Project and Production Management
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Lecture - 22
Roles of Models in Production Management

In the last lecture we have looked at the life cycle of a production system and we had identified the major decisions to be taken during the life cycle of a production system. In this particular lecture we are going to talk about the role of models and decision making. We will have a glimpse of different kinds of models that are used in practical decision making in various stages in the life of a production system. The basic idea therefore is that in order to take any decision optimally you quite often need to make a model of the situation and use this model for deriving what appropriate decisions need to be taken. So in this particular lecture we will deal with some of the issues. We will talk about the model. We will talk about the relevance to decision makers of various kinds of models. We will talk about different kinds of models which are useful and we will some examples of how models help in real life decision making.

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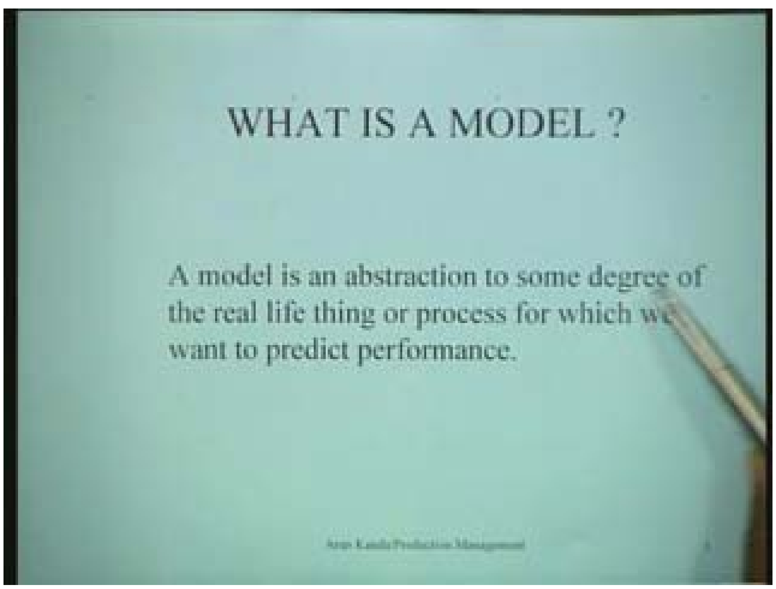

That is the intent of this particular section. Here is a working definition of a model. Model is an abstraction of some degree of the real life thing or process for which we want to predict performance in a most general sense.

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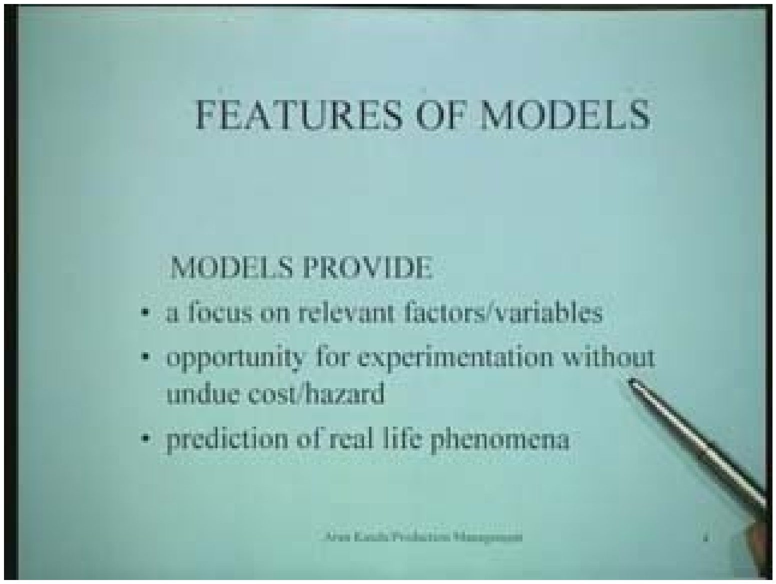

The question that arises naturally is what are the features of models and why do we make models? One must understand that models provide a focus on relative relevant factors and variables which means we have to understand reality. You must be able to pick out the relevant from irrelevant. You are essentially making models so they help us in understanding reality and picking up the right kind of factors, using them to express the relationship that we are interested in. Then models provide opportunity for experimentation without undue cost hazard. This is the major advantage of using models and production management. For instance, you might be using what is known as location models. You might want to investigate the effect of shifting the plants from Delhi to Bombay from Bombay to Chennai in terms of costs. If you actually started doing that you would be ruining all your assets. Model help us in making this analysis without undue cost hazard and help, may be using the right location.

The third feature of models is that they help in prediction of real life phenomena. There is a lot of uncertainty in the real world. The production manger has to deal with varying demands for instance; all that he has access to, is perhaps is the past historical data. You might want to use the past historical data to find out what is likely to be the demand for the next period of the (Refer Slide Time: 05:11) so that the plant production of the product might be automob$_{i}$les for the next month. So in that sense prediction is something very important for a manager. The phenomenon which is to be predicted can be predicted by using a model.

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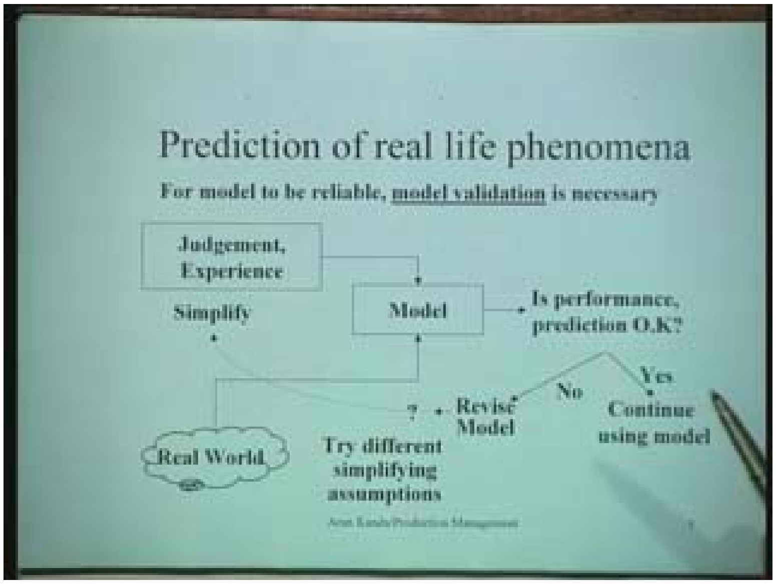

So this diagram (Refer Slide Time: 05:40) actually summarizes the process by which you would might make a model. What would happen is that there is the real world which you are interested in studying or capturing and in order to study the real world, you might have to use your judgment and experience and on the basis of these two things, you come up with model and this model is then used for process of prediction performance of the real life system. If the performance is okay you continue using the model. If not, you have to revise the model. The revision of the model would generally take place either in terms of simplifying some assumptions or introducing the additional complexities to the model and so on.

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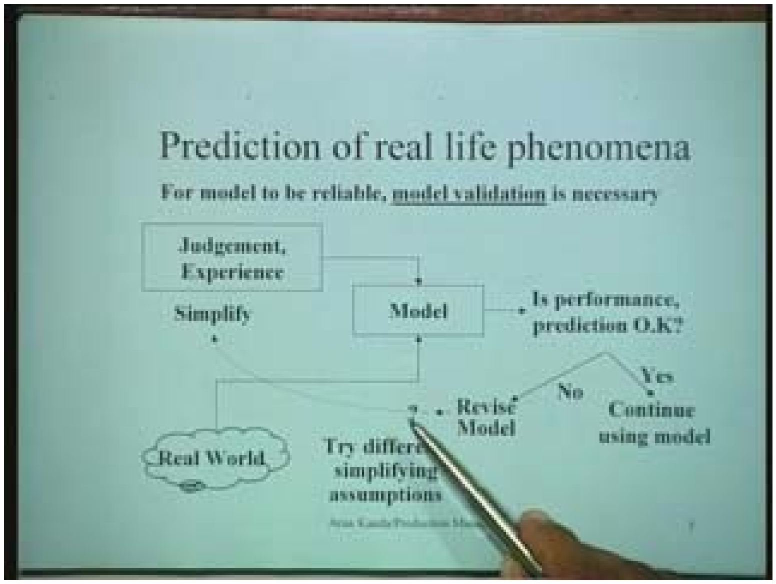

So this again refines your judgment and experience and this is the ongoing process which helps you to make models, refine models and uses them for the purpose of decision making because out intent here is to see how models are useful for decision making. Of course for a model to be reliable, model validation is necessary, this means the kinds of results that you get from the model or they are actually coming out in conformity with what real results are and this process is known as model validation. There are various mathematical ways of doing it but essentially we are trying to find out whether the results from the models are good enough or not.

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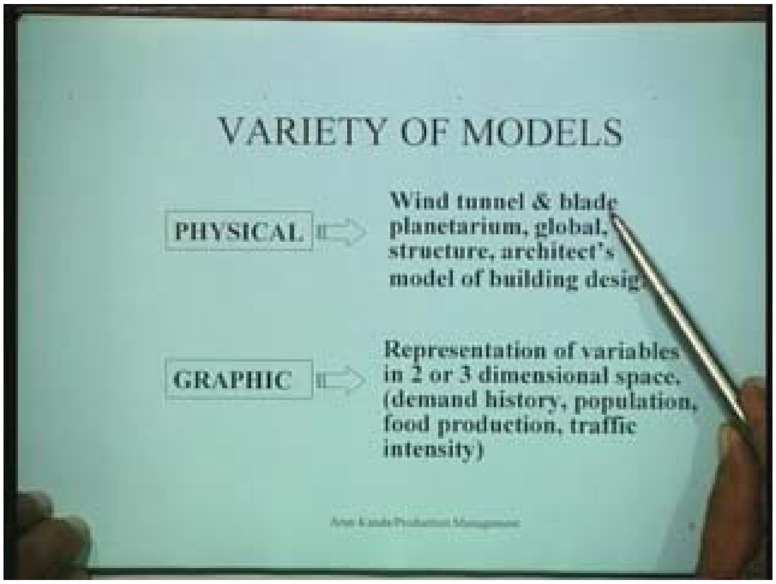

There could be variety of models for instance, model could be physical. Some examples of models are wind tunnel, wind tunnel and blade. Wind tunnel is used find out the shape of the airfoil and therefore you can do various kinds of experiments to design the airfoil in that sense it is a model. It is a physical model or a planetarium which is the model of the universe which will talk about the global structure or you would talk about the architect's model of building design which is again a physical model. The major advantage of physical model is that helps you understand and appreciate how the whole would look together and then therefore help you in forming some opinions or making decisions. Models could be graphic such as representation of variables in two or three dimensional space, such as the history of demand plotted versus time populations, food production, and traffic intensity. So all such important graphs are graphic models which give you some ideas on how the variable of interest is actually participating.

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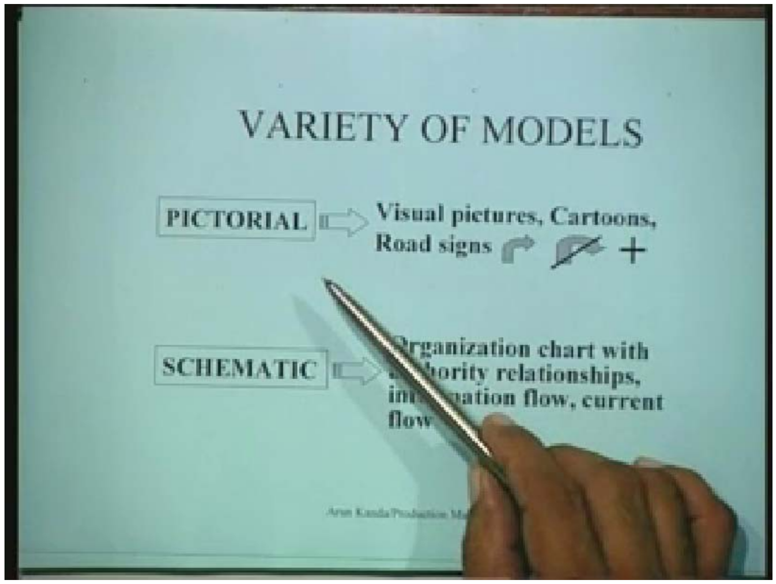

The variables could be pictorial. The models could be pictorial and you can use visual pictures or cartoons or road signs. Road signs are also nothing but pictorial models which tell you that you can turn right only or you cannot turn, you cannot have a u turn or you have various other kinds of things. Those are examples of pictorial models or a model may be a schematic which means like an organization chart with authority relationships information flow or current flow.

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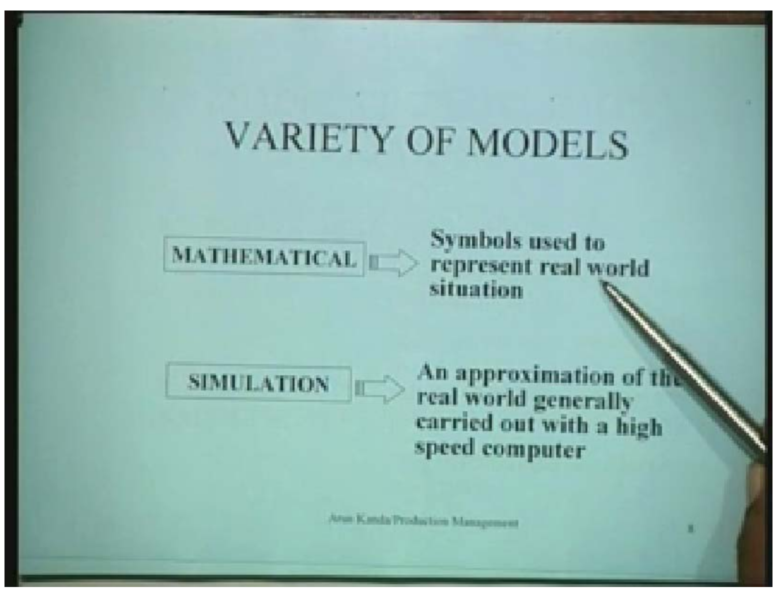

All these kinds of models are essentially schematic models of the whole thing or there could be models which are mathematical essentially. Mathematical models are those models where symbols are used to represent real life situation. You might want to represent the pressure by the variable p, the volume by the variable v and depending up on the situation you might say pv is equal to constant or pv to the power zeta is equal to constant. That is in fact mathematical model that particular situation of how gases behave. You could even set up a simulation model. A simulation model is essentially the approximation of the real world generally carried out with the high speed computer. You try to find out how the system would behave under varying conditions and try to simulate those conditions and from those simulations you can draw some meaningful conclusion. That is the intention. So these are some of the varieties of models that are present.

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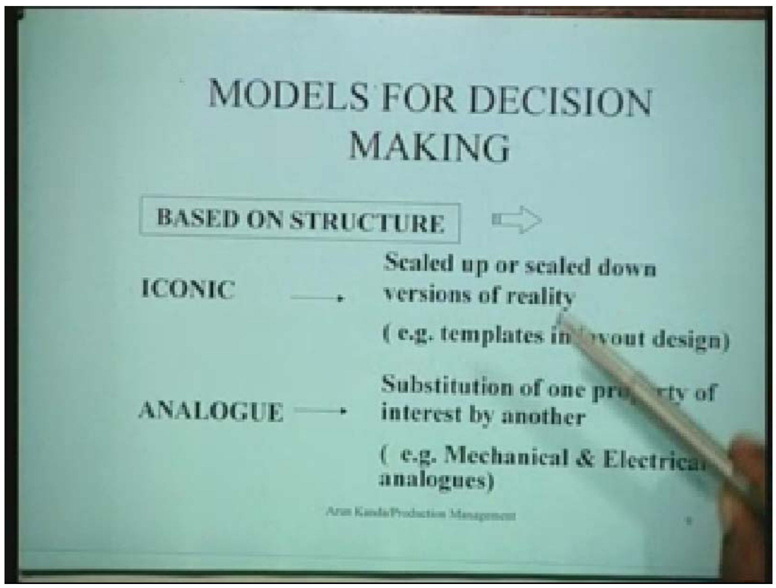

By and large, if you look at the variety of models you could classify these models in 3 different categories and especially when we talk about models for decision making, it is convenient to talk about models as iconic or analogue or symbolic as the case may be. So this is the classification that essentially based on structure of the model. What we are saying here is after all what an iconic model is? When we are talking about iconic models, these are actually scaled up or scaled down version of reality. To give an example for instance say a globe of the world is a model. It is an iconic model of the world. It has many simplifications but it performs many useful functions for instance you can use this model to understand how day and night are formed. With little bit of ingenuity you can also show how various eclipses are formed and understand how eclipses are formed so in that sense these kinds of models helps us in this nothing but scalped up or this is scale down version there could be scalped version reality. If you are talking about the molecular structure of let us say carbon for instance as a tetrahedron. You want to put a carbon atom at every node of the tetrahedron. You can have a physical model which does this and then it can help you probably calculate a variety of atomic; inter atomic properties that you might be interested. In that stage remember that iconic models are just scaled up or scaled down versions of reality and they give us understanding for the major problems. With these models for decision making, it is that they are not robust enough. You cannot keep on changing the model parameters because you have to construct a new globe and new model of the atom every time and therefore the usage for decision making is limited. The second kind of model class that we talk about is an analogue model and analogue is something in which you substitute one property of interest by another that is what it is. So you have mechanical and electrical analogues which would try to simulate or substitute the system and all considerations. A very interesting example of this was a model developed by the London school of economics which talks about representing the entire economic world of different countries by a system of pistons and cylinders with water in between. These pistons, cylinders are of different diameters and were connected depending up on the relationship on the individual countries in the world. The basic advantage of this was that if a particular major country like United States, where the policy changed and it was like moving a big piston. Energy effect would be felt in all the other countries by removing their respective small questions and so on. This is an analogue. Let us see an example by what we mean by analogue.

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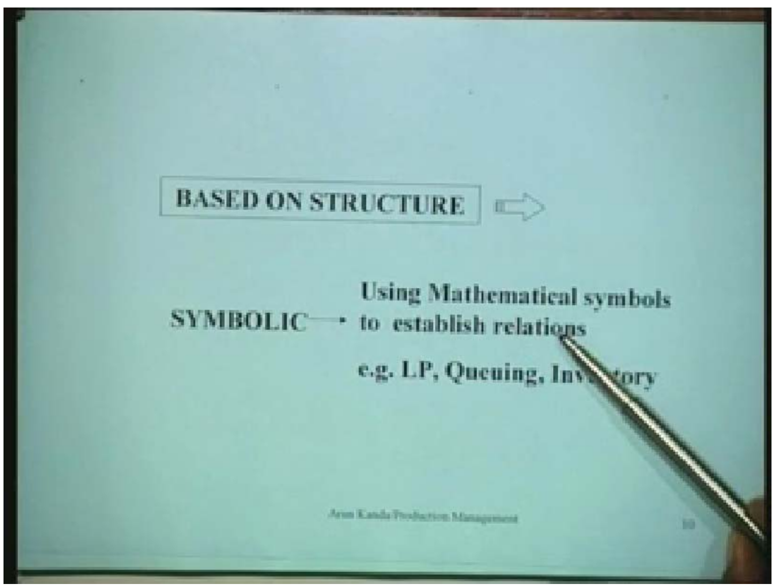

Then we have the symbolic model which is essentially based on the structure where we are using mathematical symbols to represent relations. In fact this is most commonly used in model in production system and in almost all areas of project and production management. What we are doing here is that the quantities of interest is represented by some kind of symbols, to establish the relationship between the symbols, you establish your objectives your priorities and you have a mathematical model which tries to capture the situation and some typical examples are linear programming, non linear programming, queuing theory, inventory theory. All these are basically symbolic models. The basic advantage of using symbolic model is that they are most robust. If anything changes, all that changes is the value for a particular variable or a parameter and it is very easy to run the model again with these revised changes. You do not have to make a new globe or anything of that kind. So these models are essentially very robust.

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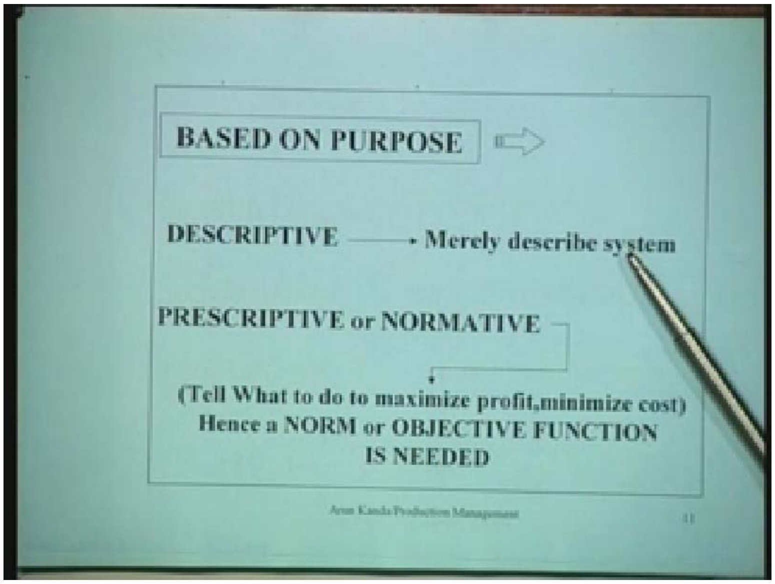

Based on the purpose we can classify models into descriptive and prescriptive. Descriptive models are those models which merely describe the system. This is what will happen if you do this (Refer Slide Time: 16:00). Typical example includes all queuing models are essentially descriptive models. They say that this is the arrival rate and this is the service rate and this is the queue discipline (Refer Slide Time: 16:17). Then the average queue length will be such and such thing or something is the descriptive model. A linear programming model is a prescriptive model. It tells you what should be of different quantity that you should produce so as to maximize your profit. So the production manager gets a prescription. He knows how much to produce each month and to maximize the profit. But both are useful and descriptive models can often be used to choose the right decisions. If you compare different decisions in a queuing situation you can always find out. For instance if you could find out at a traffic light, what should be the duration that the traffic light should be on or off? You collect distribution of the arriving cars and vehicles. Suppose you do this exercise for let us say keeping a stoppage time of 2 minutes, 3 minutes, and then 4 minutes in your computer, when you are doing this then for each case you would get different amount. Let us analyze the average waiting time for the customers of waiting cars or whatever. Based on this you would probably like to say that I would like the car to wait more than may be 5 minutes and therefore I should have a stopping time. In that sense descriptive model is being used for a prescriptive purpose and so now models can also be classified based on the environment.

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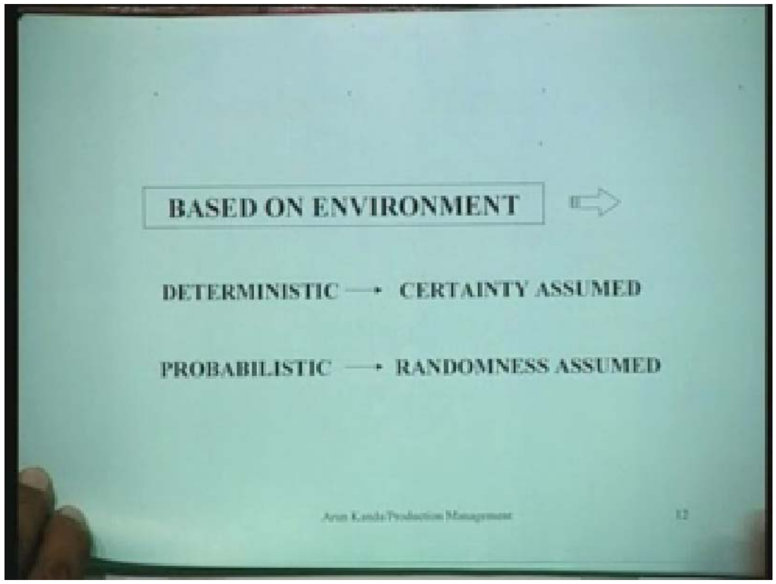

We are talking here about the decision environment. So broadly speaking you can speak with what is known as deterministic models, when you are assuming a certainty for the variables or you are assuming probabilistic models where randomness is assumed, some kind of variation is assumed for different types of variables. Since the symbolic models are generally the most popular in production management, we just look at a few examples of symbolic models.

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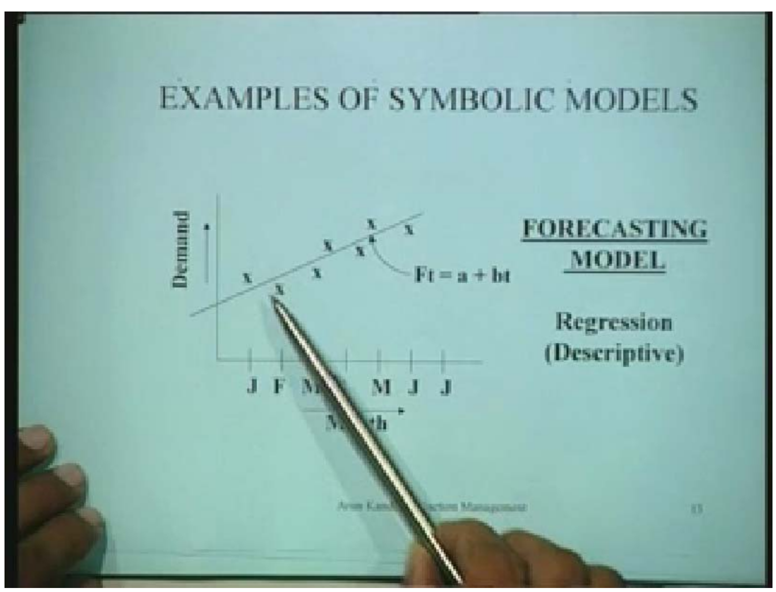

For instances what happens is that I said if I have the past history, January, February, March, April, May, June. I have the actual demand for a certain product like this and I want to utilize this information to find out the demand for the future months. One very simple example is using a regression model or a forecasting model. The regression type which is essentially a descriptive model fits some function to this. It need not necessarily be only a linear function. It could be any function and then once you fit it the function, this function then behaves as a model which will tell you how much demand would be there for future months. Depending up on the accuracy, model, validation process, you could relieve on this information to take decision pertaining to the production system. Here is an example from inventory.

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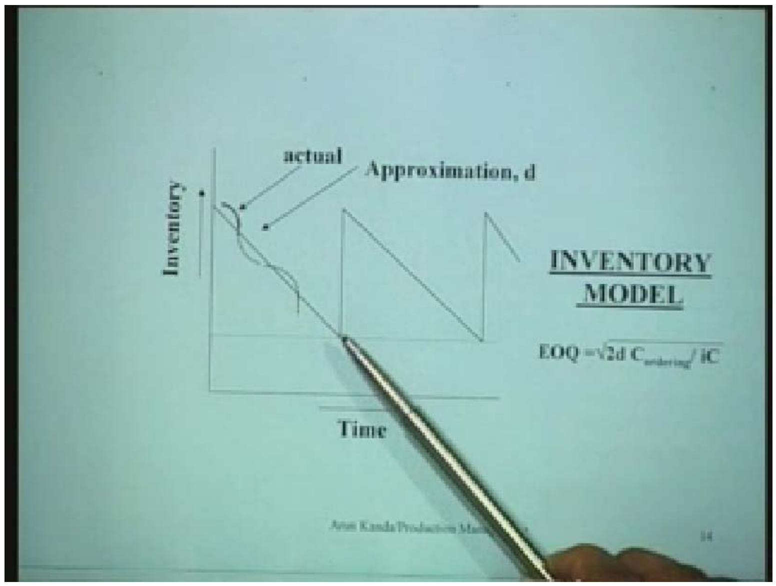

You are all familiar with inventory and when we talk about inventory management, the classical problem is that if I place an order for an item, the stock level suddenly rises and then it tends to fall gradually at an average rate. Actually the actual fall might not be smooth in this fashion but this could be this is one of the features of approximation that we bring about in the model. Then we say the average rate of consumption of this particular material is so much and then again the new order is placed here and then you get the supply here. So the stock level arises and so on. The pattern of variation of inventory typically could be modeled as the shortest curve which we know and from this by using a b_it of mathematical jugulary (which I am not going into at the moment because I am trying to discuss the basic philosophy behind models), you get the economic order quantity 2 into demand into the ordering cost, divided by i which is the interest rate into c under root. You can get the optimal ordering quantity. So it becomes the prescriptive model. It prescribes for management, what they should be ordering to minimize the overall cost.

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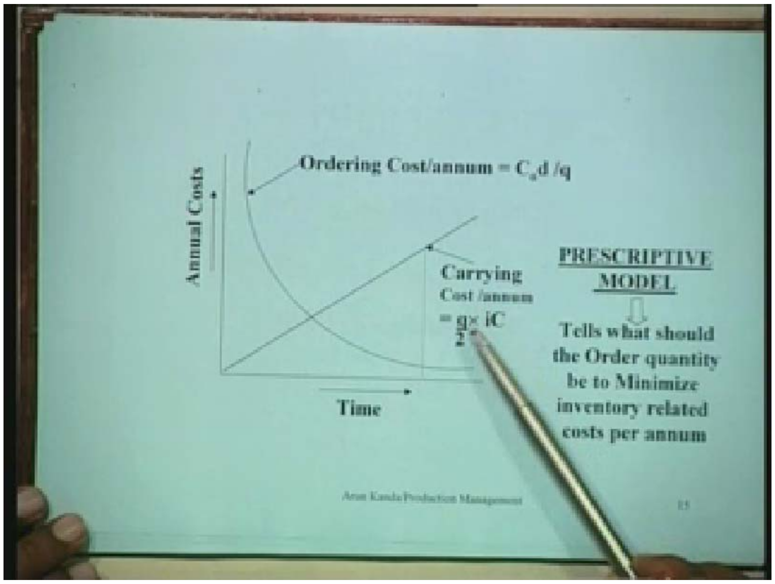

Once you have this information you can in fact see how the two major types of costs are involved. They are the carrying cost. The carrying cost is the cost of keeping the inventory in stock which would be say, the average inventory level is q/2 into i which is the interest rate into the c which is the item cost. So you have a straight line variation for the cost here and the ordering cost per annum is c_0d/q. So this particular function would be something like this (Refer Slide Time: 21:43). So what really happens is that you have two conflicting cost parameters. One increasing, one decreasing at different rate and the total cost would be something of this nature and the EOQ is the quantity which minimize the cost. But apart from this, the model tells you a lot of other useful things. For instance you might not be able to operate at EOQ but you can immediately find out if you deviate from the EOQ. What is going to be the cost penalty that you are likely to pay? Is it better to shift in this direction or in this direction and if I operate at a non optimal value, what is the cost I am carrying on these? These are all important questions; managerial issues that can be answered even through a very simplified model like the inventory cost.

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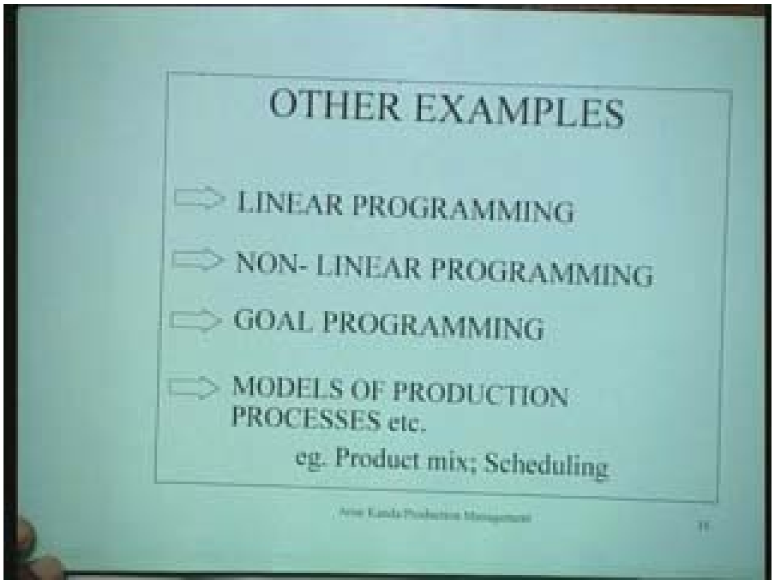

We can look at other examples of models which are very relevant in production management. We can use linear programming, non-linear programming, goal programming models of different types of production processes and we can model product mix and scheduling by using these kinds of models.

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Now we will take a small example and see how modeling for that particular product is done. Take a simple manufacturing problem. Let us say the company produces two kinds of products. These products are simply desks and tables and the manufacture of either desk or table requires one hour of production capacity in the plant. So whether you

produce a desk or a table, it means one hour of production capacity in the plant and the maximum of available production capacity is only 10 hours per week.
There is a limited sales capacity you can sell at most six desks per week or eight tables per week in that sense and the gross margin of profit is the sale of the desk is eighty rupees and from the sale of a table is 40 rupees. So if you want to model the situation, actually this is a very important prototype occurring in industry. It is a product mixed problem. Companies always make a number of different products so you are trying to find out what should be the optimal products.

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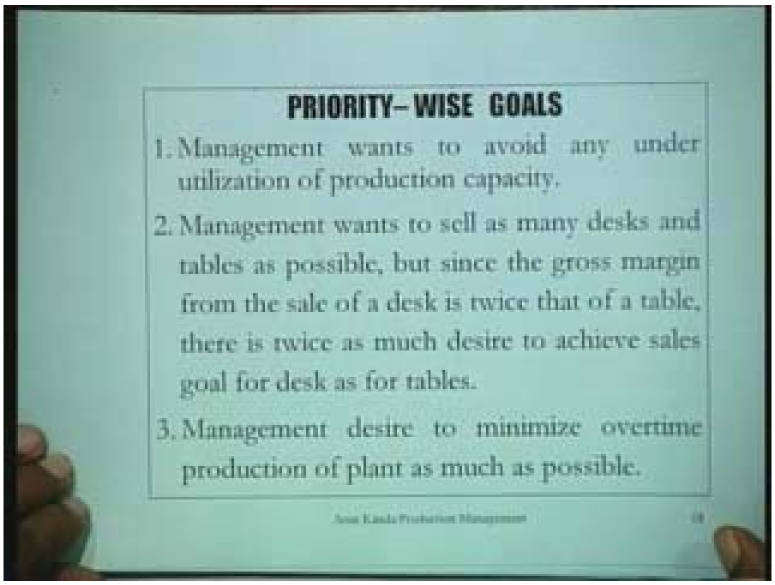

Let us suppose that the priority wise goals of the company are the management wants to avoid any under utilization of production capacity that is the first goal the management wants to sell as many desks and tables as possible. But since the gross margin from the sale for a desk is twice that of a table, there is twice as much desire to achieve sales goal for desk as for tables which is quite obvious and the third one is management desire to minimize the overtime production of the plant as much as possible.

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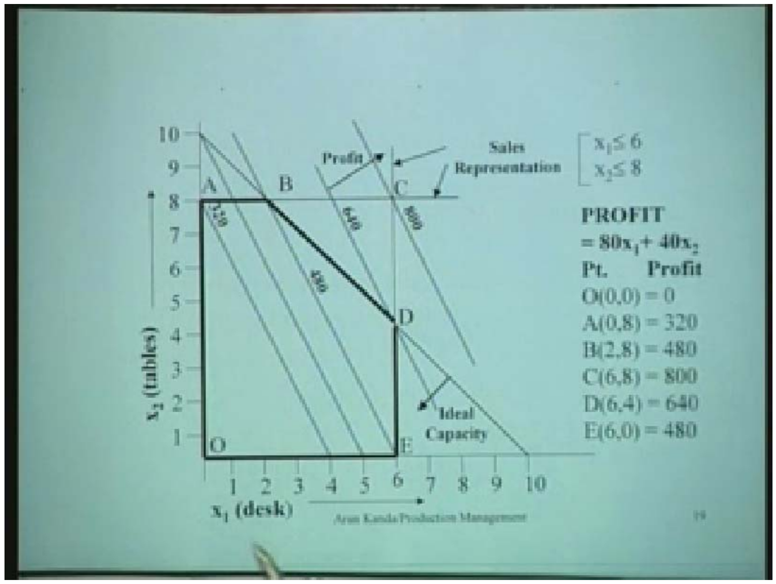

If you want to work with these priorities what can happen is well incidentally you can solve this problem graphically and what we see here is if on the x axis we have a number of desks which is x_1 and on the y axis we have a number of tables which is x_2. Then what we have basically is the total sale capacity for table is this line (Refer Slide Time: 25:23). You cannot sell more than 8. You cannot sell more than 6 of desks. So you have this line here and the ideal production capacity which we have utilized, both because each ten are available. So, one on each side and this would be just a line here with this intercept 10 and 10 on both sides. So what you find is if you would solve this problem as a linear programming problem, the space shown here (Refer Slide Time: 25:55) A B D E O would be the feasible region as far as the desks and tables are concerned you have this information itself is useful. What can be the range of product that you can make? You cannot make for instance so many tables which are outside this.

So it gives you the feasib$_i$lity range and then of course if we work out the profits at all the corners, you find that the maximum profit would in fact be, if we confine our sell to only the points A B D E O and exclude c which currently is not feasible, then of course you find that the point D gives us the maximum profit of 640. But now if you impose the priorities which we have decided for this particular problem, you notice some interesting. What would happens is that if we look at the examples, this line, (Refer Slide Time: 27:07) shows you are utilizing your capacity fully and if you go in this direction you are basically trying to over utilize your capacity. If you come in this direction you are under utilizing your capacity. So in that sense since the priority, said that if we do not want to under utilize our capacity, that means there is no implicit bar on the over utilizing the capacity through over time or through sub contracting.

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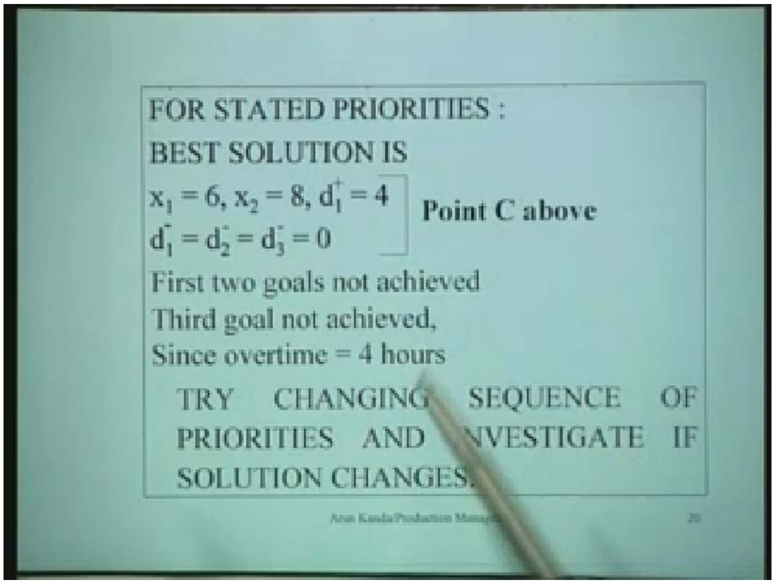

Whatever is the case, point c, which would have for stated priorities, would become the best solution. At this particular stage the first two goals are not achieved. The third goal also is not achieved and therefore since the overtime at this stage is four hours, what can happen is that you can keep on changing the sequence of priorities and investigate the solution changes. That is something that you can do and in fact what we are talking about here is the variation of linear programming. Typically known as goal programming in which we define a goal and then we talk about the deviations from the goal and then we are looking for solutions within the certain priority.

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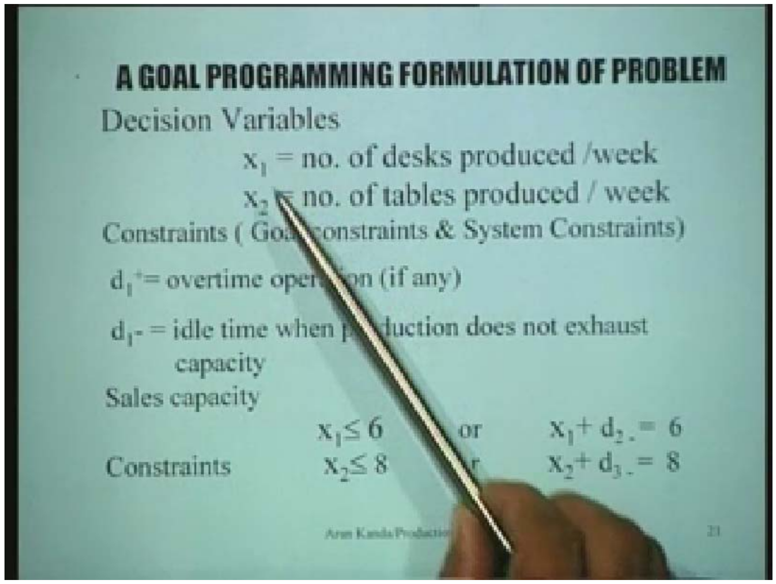

So this would be the way the goal program would look at this particular solution and when you look at a formal definition of goal programming for the problems, it would in fact be something like this. You have the number of desks and the number of tables as the two variables for the problems; we define an overtime operation if any, so if there is a d_1 +, d_1 – is the ideal time when the production does not exhaust capacity. So this is the sales restriction. X_1 is less than 6; x_2 less than 8 or we can introduce here this which is like a slag variables. So d_2 – would be the under time capacity. This is equal to 6 here. This would be 8. We have converted these two equations into two kinds of goals using these deviational variables d_2 – and d_3 –.

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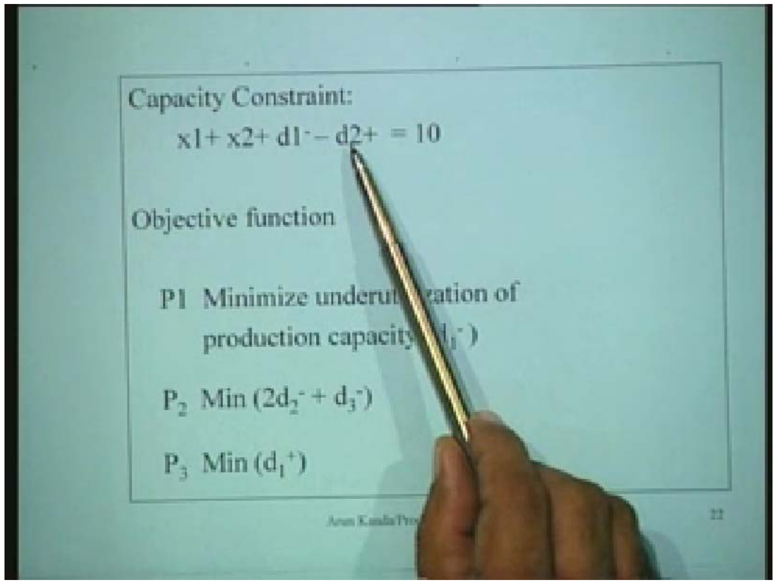

What we can then do is we have capacity constraint and the capacity constraints read like this. $x_1 + x_2$ must be generally equal to 10. But we could be producing less than 10 in which this variable will come into operation or we could be producing more than 10 in which case this variable come into operation. So since we can operate anywhere we can have both these variables coming into play. So this is like interpretation of this. This is the target and these are the under deviation or over deviation in the target because you do not need to begin with it whether we would be under deviating or over deviating from the target. So it is a very convenient device to model such situations and then of course the objective functions would have three priorities. Based on the priorities that we had minimized under utilization of the production capacity this is d1 –. This was 2 times the deviation because the profits from the desks and tables were different in the ratio of 2:1. So you have this particular goal here. You are minimizing this deviation and the third priorities is to minimize d_1 +, which is minimizing the over capacity, over utilization of capacity here. So this solution would lead to point c that we saw as the optimal solution.

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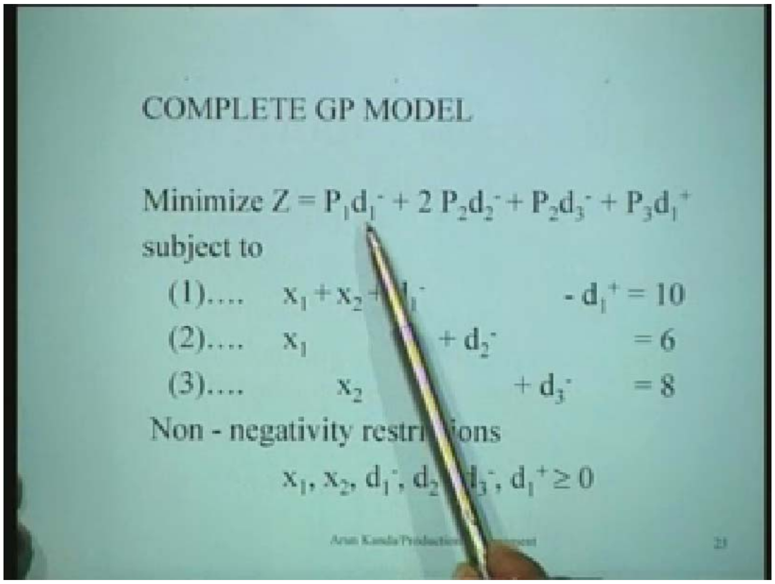

Finally this model would actually be set down in this form and you can use the goal program goal to settle down. We notice that we have basically deviational variable and constraints in this problem and the objective function is in the nature of the priority and the deviational variants. That is what happens here and this is a very common type of form which is used here. Let us look at a simple linear product mix problem, which is product mix in the sense that if we generalize this, you are talking of a situation where we have only two products. Now we have n products.

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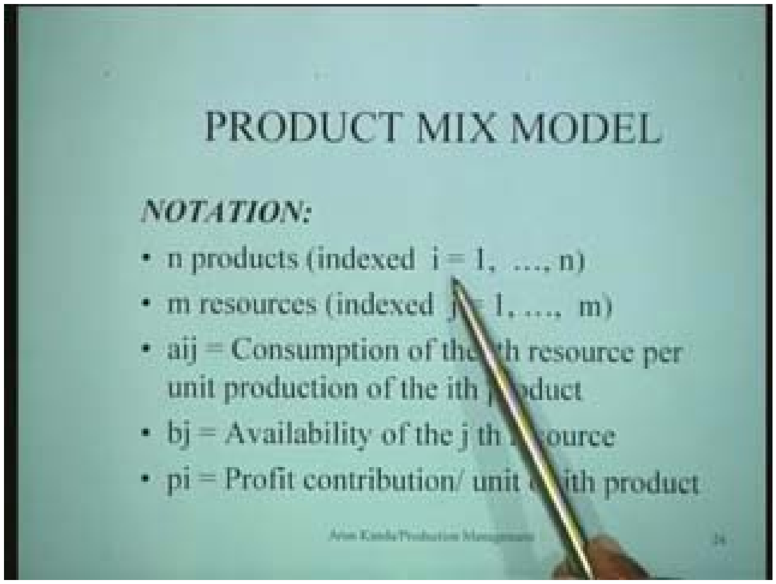

So the n products are indexed from i = 1 to n. So this is the third variation we are talking about. The simple linear programming in 2 variables, goal programming in two variables, just to indicate to you that when you have multiple goals and conflictive priorities we use that situation. This is the generalization of the first model. We have m resources a_{ij} is the consumption of the jth resource per unit production of the ith resource. bj is the availab$_{i}$lity of the jth resource in general and pi is the profit contribution per unit of the ith product.

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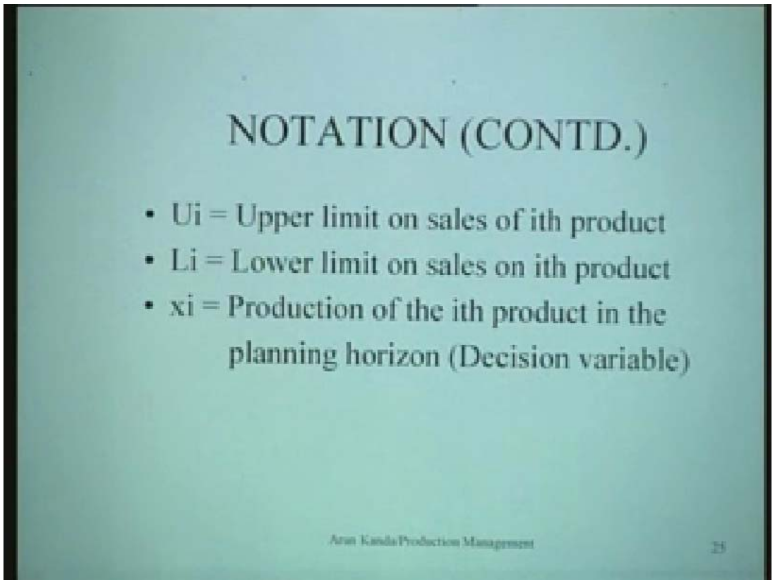

So when you have this, the various (Refer Slide Time: 32:27) other variables are ui, which is the upper limit on the sale of ith product, li is the lower limit on sales on the ith product in general and xi is the production of ith product in the planning horizon, this is our decision variable.

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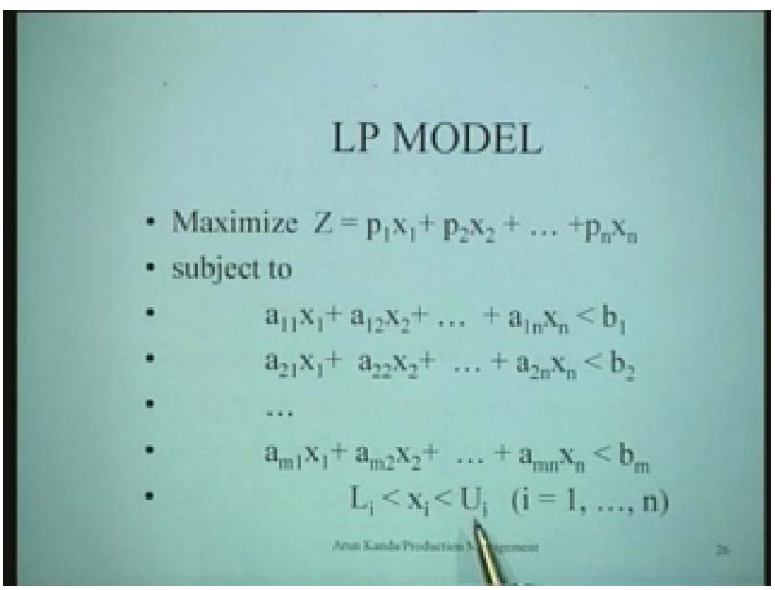

You want to find out how much of each quantity to produce and typically you have this situation this model would actually look like this. The objective is to maximize the profit so p_1 is the profit per unit from each unit of the first product, so $p_1\ x_1 + p_2\ x_2$ and so on upto $p_n\ x_n$ is the total profit that you earn is subject to various constraints. These constraints, if you see are each right hand side here, is the resource availab$_i$lity. This is the resource consumption (Refer Slide Time: 33:22). This is the resource consumption by the first product. If you are talking about labor hours, say my labor hours are 10,000. 1 unit of the first product consumes 5 labor hours. This consumes 10 labor hours and so on. So this would be then the consumption in terms of labor hours is less than this. So this could be the consumption of money, this could be consumption space and so on and you have the m resource, it is a generalized model for a product. It is a model which can be very useful to handle the l p problem, and then of course you could have restriction on sales. xi should be less than some upper bound, less than equal to some upper bound and greater than equal to some lower bound.

The upper bound could be the total capacity, rather it is the demand. So it would be the total demand for this product and the lower bound would be some minimum amount that you would have stipulated or contracted to make for that particular product, so that is the alive. Then this would easily be solved by nlp code. You notice that I am not talking about solution procedures here. Solution procedures are generally simple because you have access to available codes. You can solve them easily. What is the primary role of the production manager? It is to model a situation and set up an appropriate model. For his factory or a situation, whatever it is, it can be solved easily. Let me make digression to a model of the economy. As a whole you see we have looked at various kinds of linear programming.

A simple two product linear programming problem, a simple two product goal programming problem, and a n product mixed problem in general. Now those are all instances of models which we are actually trying to solve problems which are generally encountered within the factory and all the examples that we are talking were of that nature. We talked about the forecasting model developing, the demand then we talked about the inventory model, setting production targets of, or setting ordering targets for individual product within the factory and then we talked about the product mix which is also the decision within the factory and so on.

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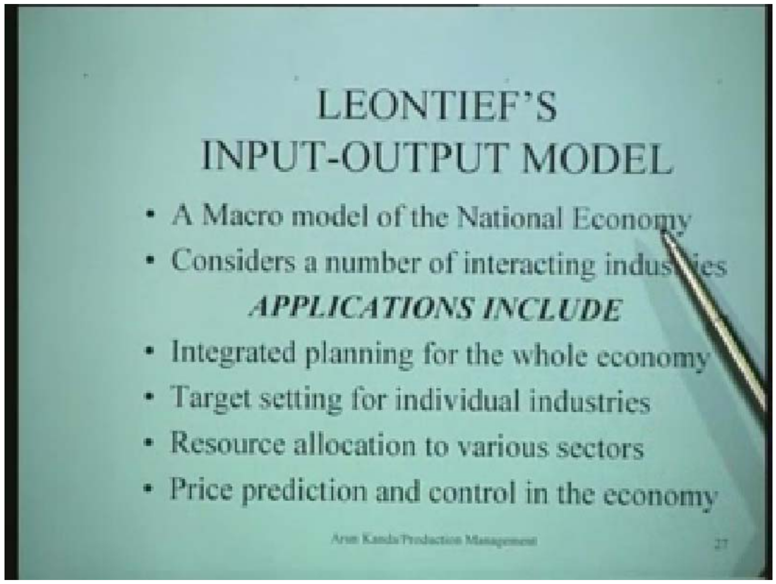

So in order to develop a broad aspect of the role of models and production systems, we will look at this Leontief's input out put models which is actually the macro model of the national economy. After all the economy is also a production system. It is producing different kinds of products and we will consider a number of interacting industries within the economy. Application of this model include integrated planning for the whole economy which means for instances whenever the planning commission wants to make the next five year plan for your country, how does it do? The Leontief's input output model is the bases for doing it. So we will see exactly how this planning can be done. You set the target for individual industries. This also comes out and the target should be that agriculture sector should be given so much and this is the target production for the next five years whatever it is. Resource allocation to various sectors is also possible through this kind of a situation and price prediction and control in the economy is also possible. You might be a little flabbergasted by saying that so many things can be done by a simple mathematical model. Let us look at the basic of Leontief's input output models and see how exactly we can go about looking at this particular problem.

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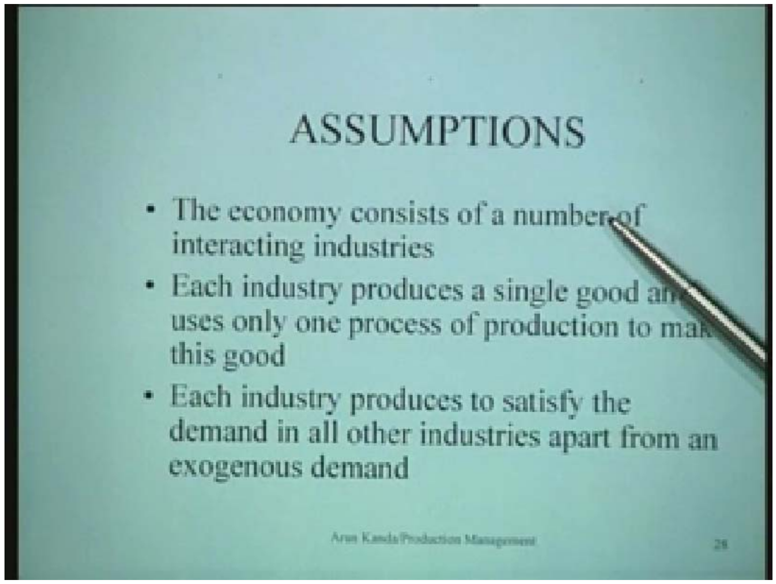

Some of the assumptions in this model are the major assumption that the economy consists of a number of interacting industries. Will you agree or not? What does this mean? In real life it means for instance that if you talk about the particular sector in the economy, let us say you talk about automob$_i$les. For its survival, automob$_i$les will have to borrow from the steel sector. The steel will probably have to be borrowed from the engineering sector and its different components and so on which are going to be used in the automob$_i$le sector and then of course all these things are going to be put together and there must be a demand for automob$_i$les. So you have people from different sectors who are actually going to use those automob$_i$les. So the important thing really is that in fact the key feature of Leontief's model is to try to model the interactions that take place between various industries.

Now one assumption that is made here is that each industry produces a single good and uses only one process of production to make this good. For instance one might say if I divide the economy into various sectors, let us say auto mob$_i$les sector is one of them then for purpose of macro planning I would not say that the auto mob$_i$le sector produces cars, jeeps, two wheelers, three wheelers and so on which are different kind of items. There are auto mob$_i$les, so I could club (Refer Slide Time: 40:05) them together I probably would say that the output of the automob$_i$le sector is in terms of general automob$_i$les. However if you want you might be able to say work in terms of standard passing the unit. This says that the maruthi 800 for instance might be taken as the 0.8^{th} standard passing the unit. A maruthi 1000, say one standard passing the unit b$_i$gger car might be taken as 1.2, 1.5 standard passing the unit. So that would give you a basic for aggregation or you might work in financial units and then everything gets comb$_i$ned very easily and normally in the Leontief's input output model, we tend to work financially. This is what we mean by saying that it produces a single good. It does not mean that automob$_i$les do not produce a single, but single good is automob$_i$le and each automob$_i$le could be different. So each industry produces to satisfy the demand in all other industries apart from an exogenous demand. Under these assumptions, we will try to see what exactly we (Refer Slide Time: 41:22).

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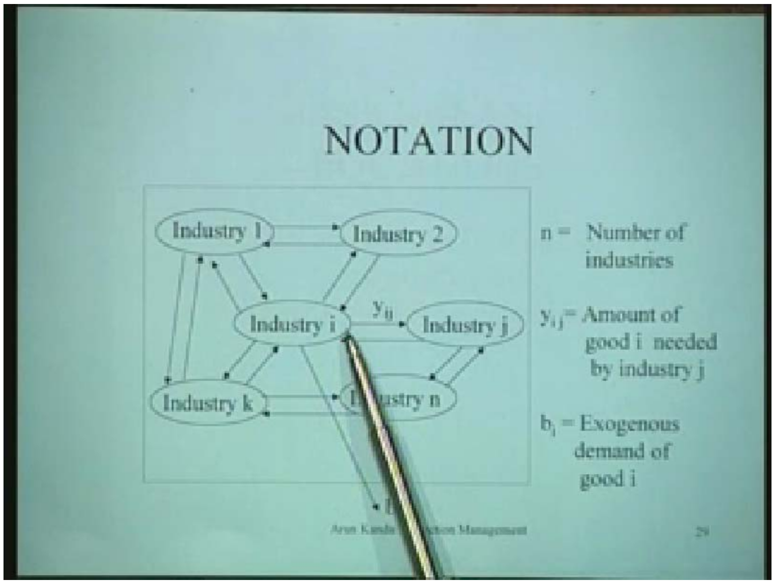

So let us see the interaction between various industries in the economy (Refer Slide Time: 41:30). Now we are looking at the economy as a whole and we have various industries which are interacting with each other. That means there is a two way interaction. Something can flow from here to here, (Refer Slide Time: 41:45), something can flow from here to there etc and there are various possib$_i$lities and so on. So we have industry 1, 2, 3 and so on up to n and let us look at 2 industries. Let us call this industry i and let us call this (Refer Slide Time: 42:02) industry j just to understand the notation which we shall be using for this particular model.

So n is the total number of industries which is there. y_{ij} is the amount of good I needed by industry j, so y_{ij} is flowing from the ith industry to the jth industry because the jth industry during the whole year or during the whole period, (mind you this entire exercise is for a certain period) and normal period is 1 year. So y_{ij} is the amount of good i flowing to the industry j, in that particular period and so on. b_{ij} is the exogenous demand of good i. So every year industry will have some exogenous demand which is either exports or whatever the consumer consumes, that is the exogenous demand. So this is the model that we are trying to look at in terms of Leontief's model. What we find here is that there are two types of things that we can do. First of all, our intention is to find out the production of each of these individual industries. Second objective is to find out the prices prevailing in the economy. If you look at these two things which Leontief's model gives, we can apply simple mass balance equation.

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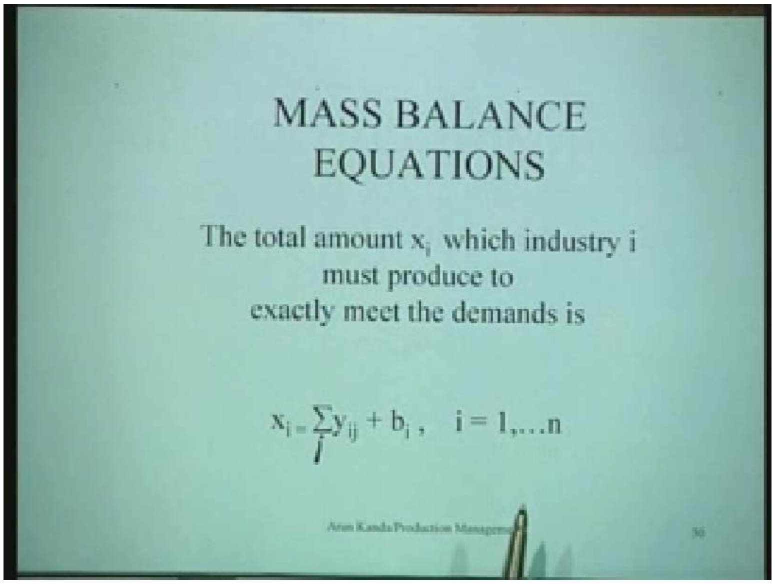

Mass balance equation for instance would be the total amount xi, which industry i must produce to exactly meet the demands will be xi = summation $y_{ij} + b_i$. b_i is the exogenous demand and y_{ij} is the amount which is going to industry j, so this particular summation that we are talking about here would be summation over ij. So you know for instance this should be j here, (Refer Slide Time: 44:27), the total amount flowing from i to all other industries plus b_i should be equal to what is production. For that it is a simple mass balance equation. This is like in electrical engineering; this would something like Kirchhoff's current law.

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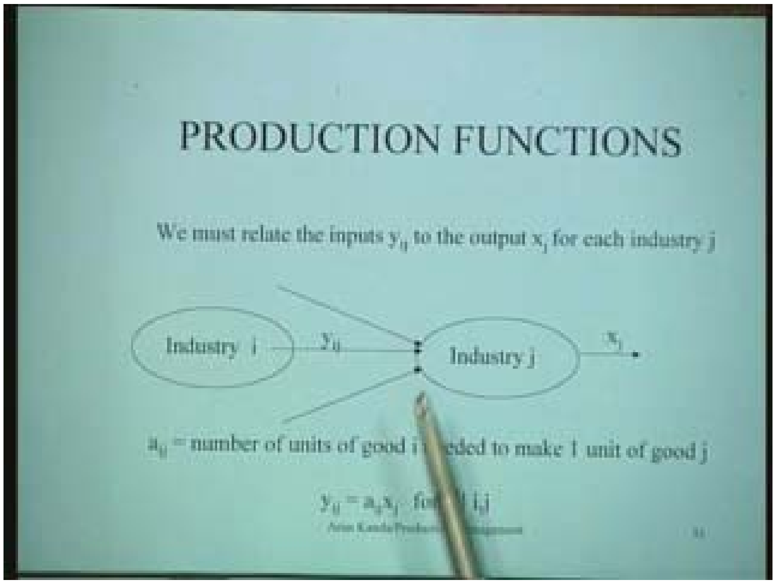

Then the notion or the sort of the approach that Leontief's took was, he developed what are called production functions. Now what is production function? If this is industry i and this is industry j and industry i is supplying a quantity y_{ij} to industry j. Industry j is producing quantity xj let us say. What does it mean? We say we relate the input y_{ij} to output xj of each industry j. So how do we do it? I have to say that $y_{ij} = a_{ij}$ into xj for all i j possible. All that this is saying is in any industry there are multiple inputs from different sources. So y_{ij} i.e., is the amount of input required for producing a unit of j = a_{ij} in that sense of term. What is the significance of this? You can see from this equation, y_{ij} divided by xj would be a_{ij} and a_{ij} would have the interpretation of the number of good i need to make one unit of good j.

If it sounds a little complicated, let us say there are numerous examples. The example is very simple. Suppose the industry makes a cake which probably your mother does in home. So if you want to make a cake, what are the various inputs you have for making cake? You would need chiefly maida. You would need baking powder, you would need sugar, and you would need other inputs depending up on the kind of flavor you want to give the cake. So the idea is to make the cake. Here is this. So the point is if I want to make a 1 kg cake, I would probably use 750 grams of maida. The point that is being raised here is that if I want to make 1 kg cake that means if I want 1 kg cake I need 750 grams of maida. You would probably need a pinch of baking powder. If I make 2 kgs cake, what could we need? You would require 2 into 750 that is 1.7 kg maid and 2 pinches of baking powder and various other things.

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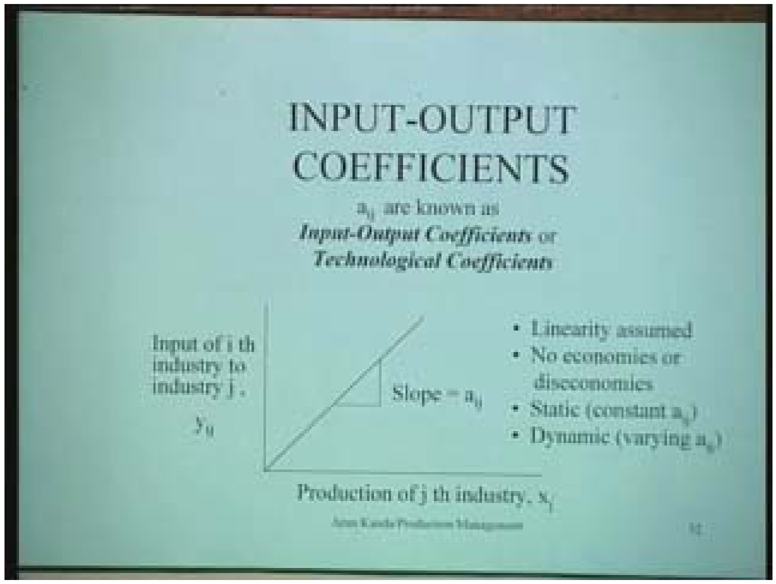

It is exactly in the same spirit that we are relating the input to the output and what we can then say is that this if you plot the relationship between y_{ij} and xj, we have said $y_{ij} = a_{ij}$. xj could be a straight line, whose slope is a_{ij}. So a_{ij}'s are known as the input output coefficients or the technological coefficients and here we are assuming linearity we are not assuming any economic or diseconomies of scale. It is a static situation and the a_{ij} is constant. If it is a dynamic situation a_{ij} would be allowed to vary that is what we are trying to say. So what is really happening is keeping the example of the cake in mind if you want to find out or predict the total requirements of a certain industry, it will be a fixed proportion. If I want a 2 kgs of cakes I use 2 into 750 grams of maida. So similarly if I need to produce one ton of something, I need this much of input (Refer Slide Time: 49:18). If I double my output then I would require more of input and this a_{ij} will capture the input you must require. This is the technology coefficient.

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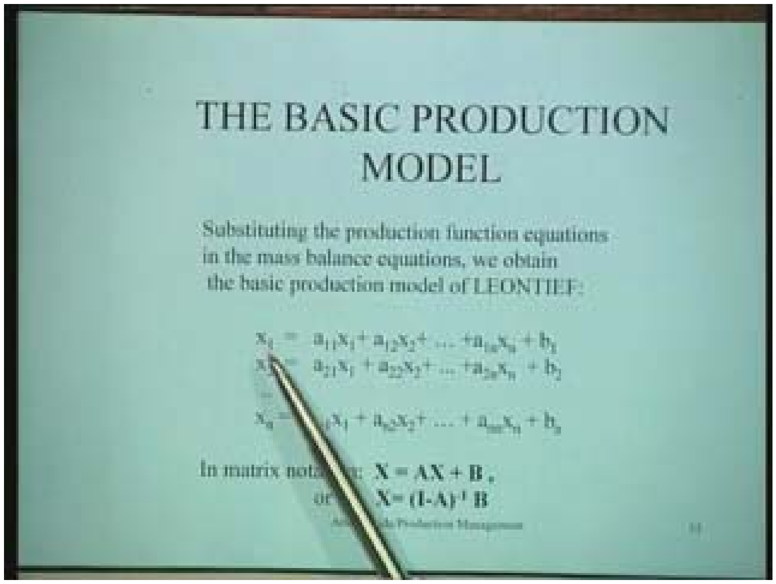

A basic production model in this particular situation comes out like this, x_1 would be equal to this particular thing which is their plus $b_1$by mass balance equation x_2 would be again a similar equation; x_n would be a similar equation. If you put it in a matrix notation you get X = AX + B and then directly you can solve this equation for xs. You get X = I – A inverse B. This is the basic Leontief's model. What it says is that you can now calculate the production quantity knowing the requirements of the individual sectors in the economy if you know the technological coefficients, that is the matrix a. This equation is the fundamental equation which is used for solving this particular problem of determining the production quantities, as a function of the (Refer Slide Time: 50:42) these exogenous demands.

What does the planning commission do basically? The planning commission finds out by to talking to different sectors as to what is likely to be the demand for the next five years. So different people converse and generate the vector b and once the vector b is known, and assuming that the input output coefficients remains unchanged, you can calculate what the production should be. That is the basic idea of the basic production model. The second aspect of this model is the prices in the Leontief's system. The prices in the Leontief's system, if p_j is the unit price of good j, then a_{ij} p_i would be the cost of the a_{ij} units of good i required to make one unit of good j. Simple, and therefore the cost of goods 1, 2, 3 and so on up to n needed to make 1 unit of good j would be the summation of this. So we have summation from i equal to 1 n of a_{ij} p_i and if the value added in the industry is important because each industry adding value It means the total cost is 20 rupees, it is making a profit of 10 rupees and selling for 30 rupees, that is the idea. So if r_j is the value added in the economy, price of good j minus this quantity should be equal to the value added in the industry. Basically these equations for i is equal to (Refer Slide Time: 52:15) 1 to n will also be the n equations that we used to solve p_j's.

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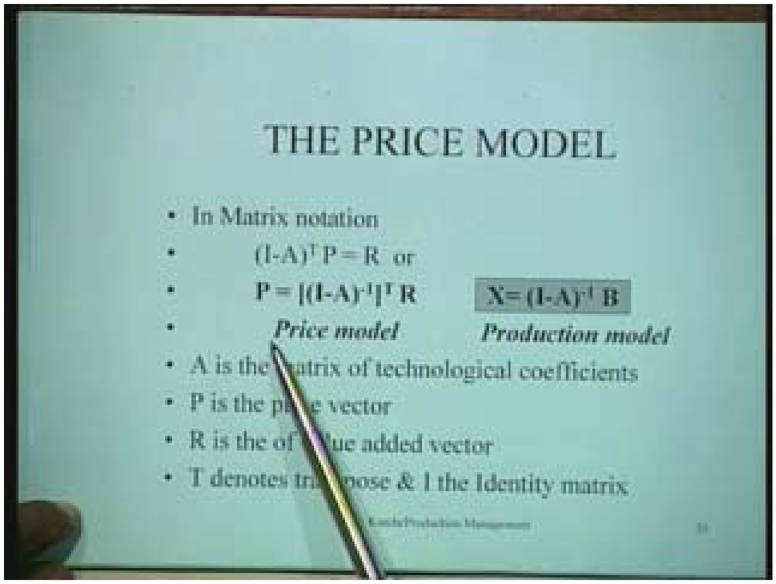

What happens is if you write them in matrix notation you get I – A transpose into P = R. That is what the equation basically leads to and therefore P can be written as I – A inverse transpose multiplied with R. This is the value added and this is the price model. So, together the price model and the production model constitute the basic Leontief's structure. This was the production model we studied a short while ago in which you are calculating the production quantities as the function of the exogenous demand b of the products. Now here you are calculating knowing the value added. You are calculating the prices of the economy; you can calculate the prices and the beautiful discovery of this particular model for which Leontief got the Nobel Prize. Incidentally, here you do not have to do additional effort to calculate this. Because this is I – A inverse which you calculate once. This is just I – A inverse, the transpose of that. The inverse is automatically calculated.

You do not have to do any matrix inversion again and in fact it was this particular discovery that led to the difference between primal problem, dual problem in the linear programming and it was about the same time that George Dancy was working on the simplest method of solving the linear programming. This was in the early fifties that Leontief discovered that this is another way of looking at duality and therefore in this, a is the matrix of technological coefficients that we have just seen. P is the price vector; this is the price which prevails in the economy. What would be the cost per unit in the automobile sector? The agriculture sector in the labor sector etc prices are determined by this model. r is the value added vector. You see what can be done for instance the government is in planning. We can give more subsidies to a certain section of the economy which means that they are playing with the value added vector of that sector. If you add to the value added vector of that particular sector for instance, the government gives conjunctions to the agriculture sector and you know how much concession have been given. You can calculate immediately the impact of this concession of the overall prices in the economy for the different sectors. I think it is a very useful way of interpreting. So ultimately this was indented to give an idea of the price model as well as the production model of Leontief which can be utilized for solving this. You know the R vector is the value added for the economy. Now the value added for the economy is actually governed by the competitive situation in, the market precisely. It is not it how much value can textile value has? How much value can automob$_{i}$le value have for its product? This is determined to a very large extent by those particular (Refer Slide Time: 56:05). Then this could be something that they would decide to set. It is their decision. If I make a product and make it for 100 rupees and decide to sell it for 150 rupees, I have actually set my value added. So value added to a large extend is the amount of what the particular industry expects from its product. You will supply the value added vector and then compute the prices. I mean if we add the value added or keep the value added; you try to work out this particular situation.

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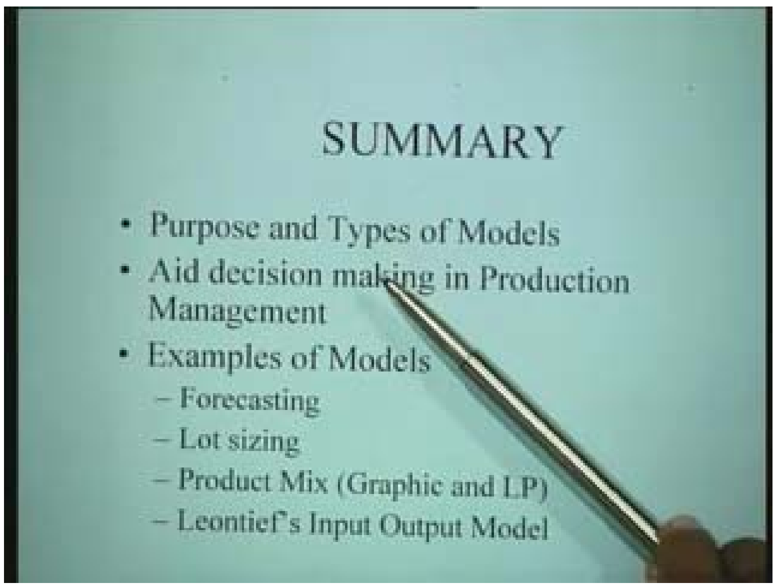

Finally let us summarize what we have tried to do in this class. We have tried to look at the purpose and types of models that are there for decision making and the basic key was that all these models aid in decision making in the production management situation (Refer Slide Time: 57:14). We took some examples of models. We looked at forecasting model and lot sizing model, product mix model for graphic and LP and also example of a macro model like a Leontief's input output model which are insisted. It would not be wrong to say that the steady state of production management is basically study of models of different kinds and these models, once you carry in your bag; you apply for real life situation.

Thank you!