

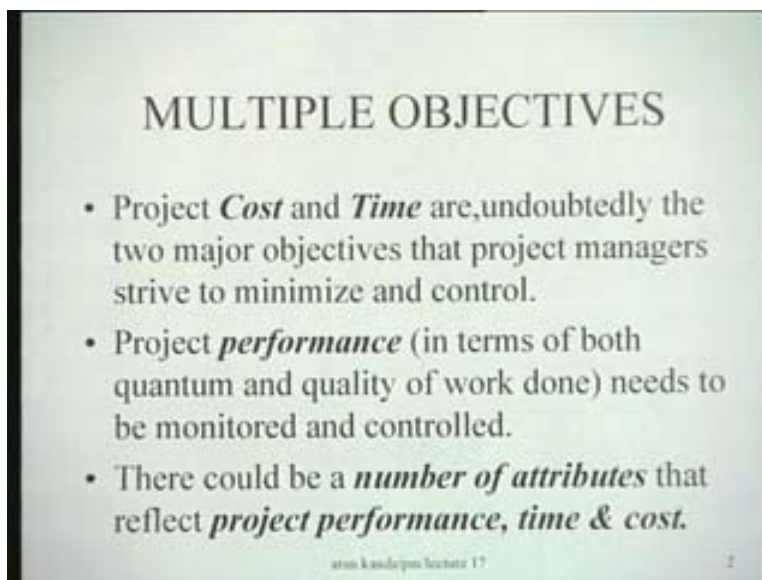
Project and Production Management
Prof. Arun Kanda
Department of Mechanical Engineering
Indian Institute of Technology, Delhi

Lecture - 13
Project Crashing with Multiple Objectives

Today we are going to be talking about project crashing when there are a number of objectives to be attained. Previously in the various project crashing models that we have considered our primary objective has been to minimize the project related costs namely the project direct costs. But in real life situations there could be a variety of other objectives related to the performance the time and the cost of a project and we are going to explore today a goal programming approach to handle these multifarious objectives. Let's first look at the variety of objectives that could be relevant for a project. First of all undoubtedly project cost and time are the two major objectives that project managers are striving to minimize and control. That's why there is so much of concern in practical project management about time management and cost management because these are recognized as the two dominant objectives in managing projects. Apart from these the project performance both in terms of the quantum and in terms of the quality of work done is important and this needs to be monitored and controlled.

There could be a number of attributes of that project which could reflect the project performance, the time that the project takes and the cost that the project actually accomplishes in terms. The various objectives that one is concerned with are related to the 3 major quantities namely the project performance, the time and the cost and in each one of these there could be a variety of sub objectives that one could try to pursue.

(Refer Slide Time: 3:13)



MULTIPLE OBJECTIVES

- Project *Cost* and *Time* are, undoubtedly the two major objectives that project managers strive to minimize and control.
- Project *performance* (in terms of both quantum and quality of work done) needs to be monitored and controlled.
- There could be a *number of attributes* that reflect *project performance, time & cost*.

arun.kanda@pm.lecture 17

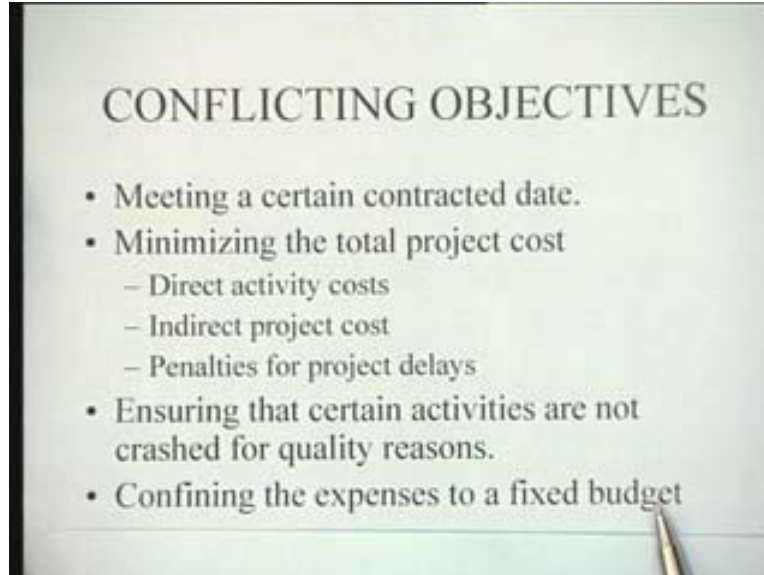
2

I think it's important to understand that many of these objectives are actually conflicting in nature. That is if you try to minimize the project duration the cost tends to go up and vice versa. Therefore we have to basically strike some kind of compromise and let's look at some of the typical conflicting objectives in a project. For instance one of the most common objectives for a project manager to achieve would be to meet the contracted date. Each project has a contracted date and it's the contractual obligation of the person who is executing the project to meet the contracted date. This is in fact one of the major objectives. Minimization of the total project cost is another objective, a very dominant objective in fact and when you talk about the project cost, this project cost could be composed of the direct activity costs.

The direct activity costs are those costs which are attributable directly to the activities in terms of either raw material costs or labor costs which go directly into the individual jobs that have to be done in the project. Apart from this there could be indirect project costs. The indirect project costs would be the costs which typically are the overheads in a project and which are dependent upon the project duration and are not attributable directly to any specific activity but these are the costs of co-ordination and control and costs of managing the project in general. These are indirect project costs. Apart from these there could be some kind of costs which are imposed for non-compliance with contractual obligations. There could be penalties for delays if a project is not accomplished by a certain date or if certain milestones in the project are not accomplished by certain dates there could be penalties for delay and these penalties could assume a variety of forms. You might say may be a cost like 500 rupees per day that the project is late is imposed or 1000 rupees per day that the project is delayed. The amount would be negotiated in the beginning and would depend upon the importance of the project and also its total cost.

Another objective might be that certain activities in the project cannot be crashed for quality reasons. Certain activities might be crucial for the performance of the project and therefore although they could be theoretically crashed but for reasons of quality and may be for reason of safety you might not want to crash certain activities. When you are developing a schedule you might want to eliminate or leave out those activities for reasons of quality and one of the major objectives of a project manager would be confining the expenses on the project to within a fixed budget.

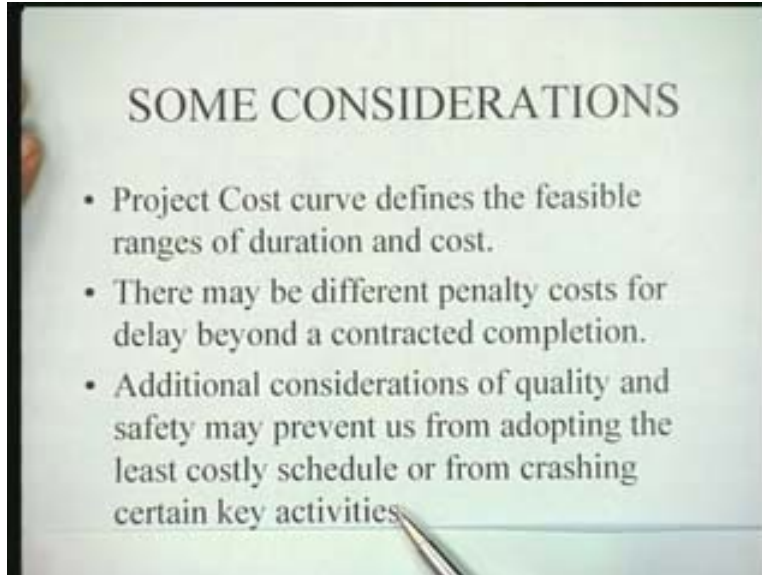
(Refer Slide Time: 6:49)



These objectives if you try to achieve one, the other could be under achieved or over achieved and they are often conflicting in nature and it becomes difficult to plan a project execution which would be able to accomplish all the objectives simultaneously. What has to be really done is basically to select a kind of a priority structure and then decide on what these objectives are. Some considerations which are important for us to keep in mind are that we already know how to generate the project cost curve. That is the cost duration efficient frontier. This is a very valuable analysis which can identify for the manager the feasible ranges of the duration and the feasible ranges of the cost and it defines in fact the entire spectrum between these two major objectives namely the duration and the cost and the kind of tradeoffs which are possible between the cost and the duration.

Apart from this there could be certain penalty costs for delay beyond contracted completion. If there are any penalty costs which are to be imposed in a certain project what can be done is that these costs could be considered in much the same way that the project indirect costs are taken into consideration. Just as we superimpose the project indirect costs on the project direct cost to determine the total cost, the penalty cost if there are any could also be superimposed on the direct cost and the indirect cost and we could determine the total cost function and use that as a basis for choosing the target duration for the project to be accomplished in. Additional considerations of quality and safety may prevent us from adopting the least costly schedule or from crashing certain key activities.

(Refer Slide Time: 9:12)

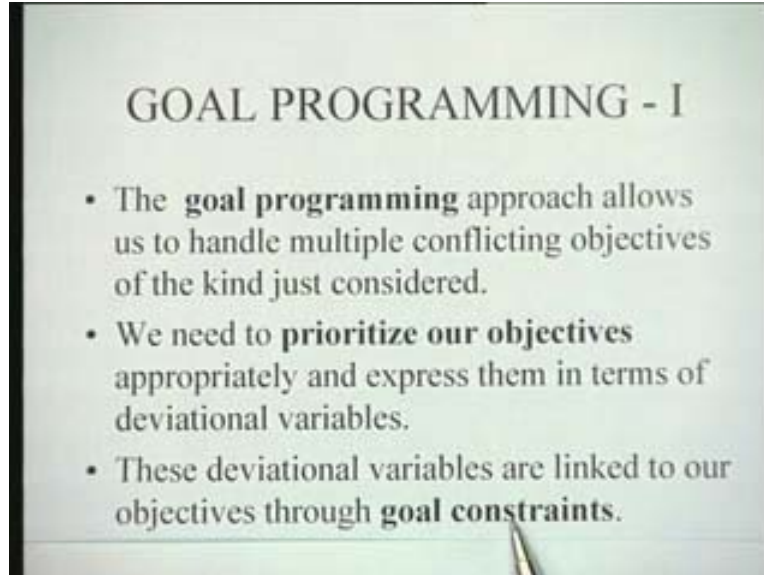


This is again a very important consideration because quality and safety being paramount in certain projects it is important therefore that we make sure in developing the project schedule that these crucial activities are not unnecessarily crashed.

To accommodate this multifarious scheme of objectives what can be done is that a goal programming approach can be used. The goal programming approach as you all know is in fact an extension of linear programming and this extension of linear programming permit us to accommodate a variety of conflicting incommensurate goals. The goal programming approach allows us to handle multiple conflicting objectives of the kind that we just mentioned. Then what we have to essentially do is we have to prioritize our objectives appropriately and express them in terms of deviational variables. Prioritization is a normal process by which managers tend to plan their activities. If they have a number of activities to be done they invariably operate under pressures of limited time and limited resources. Prioritization is in fact very, very important and the goal programming framework basically tries to cash upon that idea of prioritization in developing a practical plan or a schedule for managing the project.

This is one of the key things here and I would like to mention here that the kind of priorities that are used in goal programming are what we call priorities, what is known as a lexicographic priority scheme which means we take the first goal first and we try to achieve whatever we can for the first goal and once we have achieved the best value for the first goal we try to put a constraint that, that value is not worsened and go to the next goal and in so doing we continue this process till we reach the last goal. What really happens is that we are taking up the objectives one by one as it were in a kind of a linear programming framework. We are in fact solving a series of linear programs in this process by shifting our objective functions from the first priority to the next, to the next in that sense and the important thing here is that the deviational variables which are used in goal programming here, these are linked to our objectives through goal constraints.

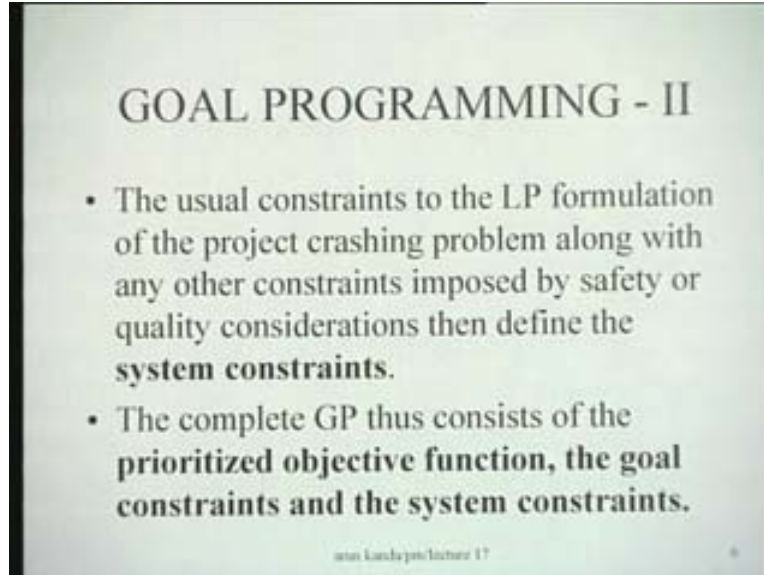
(Refer Slide Time: 12:25)



We define in goal programming a set of additional constraints known as goal constraints whose objective it is to link the targets which we have set with the deviational variables and then this permits us to express our objective function entirely in terms of deviational variables. What this really means is that we set targets and we are trying to measure the deviations from those targets and we operate in terms of those deviations from the targets and we are trying to minimize those deviations essentially. The objective function does not contain any of the original system variables that we had in mind. So goal constraints help us to link up the objectives with the deviational variables.

That's another thing and then the usual constraints to the LP formulation of the project crashing problem along with any other constraints imposed by either safety or quality considerations then define the system constraints. The original set of constraints that we had to the LP, those are nothing but the systems constraints and they are retained as such. They go in the body of the constraints of the goal program and finally complete goal program consists of number one the prioritized objective function. That's the first thing, then the goal constraints which we have introduced and the system constraints which are already there in the typical l p formulation of the problem. This is broadly speaking the philosophy of goal programming in terms of its capability to handle multifarious objectives.

(Refer Slide Time: 14:26)



It would be worthwhile to distinguish between these commonly used terms of goals and objectives. We all have certain goals in life. We are trying to pursue certain objectives but in the context of a vigorous mathematical analysis we should be clear about the distinction between the two terms. What is an objective? An objective generally specifies a general direction to pursue. That's what an objective is. Objective gives me a direction essentially. When you say a linear program has the objective of minimizing cost that is a direction, maximized profit that is a direction. You could be pursuing a certain objective in life and that objective might be to be good or to do good. That's an objective which you are pursuing which is a general direction of pursuit. On the other hand what is a goal? A goal is a specific milestone on this particular road. Goal is a specific target or a milestone on the road defined by the objective. When I say minimized cost, that's an objective. When I say my target is to bring down cost by half to 3 million rupees by the end of the year that's a specific target and that is therefore one particular milestone on this particular road. That's what it is.

(Refer Slide Time: 16:10)



In goal programming what we are doing essentially is we are setting specific targets or looking at specific milestones on this particular road and we are concerned with either the achievement or the non-achievement of those goals. We can set that we want to complete the project in 200 days. This is a specific target and you may or may not achieve it and this whether you achieve it, whether you under achieve it or over achieve it is measured by deviational variables. I think this distinction between a goal and an objective ought to be kept in mind. In order to discuss and present some of the ideas which are involved in goal programming, I will present a case study that was done by us in connection with the INSAT project.

The INSAT, Indian national satellite project was jointly undertaken by the Department of Space and the Ministry of Communications way back in 1980's. This particular project involves a geo-stationary satellite moving in orbit around the Earth at about 36,000 kilometers and this particular geostationary satellite would be able to bring back information to various Earth stations and it was envisaged that there would be a ground segment of 22 such Earth stations on a network for TV, communication and meteorological services. That was the intention. That project is now complete.

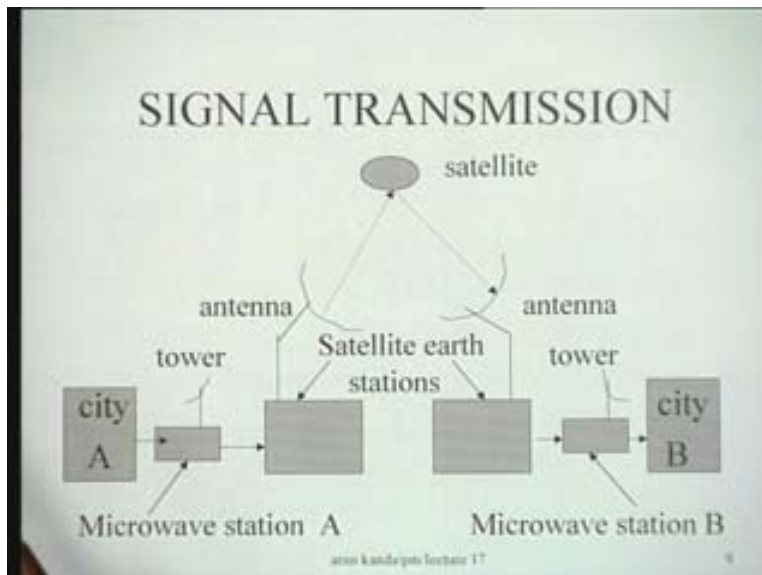
(Refer Slide Time: 18:10)

CASE STUDY

- INSAT project jointly undertaken by Department of Space and Ministry of Communications. (1981)
- Involves a geo-stationary satellite moving in orbit around the earth at about 36,000 km.
- Ground segment of 22 earth stations on a network for TV, communications and meteorological services.

The project that we are talking about is a portion of this entire project in which one of the Earth stations corresponding to these 22 Earth stations located in Calcutta was to be set up. The data for the case study refers to that. Let's see how a signal is communicated. How does communication take place via satellite? There is a city A and we are interested in sending a signal from city A to city B.

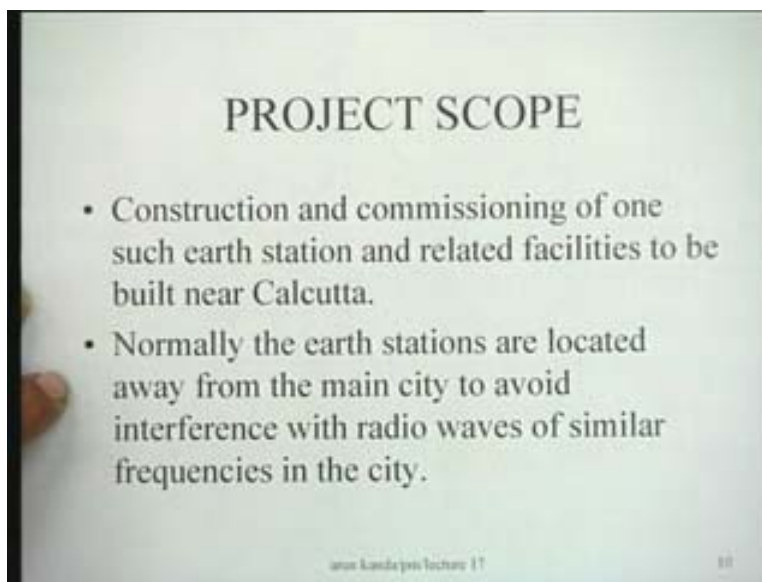
(Refer Slide Time: 18:45)



The various components through which the signal passes are something like this. From the city A, a message is sent to the tower of the microwave station A, which is located within the city generally. From the microwave station of city A, a message is sent to the satellite Earth station. The signal is sent to the satellite Earth station of the city A which is fairly close to the city about 30-40 kilometers from the city, just outside the city and the satellite Earth station has an antennae. This particular signal can be magnified and then sent to the satellite. It is the geostationary satellite and the satellite will then beam the signal back to the satellite to the antennae of the city B and this particular signal for the satellite Earth station will then go back to the microwave station for the city and finally this would then be received in city B. This is the modus operandi.

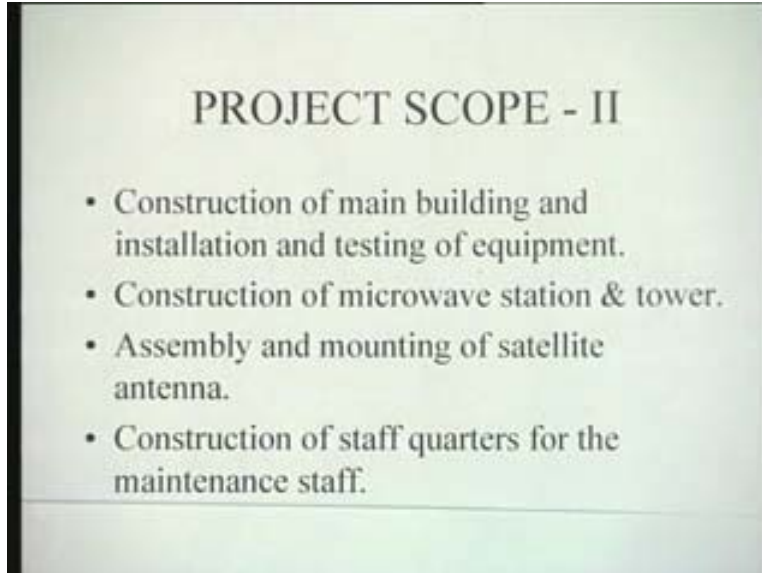
The project that we are talking about was basically the construction of the satellite Earth station, the antennae and the microwave station. That means it was basically this component at one particular city that is in Calcutta; the construction and installation of this particular component. Let's describe the scope of the project.

(Refer Slide Time: 20:21)



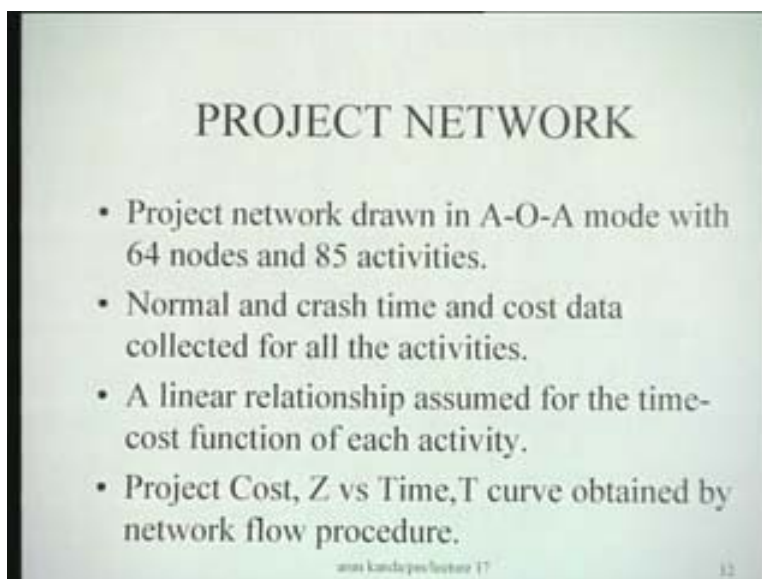
The project was really construction and commissioning of one such Earth station and related facilities to be built near Calcutta. Normally the Earth stations are located away from the main city to avoid interference with the radio waves of similar frequencies in the city. That's the reason why the Earth stations are about 20-30 kilometers outside the city. That's the reason why the Earth stations are about 20-30 kilometers outside the city. For instance if you are talking about Delhi, the Earth stations could be in a place like Gurgaon or it could be something like that. The main activities in the project were construction of the main building and the installation and testing of equipment. This is one of the major scopes of the project. Major activity was construction of the microwave station and tower, the assembly and the mounting of the satellite antenna and the construction of staff quarters for the maintenance staff which is going to stay for the satellite Earth station.

(Refer Slide Time: 21:39)



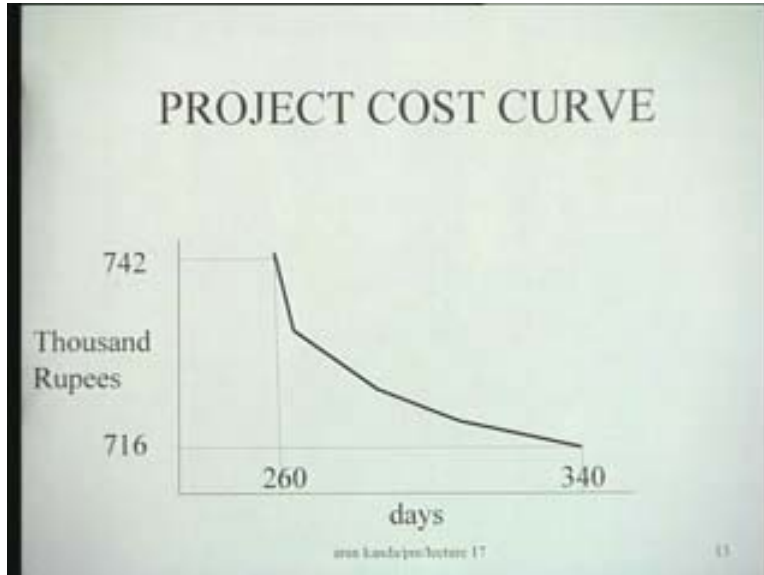
This particular project was actually modeled as a project network and the project network drawn in A-O-A had 64 nodes and 85 activities. You can get an idea that it is a reasonably sized network. What was done was that the normal and crash, time and cost data was collected for all the activities. This was the information that was collected. For each of the 85 activities the normal and crash, time and cost data was collected. A linear relationship was assumed for the time-cost function of each activity and based on this, the project cost versus time curve was obtained by the network flow procedure that we have already talked about.

(Refer Slide Time: 22:41)



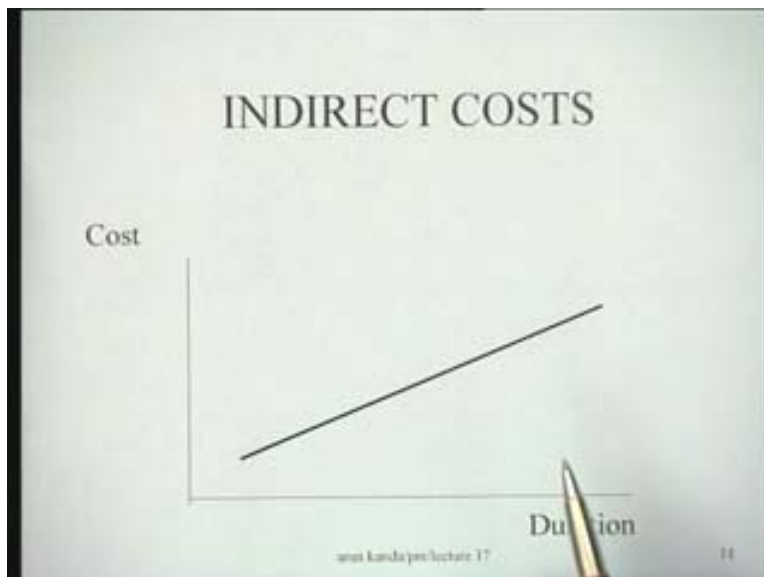
In this particular situation the Z-T frontier, the Z-T efficient frontier looks something like this.

(Refer Slide Time: 22:48)



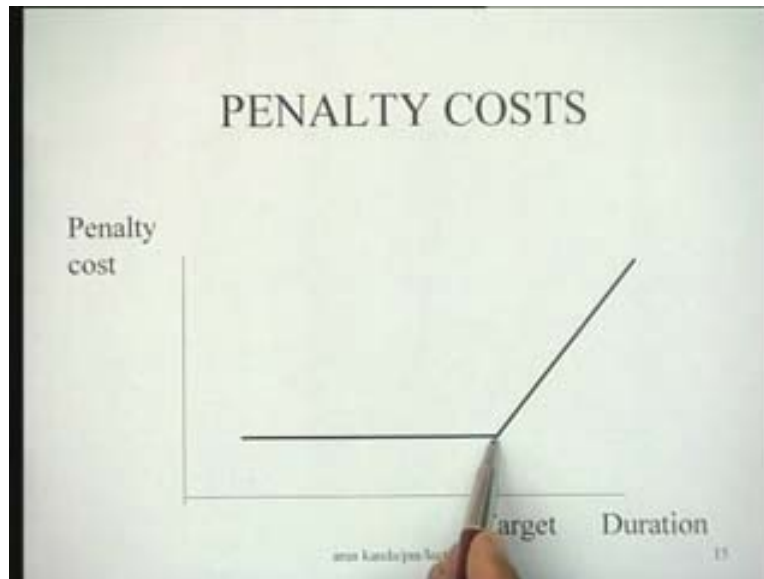
The total direct cost of it would vary between 7.16 lakh to 7.42 lakhs. Mind you I am talking about 1980 prices and the duration of this particular project could vary between 340 working days to 260 days as shown by the project cost curve here. Apart from this the project indirect costs would have a behavior of this nature.

(Refer Slide Time: 23:33)



The cost versus duration, the larger the duration the higher would be the indirect cost and similarly if there were any penalty costs, the penalty cost behavior would be that if this is my target duration or my contractual duration then there is no penalty if you complete the project by this date.

(Refer Slide Time: 24:02)



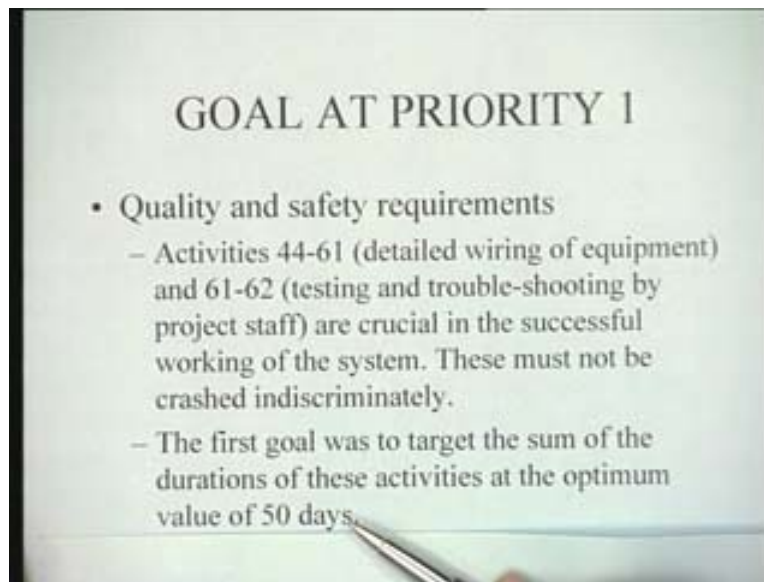
However thereafter the penalty cost tends to increase and normally if you assume a say a penalty cost of let's say 100 or 500 rupees per day delay, the slope of this would be 500. The slope of this line would in fact be negotiated between the two parties before the project is actually accomplished. This is the nature of the penalty cost function. Both the indirect costs and the penalty costs would in fact be superimposed on the direct costs to obtain the total cost function for the project. You could handle the multiplicity of costs including the penalty costs as well as the project indirect costs together in this particular framework right away.

We specify now the broad goals that were there for the management in this particular case and try to develop a project schedule so that all those goals could be accommodated as far as possible. That was the intention. This is the problem setting that we have. As we said in this particular situation, the management thought of 4 different goals. We will discuss those goals now. The top most goal, at priority 1 basically was to do with quality and safety requirements. The management felt that this was the most important goal for them to achieve. The interesting thing here is that the quality and safety requirements of the project have to be ultimately converted in terms of variables which are related to the time and cost of the individual activities or the project. That is the intention.

How this is done is something like this. In fact by quality and safety requirements what the management felt was that there were 2 major activities. Activity 44-61 on the network which was detailed wiring of equipment and activity number 61 and 62 which is testing and trouble shooting by project staff. It was felt that these 2 activities were crucial in the

successful working of the system and much of the success of the project or the successful working of the entire system was dependent upon proper wiring of all the equipment and subsequent testing and trouble shooting by the project staff. These two activities were identified and it was felt that these activities must not be crashed indiscriminately in developing a schedule for quality reasons and also for reasons of safety. In order to translate this into a workable scheme it was felt that the first goal that management had to achieve was to target the sum of the durations of these activities at an optimum value of 50 days.

(Refer Slide Time: 27:39)



It was felt that 50 days was perhaps the optimum time that both these activities should be accomplished in. Though if you want to crash the project this could go down to something like 35 days and in the normal duration you would also have may be something little more than this. But here we are talking of an optimum duration for both these activities which would be best for the performance of the activity. The first goal at the first priority was to ensure that the sum of the durations of these two activities is at an optimum value of 50 days. This was taken as the first priority goal.

Similarly let's see what the other goals were. The goal at the second priority was basically a goal to stay within the budget which is quite a natural goal for management. That means there was a specified budget for performing these activities and the objective was to minimize the upper deviation of the direct activity cost from the specified budget. That was the intention. For instance in this case the total direct cost for the project, if you are assuming linear cost-time tradeoffs, will be $b_{ij} - a_{ij} x_{ij}$ for all ij belonging to the set of arcs where you know that this is the cost intercept and this is the cost slope. a_{ij} is the cost slope and x_{ij} is the duration of that particular activity ij which lies between the normal and the crash limit. This we already know from our previous analysis of this. We want that this total direct cost should not exceed the budget which was specified and the budget was 7 lakh 25,000 for the various activities.

The value of the summation b_{ij} for all the activities was computed and this value was found out to be 11,77,253 as shown here and therefore the goal value for summation $a_{ij} x_{ij}$ would be nothing but summation b_{ij} which is this particular value minus the total direct cost. This value summation b_{ij} , which was worked out to be this for all the activities in the project minus the total direct cost budget which is **seven hundred or lakhs and twenty five thousand**. The goal value for this that is $a_{ij} x_{ij}$ summation for all the activities worked out to be this particular value. The point that is to be understood here is that we are now translating a goal pertaining to the budget which is 7,25,000 into a goal for some of the other variables which we are going to work with. We had specified the goal value for this to be something like 4,52,253. If we want to stay within the budget of 7, 25, 000 what are we trying to show? If we exceed this value here your cost is going to be lower because this is a negative sign here. The higher the value of this the lower will be the cost. It's as simple as that and therefore this is in fact a lower bound on the value of $a_{ij} x_{ij}$. That's what we are trying to say. It can be higher than this but it should not be lower than this because if it's lower than this the total cost will be more than 7, 25, 000. Is that clear? Although we want to restrict the cost to this budget of 7, 25,000 here what it implies is that if this is not to exceed this, this must be less than equal to this particular value. This is the goal at the second priority which is essentially staying within the budget goal which is a major goal for all projects.

(Refer Slide Time: 33:04)

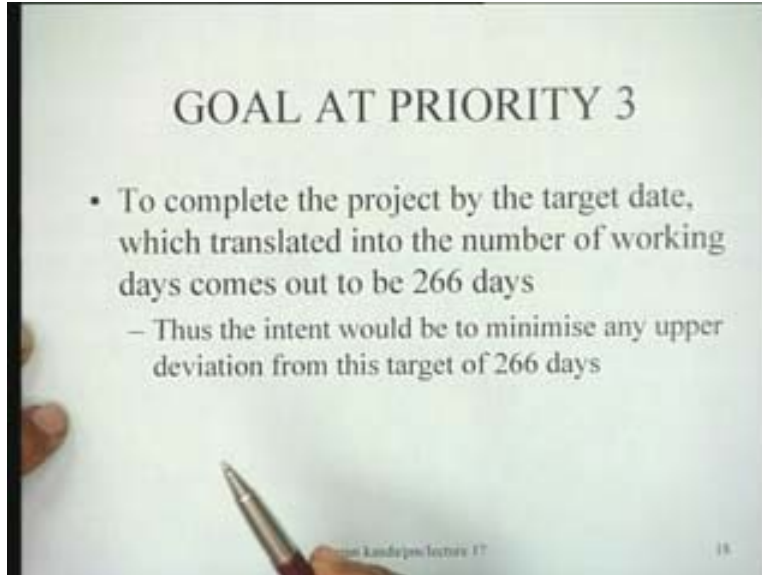
GOAL AT PRIORITY 2

- To minimize the upper deviation of the direct activity cost from a specified budget.
 - Total direct cost = $\sum (b_{ij} - a_{ij}x_{ij}) \quad (ij) \in A$
 - Budget specified = Rs 7,25,000
 - Value of $\sum b_{ij} =$ Rs 1177253
 - Goal value for $\sum a_{ij}x_{ij} = \sum b_{ij} -$ Total direct cost budget = Rs 452253

aran kanda/ppt/lecture 17 17

The goals that we are talking about are generic in nature. Almost all projects would have these kinds of goals irrespective of whatever the setting is. The goal at the third priority was again to complete the project by the target date and the target date which when translated into the number of working days comes out to 266 days. This was the target date for the project. Thus the intention was to minimize any upper deviation from this target of 266 days.

(Refer Slide Time: 33:43)

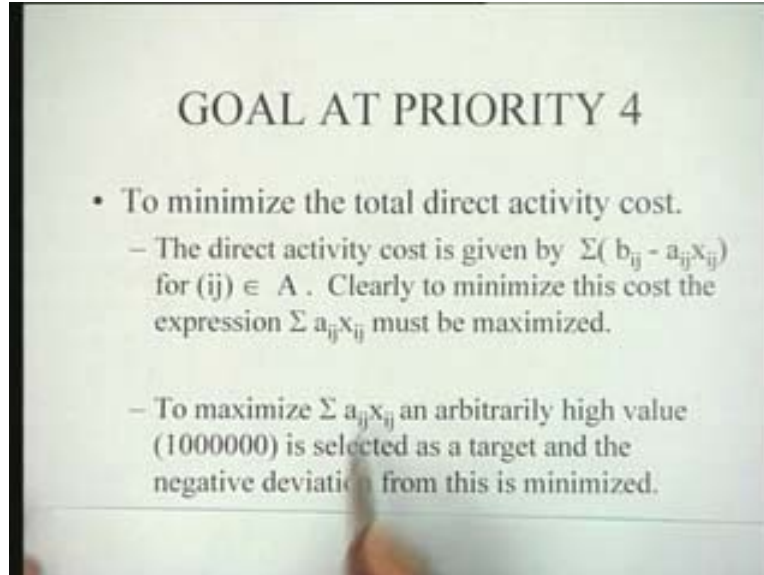


We don't want to minimize any upper deviation. We want to minimize any upper deviation from this target. We want to be below 266 days. If you recall I had shown you the Z-T frontier. The range of project durations was something from 350 to something like 260. 266 was very much within the feasible range of durations and this was the target date for the project.

The goal at the fourth priority was to minimize the total direct activity cost. You might tend to think here that this is probably in conflict with the goal at priority 2 where we were trying to stay within the budget and the budget was 7,25,000 but that was just staying within the budget and fourth priority is to minimize the total direct activity cost. Again the total direct activity cost is given by this linear function which is b_{ij} minus $a_{ij} x_{ij}$ for all (ij) belonging to the set of arcs and clearly to minimize this cost this expression must be maximized; that's what we had said that this expression must be maximized. In fact what we had said at priority two was a lower bound on this particular function.

You are saying that this must be maximized. How do we maximize this particular function? Because everything in goal programming has to be in terms of deviational variables from targets we can set an arbitrarily high target for this value and then try to minimize the underachievement from that target. That's how you will maximize it. That is precisely how we do it. To maximize this quantity an arbitrarily high value of say 10 raised to the power 6 is selected. You can find out what is the maximum value that this function can take? You know the slopes and you know the x_{ij} 's and the maximum x_{ij} value can be u_{ij} 's. That's the normal duration. You can calculate what this higher value is? Take any value which is higher than that. It's like taking infinity but you have to choose the infinity appropriately. In this case this value was much higher than the value that you had.

(Refer Slide Time: 36:25)



So 10 raised to the power 6 is selected as a target and the negative deviation from this is to be minimized. **because minimizing the negative deviation from a target is equivalent to** If I set a very high, unachievable target and I try to achieve it I am trying to maximize my own achievement. That's the idea. Because these goals were specified by management in the order of priority that we have just discussed these goals are now to be translated into goal constraints.

How do you do that? Let's look at these goal constraints for instance. Goal constraints are pretty simple. Let's look at the first goal. In the first goal we had 2 activities. This was detailed wiring and trouble shooting. In the activity network this was the 39th activity and this was the 41st activity, just a numbering scheme. We had a corresponding variable for this and a corresponding variable for this and what we want to ensure is that the sum of the durations of these two activities should be equal to 50. That's what we are trying to say. The right hand side is 50 days. But this sum can be anything. It can be less or it can be more. If it's less we have s_1 minus which is a negative deviational variable and s_1 plus which is a positive deviational variable. In goal programming you always have the negative and the positive deviational variables and what we are trying to simply say is that if for instance this sum is equal to 40, this particular variable will be 10 and this will be zero. That means I am underachieving 50 by 10.

However if this particular sum is greater than 50, suppose it's 55. Then what it would mean is that this is 55-5. So s_1 plus would be 5 and this would be zero. This is an over achievement and this is an under achievement. Both of the variables cannot be positive at the same time. Only one of them will be positive at one particular time or if the goal is exactly achieved both will be zero. The structure of these goal constraints is like this. This is the variable of interest. This is my target and these are the deviations from the target. This is the underachievement and this is the overachievement from the target. The solution will then give me values for these two variables which will specify exactly how

much I have underachieved or over achieved that particular goal and the formulation allows for both the variables because you do not know whether you are going to underachieve or over achieve the particular thing.

This is the first goal constraint corresponding to the first goal at the first priority. In fact you have one goal constraint corresponding to each goal. We had four goals. So we will have 4 goal constraints. We will just generate those constraints. The second constraint was we had worked out that summation $a_{ij} x_{ij}$. Summation a_{ij} plus again the deviational variable, the negative and the positive deviational variable would be equal to the target value which we had calculated for this particular thing. This is that target value and we would like this particular thing to be as high as possible to minimize the cost. But it should at least be equal to this value. That's what we want if we want to stay within the budget of the direct cost. That was the intention.

Similarly look at the third goal constraint. The third goal constraint was the project duration. t_{149} here refers to the node time of the last node. That's all, that's the indication. t_{149} minus t_1 is the project duration and we want this to be equal to 266. Since we want this to be 266 we allow for negative and positive deviational variables here corresponding to the third goal which is s_3 minus and s_3 plus and we are able to get this particular goal value here. This is 266 days here. Then finally the fourth goal; what is the fourth goal? Fourth goal was minimization of the direct cost. Again we had this particular thing. In order to minimize the direct cost we are interested in maximizing this.

(Refer Slide Time: 41:56)

GOAL CONSTRAINTS

$$x_{39} + x_{41} + s_1^- - s_1^+ = 50$$

$$\sum a_{ij} x_{ij} + s_2^- - s_2^+ = 452253$$

$$t_{149} - t_1 + s_3^- - s_3^+ = 266$$

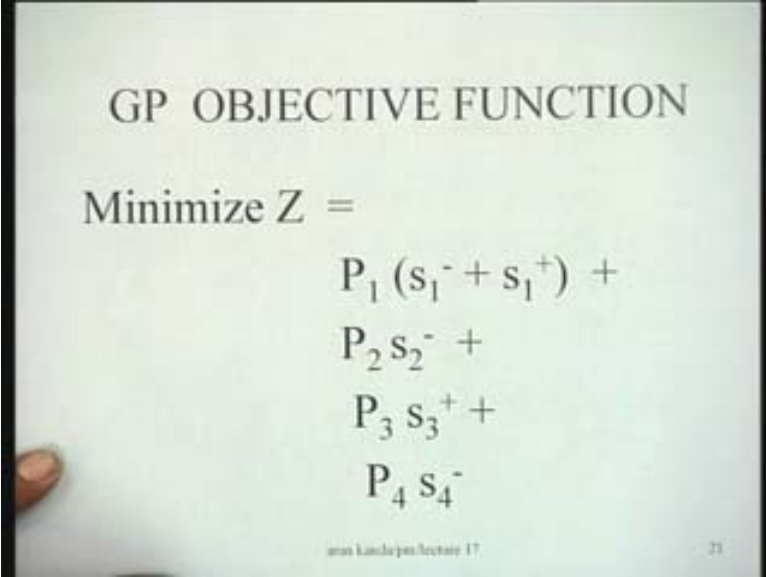
$$\sum a_{ij} x_{ij} + s_4^- - s_4^+ = 1000000$$

How do we maximize this? We set a target which is very high, 10 raised to power 6 and what are we going to achieve? We will strive to achieve or to strive to minimize the negative deviation that is s_4 minus for this particular goal but the goal constraints are nothing but accounting procedures. It's like saying that this is my achievement, this is my target. I could either under achieve or over achieve it. I can measure exactly what my

under achievement or over achievement is likely to be? What we do is we set up the goal constraints in this particular fashion.

Let's try to construct the goal programming objective function. The goal programming objective function is sort of lexicographic and it's sort of non pre-emptive as they say. A non pre-emptive lexicographic objective function means that you have the first priority level, then the second priority level, then the third priority level and then the fourth priority level and what we try to do is at each priority level we try to define what is to be minimized?

(Refer Slide Time: 43:20)



GP OBJECTIVE FUNCTION

Minimize $Z =$

$$P_1 (s_1^- + s_1^+) +$$
$$P_2 s_2^- +$$
$$P_3 s_3^+ +$$
$$P_4 s_4^-$$

aman kanchan/lecture 17 21

For instance if you look at the first priority level, our objective was to ensure that the sum of the activity durations was exactly equal to the optimum level which was 50. I put here s_1 minus plus s_1 plus both because when this is zero, as well as this is zero this is the theoretically minimum value for this particular objective and that would correspond to my desired objective of keeping the sum of the durations of the 2 crucial activities to a level of 50 days. That's why I have kept both the variables here. There could be only 3 things that you can want in optimization problem. You either want exact achievement of an objective or you want to under achieve it or you want to over achieve it. You could either be trying to minimize the upper deviation or you could try to minimize the lower deviation or you could be trying to minimize the sum of the deviations of both. These are the only three things which can possibly take place and we have instances of all the three in this example. In the first priority goal we are trying to ensure that the total sum of the durations of the two activities comes out to equal to 50. Here we have s_1 minus plus s_1 plus, both of them coming into play.

Look at the second goal. What was the second goal? The second goal was trying to stay within the budget. Staying within the budget means trying to exceed or this target of 4,52,253 for summation $a_{ij} x_{ij}$. You were trying to actually exceed this value. Exceeding

this value is like trying to say you are minimizing the under deviation from this. You don't want any under deviation from this. That's why we have here s_2 minus. At the second priority we have minimization of s_2 minus as the objective. This is in fact one of the major things in goal programming. How to define your prioritized objective functions? At the third goal what were you trying to do? You were trying to make sure that 266 which is the target duration preferably should not be exceeded; preferably should not be exceeded means the over deviation should be minimized. s_3 plus preferably should be zero otherwise it should be minimized. Here you are minimizing the upper deviation. Here you are minimizing the under deviation. Here you are minimizing the sum of both of them and similarly when you look at the fourth objective, fourth objective was minimization of the total direct cost which was equivalent to maximization of $a_{ij} x_{ij}$. Maximization was actually equivalent to minimizing the under deviation from a very high, arbitrarily high target. Again we have a negative deviational variable which is to be minimized. This completes the identification or the development of the total goal programming objective function.

Notice that the objective function is entirely in terms of deviational variables. No problem variables are there in the objective function at all. These are deviational variables corresponding to first goal, corresponding to second goal, corresponding to third goal and corresponding to the fourth goal. We have to develop in a goal programming situation this kind of an objective function totally. Once this is attained our next job is in fact to complete the goal program and look for a solution. As I just told you the goal programming objective function, along with the goal constraints has now to be appended to the usual system constraints that we have. Which are the typical system constraints for this problem? The system constraints as you know for a typical project crashing problem are these are the usual 3A+1 constraints appended in the LP formulation for the crashing problem. What are these constraints? These constraints are shown here.

(Refer Slide Time: 48:20)

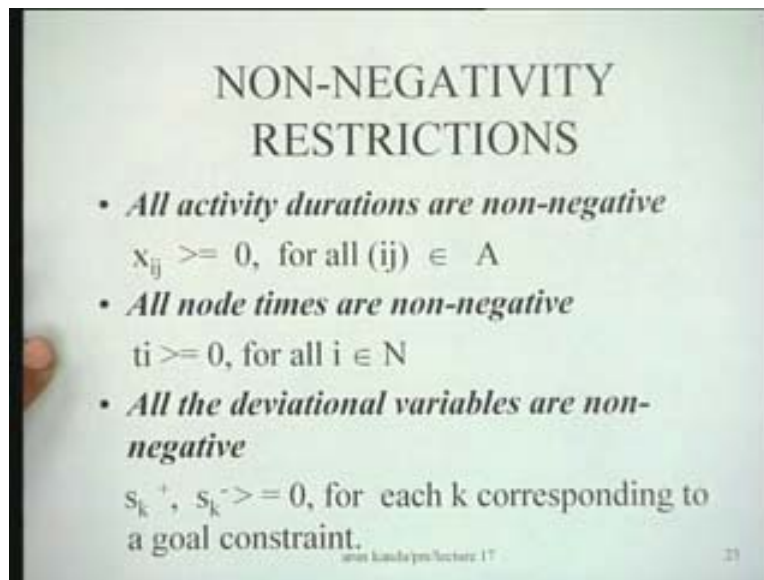
SYSTEM CONSTRAINTS

- These are the usual 3A+1 constraints appended in the LP formulation for the crashing problem.
- $t_i - t_j + x_{ij} \leq 0, (ij) \in A$
- $x_{ij} \leq u_{ij}, (ij) \in A$
- $x_{ij} \geq l_{ij}, (ij) \in A$
- $t_n - t_1 = T, \text{ where } T \text{ lies in } (T_C, T_N)$

These are precedence constraints. t_i minus t_j plus x_{ij} must be less than equal to zero for all the arcs. We have this constraint. Then you have x_{ij} must be less than equal to the normal duration for each arc. Then you have that x_{ij} must be greater than or equal to l_{ij} which is the lower or the crash duration for each arc and then there is a constraint saying that t_n minus t_1 is equal to t where t lies in this particular interval. You impose this constraint by fixing a specific value for t . You say that the project duration lies between the crash duration and the normal duration. This is a parametric program but if you are going to have a duration goal or a budget goal then this constraint need not be appended in the systems constraints in the goal program. If you have a goal constraint which says that the project duration should be 266 days as we say here, there is a constraint there. This need not be imposed in this particular case.

Then we have the non-negativity restrictions. What are the non-negativity restrictions?

(Refer Slide Time: 49:40)

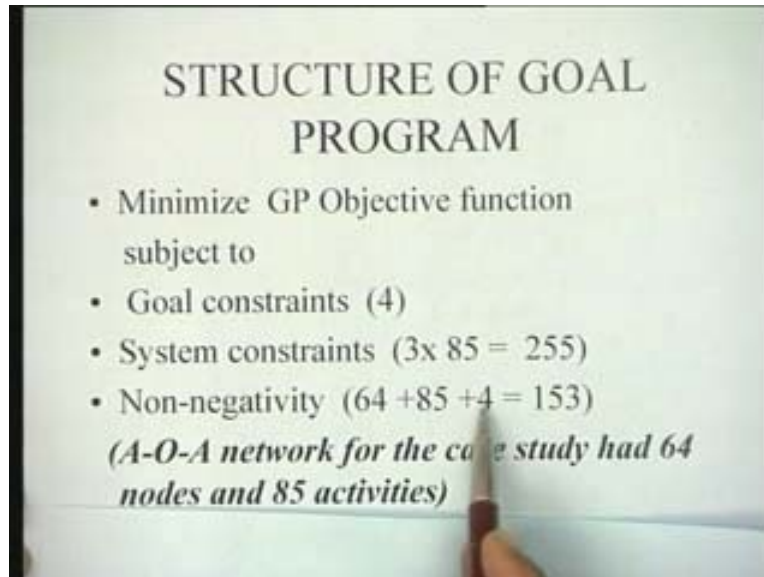


The non-negativity restrictions for the problem are that all activity durations are non-negative. That is x_{ij} should be greater than equal to zero for all (ij) belonging to A . All node times are non-negative. This we know. All the deviational variables, there are 8 deviational variables 2 corresponding to each goal. s_k plus and s_k minus are both non-negative for each k belonging to a goal constraint. This completes actually all the set of problems that we have for defining the various types of non-negative.

The structure of the goal program, the complete problem which we are discussing is to minimize the goal programming objective function which we have developed in terms of the 4 priorities subject to: number one goal constraints. There are 4 goal constraints in this particular case, one corresponding to each goal. Those we have developed in detail. Then apart from this there are system constraints. Ignoring t_n minus t_1 is equal to t because there are 85 activities there are 255 constraints corresponding to the system here. Similarly there are the usual non-negativity restrictions on $64+85+4$; that is 153 variables

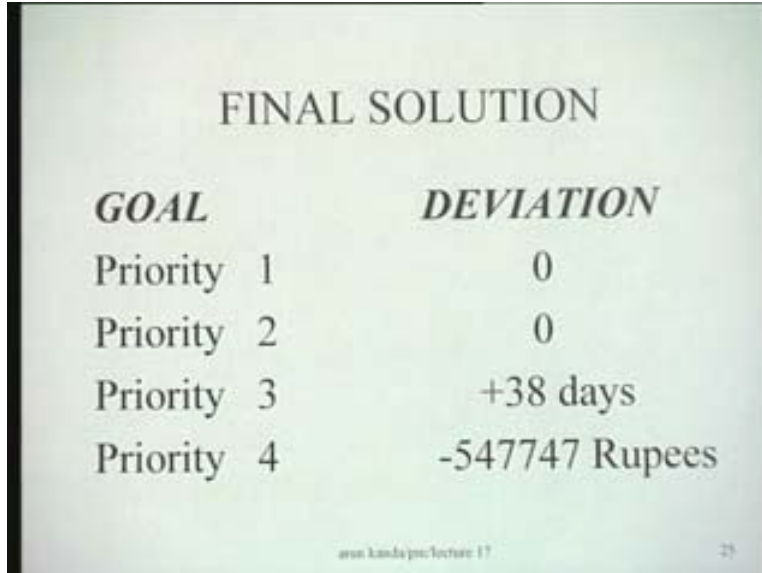
for this problem. This is the complete structure of the problem and remember that the A-O-A network for this case study had 64 nodes, 85 activities and 4 goals.

(Refer Slide Time: 51:19)



The total number of variables was 153 for this particular case. You are already familiar with system constraints non-negativity which is used in the LP formulation. The only additional constraints that we have appended are the goal constraints which are 4 goal constraints and the objective function for this particular problem. This particular goal program with this structure was solved for the case study that we just presented. The final solution was something like this. The final solution was that the goal at the first priority the deviational variable was zero. Goal at the second priority also the deviation was zero. So the goal at the first priority and the second priority were fully met. What was the goal at the first priority? Making sure that the sum of the durations of the two activities must be equal to 50. This was attained. The first goal without any constraints is easy to The second goal was staying within the budget. This was also met fully. The third goal was staying within the time constraint of 266 days. This was over achieved by 38 days. That means it was not possible to do the project in the target duration of 266 days. It took 38 days more for this particular solution to take place and the fourth priority, the negative value was this. In fact this negative score only shows the under deviation from the arbitrarily high value of 10 raised to power 6 values that we had set for the direct cost. The direct cost was so much less than 10 raised to power 6. That is the interpretation of fourth priority.

(Refer Slide Time: 53:06)



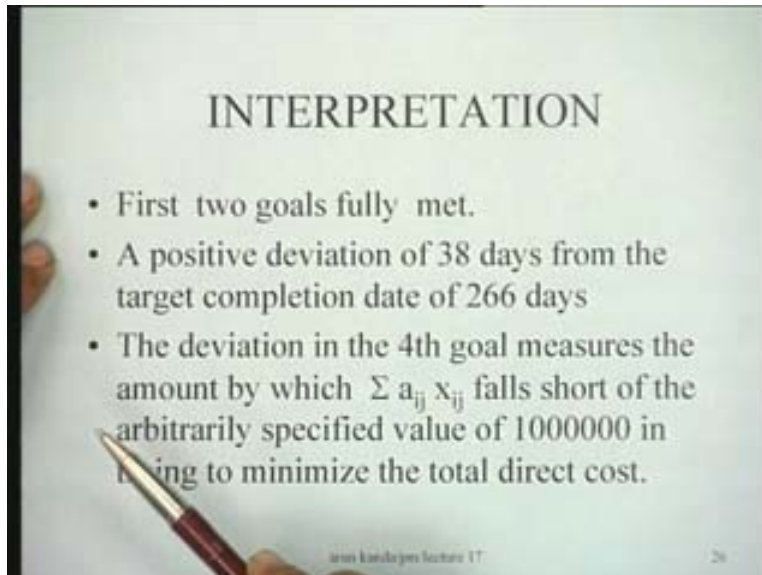
A slide titled "FINAL SOLUTION" with a table showing goal deviations. The table has two columns: "GOAL" and "DEVIATION".

<i>GOAL</i>	<i>DEVIATION</i>
Priority 1	0
Priority 2	0
Priority 3	+38 days
Priority 4	-547747 Rupees

Small text at the bottom: arun kanda jee lecture 17 25

Let's quickly look at the interpretations. The first two goals are fully met in the solution; a positive deviation of 38 days from the target completion date of 266 days.

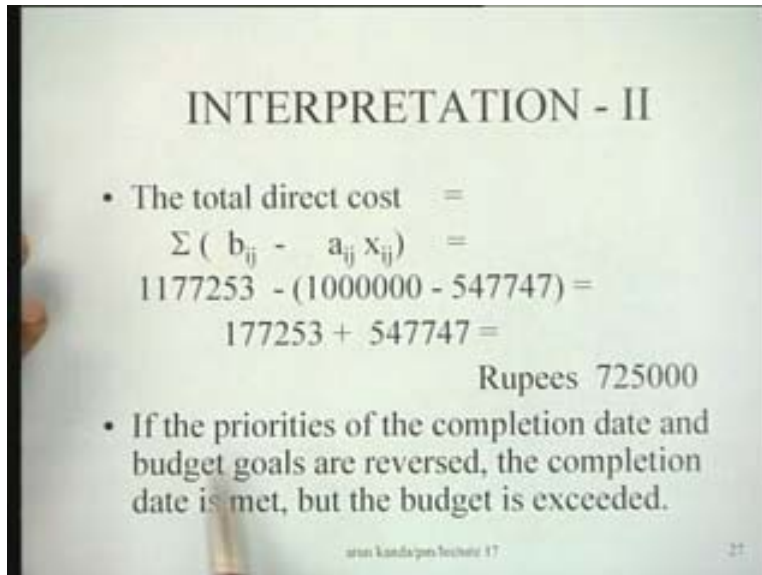
(Refer Slide Time: 53:19)

- 
- A slide titled "INTERPRETATION" with three bullet points. A hand holding a pen is visible on the left side of the slide.
- First two goals fully met.
 - A positive deviation of 38 days from the target completion date of 266 days
 - The deviation in the 4th goal measures the amount by which $\sum a_{ij} x_{ij}$ falls short of the arbitrarily specified value of 1000000 in trying to minimize the total direct cost.
- Small text at the bottom: arun kanda jee lecture 17 26

The deviation in the fourth goal measures the amount by which summation $a_{ij} x_{ij}$ falls short of the arbitrarily specified value of 10 to power 6 in trying to minimize the total direct cost. This was what we had seen. The interpretation could be that we could calculate the total direct cost which is this quantity, which is actually this value; summation of this value is known to be this and this is falling short of this by so much.

This was the value that you got. You know this particular value. You know for $a_{ij} x_{ij}$. The total direct cost was actually equal to 7,25,000. That's what it was exactly.

(Refer Slide Time: 54:01)



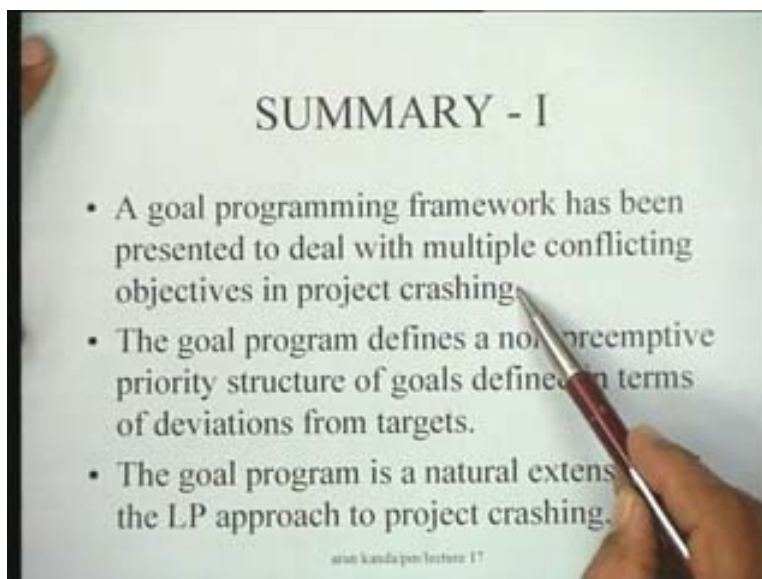
INTERPRETATION - II

- The total direct cost =
 $\Sigma (b_{ij} - a_{ij} x_{ij}) =$
 $1177253 - (1000000 - 547747) =$
 $177253 + 547747 =$
Rupees 725000
- If the priorities of the completion date and budget goals are reversed, the completion date is met, but the budget is exceeded.

aran kanda/pes/lecture 17 27

If the priorities of the completion date and the budget goals were reversed the completion date is met but the budget is exceeded. You can see very clearly that in this case we are exceeding the target duration by 38 days but confining ourselves to within the budget. When we reverse these priorities the reverse thing keeps happening. Finally we can summarize our approach by saying that the goal programming framework has been presented to deal with multiple conflicting objectives in project crashing.

(Refer Slide Time: 54:40)



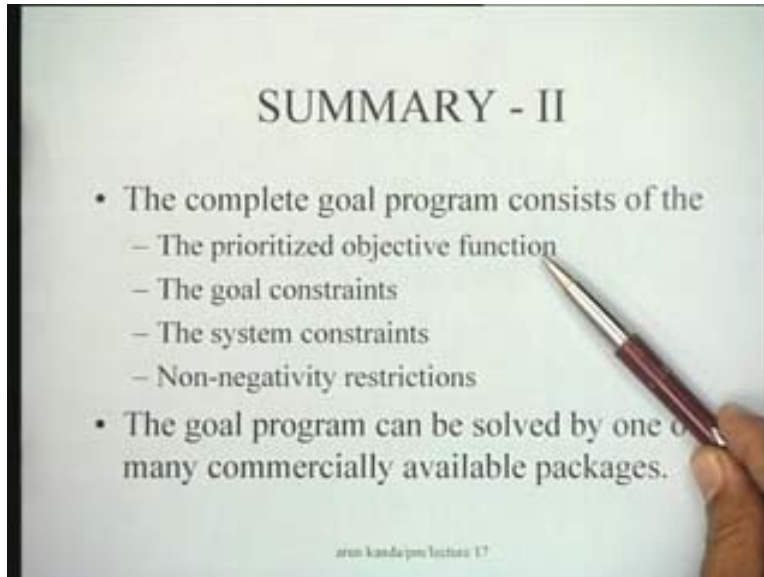
SUMMARY - I

- A goal programming framework has been presented to deal with multiple conflicting objectives in project crashing.
- The goal program defines a non-preemptive priority structure of goals defined in terms of deviations from targets.
- The goal program is a natural extension of the LP approach to project crashing.

aran kanda/pes/lecture 17

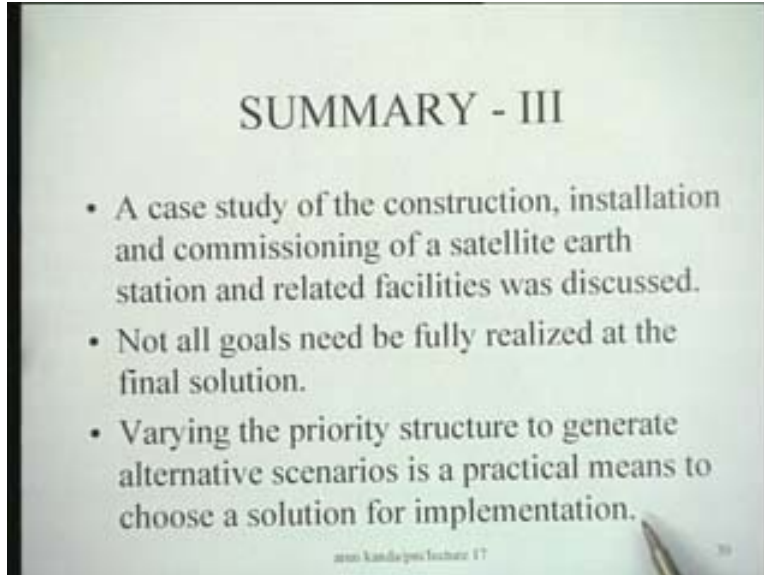
The goal program defines a non-pre-emptive priority structure of goals defined in terms of deviations from the targets. The goal program is a natural extension of the LP approach to project crashing. That is one thing we have seen. The goal program consists of the following: it consists of a prioritized objective function.

(Refer Slide Time: 55:01)



It consists of the goal constraints. It consists of the system constraints and it consists of the non-negativity restrictions and the goal program can be solved by any one of the many commercially available computer packages for solution. So solution is really not a problem. It is trying to formulate your problem in the goal programming framework which is the major thing. A case study of the construction, installation and commissioning of a satellite Earth station and related facilities was discussed. Not all goals need be fully realized at the final solution. This is **enough** and varying the priority structure to generate alternative scenarios is a practical means to choose a solution for implementation.

(Refer Slide Time: 55:46)



One can really do a kind of a sensitivity analysis by varying the priority. Just as I indicated that if you interchange the cost and the duration goals, the duration can be met but the cost could be exceeded and then you can see whether you are really in a position to spend additional money for achieving the duration or not. These are therefore very practical means for dealing with multiple conflicting objectives. Thank you!