Project and Production Management
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Lecture - 27
Product Mix Decisions

We had looked at some of the important principles that have to be considered while designing a new product. Today we are going to talk about product mix that means no company would ever be talking about marketing a single product alone. It would be marketing a number of products together and therefore the question of finding out how many units of which products should be marketed is an important question. So this is what we refer to as the product mix problem in the context of designing and introducing a new product.

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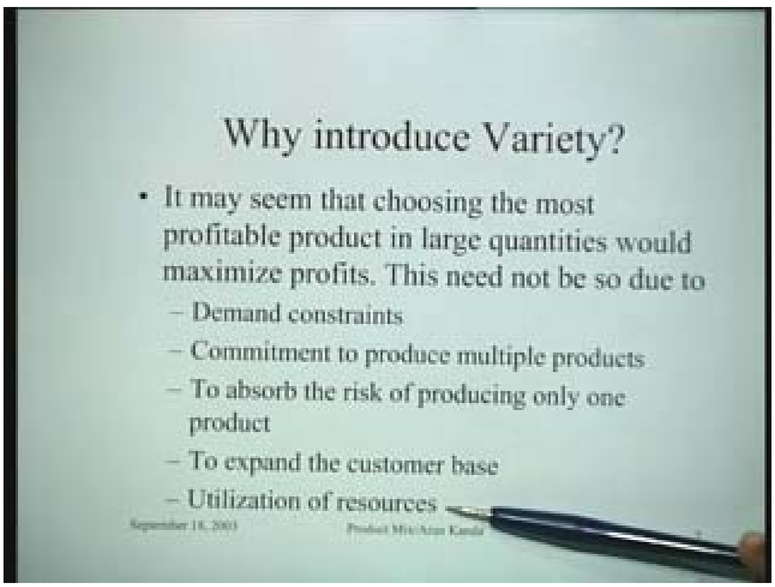

I think a question that normally comes to our mind is that if there are a number of products and if that particular one particular product is the most profitable then why not keep on producing quantities of that particular product and ignore the product which is less profitable? So this is the reason for this observation. It may be seen that choosing the most profitable product in large quantities would maximize profits. However in practice this need not be so. Why? There are a number of reasons to why this will not happen in practice. First of all there would be a large number of constraints which would have to be considered before introducing any new products. Typical constraint could be, for instance demand constraints. When we talk about demand constraints what we are talking about is that the quantities to be sold for individual products could be limited by the market demand for those products and therefore it is not entirely in your hands to be able to sell as much of the products as you want, even if the product is most profitable. These are very significant constraints which have to be considered. Commitment to produce multiple products made by the company might have already committed to a large number of customers that it would supply some products. Some of the products may even become absolute, but for the sake of goodwill of the customer, you are committed to producing those products and therefore you would have to produce multiple products in the whole mix of products that you make. To observe the risk of producing only one product, you all have heard the phrase; do not put all your eggs in one basket. So most companies are not riskovers and they would like to make sure that they do not put all their eggs in one basket. So if this is so, this means that the option they have at the disposal is to produce in a large number of products and so that the risk can be shared by those products. Another common reason for introducing variety as far as product mix is concerned, is to expand the customer base. So what happens typically is that one product captures a certain fraction of the market.

If you introduce another product, that product will capture another segment or an overlapping segment of the market. So if you have a large number of products, you probably created to a large number of customers. Companies like Fanta attract customers and catches their attention and introduce product variety as a consequence. Another reason for the introduction of new products in the market is utilization of resources. Quite often with one product you might not be able to utilize all the existing sources that you have and therefore you might want to introduce some related product so that your resources are better utilized. This could also be a consideration. So it becomes almost imperative for a company to introduce a variety, or introduce a large number of products together and talk about developing a product mix. So that is the problem that we are addressing today. How to determine the optimal product mix for a company and we are going to examine a number of different situations and number of mathematical models which can be used for answering this particular question.

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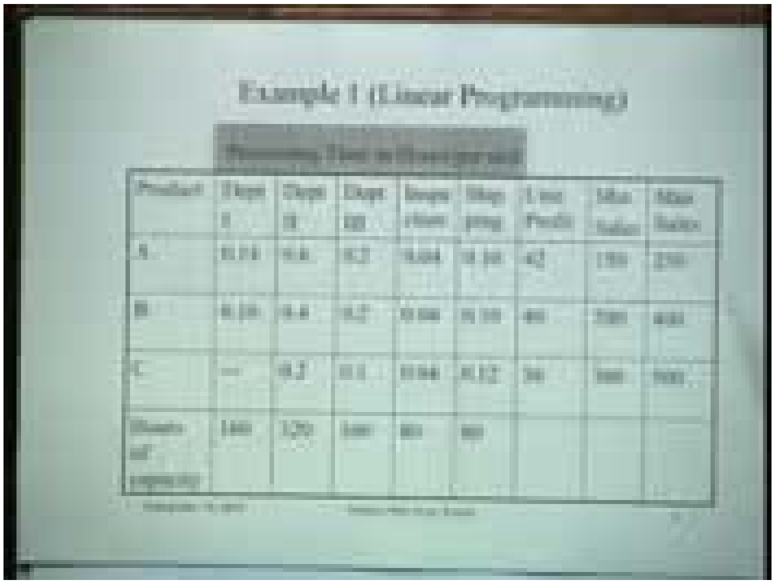

Let us first take an example of situation with which we are already familiar and the problem is that suppose the company is producing three products, we call them A, B and C, these three products require processing in 3 different departments. Different departments could be lathe machine, it could be grinding machine, it could be milling machine. In general we are talking about three different departments and apart from these three different departments the products require it to be inspected. So there is inspection department and then there is a shipping department. So there are five departments which are coming into play in the manufacture of these three different products. The information that we have at the disposal is processing time in hours per unit. So we have estimated for instance that if a unit of product A is to be estimated, it would require 0.14 hours per unit in department one. Similarly it would require 0.6 hours per unit in department two and so on in different departments. Of course the product c does not require any processing in department one and the unit profit from sales of these three products are given here. 42 rupees, 40 rupees, 36 rupees is the profit that you earned by selling one unit of product A B and C respectively.

It is given that the sales for these products vary and historical experience shows that the minimum sales for this product are 150 units for the planning period which we are considering. It can at most go up to 250 pieces per period, similarly the figures for product B are 200 minimum sales and 400 minimum sales for the product B and for product C, the corresponding figures are 360 and 500. This would be typically the kind of data that you would require for doing any product mix problem using, let us say linear programming we are talking about the case where we handle this through linear programming. Obviously the assumption that we are making is that the cost varies linearly which means that if it takes .10 unit hours per unit to make 1 product, 2 product takes 0.20, 3 products takes 0.30 and so on. So that is the assumption that we are making in effect, what we are ignoring here is the fixed cost or you may assume that the fixed costs are actually included in these and we are making the assumption that the costs are linear. So in this data that we have collected, for 3 products, we have the processing time in hours per unit. Then we have for each of the departments, the hours of capacity. For instance department one has the capacity of 160 hours, department two has the capacity of 320 hours, department three has the capacity of 160 hours and similarly the inspection and shipping each have 80 hours. So the differences in the capacity that I mention could be because of the varying number of ships that you operate these various departments. Let us try to formulate linear programming problem. In this particular example our decision variables are clearly X_1, X_2 and X_3 the quantities of A B and C to be produced.

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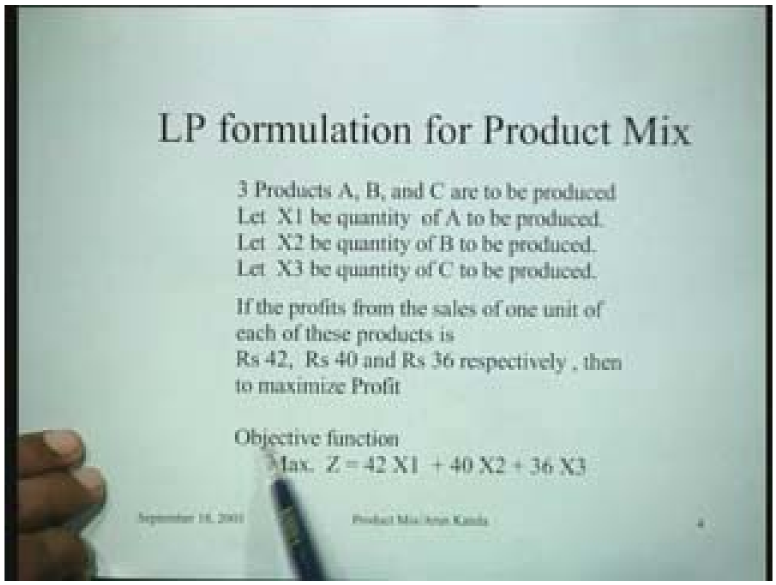

Since the profits are 42 rupees, 40 rupees and 36 rupees respectively from unit of sales of each of these products, then we can easily write down the expression for objective function which is to maximize the profit which will be 42 X_1 + 40 X_2 + 36 X_3, so this is what is to be maximized subject to various constraints. Now the constraints in this problem are essentially the capacities of the various departments. For instance the first department has a capacity of 160 hours and it processes products 1 & 2 and does not process department 3.

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So if you look at the column in that particular data that we were examining a short valuable, you find that 0.14 hours per unit is the requirements of producing a unit of A and .10 is the requirements of producing a unit B, as far as the first department is concerned, so the total time required for the production of these items is 0.14 X_1 + 0. 10 X_2 and this should be less than or equal to 160 which is the number of hours available. We would have a similar constraint for each of the departments. We have five departments so 1, 2, 3, 4, 5, the right hand side here is the hours of availability and this is the total consumption (Refer Slide Time: 12:00) in terms of time, in the various departments depending up on the processing times and the quantities to be produced. Then we have sale restrictions, the typical sales restrictions for this problem are the X_1that is the quantity demand for the first product should lie between 150 and 250. So we have the constraints that have X_1 greater than or equal to 150 and less than or equal to 250.

We would have similar constraints for the other two products; product two for instance has the sales which will lie between 200 and 400, so we say X_2 should be greater than equal to 200 and less than equal to 400 as shown here. Similarly the third product, the demand lies between 360 units and 500 units. So what we are basically interested in is developing these constraints. So we have the total number of constraints for this problem. We have 5 of these constraints, 1 for each department then each of these 3, we have an upper bound and lower bound constraints. So we have 6 such constraints. So the total number of constraints that we have is 11 and we have 3 variables, 3 decision variables X_1, X_2, X_3. Now the solution to such a problem can easily be obtained by using any one of the computer packages which are available. You may use for instance lindo or micro manager, which is available in the industrial engineering laboratory and we will therefore not go into solution techniques because we are already aware of the various ways of solving linear programming problems.

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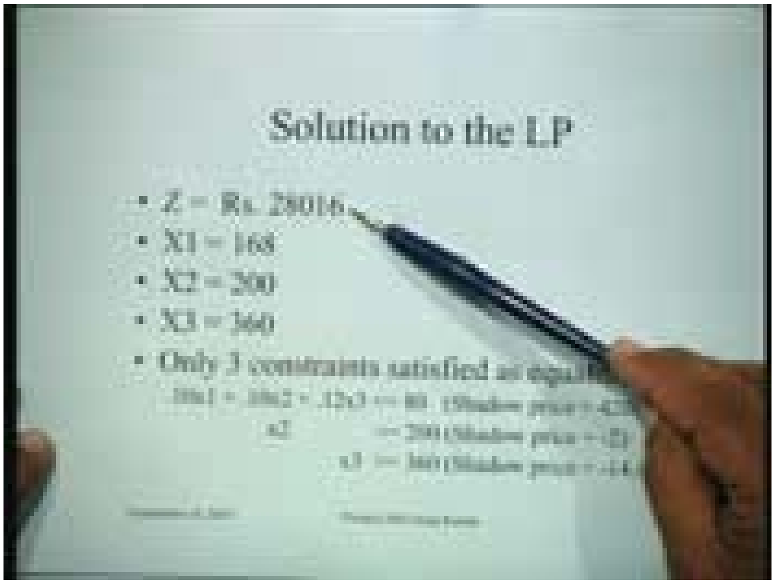

So if we look at the solution from a package, we find the following. We find the total profit is 28,016 rupees and the optimal values of X_1, X_2 and X_3, workout to 168, 200, 360 units respectively. Now this is the optimal product mix and this (Refer Slide Time: 14:25) is the profit that you get from this particular product mix. There is some useful information that you get from the output of a typical l p package and I am just pointing out here something that needs to be emphasized. For instance in the optimal solutions it is found that only 3 constraints are satisfied as equalities and these (Refer Slide Time: 14:48) are the 3 constraints which are satisfied as strict equalities. The other constraints that means out of the 11 constraints only 3 constraints have satisfied as strict equalities. That means the left hand is exactly equal to the right hand side and so on. This (Refer Slide Time: 15:11) particular constraint as you can see is nothing but the utilization of particular department. Let us check up from the data to which department it refers to.

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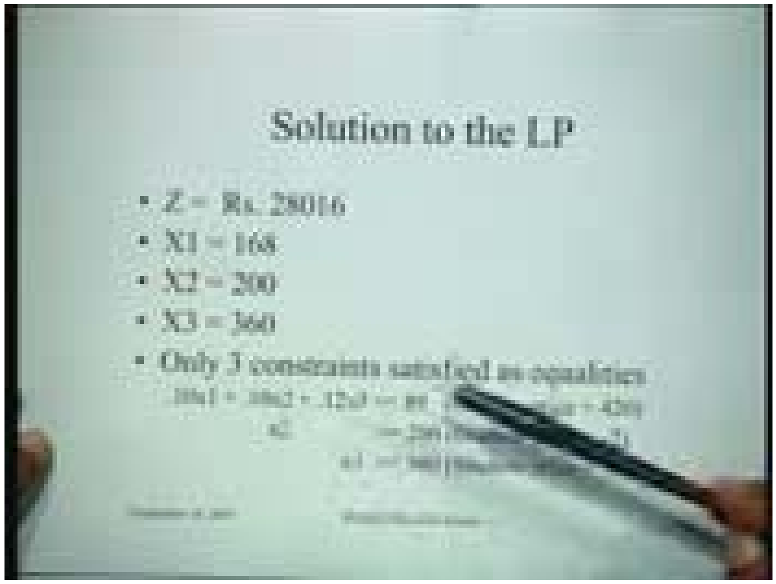

You would see for instance that if you look at the problem, we are referring to the shipping department here. So this is the constraint corresponding to the shipping department which says $0.10X_1$ + 0.10 X_2+ 0.12 X_3 should be less than or equal to 80 which is the capacity of the shipping department. Since this constraint is satisfied as an equality and similarly we have X_2 is equal to 200 and X_3 is equal to 300 for all such constraints which are satisfied as strict equalities. You can calculate a shadow price or a dual variable and these shadow prices or dual variables are available in the solution that you obtain from any package. The interesting thing about the dual variable really is that here corresponding to this constraint which pertains to shipping, the entire 80 hours of shipping are currently being utilized and the shadow price for these particular constraints is 420. What it shows is if you increase the capacity of the shipping section by one unit, that is make it from 80-81, the contribution to the profit is likely to be 420. So the dual variable or the shadow price gives you very valuable information as to which capacity to increase and which not to touch.

The other capacity which has all equalities at the moment need not contribute to the objective function for instance, this being utilized fully, so there is an increase in one unit. Take this particular constraint, the constraints says that this is greater than equal to the constraint which is 200. It is a negative value; the significance says that if I reduce this by one unit the profit will go down by 2 units. This is the market demand currently, you are selling 200 units, if the market demand falls by 1 unit, and your profit will fall by 2 units that is the implication. Similarly this one is greater than 360 units, if this falls by 1 unit, the profit is likely to fall by 14.4 units, so linear programming is a very worthwhile tool which can not only determine the optimal product, makes the corresponding maximum profit but also give you very useful hints on how to improve your profits and where to invest for improving the profits. I think this is another aspect about linear programming which needs to be carefully understood and applied when you are dealing with real life problems.

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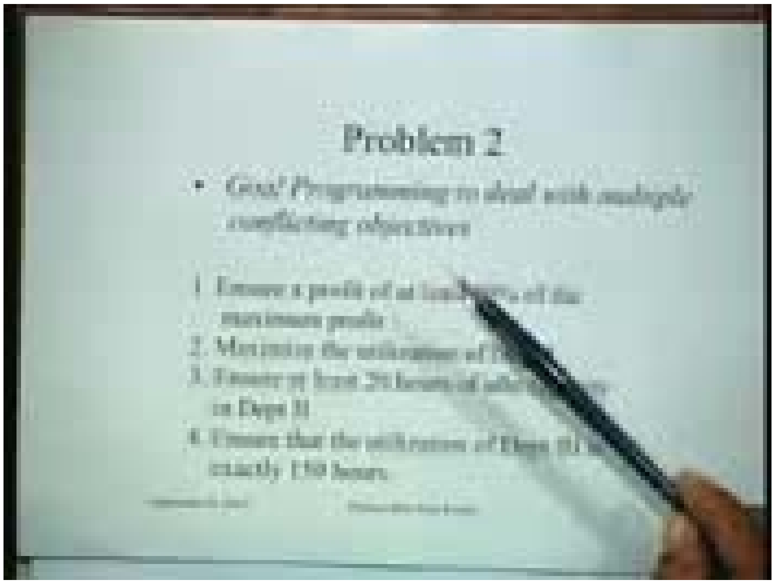

After having looked at the simple linear problem you know in real life there can be a number of conflicting multiple objectives. So let us say that the same company which is dealing with the same 3 products has these 3-4 considerations or goals to consider. The first goal in the order of priority is to ensure a profit of at least 80 percent of the maximum profit is realized. 2. You want to maximize the utilization of department one. You know department 1 has 160 hours of capacity in the current optimal solutions. The department 1 is underutilized, so they want to maximize the utilization of department 1 at second priority. Third thing is to ensure at least 20 hours of idle capacity in department 2. What could be the motivation for an objective like this is, it may so happen that the company is interested is already committed certain capacity of this particular department to other users. It would like to make sure that at least 20 hours of idle capacity are retained in department 2 and the fourth constraint is to ensure that the utilization of department 3 is exactly 150 hours.

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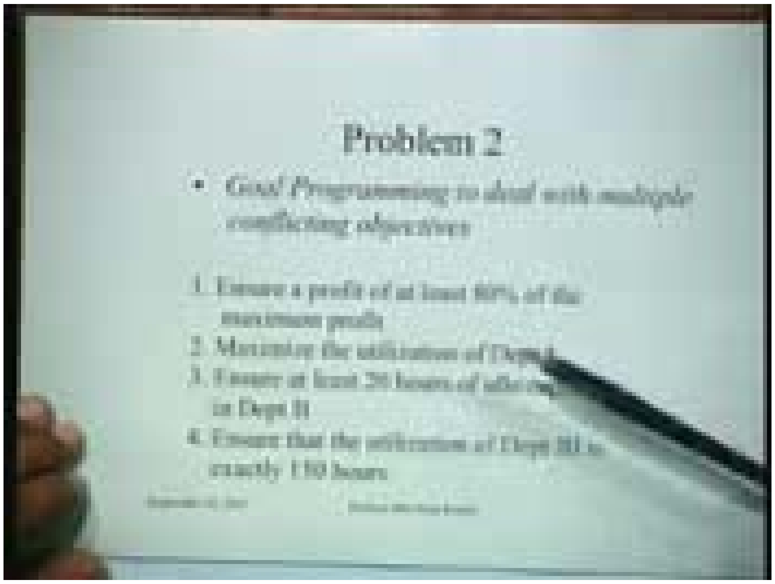

So we are using this example to see how a goal programming formulation could be set up for this case and I think that would give us some insights into how to handle multiple constraints.

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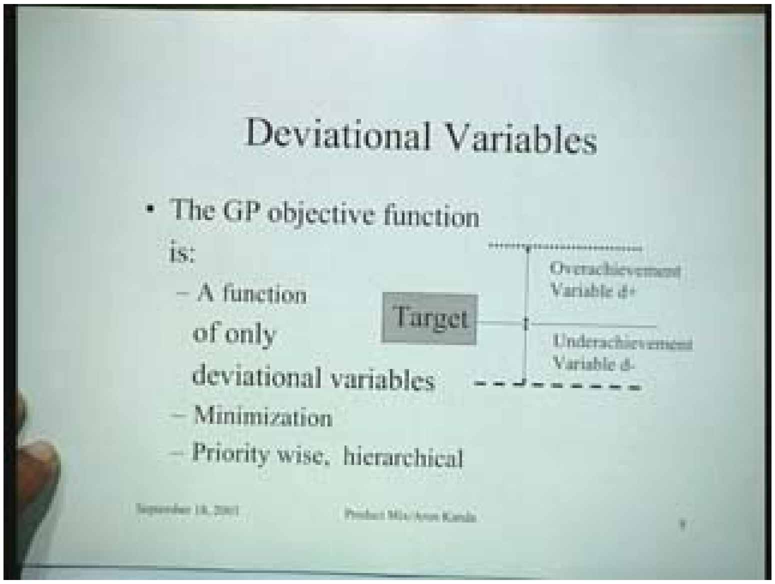

The essence of goal programming is the use of deviational variables and what happens is that, what is the deviational variable? A deviational variable is, if you set up a target for something, this is my target. In an actual situation I might over achieve the target or under achieve the target. So we use 2 kinds of variables called overachievement variables and underachievement variables denoted by d + and d – respectively, which tell you how to achieve the target. For instance suppose you are aiming to get marks of 75 in a particular course and for instance if your score is 90, you have overachieved your target by 15 marks. However if you get 50, you underachieved your target by 25 marks, so basically these deviational variables measure whether you have overachieved or underachieved the target and mind you even in goal programming, we are always dealing with targets. In linear programming you are always trying to choose a favorable direction anyway, trying to maximize or minimize your travel on that particular direction. If you choose profit and you say I want to maximize profit and when you are maximizing profit in linear programming, you are essentially saying that this is the road to profit. I want to go as far as possible without saying how much you want to go on that road.

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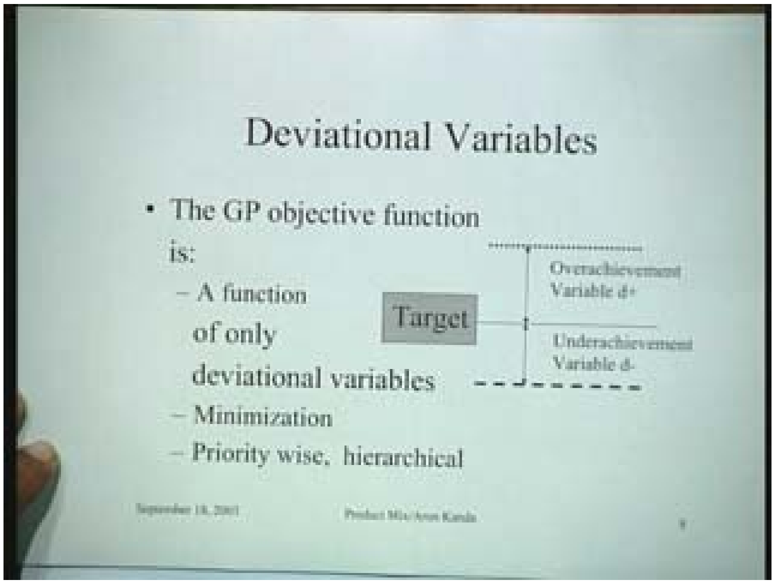

Whereas in goal programming, we step up milestones on that road and we say you would like to have a profit of at least 2 million rupees. So we have set up a mile stone on that road and goal programming. We are more concerned about achieving those milestones rather than trying to maximize or minimize as the case may be. The goal programming objective function then is set up in terms of these deviational variables. So this goal programming objective function is a function of only deviational variables.

The objective function is always a minimization function because you are talking in terms of minimizing the deviations from targets and it is a priority wise hierarchical in nature, that means you choose a set of priorities and work according to that particular set of priorities. So these are some of the basics that are required for understanding a goal programming formulation.

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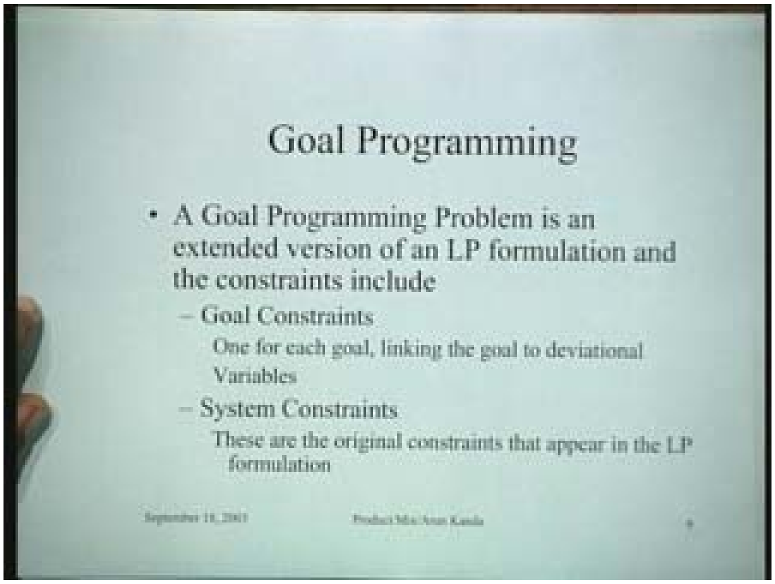

In goal programming which is actually an extended version of an LP formulation, there are 2 types of constraints. The first type of constraints is called goal constraints. There is a goal constraint, one for each goal, so this links the goal to the deviational variables and then of course there is a system constraint which is the original constraints that appear in the LP formulation. So we will try to develop for our example both the goal constraints and the system constraints so that we have the complete goal programming formulation.

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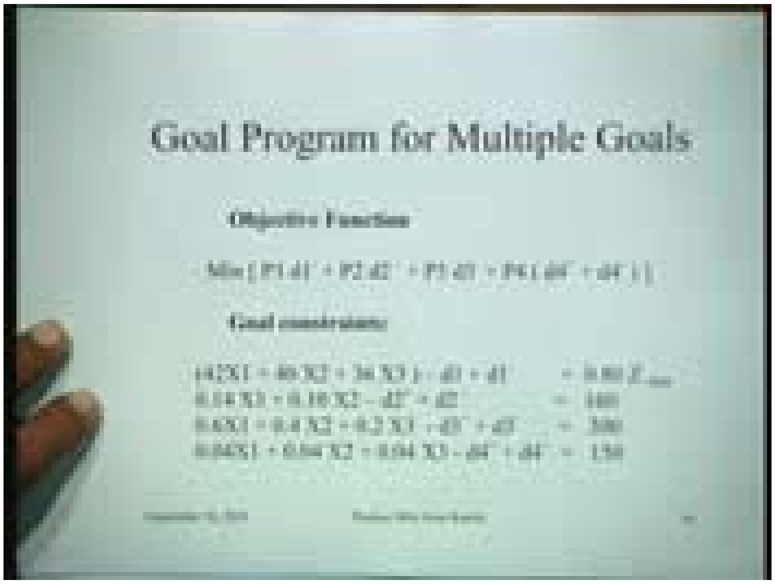

What we have here is the objective function and the goal constraints for the problem. So we have 4 goal constraints and the right hand side here is the target values that we have set. We have set a target of 80 percent of the Zmax. Zmax is the value that you obtain through the linear programming problem and then what you notice is that this is the total profit this function minus d_1 + (+ d_1 –) that means – the overachievement + the underachievement should be equal to the target and exactly similar considerations are applied to the other four. For instance here (Refer Slide Time: 24:48) this is the total hours consumed in the first department – the overachievement + the underachievement will be equal to 160. So this is the same concept that you can either overachieve or underachieve, you do not know.

That is we have both the variables. Remember these variables are such that only one of the variable will be in the basis at an optimal solution because if you are aiming to achieve 75 marks, you will either get more than 75 in which case d + will be in effect or you will get less than 75, in which case d – would be in effect. The other deviational variable will be 0. So we write down these deviational variables corresponding to the first constraint, the second constraint, the third constraint and the fourth constraint and we write these and these are nothing but the goal constraints. That is what we mean by the goal constraints. Now look at the objective function. If your objective function P_1, P_2, P_3, and P_4 refers to the priority levels, the first priority goal is to ensure that you get at least 80 percent of this profit. So what we want to do is that the negative deviation from this should be minimized. We would like to have the negative deviation from this to be 0 ideally. That is why we include the objective function similarly the second one; we want to maximize the utilization.

Now maximum utilization possible is you know 160 hours and plus if is you overachieve and minus is if you under achieve it. So d_2 – is what you want to minimize, so d_2 minus is taken as the objective function and similarly p_3. This is the utilization of the third department and what we want to do is, we want to minimize this under utilization of this particular department and what we have done is you know the capacity was 320. We have already subtracted 20 units, so that our target performance for this particular utilization of department three is only 300 hours after accounting for 20 hours of ideal time which you wanted on this particular department. Then you want to minimize d_3 –, so you have the fourth priority goal was that we would like in the fourth department, the exact utilization should be 150 hours so this is the situation where you would like to minimize the sum of both d four plus and d four minus when you are exactly trying to achieve a goal this is the mechanism that you adopt for ensure because both of these are zero then this will be minimized that would mean exact utilization of 150. In a nutshell the goal programming formulation confesses as far as the objective function is concerned, 3 types of deviational variables.

You could be either be minimizing some d – or minimizing some d + or minimizing the sum of both of these. So it all depends upon whether you want to minimize the overachievement or the underachievement or you want to exactly strict to the goal. So this is our objective function and then the goal programming problem will be now completed if we add to the system constraints.

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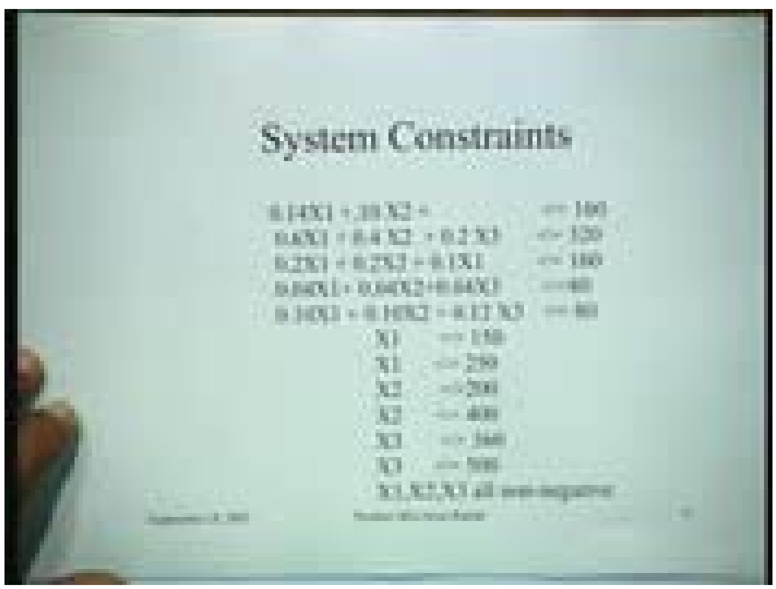

The system constraints are the same constraints that we used in linear programming. All those constraints as it is come here so now the problem is completed. We have a goal programming objective function. We have a set of goal constraints and the system constraints and we can use this to solve the problem. You can use the package of lee and moor for solving this particular problem and ultimately the solution to this particular goal program is given here.

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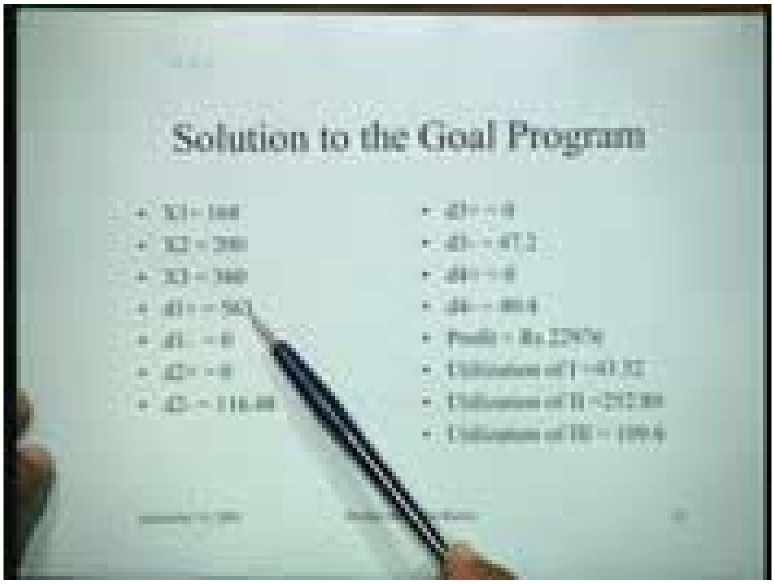

We find that we have found out the product mix that is 168, 200, and 360 and then we have found out the original deviational variable. What is the significance of these deviational variables let us see. Our target was 80 percent of the goal. So I have taken that as target d_1 + is that we have over achieve the target by 563, so the total profit is the original profit goal + 563 which is this profit 22976. So this is the total profit similarly the utilization for department one utilization for department 1 or d_2 + was 0 and d_2 – was underachieved. The utilization of the department which was 160 by so much this much was the unutilized capacity so utilization was 43.52 only and so on for utilization of other two. So you can accommodate multiple goals and find out values of the objective function and other utilization and other goals that you want by treating this for anything. So goal programming is basically as I said an extension of linear programming in which we could accommodate a variety of goals and solve the problem. Let us look at the third problem in the context of product mix.

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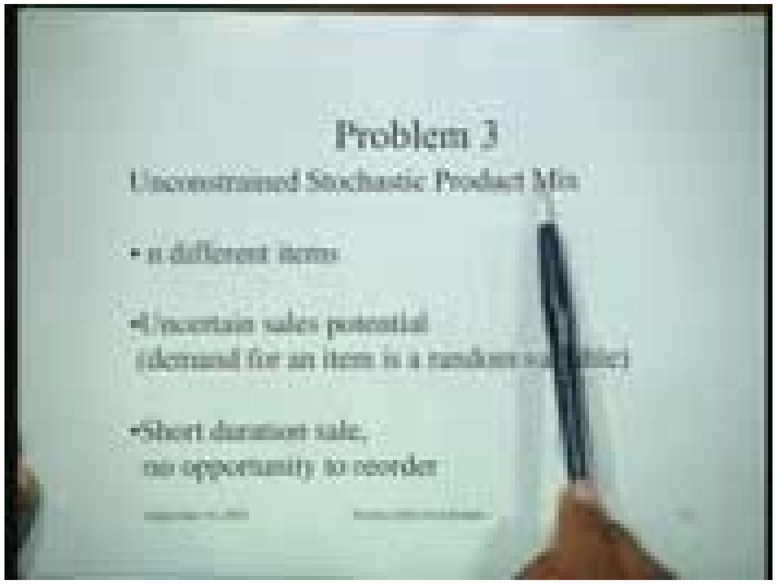

Let us look at this problem of what we call unconstrained stochastic product mix. So far we have assumed that the product demands are deterministic. Now we assume that the product demands are not deterministic but they are probabilistic and may be followed by certain probability distributions. How we are going to determine what is the optimal product mix in such a situation? Just to keep the problem simple, we will consider the following assumptions. We assume here that there are n different items. These n different items have uncertain sales potential. We do not exactly know the number of items to be sold and the demand for the item is a random variable with the specified probability density function. That is what we will assume and this would be something that would come to you from experience. You would know for instance that if you have been selling everyday say certain medicines, the chemist knows that the demand for Crocin is likely to be anything from 10 to 400 million , may be, depending upon the season. You could probably fit some suitable distribution to that and we will assume that it is a short duration sale just to simplify things. So you do not have to consider other types of costs and there is no opportunity to reorder.

Let me give you some examples of the situations. For instance if it is known to a company, suppose a number of companies are preparing to set up an exhibition at Pragathy Maidan and this is going to be a short exhibition of let us say 3 days or 4 days. They want to keep different items for sale and each item has the probability distribution for sales. The question that we are trying to address is how much of each item should this company stock in the sale such that it’s expected profit is maximized and the maximization of the expected profit should be subjected to the consideration that, during the period of the sale which is the short term sale, we are not allowing fresh consignment to come in. It means whatever was brought in the beginning will be sold. They will have shortages as the case may be but we are not allowing new replenishment during this period. This is the basic idea. In fact jocularly, this is the problem of the vegetable vendor who goes to main market every day. In the market he has to buy different vegetables and he knows that these are the products that are in great demand from his experience. So he decides what vegetables have to be bought every day, each day such that this is ongoing affair. Every day he will have to take this decision such that his expected profit is being maximized and once he brings the vegetable to this shop, he will not able to go to the market during the course of day to bring the additional vegetables. So we are basically talking about that situation. This is the second example, the third example that I can give you is for instance, take the case of perishable commodities.

A super market might be interested in finding out the quantities of different items that includes all perishable commodities like medicines, foodstuffs etc in order to stalk for a certain period such that their expected period is maximized assuming that there are no further replenishment during that period. This is because replenishment from the chemist or something generally comes only on a particular day on Mondays when the suppliers comes and gives you the supplies. So this is the kind of scenario that we are trying to module here. So let us look at this problem. Let us first introduce some notation to formalize the problem.

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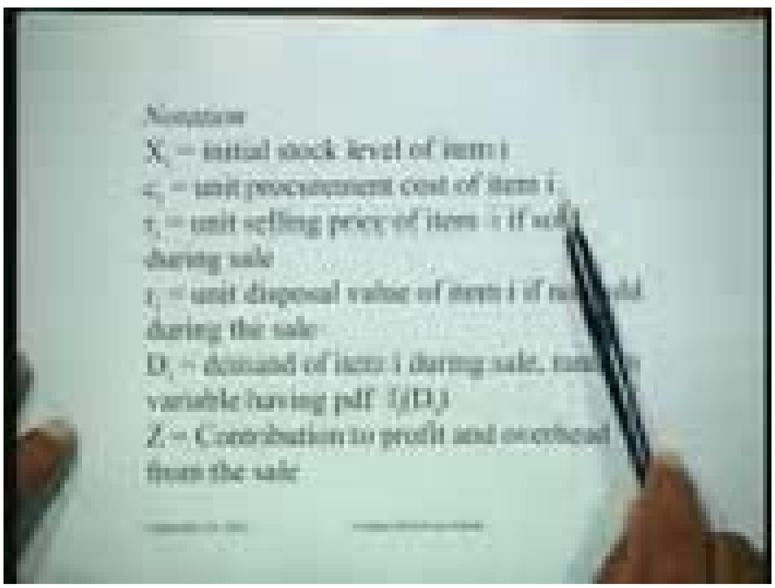

Let us say x_i is the initial stack, level of item, i. So this is our decision variable we need to find out how much of each item to stock, c_i is the unit procurement, cost of item I, ri is the unit selling price or the revenue that you get if item i is sold during the sale and ri prime actually, what we are talking about here is not r_i but r_i prime. So ri prime is actually the unit disposal value of item i, if it is not sold during the sale. So what we are saying is take the example of a vegetable vendor again if he sells potatoes, during the day we will probably be able to get 10 rupees per kg. But if they stay on till end they have to dispose them for 2 rupees per kg or 3 rupees per kg so that is the disposal value and this is the unit selling price. Capital D_i is the demand of items i during the sale and this is the random variable which we are assuming as a pdf of f_i D_i. So this is an arbitrary pdf. It could be a normal, exponential, erlang any distribution that you want. In general f_i D_i and Z is the contribution to profit and overhead from the sale. It that means our objective functions is this Z but Z is the random variable so we will talk about the expected value of Z and that is what we are interested in for this particular case.

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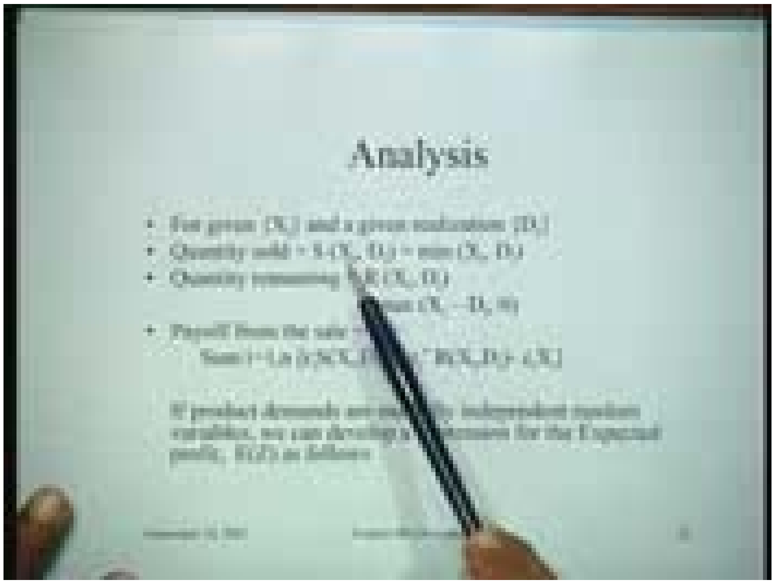

Let us try to do a bit of analysis for a given set of stocks that you have different items which are x_1 x_2 and so on, x_n and given realization of demand which is D_1 D_2 and so on up to D n. The quantity sold is denoted by S which is the function of x_i and D_i obviously this quantity sold will be the minimum of x_i and D_i. If suppose I stock 20 items and the demand is 50 items during the day, I will be able to sell only what I have. So it will be only x_i. However if I have 100 items and the demand is only 20 items then I will able to sell only 20 items. The demand is always the quantity sold is the minimum of x_i and D_i and what will be the quantity of the remaining? At the end this is nothing but R which is again a function of x_i and di and this quantity will be the maximum of x_i – D_i, 0. Because if you have 30 items in stock and the demand is only 10 that means 30 –10 = 20 items will be basically remaining at the end. On the contrary if you have 20 items and the demand is 50 more than what you have, and then all the items will be sold away. They will somehow satisfy the demand and you will therefore not be left with any quantity which is 0. So the payoff from the sale is nothing but the summation of I = 1 to n that means summation for all the products for I = 1 to n and this is the revenue that you get by selling the item. So revenue multiplied by the quantity sold plus this is the disposal value multiplied with the quantity remaining and this was your cost – ci x_i. This is the payoff from the sale because the demands are random variables, Z is also a random variable and therefore our consideration is to find out the expression for the expected value of z that is EZ which can be found out by integrating this function over all possible values of the demand.

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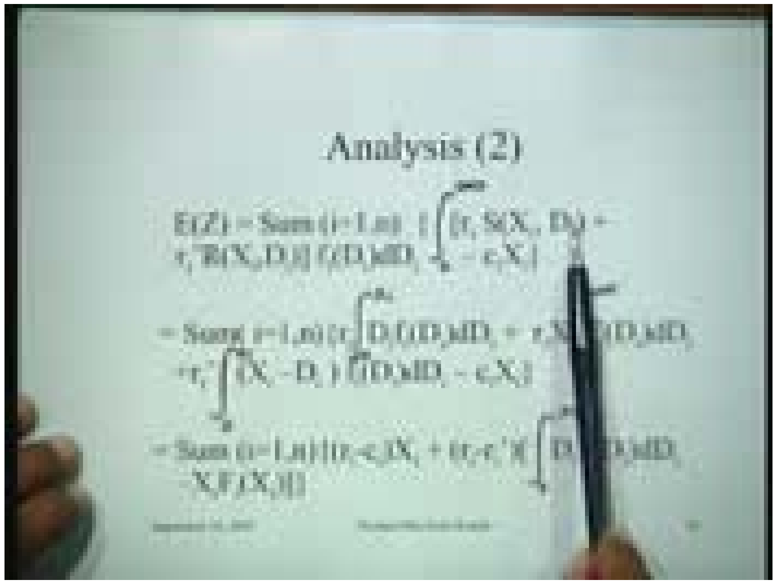

So what we do now is we say that the expected value Z is the sum from I = 1 to n. We have this entire expression integral from 0 to infinity. For all possible values of demand, this is the revenue that you earn from sales this (Refer Slide Time: 39:16) is the revenue that you earned from disposal of the item into fD which is the f_i D_i which is the pdf multiplied by dD_i. This is the integral which you have to evaluate here – ci x_i which is the cost of the item. Now this integral can be very conveniently split into ranges, for instance what we can say is that this integral can be from 0 to x_i and from x_i to infinity. We can take this particular value here. The revenue from sales can be taken out D_i f_i D_i into dD_i+ r_i x_i can be taken outside integral from xi to infinity f_i D_i dD_i. This is nothing but 1 minus the cumulative density function of the demand. So this particular expression on the other hand is the quantity remaining. This was the first integral and we will be talking about the quantity sold which can be depending upon whether the demand is less than xi or the demand is greater than xi. You have different function for that and the quantity remaining is x_i – D_i. This is the quantity and this will happen only if the demand is between 0 to xi if the demand exceeds xi, this quantity will be 0.

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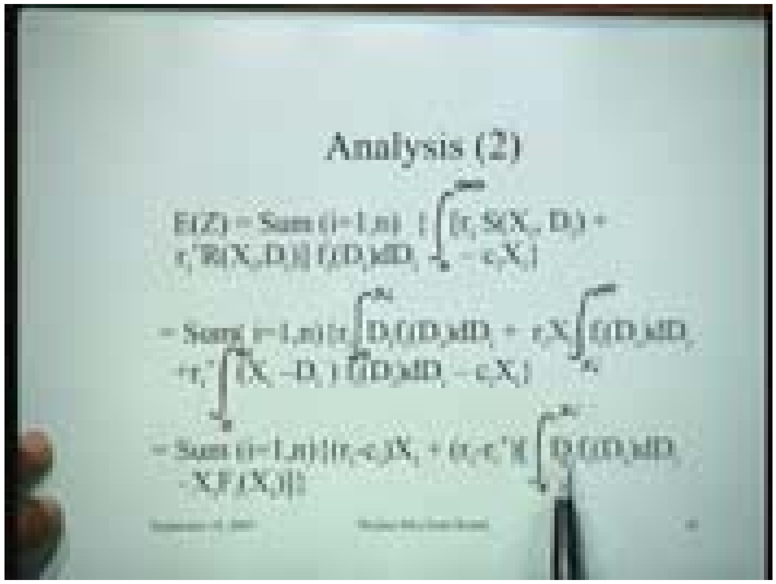

So in the second case the second part of the integral will not be there so if you rearrange the term you get this expression that sum of all the products from 1 to n r_i – c_i into x_i + r_i – r_i prime integral from 0 to xi D_i fi D_i d D_i – x_iEix_i is basically the expression for the expected value of Z. Now what we are interested in doing from this expression is for the expected value of Z which we have developed, we are interested in that particular value of Z.

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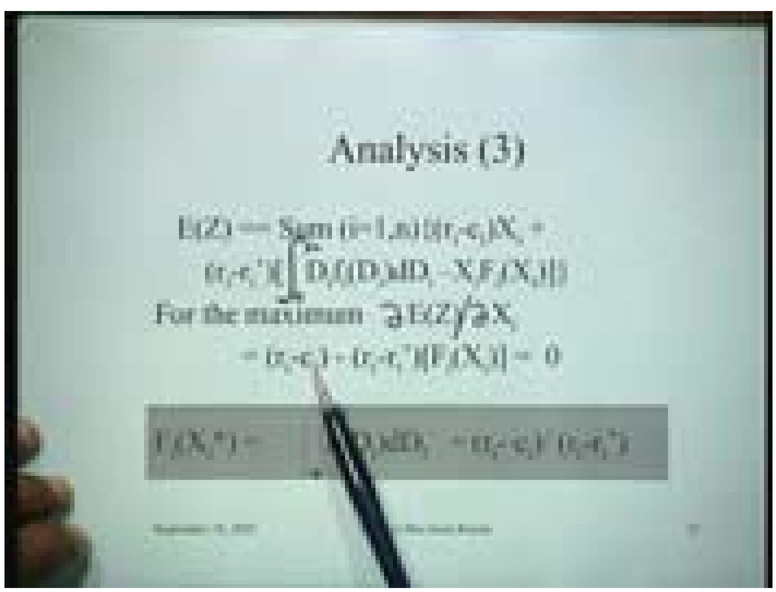

We are interested in maximizing the value of the expected value of z, so if you take the partial derivative of the expected value of Z with respect to x_i, what you would have is this expression $r_i - c_i - r_i - r_i$ prime into capital F is the cumulative density function of the demand. This is equal to 0 and if we just rearrange the function and our final result comes in a very compact form. This says that F_i x_i star star shows the optimal value is in fact nothing but this F_i x_i star will be 0 to x_i star f_i $D_i$diD_i. This is the cumulative density function up to the value. X_i star is nothing but $r_i - c_i$ divided by $r_i - r_i$ prime and there would be one such equation for each product, so I is = 1 to n, so you have n such things and these equations would in fact pretty simple to evaluate. Because what you would have to do is the right hand side is the constant $r_i - c_i$. This is the revenue (Refer Slide Time: 42:28); this is the cost that you get for item. This is the revenue and this is the disposal value, so this is the constant. This particular value of the constant, if equated to the **cdf** of the demand, you know the value and you get find out what the corresponding value of the x_i star would be from the pdf of the distribution or cdf of the distribution. Let us take an example to illustrate this approach.

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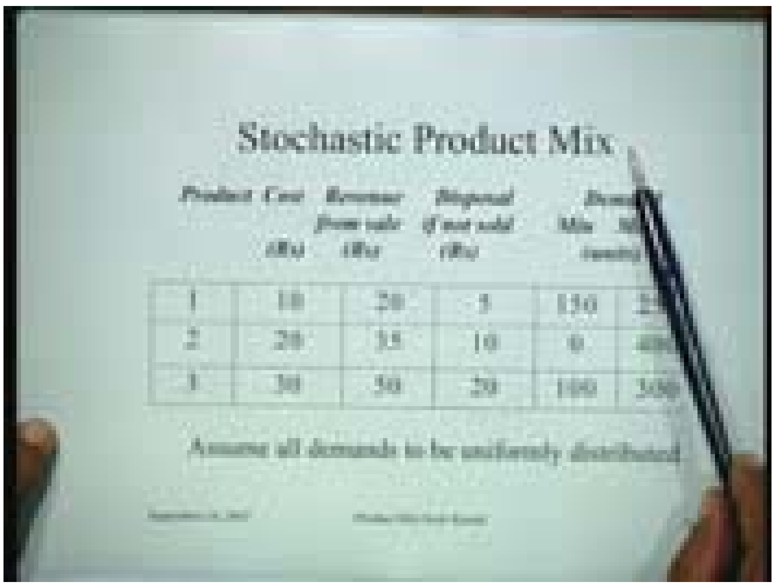

Let us say we have a simple problem of the stochastic product mix in which we have 3 products 1, 2, and 3, these three products have a unit cost of 10 rupees, 20 rupees and 30 rupees and the revenue from the sale (If you can sell it during the sale during the Pragati Maidan sale) will cause 20 rupees, this will earn 20 rupees, 35 rupees and 50 rupees respectively. However these are not sold during the sale then you will probably have to sell them off for a lower value. The costs are 5 rupees, 10 rupees and 20 rupees. In fact classically this problem is also known as the News boy problem. This is because the newspaper man who has to buy newspaper everyday and sell them the demand for newspaper is also a random variable. He does not know how many to buy such that during the whole year there is an expected profit that is maximized. So here we are taking a situation where the news vendor buys the Hindustan times, the times of India and India today. Each one of them is priced differently. Each one has the demand which is given here, This is the case and what we are saying here is each product has a range of demand. It has 150 to 250, 0 to 400 and 100 to 300 and we assume that all demands are uniformly distributed. They could follow any distribution and we assume that they are uniformly distributed.

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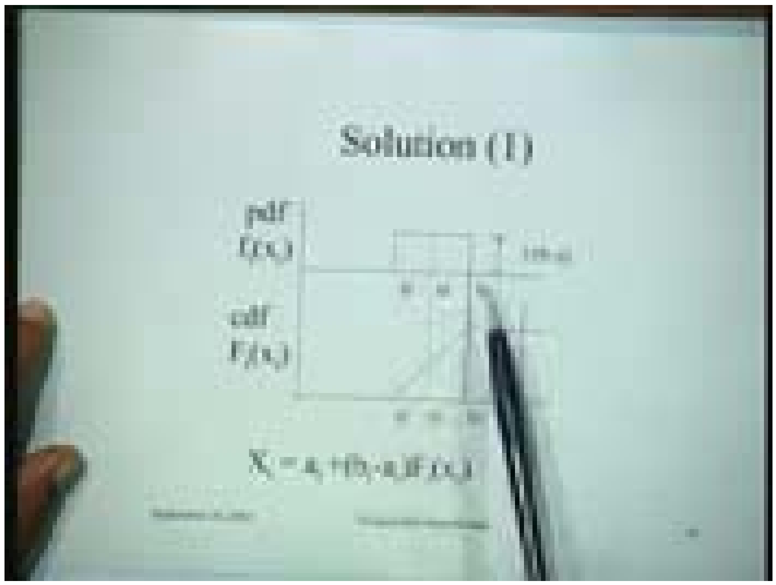

In order to solve this problem what we need to do is look at is the uniform distribution first. So uniform distribution is like this. We are saying that the minimum demand is ai for the ith product, bi is the largest demand for the ith product, so the pdf of this particular demand is the rectangular function and this particular value will be 1upon b – a because in a valid pdf, the total area must be 1. The cdf for this particular function will be a ram function going from ai to bi and any particular arbitrary value of x_i line between ai and bi. You can from similar triangles develop this particular relationship because you estimate at this particular point the value of the pdf. It will be this area. So this area is nothing but 1 up on b – a into xi – ,if you then so this is my cumulative density function F_i x_i which is what it is this area here I can develop a relationship directly for x_i is equal to ai + bi – ai into $F_i x_i$ which will be valid for all uniform distributions. Now you might be wondering why I am doing this. You will have to do this because if you want to solve that particular model, then you need the values of x_i corresponding to a given value of the cumulative density function so this is the equation which will very easily help us to do that. Let us try to solve the problem. All that we need to do is we have to calculate F_1 x_1. What is F_1 x_1? F_1 x_1 is nothing but r_1 – c_1 divided by r_1 – r_1 dash. 20 rupees is the revenue that you get from this item. So 20 – 10 in the numerator divided by 20 – 5; 20 is the revenue and 5 is the disposal value so this is just 10 by 15 and x_1 by using, we have just developed this equation I am referring to this equation (Refer Slide Time: 47:02), x_1 for a given value of $F_i x_i$ can be very easily computed from here.

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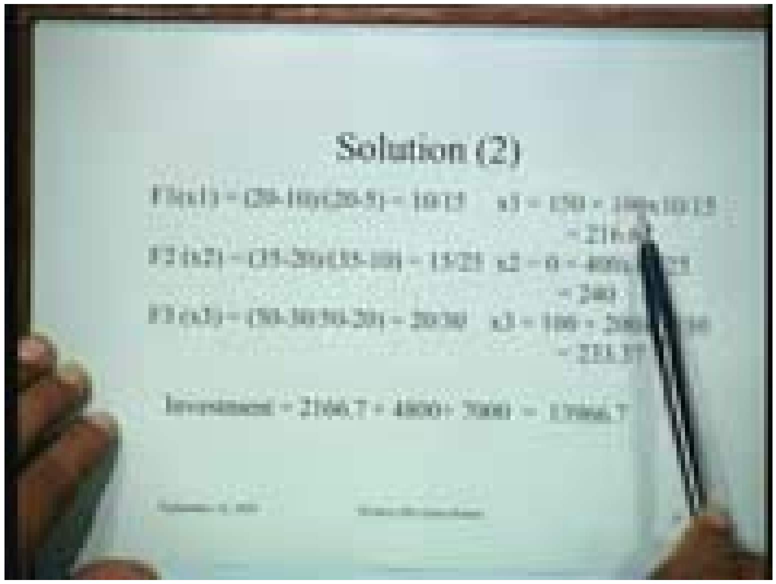

You have x_1 is = 150 plus because this is the minimum value of the range ai + 100, 100 is the range of this into 10 by 15 is the value here. So you get x_1 is = 216.67. Similarly the value for F_2 x_2 can be easily computed and this value is 240. Similarly F_3 x_3, this value is 233.7 corresponding to the values of the cdf value which is so much, that you can easily determine the values of x_1 x_2 and x_3 are going to be so. This determines the optimal product mix for this stochastic problem. So it is a very simple and elegant solution technique that you have for finding out the quantities of x_1, x_2 and x_3 which will in fact give us the maximum expected profit. Incidentally how will you compute the maximum expected profit? We had developed the expression for EZ, so these values of x_1 x_2 and x_3, if you substitute in that expression you will get the expression for value of the EZ.

Another interested thing that can be computed from here is, the practical significance is that if you buy 216.67 items and each item costs rupees 10. The total investment in purchasing item is so much, the total investment in purchasing 240 units of this particular item is 240 multiplied by 20 which is 4800 and finally the third thing is if you purchase 233.37 items and your cost is 30 rupee an item, your cost is going to be 7,000 rupees. So this is like the bill that the vegetable vendor has to pay and he finds that he is spending the maximum in product 3, out of his total budget of about 14,000 rupees which has to be spent. Roughly half of it has gone towards purchasing the 3rd item and now these are very useful insights that you can get into determining the optimal product mix under a situation where the demands are random variables in that case, what we have considered in this particular example is the situation where there are no constraints on the budget. For instance if the person who has to buy these 3 items had more than this, he had 15-20,000 rupees in his pocket, he would not be concerned because he can easily buy all these items and that would be (Refer Slide Time: 50: 18) and suppose he had a limited budget of only 10,000 rupees, he would have to curtail some of these items so that everything is confined to his budget of 10,000 rupees. So that would be the case of stochastic product mix with his single budget constraint. We have not addressed that problem. The problem will be addressed in the tutorial classes but what you can see is that ultimately you will have to cut something everywhere and how much you cut each of these places is actually determined by the process of optimization. So that problem can be solved by using Lagrange multipliers super imposed on this particular problem and we will see how the constraint problem can be solved.

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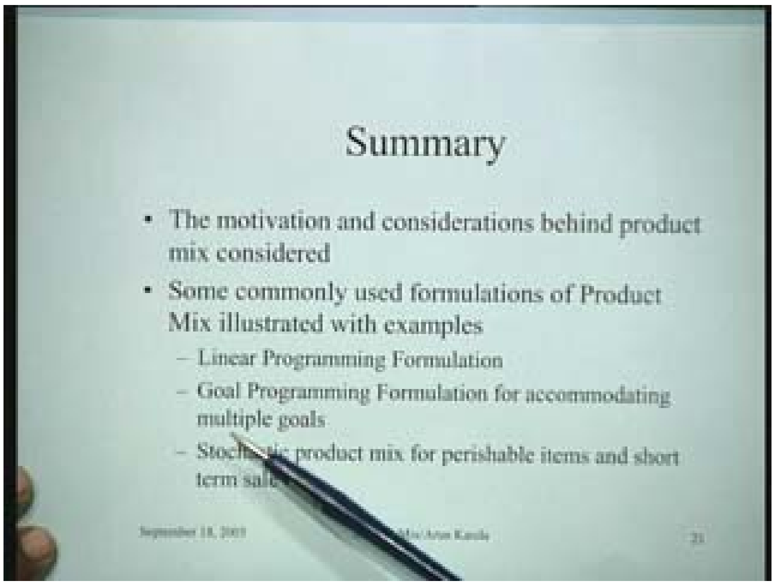

Finally let us look at what we have done in today‘s lecture. We have basically looked at the motivation and consideration behind the product mix problem. We had seen for instance as to why product mix is done because you do not want to put all your eggs in one basket. This is because there are different kinds of constraints which we enumerated, so you have to be careful and aware of all those things as to why that product mix problem becomes important. Then in this lecture we looked at some commonly used formulations of product mix illustrating them with examples. We look at,

1. The linear programming formulations which is the classical formulation. It is still a very useful mode of solving this particular problem.
2. Goal programming formulations for accommodating multiple goals.

The major limitation with linear programming is that you can take into consideration only one goal or one objective. So goal programming formulation which takes a number of deviational variables can accommodate multiple goals and finally we saw a stochastic product mix problem in which there were demands which are modeled by the random variables and you can have any arbitrary distribution for the demand and number of possible applications of these perishable items and short term sales which were actually highlighted and also possible extension of this model to situations where there are constraints on the budget or space could in fact be behind it by a similar kind of frame work with extensions using Lagrange multipliers.

Thank you very much.