends to fall and the minimum slope is here. So it is like saying you are penalizing the candidate most. That is capital cost. There is a fall in capital cost. You are penalizing here and relatively at this point, the rate of penalty is decreasing. It is the reverse in the case of the other function. Now what I am simply trying to emphasize here is if you give careful thought to normalization, this is a very important strategic decision location and what is normalization? Normalization curves represent your attitudes towards the particular object.

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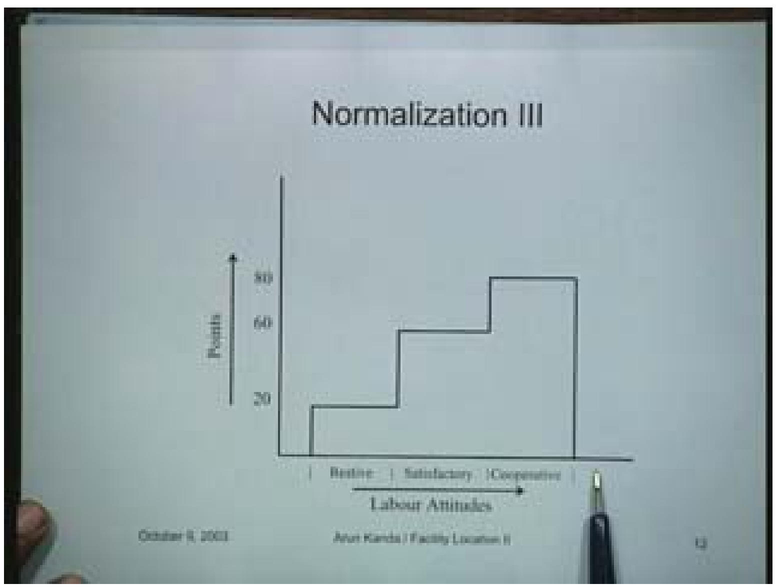

For instance, if you have a factor like labor attitude, you might want to classify labor attitude into three categories, bad, medium and good. You might say rested where the labor is so restive. It is willing to pick up agenda for every small provocation. So it is a restive labor. You have a satisfactory labor. The labor is willing to be cooperative and you might have a cooperative labor which is willing to do everything. Obviously this is the best. So you give maximum marks. Let us say 80, between these you give 60 between these you give 20. So essentially the normalization curve for such subjective factors will be a staircase function of this kind where you can decide the length or the height of the stair in each case. It is not necessarily true that the normalization curve is linear or be portions of linear functions.

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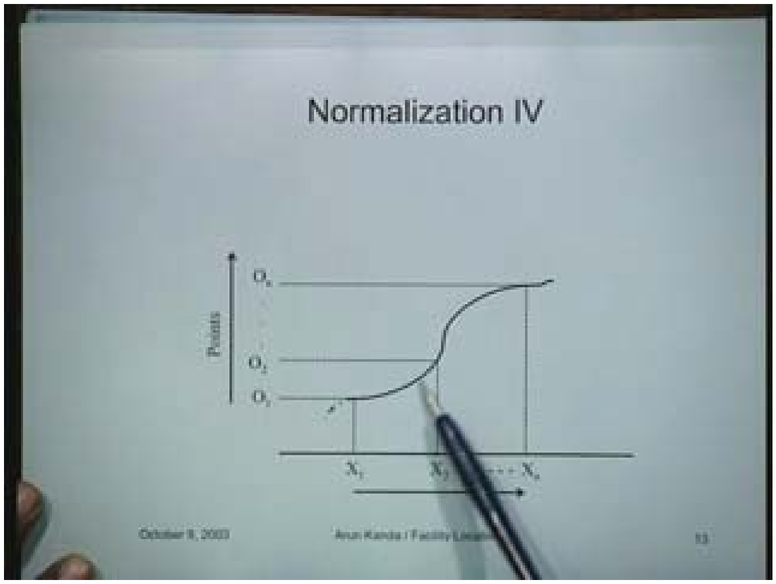

A normalization curve could also be nonlinear. So a nonlinear function is Present, so this is the least desirable. This is the most desirable. The most desirable aspect is given the maximum number of points. The least desirable is given the least number of points and you can think of what would be the nature of this curve for your case. So the point is that once we have developed the normalization curves for individual objectives, this process becomes very objective. All that you need to do is develop the weights for the objectives like we did through the triangular matrix, determine the normalization curves and feed them to a computer and then feed the data corresponding to individual alternatives locations into the program and then you can get the final solution for various options. So the idea here was that we were dealing with the practical plant location problem. Practical plant location problem has both subjective and objective factors. We had here a mechanism to deal with both subjective and objective factors but there was no attempt at optimization.

What was the kind of problems that we were considering? We looked through the feasibility report or identified a number of locations and we were trying to choose the best location. So it was the problem of choosing the best out of the available options. That is the kind of the problem that we were considering, however in facility location we have certain mathematical models where the intention is to determine the best or the ideal location and that particular ideal location can then motivate you to locate your facilities wherever they are. Let us see the models and we looked at some simple models, for facility location in this context.

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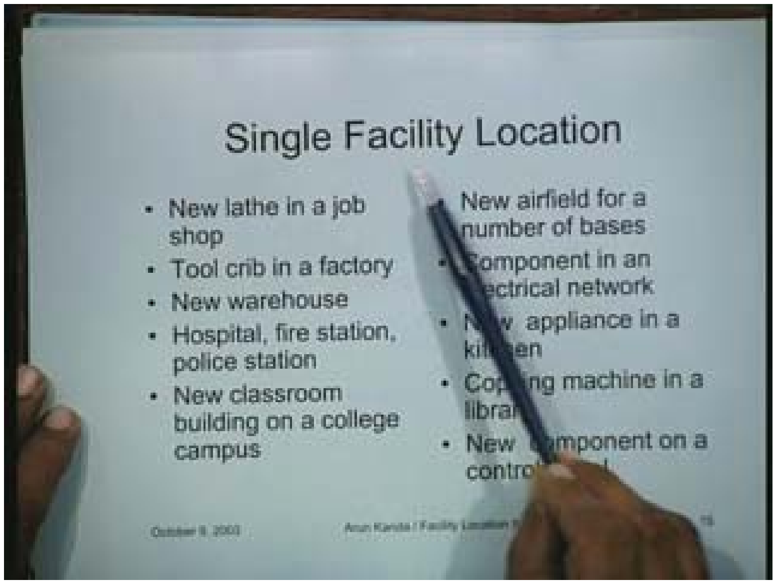

Let us talk about the problem which is known in lecture as the single facility location problem. The single facility location problem is essentially the problem where you want to locate one new facility and this problem occurs very frequently in life. Here are some examples of what we call the single facility location problem. For instance you want to determine the location of a new lathe in a job shop and you already have 23 lathes. You bring in another lathe, the 24^{th} one and you are trying to find out where it should be located. It is a single facility location problem because you are trying to find out the location. A tool crib in a factory is the place where we have the workers taking their tools and deposit their tool every time.

So it is a common facility for all workers. You might want to find out what is the best location of the tool crib such that the total travel distance of all these workers is minimized which means total time is minimized. This means their productivity is maximized because they are wasting less time in computing, in that sense you could talk about the location of a new warehouse for a company warehouse. You could talk about the location of the hospital or a fire station or a police station. All single you could talk about is the location of the new classroom building on a college campus, single facility location problem, a new airfield for a number of bases and you can talk about the component in an electrical network. You know electrical network these days have a large number of electrical components put onto one board. Suppose it is decided that you improve the design characteristics, you want to put an additional resistor somewhere. Where should that additional resistor be and it is going to be connected to some of these components. So you are trying to say where should I put up the additional resistor such that the total length of the cable or the length of wire is minimized? This is again a problem of a single facility location.

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You want to talk about installation of a new appliance in a kitchen; this is again a single facility location problem. You want to find out where copying machine should be placed in the library, assuming that in a library, different readers from different locations may have statistical data on an average 20 percent of the users, from first floor and 30 percent from this floor, come here for this particular thing. So you can determine where the optimum location for the copying machine should be or if there is a new component on a control panel you have a big switch here Larson and Tubro switch here. It is huge one and we are trying to find out where new component should be installed which also happens to be a single facility location problem. So the single facility location problem is actually a problem, in fact that is why it is called single facility. We are not talking about single plant location because when you recall, when we were discussing the issues in locations were specified that often tend to speak in terms of facilities and depending on whichever facility it is you have to determine the optimum location.

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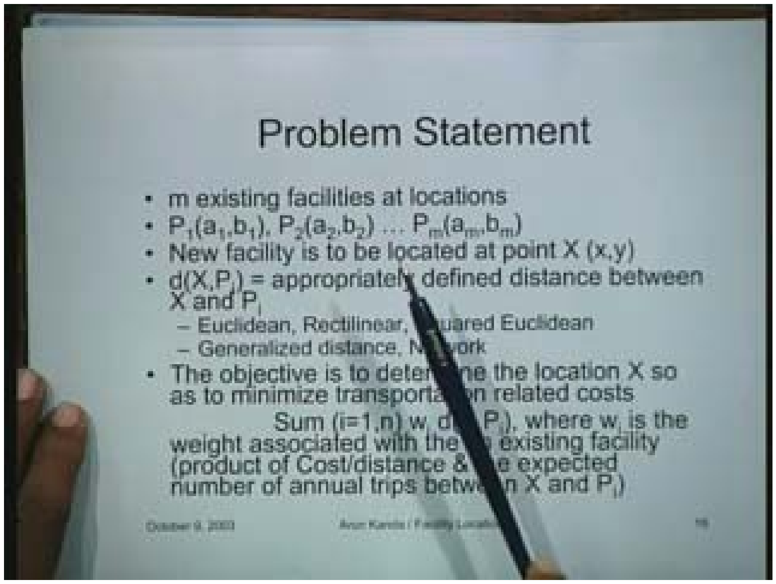

For this problem the statement mathematical statement can be built up as follows: we say that we have m existing facilities at locations P_1, P_2, P_m whose locations are a_1b_1, a_2b_2 and so on up to a_m b_m, so we know the location of the m existing facilities and the new facility is to be located at some point X which is x y. You want to determine this (Refer Slide Time: 28:44) and where it should be located. The new facility, the other existing facilities are fixed that is their assumption and dX P_i is an appropriately defined as the distance between X and P_i. X is the new facility you want to locate and P_i is one of the existing facilities. So the distance between them can be called as dX, P_i and this distance could be defined in various weights.

It could be Euclidean distance which is a straight line distance. It could be the Rectilinear distance which is the Manhattan distance between these two points, it could be the square Euclidean distance, it could be what we call a generalized distance or it could be a distance on a network. In fact this would define a variety of models which are available even for the simple problem, the single facility location problem. What is the objective of this exercise? The objective is to determine the location X for the new facility so as to minimize transportation related costs, that is what you want to do and what are these transportation cost related to? The transportations related to cost are the sum from i = 1to n, the summation of w_i into dX, P_i where w_i is the weight associated with the ith facility which is the product of the cost per unit distance and the expected number of annual trips between X and P_i. If you recall our discussion of the varianon frame where we had determined weights corresponding to each of the points, the interpretation of w_i is the same. It is a weightage given to the ith point. How important it is in terms of the interactions that take place between new facility and that particular facility. That is the objective function. This is general problem; a general statement of the problem is this.

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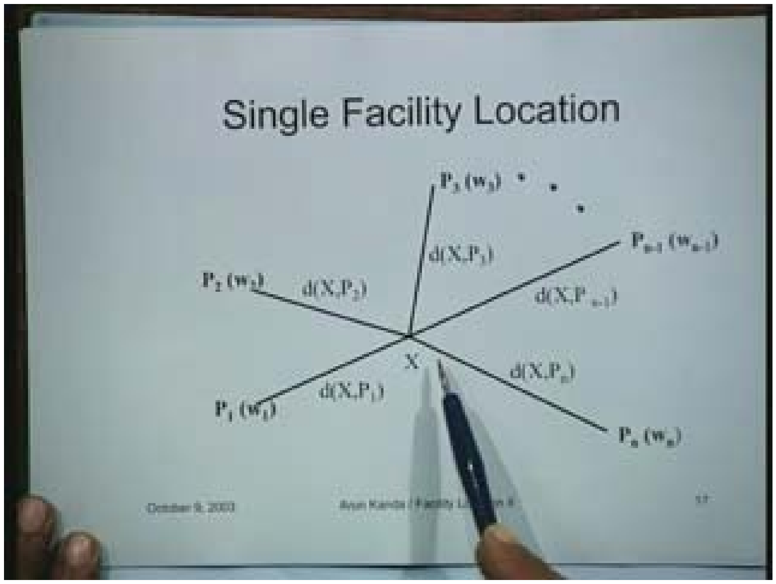

So essentially if you look at the problem in a graphical perspective what is really required is that we have points P_1, P_2, and P_3 and so on up to P_n or P_m. This is the case and it may be spread out. These are the existing facilities. We are interested in finding out an optimum location X. This optimum location X should be such that the total cost of movement between all these should be minimized. That means what we are trying to say is that we are trying to minimize the total cost of travel or transportation between the various facilities. There could be many examples which we can construct for instance, if these facilities are hospitals in the city, let us say Delhi has 10 hospitals, these are the 10 hospitals that you have in the city and the problem is that you are trying to find out the location of a centralized blood bank facility. This can be called upon by any hospital in case of emergency or whatever it is.

So what we are trying to say is that we should determine the location such that if you know roughly that during a particular day there are roughly 5 trips, here 20 trips, here 10 trips, here 10 trips, here 30, and so on which you, from the past historical data use the information to determine the optimum location of the new service facility. Similarly you might say for instance if I want to find out the location of a hospital which will cater to the needs of different people in the localities and these localities have these weights in terms of number of people falling sick. Let us consider an order of magnitude proportional to population, assuming that all people are subjected to same kind of health conditions. So if you do that then again the problem of finding out the location of this hospital is basically a single facility location problem. Now what we would like to do is first understand the various commonly used distances that we talked about.

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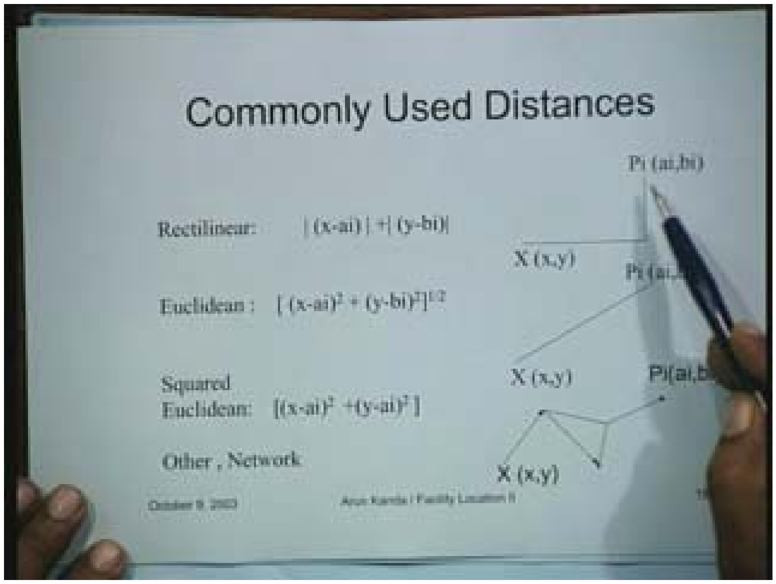

For instance if the location of the new facility is at X and the ith existing facility is located at the point Pi which has coordinates ai, bi then the rectilinear distance between these 2 points is actually this (Refer Slide Time: 33:52) distance. This distance and then this distance (Refer Slide Time: 33:54) and mathematically the rectilinear distance will be represented by x – ai mod + y – bi mod. Now you must understand that there are a large number of situations in real life where the rectilinear distance would be an appropriate one to consider, for instance all movements within the factory are generally rectilinear because in a factory on the shop floor you can typically move along a set of perpendicular heights. So if a worker has to go from one place to the other he has to take of the perpendicular route this way and then go this way.

That is the rectilinear distance and therefore if you are dealing with a problem of let us say machine location within a factory, or you are talking about location of some other tool crib in a factory or if you are talking about the location of a let us say even canteen within a factory. The distance measure that you think that is most appropriate is a rectilinear distance measure and therefore this is a very important distance measure especially in the context of location on the shop floor, locations within the factory. Of course if you happen to travel to New York for instance the distances are rectilinear that is why it is called Manhattan. What you have is you have a large number of parallel streets and these are intersected by a number of other parallel streets and all that you have therefore is the movement is always rectilinear and in fact that is the word Manhattan and rectilinear comes from it. In our context Delhi would not be rectilinear. If you go to Chandigarh for instance the movement from one place to other is generally rectilinear because the layout of the city is such that you move about one round about another and there are parallel streets and so on.

So it is essentially rectilinear if you are talking about location of a facility in Chandigarh. Rectilinear distance could be any problem which is an important distance measure. The next important distance that we consider is what we call Euclidean distance measure. Euclidean distance measure is the shortest distance between two points.

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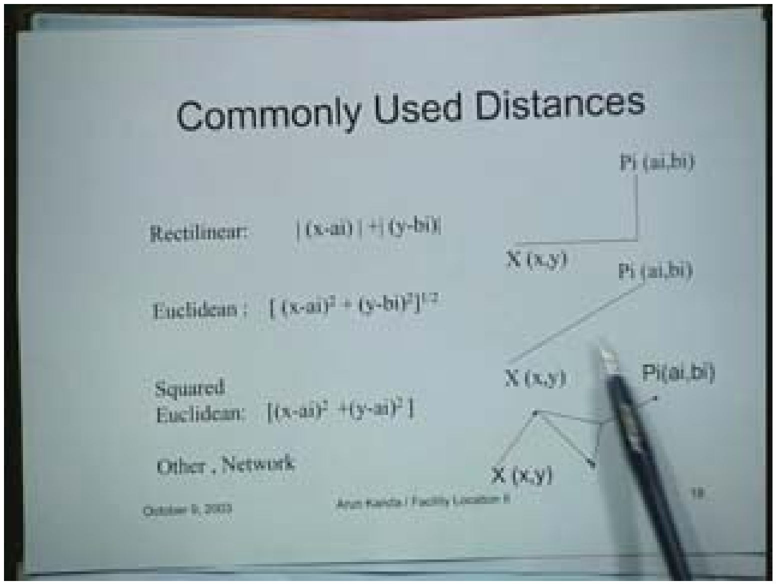

The Euclidean distance from the point X which you want to determine to the existing facility Pi, ai, bi, you have this particular distance which is the shortest distance between 2 points. It is governed by this function x – ai whole square + y – bi whole square and the square root of this. This can be conveniently calculated in this fashion. Where do you think Euclidean distance will be appropriate in the location problem? Where do you think you could consider where things take place? Let us say generally straight line generally and if you are trying to talk about this, what would happen? For instance if you are talking about the location of an airport, you know that you are going to expect flights from a large number of cities in India and abroad here and flights going out. So that would determine the weights for the cities and since you travel almost a straight line, you could say that the determination of the optimum location of the airport could be (Refer Slide Time: 38:00) but there are other considerations like determination of the airport. But definitely from the point to view a minimization of the total travel, this would be an optimum situation.

Then where else you think this would be appropriate? It all depends up on whether naval applications, whether you travel straight or you travel all over the sea, how you travel and so on. For instance suppose you travel by this trolley car or a ropeway, suppose if in the mountains, the government wants to decide up on picnic spot and it was to find out the location of a ropeway and this way you could probably visit location on one hill and the other hill wherever it is and different locations as it is. Then which particular location will minimize the total cost of the cableway?

Obviously it will be Euclidean distance. So you could have a very large number of situations wherein if you are talking about electrical circuit, you want to fix a particular component and the component is to be connected by a wire to a number of different components assuming of course that if it could be done on a Euclidean way in that particular component, then the location which tries to minimize the cost of wire, the cost of cable etc but also be using the Euclidean. Finally let us look at the problem where you are talking of squared Euclidean. This under root sign is missing so we are talking about the square of the Euclidean distance. Square of the Euclidean distance is just the distance but the square of it.

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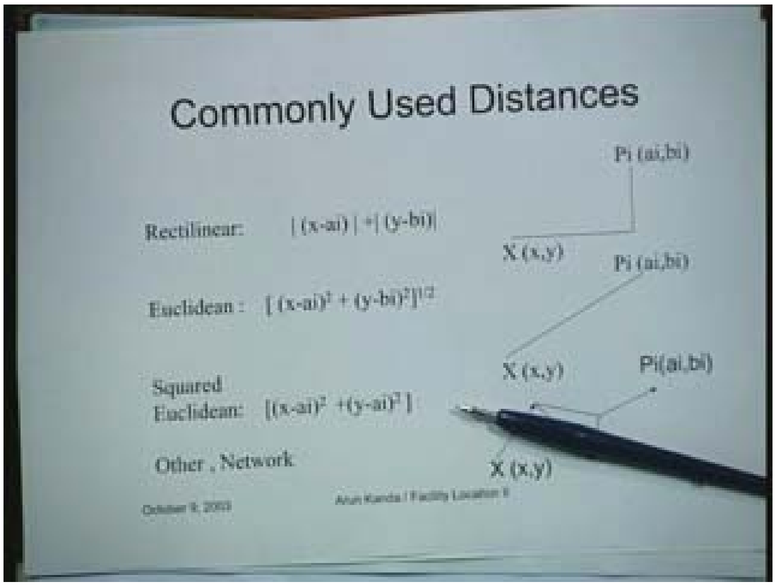

Square Euclidean distance location problems are also very relevant in many situations. They are relevant mainly in problems which involves radiations. The radiation loss is typically proportional to the square of the distance. You may remember your thermodynamics and see that the heat loss from a particular source is proportional to the square of the Euclidean distance from that particular source. Now this is something very important today because this is in the context of let us satellite location problem or in the context of suppose you want to locate a satellite in space and the satellite has to serve a number of existing location on the earth, it has to serve New Delhi, Kolkata, Hyderabad, Singapore etc. So you have a number of m points which it has to serve and let us say you know the weightages for these points so the idea is that the optimal point for the satellite would be the one if you want to minimize the total radiation loss.

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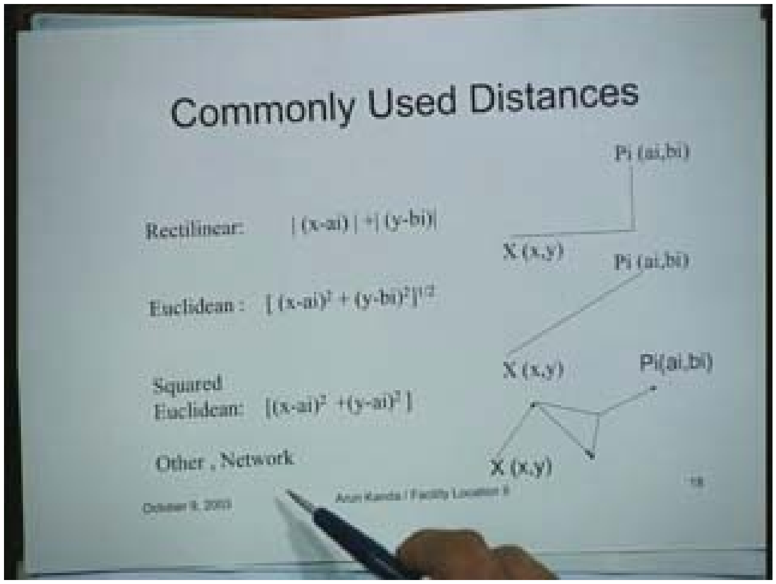

You want to minimize the total radiation loss; it should be located, such that this particular distance criterion is actually this particular criterion. So the distance measure may be any other because you travel anyway so I have to go from here to here, or I can go this way, I can go this way, I can travel on a network in any particular manner and so on. So it could be other or any other network. Those can be commonly used distances so what we do is normally the network location problem is a practical problem because there you define exactly the distances from one node to the other on the network. You can look at the network and try to solve the location problem on the network.

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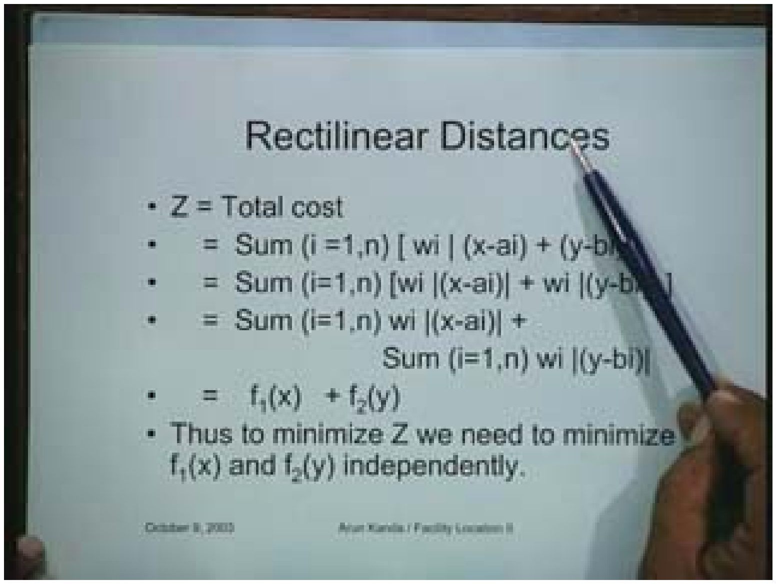

Now let us try to look at this particular problem of rectilinear distances. In a rectilinear distance problem our objective function is simply Z which is the total cost, which is the summation from i = 1 to n, wi into x – ai + y – bi mod. That is what it is, that is the rectilinear distance. The only interesting thing here is that this particular function could be sort of broken up into wi into x – ai mod + wi into y – bi mod and this can then be written as two separate summations. The summation for x – ai mod and the summation for y – bi mod and the interesting thing here is that this is the function of x alone. This (Refer Slide Time: 43:30) is the function of y alone. So the problem of rectilinear distances is that if you want to minimize Z what you have to basically do is you have to minimize f_1 x and f_2 y independently that means optimum location for the rectilinear distance location problem is determined simply independently minimizing f_1 x and f_2 y. Now you can determine the procedure for doing this and rather than deriving the procedure I would state the procedure here in fact in this particular situation, the f_1 x and f_2 y are minimized, if you locate at the medium location, that is the result. That actually comes from duality theory. We can look at the proof in the tutorials.

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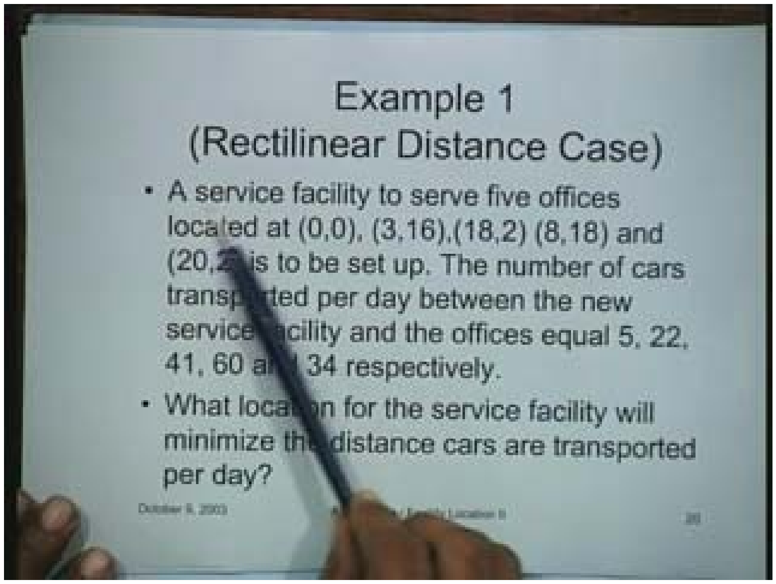

Let us take an example to illustrate what we have been talking about and we will use this frame work of the example on different distance measure. For instance let us assume that service facility to serve five offices located at (0, 0), (3, 16), (18, 2), (8, 18) and (20, 2) is to be set up. It is like saying the number of cars transported per day between the new service facility which you want to set up and the offices equal 5, 22, 41, 60 and 34 respectively. So the question is what location for the service facility will minimize the distance, the cars are transported per day? It is like saying that you have collection centers for cars at 5 places and cannot place south Delhi etc. You also want to have central service facility. Where will you avail the service? The people can deposit their cars here and their experience shows that the number of cars deposited per day is so much so where should your service facility be? This is the problem and we are assuming that the distance involved is rectilinear.

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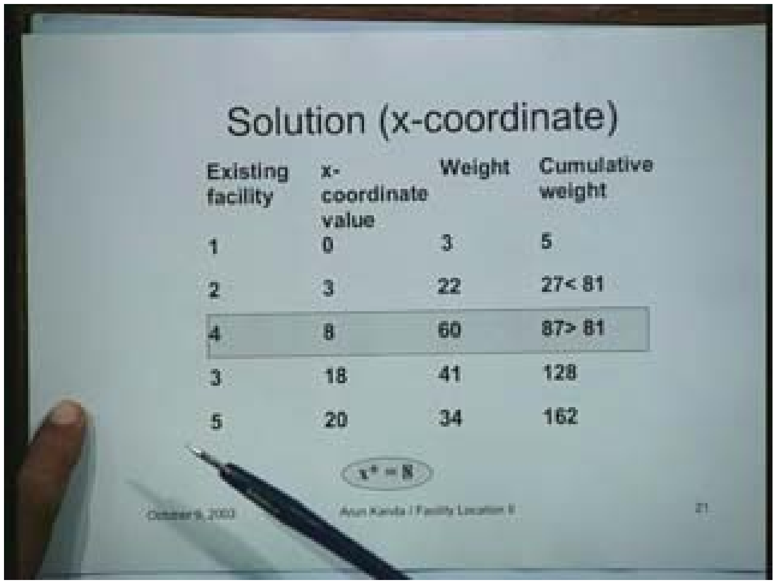

So as I indicated to you, the answer to this problem is that we can determine the x coordinate and the y coordinate independently and the x coordinate is nothing but a median location. So this simply means if we arrange the entire x coordinates in ascending order, these are 0, 3, 18, 20 and this is corresponding to the existing facility 1, Corresponding to the existing facility 2, this is corresponding to existing facility 4, this is corresponding to the existing facility 3, and 5. So ascending order of the x coordinates and we determine the weights. Weights are the number of cars that come here, 3, 22, 60, 41, and 34. Determine the cumulative weight, this should be 3, rather this should be 5. This is error somewhere, so misprint. So you have 5, let us say 5 is the cumulative weight here and then 27 and then 87 and then 128 and then 162. What needs to be done is if the total cumulative weight i.e., 162 is the sum of the weights, half of that is 81. So you find out where the cumulative weight changes, that is this is less than 81 and this is greater than 81.

This becomes greater than 81 here for the first time. So this row determines the optimal solution. A very simple straightforward and therefore x star = 8. So we have determined the optimum x coordinates for this particular problem. The rectilinear distance location problem is merely a median location. So this is how you determine the median location then we can do a similar procedure for the y coordinate. When you do the y coordinate what happens is that if you arrange the y coordinate in ascending order 0, 2, 16 and 18, you find that this corresponds to the first facility. Both the facilities 3 and 5 share this coordinates so we write both of them here and 2 and then 4 in that order and then the weights which are there are 5, 14, so this is 5 in the previous slide, it should have been 5. This is 5, so this particular value should be 5 (Refer Slide Time: 48:19). Now we have 80 again, you have the total cumulative weight which is 162, half of that, this is less than this, this is greater than this (Refer Slide Time: 48:30), so where the cumulative weight first exceeds half the value, that row is here, so corresponding to this we can identify that y star = 16, it is very simple. So you observe that we can determine the x star and the y star value independently by similar procedures by just identifying the median and we have identified the median. So in this particular case we now have in fact determined both x star and y star. So x star is 8 and y star is 16, so 8,16 becomes the optimum location for the service facility, that means this will minimize the total cost of travel, that is the optimum solution or the ideal solution for the location of the service facility for this particular problem.

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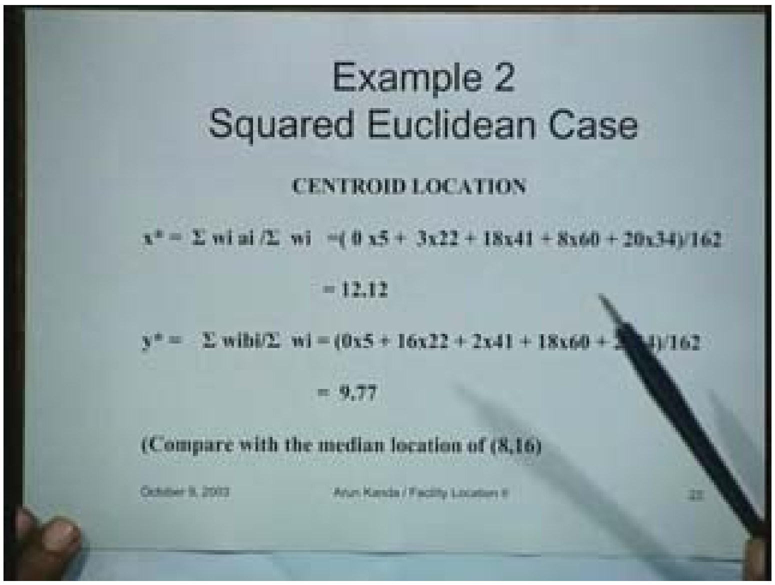

Now let us look at the case where you find that the distance measure is, let us say square Euclidean. We will not talk about the Euclidean case now. We shall talk about it later but let us talk about the squared Euclidean case that is the satellite location problem. So the location is also another interesting problem here. You know it would be like trying to say that suppose in the squared Euclidean problem, the optimum location is in fact in this case, centroid location. This can be proved very easily. So the centroid location for this problem is simply x star = summation wi ai divided by summation wi, y star is equal to summation wi bi divided by summation wi. So for the same problem that we considered earlier for the Euclidean distance, for the rectilinear distance case, we can easily determine this summation wi in the denominator. It is 162 which is the total weight. Sum of the weights and x star is 12.12; y star is 9.77. So if your distance measure is a squared Euclidean, it would be best to locate the new facility at 12.12 and 9.77 and obviously this differs from the earlier median location that we determined of 816. I am just trying to tell you that if you are still not very clear about when you should use the squared Euclidean case, suppose your set of 5, 6 friends are celebrating a bonfire in winter, imagine the situation and they are standing or sitting in a certain location around bonfire, then the question is which is the best position for the bonfire such that the total loss of heat energy is minimum for all these guys. So if that is to happen, the bonfire should be at the centroid of all these points, where the people are sitting. That is exactly where the satellite location problem is in terms of whatever it is.

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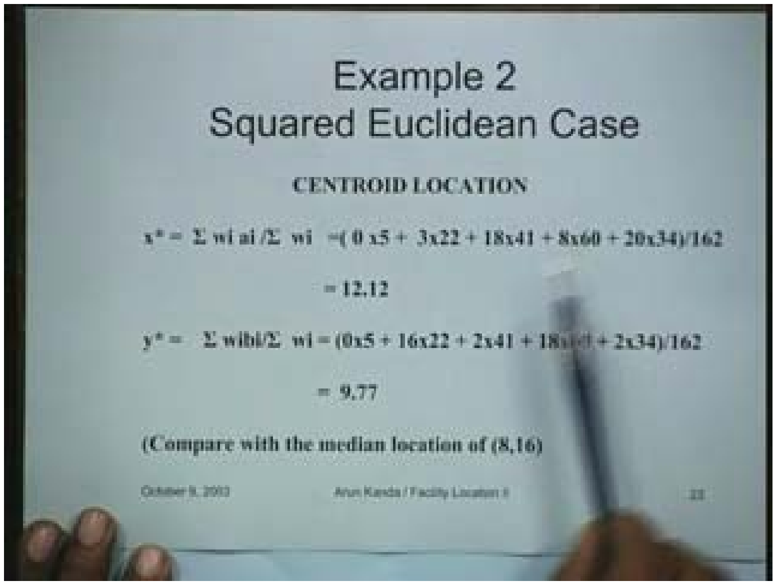

So this can be determined very simply by saying 12.12 and y star is also is equal to 9.77 and therefore you can determine the centroid location very easily.

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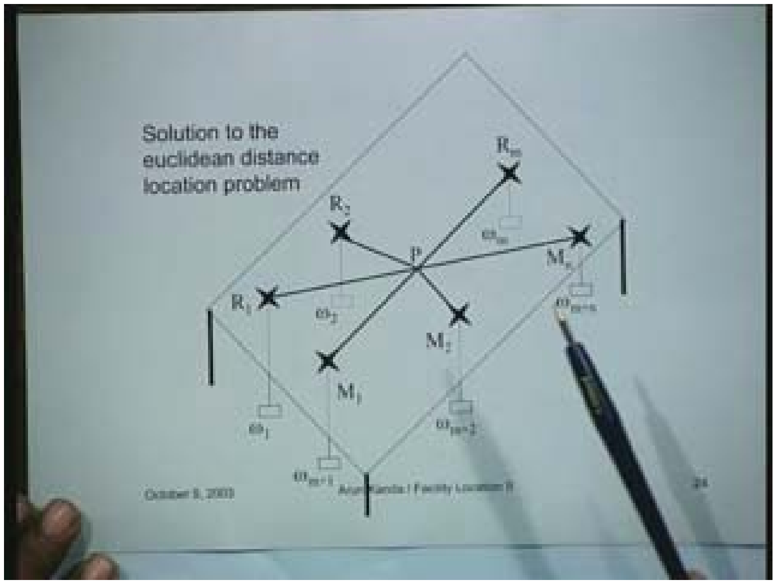

In this particular case if you now look at the Euclidean distance locations which is the third problem that we are trying to talk about, you recall that one simple way of solving this problem is by using the mechanical analogy that was developed by varianon in the frame which we call a varianon frame. All that we have done was all these points are raw materials sources and markets so all the total points are actually plotted here to scale and then you take the required number of strings, tie them to a common knot, and pass one string through each of these holes which you have drilled here. Then you can hang a weight. This is equal to the weight corresponding to that particular point. You allow the system to come to equilibrium and when that happens, the location of P is actually the solution to the Euclidean distance location problem.

As we had seen the other day, normally this method of constructing the varianon frame and the strings models is quite cumbersome. So mathematical procedures are available by means of which you can compute this particular point but there are certain mathematical complications in this analysis and therefore the resort has to be made to iterative heuristic procedure for getting this particular point. So getting this point mean that you start with a trail point P and the trail point P most of the solutions is the solution to the centroid problem you can start with 12.12 and 9.77 and then keep iterating in certain fashion. We will then talk about the algorithm in the tutorials, how it is done. But essentially this is an exact solution of the Euclidean distance location problem which you can see from here and I think that gives you a broad idea of how these problems are. Yes, I think so far all the problems that we have considered; it can be termed as minisum problem that means it comes to the objective function.

In all cases the objective was to minimize the sum of the total travel from the facility to all other facilities. If you are talking about locating a central service station, so you are talking about the cost of movement from the service station to the first facility from the service station to second facility and so on. This sum was there. So all the three problems that we have considered so far were actually minisum problems where you are trying to minimize the total sum. Now in many instances especially in emergency location problem it is not appropriate to think of a mini sum objective.

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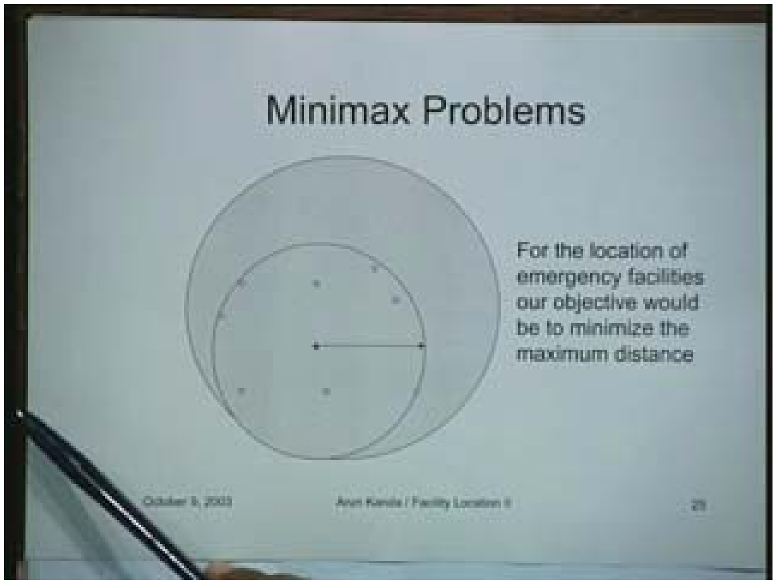

It is better to think in terms of what we call a minimax objective. For example it is like saying if these are the resistances of different people in different locality and you are trying to find out the location of a fire station, The location of the fire station should not be such, the total cost to this plus this plus this plus (Refer Slide Time: 55:50) this should be minimum because when a fire breaks out, the fire tender has to run to that particular place. So really speaking, the location of the fire station should be such that it should be nearest to the farthest point.

You are trying to minimize the maximum distance from that particular in all emergency location, mimimax problems are quite common. For the location of emergency facilities our objective would be to minimize the maximum distance just to give a motivation without going into details. For instance these were the points and you want to locate a facility. The idea is that what would be a minimax location? A minimax location will be we try to enclose these in a circle and the circle which has the minimum diameter. The center of that circle which is let us say this one (Refer Slide Time: 52:61) encloses these points. This is the circle which encloses all these facilities with the minimum diameter. This circle is larger and there could be other larger circles. Basically what we are seeking is this circle and the center of this is optimum location. Now graphically it is very wonderful to look at this particular procedure and there are algorithms for instance there is a beautiful algorithm by Elzinga and Hearn which is actually based on this analogy. So this is just to tell you that if the objective function changes, the problem is no longer a minisum problem but a minimax problem. Finally let us look at this complication.

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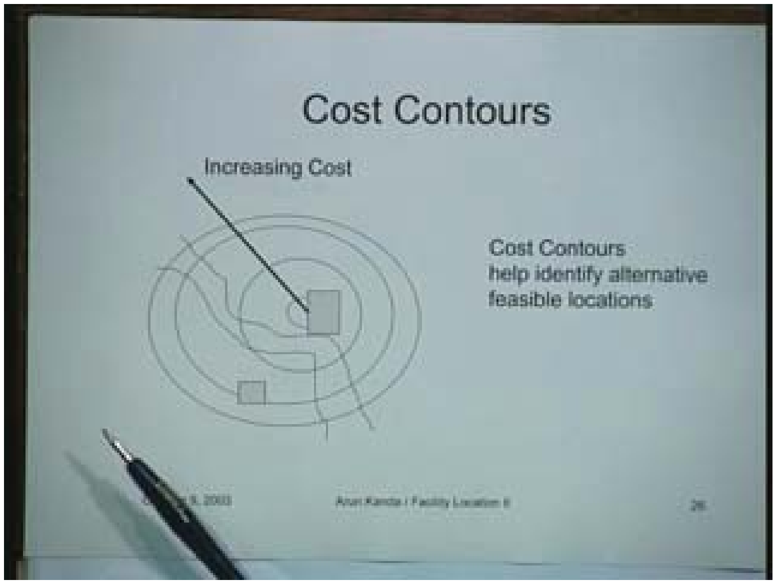

For instance we determine an ideal location by these mathematical procedures, if we determine an ideal location what may happen is that the ideal location may not be available. There might be a building, there might be a river running through it or there might be other complications and you cannot locate because in solving the problem, in determining the ideal solutions, we have taken into consideration the feasibility of these locations. So what is interesting is that we can build cost contours. These are lines of constant cost. So this is the minimum cost of optimal location. Cost increases by 5 percent, by 10 percent, by 15 percent, by 20 percent whatever it is and you have lines of constant cost. This will help you to choose an alternative location. These are very much like lines of constant height on a map; on a geographical map you have lines of constant height. Similarly you have lines of constant cost and this mapping once is done would give you ideas as to which side you should shift, where is the cost lesser, where is the cost more and there are procedures to develop the cost contours. In fact in the tutorial exercises you will have an opportunity to develop these cost contours. Finally let us see what we have done in this particular lecture.

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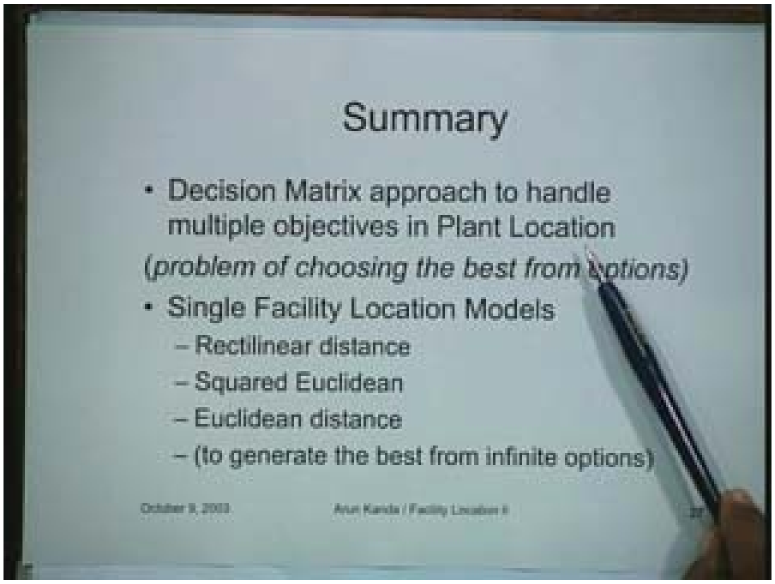

We started with looking at a practical problem, a case study and we saw that the decision matrix approach to handle multiple objectives in plant location was in fact very worthwhile approach to consider all the objectives and consider the feasible solution. Basically this problem could be termed as a problem of choosing the best amongst options. Then we have the single facility location models. We talked about the rectilinear distance, the squared Euclidean distance, the Euclidean distance where the objective was to generate the best from the infinite options which are available and finally we talked about the notion of minisum and minimax problems where the objectives could change depending up on the context. Finally we saw our cost contour that can help to accommodate practical constraints that is moving from ideal to a feasible solution. So with this we conclude this particular lecture and in the next lecture we will go over to the facility layout problem which is the next important problem in the context of facility location.

Thank you!