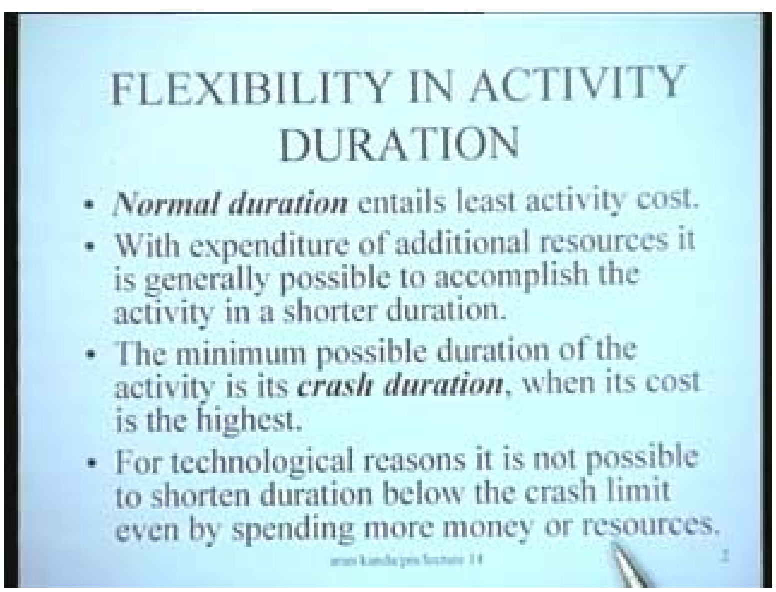
Project and Production Management
Prof. Arun Kanda
Department of Mechanical Engineering
Indian Institute of Technology, Delhi

Lecture - 12
Linear Time-Cost Tradeoffs in Projects: A Heuristic Approach

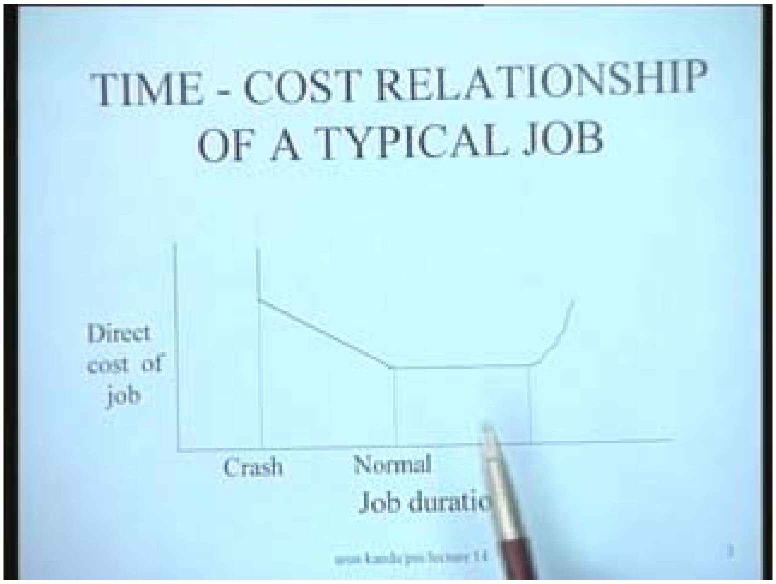
So far we have been talking primarily about time scheduling in projects and in time scheduling we have looked at apart from basic scheduling, aspects of simulation and other things. But apart from time the next major important parameter in a project is its cost. Today we will talk about this important aspect of time-cost tradeoffs in projects and to begin with we are going to be talking about linear time-cost tradeoffs in projects. To introduce the subject we must appreciate that different activities have a certain flexibility in their durations and this flexibility to a very large extent is in the duration of the activity which means that there isn't always a fixed duration in which the activity can be carried out. Depending upon the urgency, depending upon the amount of resource that you are willing to invest the duration of the activity can vary. We always define the normal cost. The normal cost and the normal duration is that duration which entails the least activity cost and then what we say is with expenditure of additional resources it's generally possible to accomplish the activity in a shorter duration. This is quite logical and we have all experienced this. The minimum possible duration of the activity is its crash duration. That's the definition and that is the duration when its cost is the highest and for technological reasons we say it's not possible to shorten the duration below the crash limit even by spending more money resources.

(Refer Slide Time: 3:04)



There is a limit to which an activity can be crashed. You cannot reduce the activity duration to zero because there are some physical limitations and you cannot keep on reducing the duration of the activity. What we are essentially trying to say is that a typical time-cost relationship of a job would be something like this.

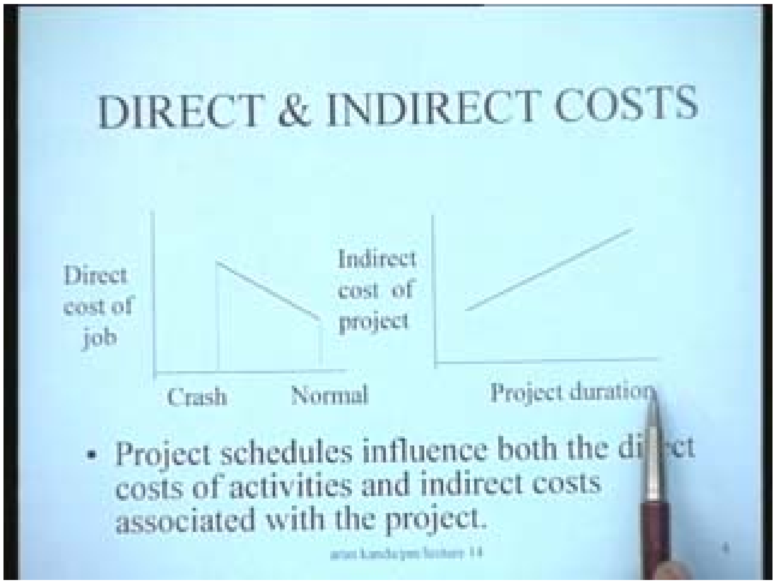
(Refer Slide Time: 3:30)



You would normally have a normal duration for an activity and this would be the minimum cost of the job which is attainable which is what we call the normal duration. If you want to reduce the duration below the normal duration you will probably have to spend more money on it and you can keep on reducing the duration up to a certain point which we call the crash duration and beyond the crash duration even if we spend more money on it, this line is vertical. That means it's not going to reduce the duration of the activity in anyway. However it's quite likely that the activity may take duration longer than the normal duration. I mean if you are pretty …… about it, you can keep on doing this but this would be so. However if you delay it unnecessarily the costs tend to rise; that's it.

What we are seeing is that out of this, it's uneconomical to operate in this range up to here because we can get a lower duration at the same cost by operating here. For all practical purposes, the operating part of the time-cost tradeoff curve for a job is this one which lies between the normal and the crash durations. We will assume that every activity can be performed at duration between the normal and the crash duration and there can be a variety of relationships. It's not necessarily true that the relationship between the activity and its duration be linear. It could be other kinds of relationships. This cost that we were talking about for the activity is a direct cost of the job. The direct cost behaves in this fashion between the normal and the crash limit you have this particular variation of the cost of the job. However there is an indirect cost of the project as a whole and this cost is increasing with project duration.

(Refer Slide Time: 5:47)



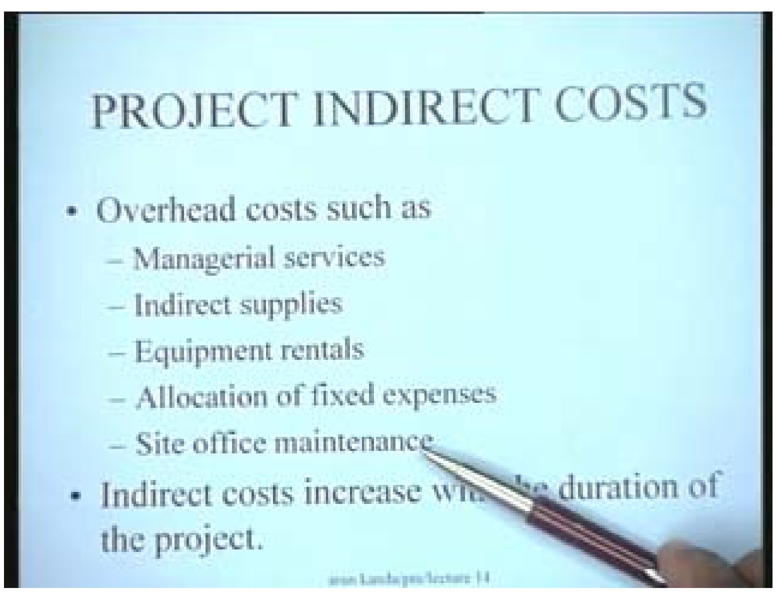
What we have seen is that the project schedules influence both the direct cost of the activities and the indirect costs associated with the project and what we are in fact seeking in a time-cost tradeoff exercise is may be the best compromise or the best way where the total cost is going to be a minimum. We look at the previous graph for instance. We have said here that this is the best way or the best manner in which the activity is accomplished between the normal and the crash duration. It's quite likely that if we delay the job for some time the cost is not going to change but thereafter there are going to be penalties of delay and the delay could be the material you bought goes bad. The costs keep on increasing. If you keep on delaying the activity beyond a certain limit the cost of performing that activity because of the costs associated with the delay, the deterioration of the parts that you have obtained and various other things there could a price escalation and things of that kind. All these things mean that if you are delaying it beyond a certain level it becomes uneconomical to do the activity. That's the idea.

Coming to this distinction between direct and indirect costs, direct costs are actually activity related costs and the indirect costs are actually costs associated with co-ordination and control of the project as a whole but let us see what are the typical components that build up the activity direct costs? The activity direct costs are those costs which are associated with the performance of the specific activity and specifically these costs will include the cost of planning and design of that activity, the raw material procurement for that activity, the labor costs for that activity, in manufacturing or the processing costs involved in that activity, the travel, the communication and the transportation of both men and goods involved in that particular activity, the fees of any consultant that you hire to give you either technical advice or some other advice and those kinds of fees. All these costs are costs which are directly attributable to the activity itself and you could for purposes of accounting trace them to the individual activity.

(Refer Slide Time: 8:38)

Let's look at what we mean by the project indirect costs? The project indirect costs are overhead costs such as managerial services which would be common. For instance a manager would be responsible for the co-ordination of not just one activity but all the activities in the project. His wages come from this pool of indirect costs. Indirect supplies which are needed here, all the stationery which goes into the project. The electrical utilities, the power and the water supplies and all these kinds of things would be part of the indirect costs; also equipment rentals, allocation of fixed expenses and site office maintenance.

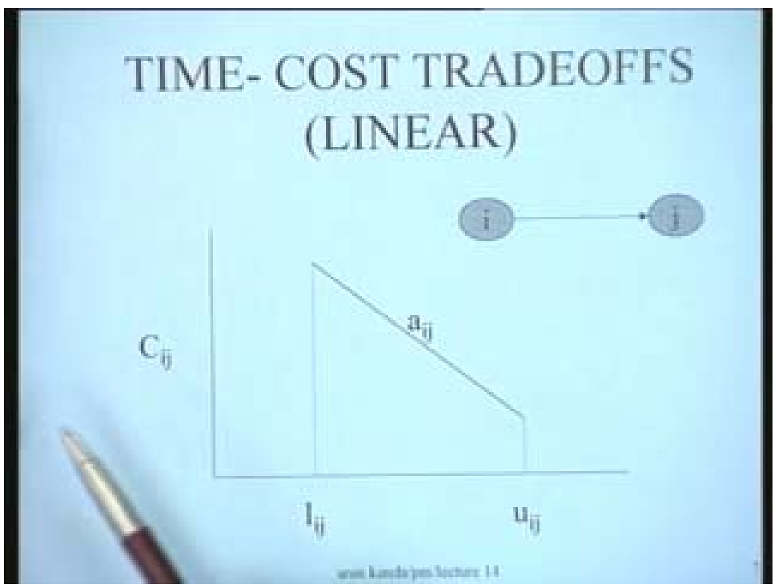
(Refer Slide Time: 9:29)



As long as the project is in progress you would have to maintain a site office and the cost associated with that site office are dependent upon factors like how long the project is in operation because you will have a per day cost of operating the site office and therefore you would have these kinds of indirect costs. The important thing to notice is all the indirect costs increase with the duration of the project. The longer the duration of the project more you have to maintain the site offices. There is a per day cost associated with that.

Now we are going to be dealing with, as I said, primarily in this particular lecture with linear time-cost tradeoffs. For an activity let's say (i j) in the A-O-A framework, the cost of the activity is c_{ij} and it has an upper duration u_{ij} which is the normal duration and it has a lower duration l_{ij} which is the crash duration and the magnitude of the slope of this is a_{ij} although slope is actually negative for this particular activity.

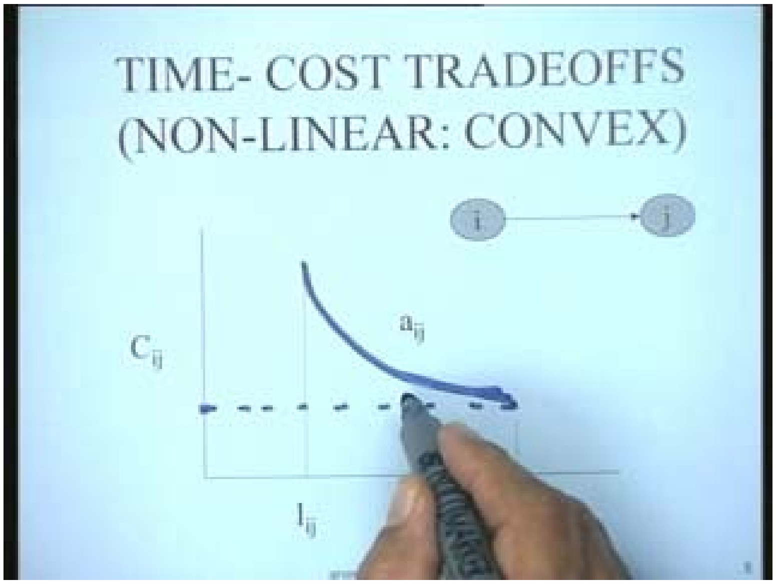
(Refer Slide Time: 10:44)



Linear time-cost tradeoffs would mean that the cost time relationship for the activity is of this nature.

However there could be other types of tradeoffs. You could have a non-linear time-cost tradeoff. In non linear time-cost tradeoff of this nature what does it imply? It implies for instance that let's first we are talking about a convex function. Convex function means that in the beginning this is the normal duration and the normal cost, the minimal cost of performing the activity. The minimum cost of performing the activity is somewhere here. This is the minimum cost of performing the activity. As you go along this curve, this particular curve here exhibits this kind of behavior. Initially the cost slope is less. Progressively as the duration is decreased the cost slope becomes higher. The cost slope is nothing but the cost in rupees per unit reduction of the duration of the activity. The implication is that in the initial phases it's less costly to crash the activity.

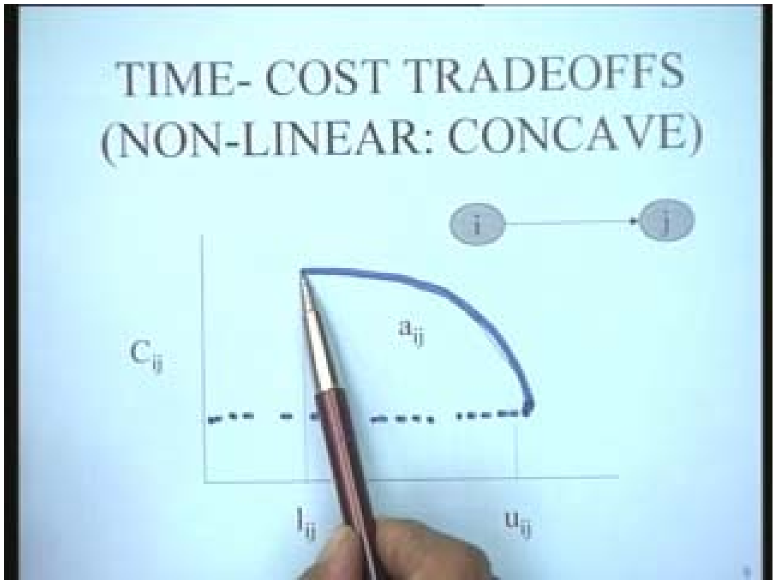
(Refer Slide Time: 12:08)



As the activity is crashed more and more, it becomes increasingly more costly or more difficult to crash the activity. This kind of a behavior is found in most activities which you want to crash. For instance if some particular activity takes 15 days trying to reduce its duration to 14 days might be easy. People would not even notice what's happening. If you reduce it to may be 12 days people would start noticing and if you reduce it further down to 10 days some people might even react violently to what you are trying to do. There would be increased resistance to reduction in the duration of the activity as you keep on compressing the activity. This to some extent tries to model the typical human response that is there to accommodating any changes or any reductions in a particular activity. This would be the kind of thing that would happen here.

On the contrary let's see what would happen if we have a non-linear time-cost tradeoff, a non-linear time-cost tradeoff which is concave. A concave function is actually the reverse of the convex function. It would look something like this. In this particular situation what we are seeing is if this is the minimum duration, the slope here is the highest and as you progressively go down reduce the duration, the slope becomes smaller and smaller. But the slope is the maximum at this particular point of time. The implication really is that initially to reduce the duration is difficult but once the duration gets reduced slightly people become more and more sort of addicted to that reduction and they to want more and more of that reduction. till whatever.

(Refer Slide Time: 14:12)

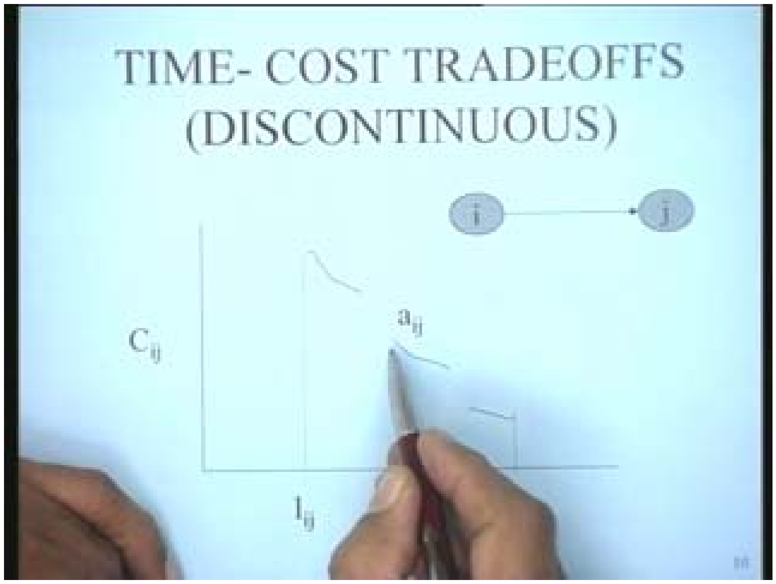


This is very much like the introduction of computers in banking for instance. Initially there was reluctance and all the employees of the banks went on a strike and they said we don't want computerization. It will throw There was an initial reluctance but then gradually what happened as computers were introduced more and more learnt the use of computers. They said yes; we want more of it, we want more of it. They sort of were sucking in more computers and ultimately that's the kind of thing. This is like a typical response that you have in human beings of a resistance or reluctance to change where this is high and then subsequently as the technology, the new technology becomes more and more familiar you have easier and easier absorption and you have lower and lower deviation. That's the kind of interpretation that you could give to this.

To give you a simile from strength of materials, may be, you know this is very much like the plastic flow of metals. The amount of stress required initially till the yield point is very high but once the yield point takes place then the material just tends to flow and the cost required to deformat is smaller and smaller. This is that kind of a behavior that you find here in time-cost tradeoff for many activities. For all administrative activities this would be a kind of a situation where there would be a reluctance to change by people and you can model this as a non-linear time-cost tradeoff problem.

However there could be time-cost tradeoffs which are discontinuous. They could be up to here and then up to here and then up to here. This is an example of a discontinuous time-cost tradeoff. When do you think this can happen? This can happen for instance when you have the same activity and this activity could be done by 3 alternative technologies. Suppose this particular activity is tilling a plot of land. It could be done manually. The cost would be low but it could be done may be in 1 month and may be if these people work very hard they could do it in about 25 days. There is range in this technology. Next we go to a higher level of technology. May be you start using bullock carts. So you have reduced the duration and you have a certain range.

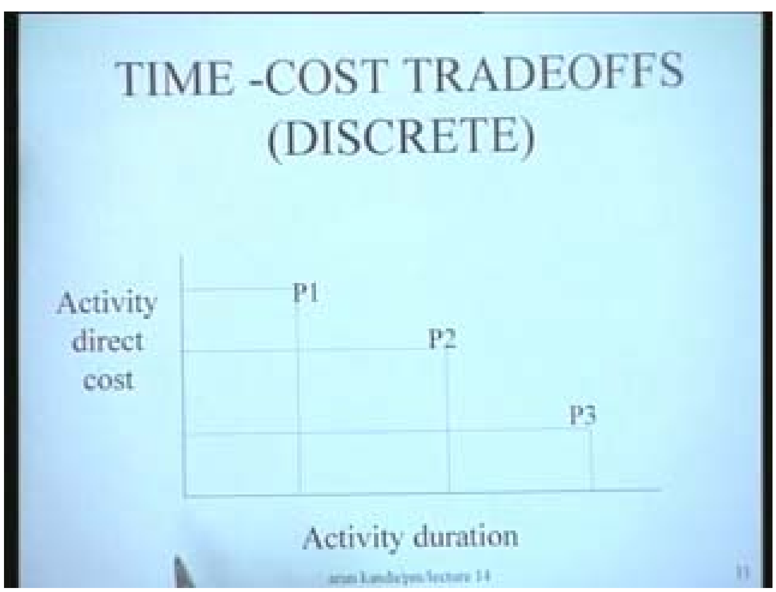
(Refer Slide Time: 17:14)



The next stage of technology would be to start using a tractor. When you use a tractor your duration is going to be much smaller and there also depending upon the usage of the tractor you could be having a range of durations. This kind of typical discontinuous time-cost trade off can be utilized when you have different choices, multiple choices as you have in this case. It's like changing of technology from one to the second to the third and you have this kind of change over.

Similarly the time-cost tradeoffs may be discrete. By discrete time-cost tradeoff we mean one point, another point and another point.

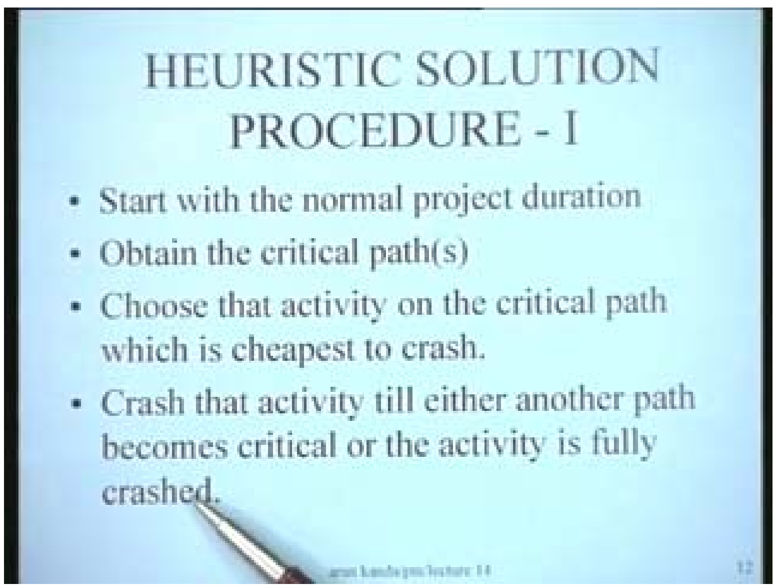
(Refer Slide Time: 18:03)



Suppose the activity is to transport goods from Calcutta to Delhi. This is one activity in the project network. How could it be done? This could be done by rail which is one option. If it's done by a rail the duration is going to be let's say 15 days or whatever it is and the cost is going to be so much. The second option possibly could be I could bring it by my own truck. Time would be probably less and it will be higher cost and the third possibility again could be to air lift them. The duration will be less and the cost is higher. Even in this particular situation discrete points would represent alternative technological choices which you can exercise to perform that particular activity.

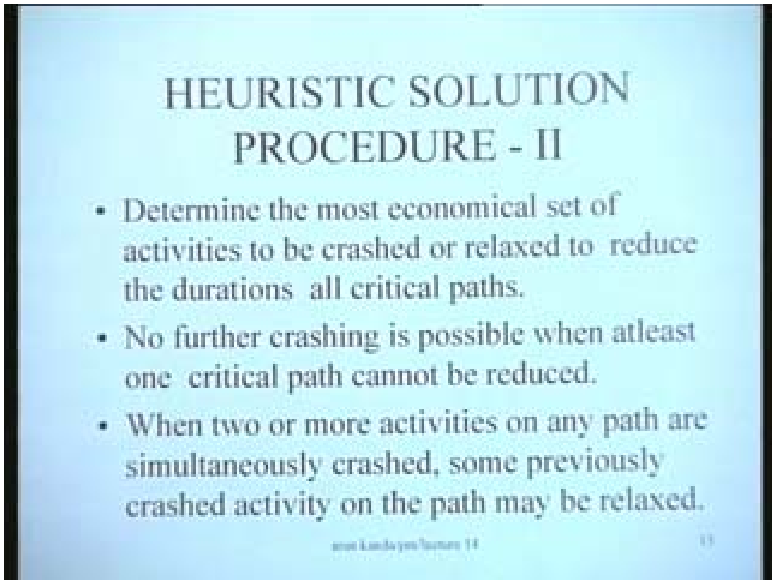
There are therefore a variety of options in time-cost tradeoffs. To begin with we look at only the linear case and subsequently see how these other cases can be dealt with. We will today look at a heuristic solution procedure in which the broad aim is to start with the normal project duration, obtain the critical paths, choose the activity on the critical path which is cheapest to crash and then crash that activity till either another path becomes critical or the activity is fully crashed.

(Refer Slide Time: 19:25)



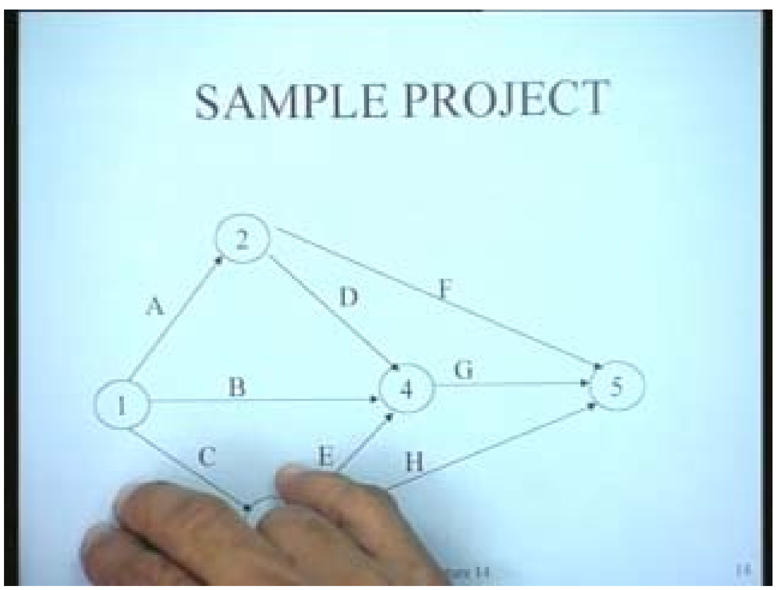
This is the kind of behavior and then we would determine the most economical set of activities to be crashed or relaxed to reduce the durations of all the critical paths and then no further crashing is possible when at least one critical path cannot be reduced. That would be our termination criterion and when two or more activities on any path are simultaneously crashed some previously crashed activity on the path may be relaxed. That is another feature that we have seen.

(Refer Slide Time: 20:06)



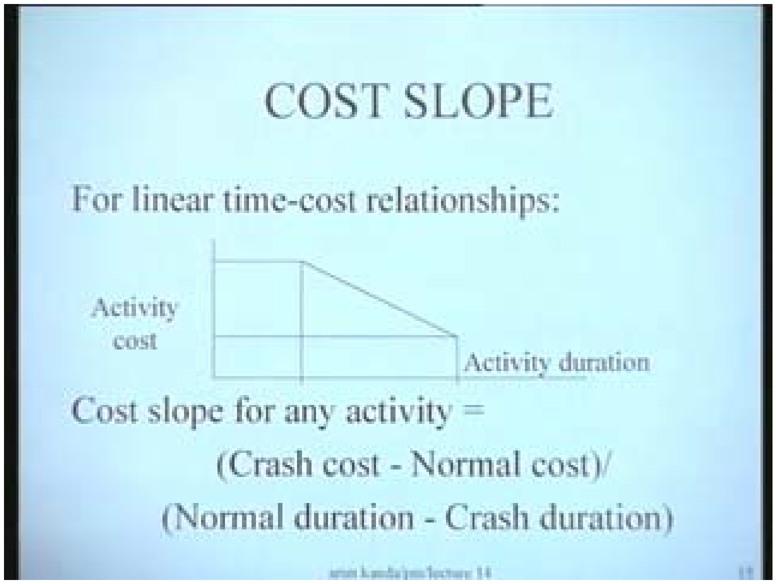
Let us try to use these intuitive concepts in solving a problem. Let this be our sample project. This project contains activities A, B, C, D, E, F, G and H and this is the project network for this particular example that we are talking about.

(Refer Slide Time: 20:27)



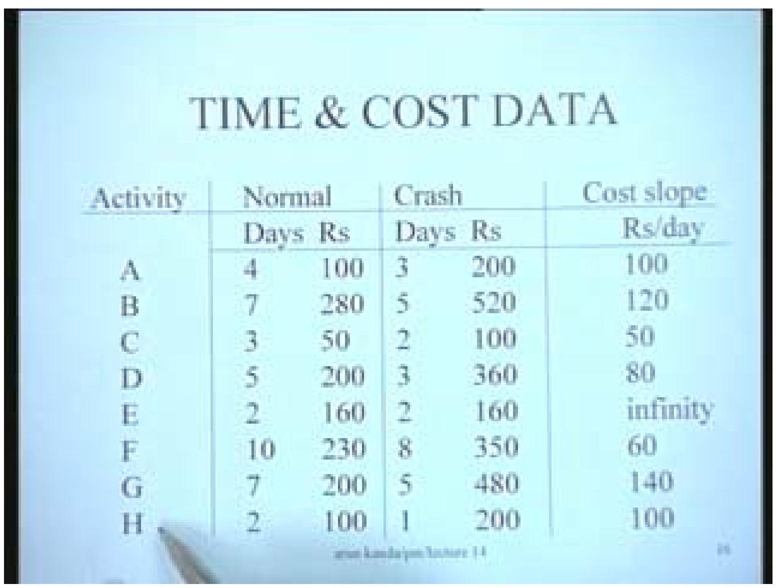
Let us now give the data for this problem. Before giving the data we just like to specify how the cost slope is computed. For linear time-cost tradeoffs you have the activity cost and the activity time on this side. Cost slope for any activity is nothing but the crash cost. It's actually this upon this. It's the crash cost minus the normal cost which is this distance divided by the normal duration minus the crash duration. It's as simple as that.

(Refer Slide Time: 21:03)



The time-cost data for the example is as shown here. These are the activities.

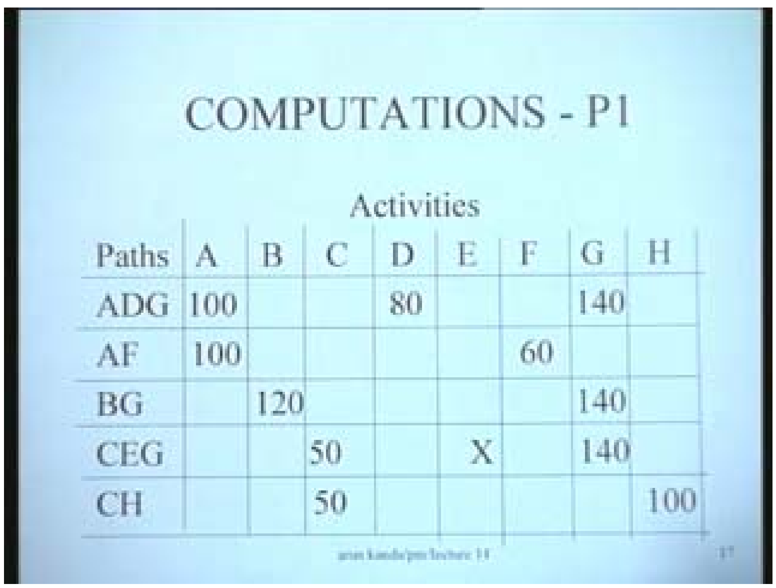
(Refer Slide Time: 21:15)



The normal duration in days and the normal cost in rupees is given here and the crash duration in days and the crash cost in rupees is given here and from here the cost slope can be computed which as we have seen is simply 200-100 divided by 4-3 which is 100 rupees per day. We have in this manner computed the cost slope for each of the various activities in this project which go from A to H and we have the normal duration, the crash duration and the various cost slopes.

Now let’s see how exactly we could organize our computations. What we will do is we have this particular project network. Basically we have to keep track of all the paths. Which are the various paths? AF is a path, ADG is a path, BG is a path, CEG is a path and CH is a path. These are the various paths in the project, all the paths. You can easily identify them. We look at all the paths and make a matrix like this which says that these are the paths. ADG, AF, BG, CEG and CH and these are the activities on the path and what I have written down here are the cost slopes of the various activities. For instance ADG involves A which has a cost slope of 100, D which has a cost slope of 80 and G which has a cost slope of 140 and similarly AF, BG and CEG. E is an activity which cannot be crashed. So I put a cross there or you can put an infinity there if you like. This is an activity which cannot be crashed at all. In order to begin the computations we know the durations of all the activities at their normal duration.

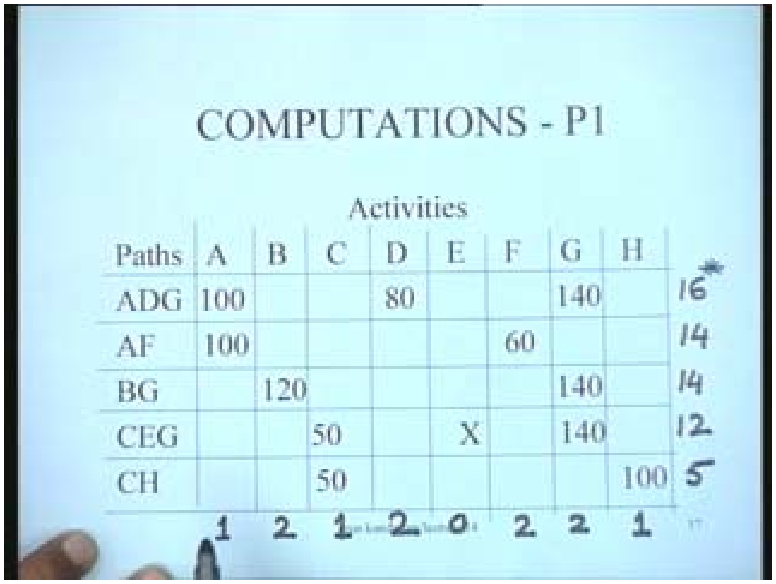
(Refer Slide Time: 23:29)



Paths	A	B	C	D	E	F	G	H
ADG	100			80			140	
AF	100					60		
BG		120					140	
CEG			50		X		140	
CH			50					100

At the normal durations for instance activity A is 4 days. Activity D is 5 days and activity G is 7 days. That would mean 4+5 is 9, 9+7 is 16. 16 would be the length of this particular path. Let me write it there. Similarly for the path AF the length is going to be 14 days. So this is the length of this path. The length of this particular path BG is going to be 14 days. Length of CEG is 3, E is 2. That makes 5 and g is 7. That makes 12. The length of this path is 12 days and CH has a path length of 3+2 which is just 5 days. I note down the current durations of all the paths in this particular way. I can also note down for instance that activity A that we had seen can be crashed from 4 days. It has a normal duration of 4 days and crash duration of 3 days. It can be crashed by 1 day. B can be crashed by 2 days. C can be crashed by 1 day. D can be crashed by 2 days. E cannot be crashed at all. F can be crashed by 2 days and G can be crashed by 2 days and H can be crashed by 1 day. To begin with these 5 paths in the project, at all normal starting point has these durations. This is the longest path. This particular path I put an asterisk on it. The critical path now is ADG.

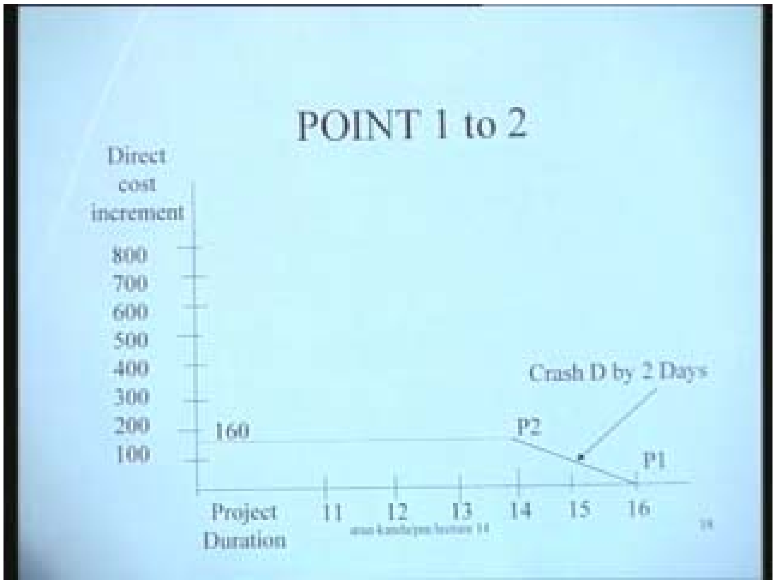
(Refer Slide Time: 25:36)



The other paths are smaller. We find out what is the cheapest way to crash or bring down the duration of this particular path. We look at this row and we find that D is the smallest value here. D can be crashed by 2 days and that will reduce the duration of this path from 16 to 14 and that is the cheapest because other ways of reducing the duration of the path are more costly. That was the logic for putting the cost …. here. These durations are nothing but the durations of these activities. They are not crash durations. These are the normal durations of the jobs. These are nothing but the amounts by which the activities can be crashed. For instance activity A has a normal duration of 4 days and crash duration of 3 days. It can be crashed by 1 day. This is 1 and this is the amount by which the activity can be crashed. We want to keep track of this information because subsequently as we keep on crashing activities we would like to know how much of each activity has been crashed. It's like a bank balance. I must know how much I have in the bank. If I use some money what is left? I am going to that accounting process.

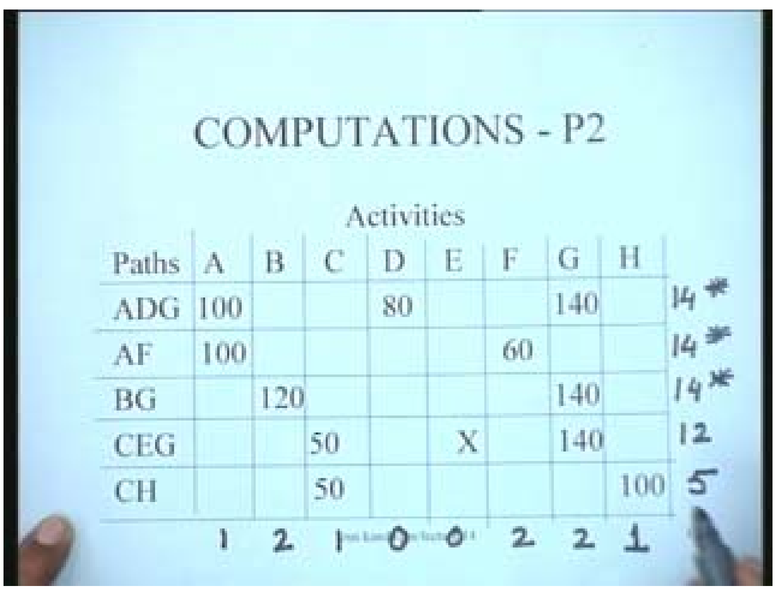
We have identified what we can do at this particular stage. We have said that the normal project duration is 16 which is the point P1 which is where we work. If we crash D by 2 days the project duration will reduce to 14 days and we will move from the point P1 to P2 and since D has a cost slope of 80 rupees per day, the cost increase will be 160 rupees because we have reduced by 2 days, so 80 into 2. The basic increase in cost is 160 rupees above the datum level of all normal durations. This is what it is.

(Refer Slide Time: 27:59)



Having done that we want to find out what we are going to do now? Now we have moved from point 1 to point 2. Let us try to see what is the status at point 2? Let us update our status. Since D was crashed this path is 14 days. This path is 14 days, this path is 14 days, this path is 12 days, this is 5 days and since D was crashed, it will not be possible to crash it further. The amount of availability of D is now 0. It could be crashed by 2. It has been crashed by 2 whereas the others remain the same. So 1, 2, 1 and then this is 0 and this is 2, 2, 1. This is the kind of information that we have at this particular stage. The interesting thing now is that originally there was only one path which was critical. Now all these 3 paths have become critical.

(Refer Slide Time: 29:06)



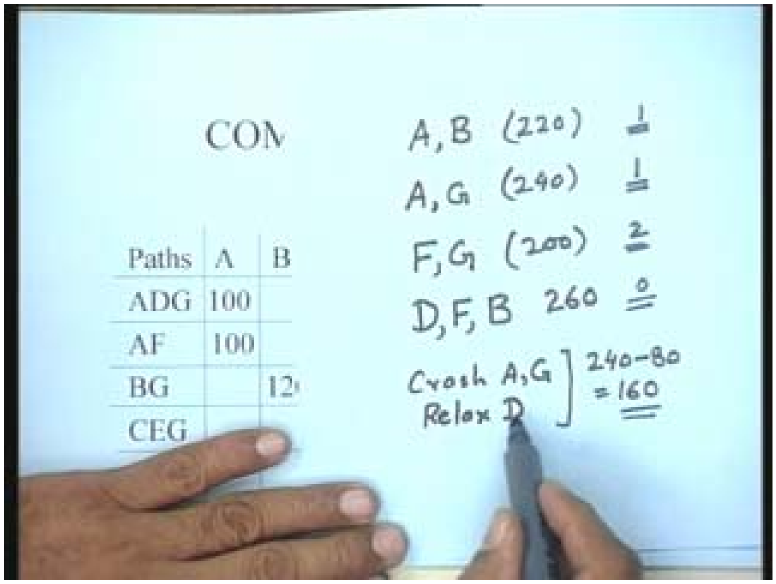
Paths	A	B	C	D	E	F	G	H	
ADG	100			80			140		14*
AF	100					60			14*
BG		120					140		14*
CEG			50		X		140		12
CH			50					100	5
	1	2	1	0	0	2	2	1	

Let us now try to use our heuristic reasoning to find out because if we want to reduce the duration of the path we must be able to reduce the durations of all these 3 paths together. Let us try to find out what are the various ways in which we can reduce the durations of all the paths. At this stage we have the top 3 paths to reduce. If we reduce A both the first and the second path will be reduced. One option that we have at our disposal is to reduce A and reduce B at the same time. The other option that we can have is reduce A and reduce G because A will reduce both these paths and this path will have to be reduced. This is the first option; A with G is another option. But which are the other options that are possible? One other option that could be there is you could reduce G here and G will reduce both these paths. So you can have F and G. F and G will reduce all these paths. Is there anything else that we can do? Yes, there could have been other options. For instance you could have reduced D, reduced F and reduced B. You could have done that and this would have reduced the durations of all the paths.

Let's now find out what would be the cost of doing this. A has a cost slope of 100 and B has a cost slope of 120. A and B option is going to cost you 220 rupees per day. Look at the A and G option. It is going to be 240 rupees per day. F and G option is going to be 200 rupees per day and BDF option 120+80 is 200; 200 and 60 is 260. Notice another thing that this would be from the point of view of cost. We have to find out whether this is possible. For instance A can be reduced by 1 day, B can be reduced by 2 days. This option can be done for only 1 day the minimum of the two; A and G again by only 1 day. This would be possible to do only by 1 day. What about F and G? If you look at F and G this option is available for 2 days and DF and B, D cannot be crashed at all. This option is actually not available to us. We cannot do anything here at this stage. On the face of it, it would appear to us that the cheapest option available, what is the cheapest option available? F and G which is 200 rupees per day. But a little reflection will show that this is in fact not the optimal solution.

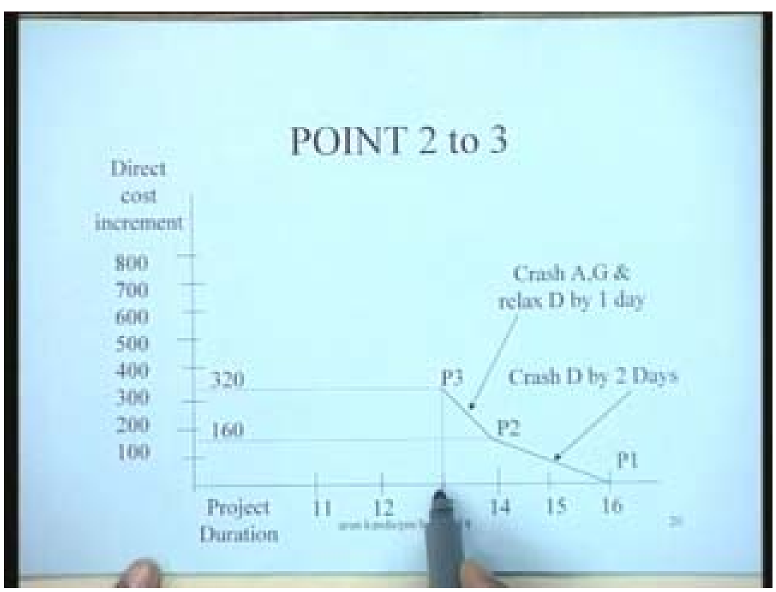
The optimal solution in this particular situation would be if you look at this problem a little more closely you see that if I crash A and G this path is reduced. A is crashed, G is crashed. This path will be reduced by 2 days. This path will be reduced only by 1 day. This path will be reduced by 1 day with A and G. What can be done is that D which was previously crashed can in fact be relaxed because we don't want to reduce it by 2 days. We want to reduce it only by 1 day. If I for instance say that crash A and G by 1 day and relax D by 1 day, relaxing would mean 240-80 which is equal to 160. This option would be possible for 1 day and we could exercise this option for 1 day only because AG can be exercised for 1 day although this can be relaxed by 2 days.

(Refer Slide Time: 34:01)



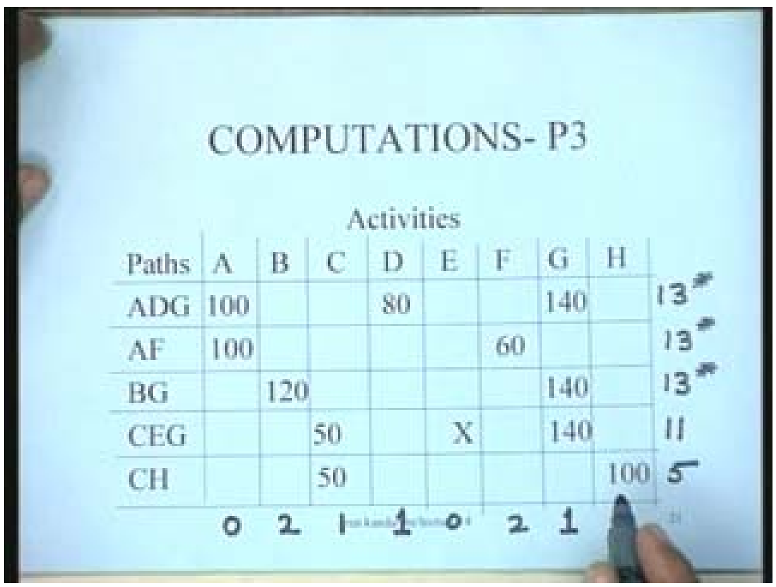
The cheapest option is crash AG. This is the cheapest option. This is where one has to realize that identifying the optimal cost option is really not a simple thing even for a very simple trivial network like the kind we are trying to explore in this particular example. At this particular stage we have said that the best thing to do would be to crash A and G and relax D which will cost us 160 rupees a day and if we do that let's see what happens. We can for instance see now that in this particular situation we would in fact be moving from point 2 to point 3 in the curve that we are generating. We had reached from P1 to P2 by crashing D by 2 days. Now from P2 to P3 we are moving by crashing A and G and relaxing D by 1 day and this cost slope is 160 rupees. This is only 1 day reduction.

(Refer Slide Time: 35:13)



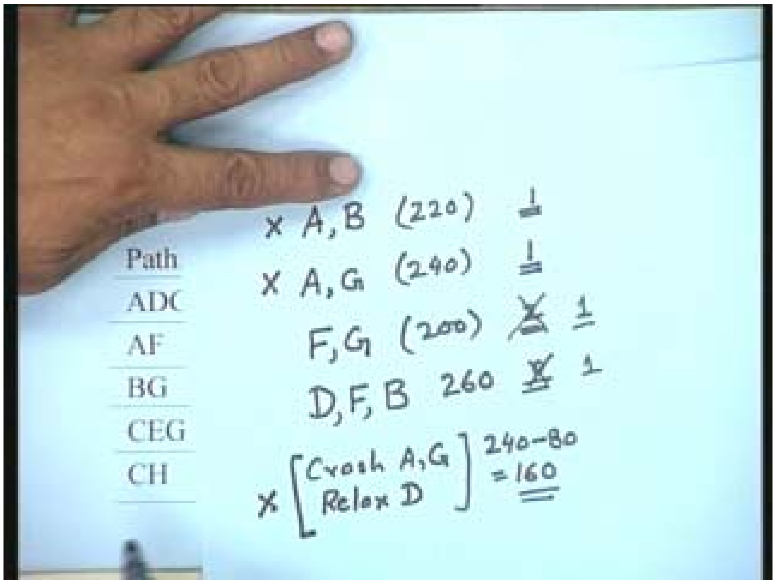
We have reduced the project duration from 16 to 14 and then to 13 now and in 13 the total cost has become 320. This is how we make a transition from point 2 to point 3. After we move from point 2 to point 3 we are at point 3 now. Let us continue the logic. What we have done is basically we have crashed AG by 1 day and relax D by 1 day. This path has become of duration 13. This path has become of duration 13, this path has become of duration 13 and as far as CEG is concerned because G has been crashed by 1 day, this path has become 11 and this particular path CH remains unchanged at 5 and since A and G have been crashed there is no slack available on A. G could be crashed by 2 days so only 1 unit of slack is available and activity D mind you which had a zero slack it now has 1 slack because it has been relaxed by 1 day. We update these figures, rest of the figures are the same. I put them down here. I have updated the status at point 3 as far as the whole thing is concerned and at this stage I find that there are 3 critical paths once again but they are now of length 13 and if I want to reduce the duration of the project further what do I have to do?

(Refer Slide Time: 36:57)



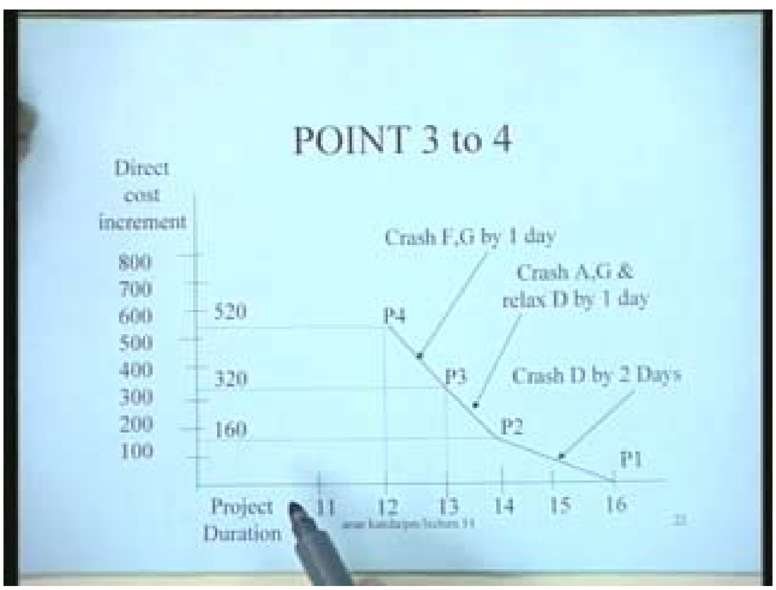
I have to again reduce the durations of these paths in some manner. Since these are the same paths I have actually the same options which I had discovered earlier. In these options what has been exhausted? A cannot be crashed anymore. This option is not available. This option is also not available. All options which involve A are not available. So which of the options available to me at the moment and moreover at this stage although earlier FG could have been crashed by 2 days but after the previous crashing of G by 1 day, G can be crashed by 1 day and F can be crashed by 2 days. So this option can be extended at this stage not by 2 days but by just 1 day and what about this option? Is this available? This option has become available because D can be crashed. D can be crashed by 1 day. F can be crashed by 2 days and B can be crashed by 2 days. The minimum of all this is 1 day. It's possible to do this crashing by 1 day now something which was not open is now open. It's like a window. Window keeps opening and closing like a shop. Sometimes it's open, sometimes it's closed. What you are finding here is that this particular option is now available out of these two and this option is not available at this point of time.

(Refer Slide Time: 38:35)



The cheapest option is to crash F and G by 1 day now because if we crash F and G by 1 day all these paths will be reduced. What's going to happen now is we are going to move now from this point. Let's see how we go further. We now move from point 3 to point 4 and in the movement from point 3 to point 4 what has happened is that we are crashing F and G by 1 day and crashing F and G by 1 day is equivalent to a cost of rupees 200 per day.

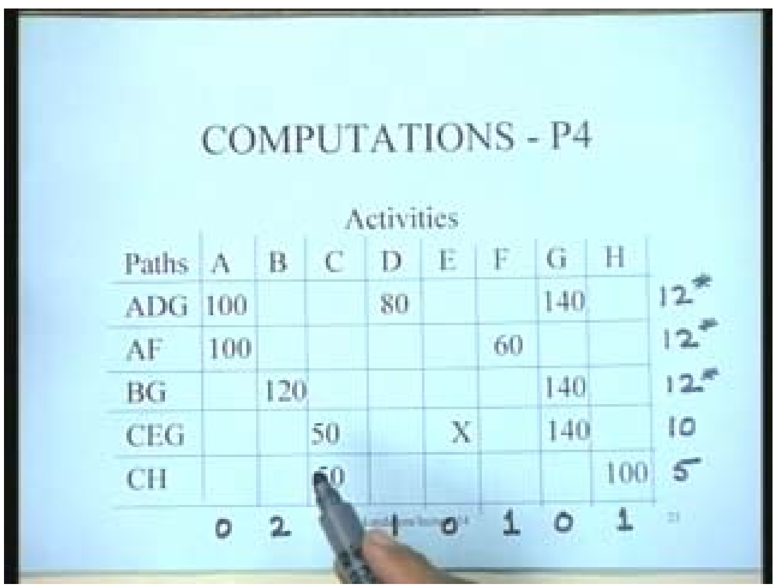
(Refer Slide Time: 39:18)



The total cost goes up from 320 to 520 corresponding to point 4. We have moved from point 3 to point 4 and we have come to project duration of 12 from 13 when we were at point 3. This is the manner in which we have moved.

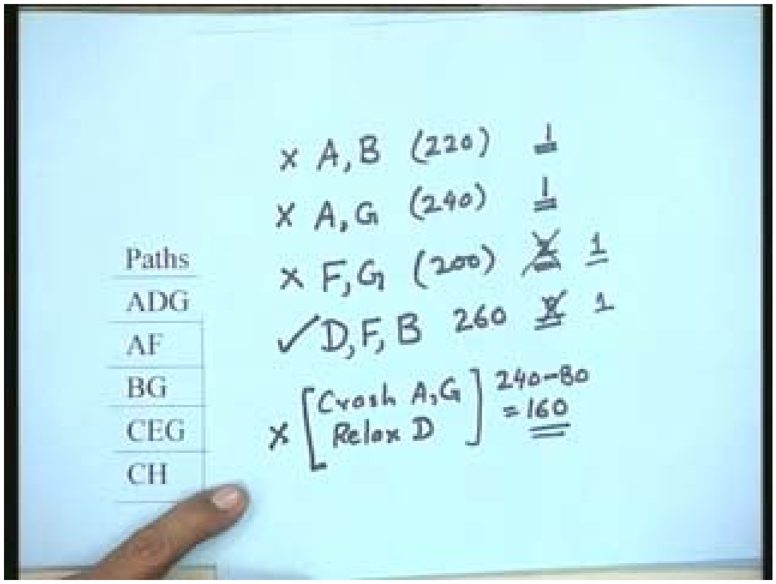
Let us now try to see what would happen further. When we reach point 4 the first thing that we have to do is let's update the status at point 4. What we had done was we had crashed F and G by 1 day. Crashing F and G by 1 day would have meant simply that these particular paths are now of duration 12 and this particular path CEG would now be of duration 10 because G had been crashed by 1 day and CH was untouched at 5 and let's see how these slacks had been affected. In fact since F and G had been crashed, F had a total slack of 2. So this becomes 1. It has been crashed by 1 day and G had a slack of 1. It simply became zero. F and G cannot be crashed any further. Rest of the slacks we had in these various activities remains the same. 1 0 1 and then we had for the other 0 1 1 2 0 this remains. This is now the status at point 4 and at this point you try to see again. It so happens that these 3 paths which have the longest duration they are the critical paths and because these paths are critical we have to again look for means to reduce their durations.

(Refer Slide Time: 41:26)



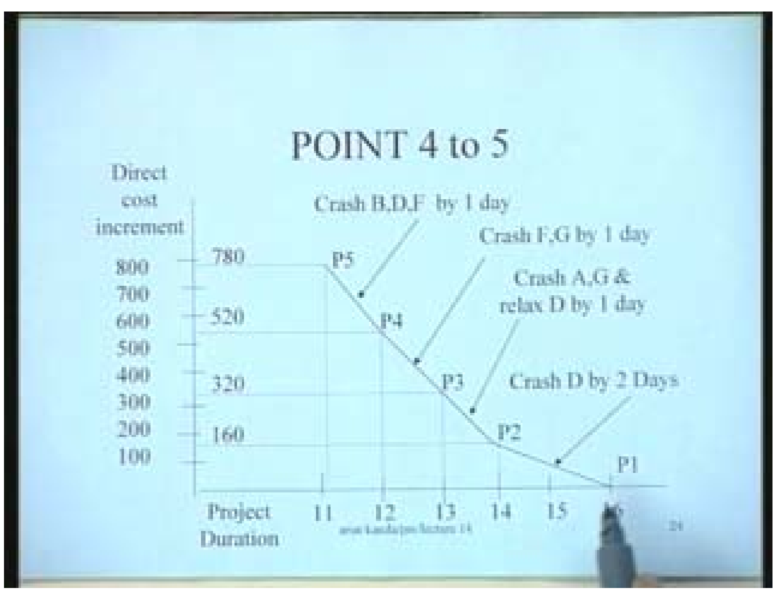
Since these are the same paths we have nothing but the same options. If the path set was different we have to generate new options. These are again the options and as you see this option is not available because A has been fully crashed. This is also not available. Can we crash F and G? We cannot crash F and G because G is fully crashed although F can be crashed by 1 day. This option is not available to us anymore but this option of D F and B. D can be crashed by 1 day, F can be crashed by 1 day and B can be crashed by 2 days. The minimum of this is 1. We can exercise this option. In fact this is the only option available at this particular point of time. We will take this option. We will crash D F and B and crashing D F and B will mean a cost of 260 rupees per day.

(Refer Slide Time: 42:26)



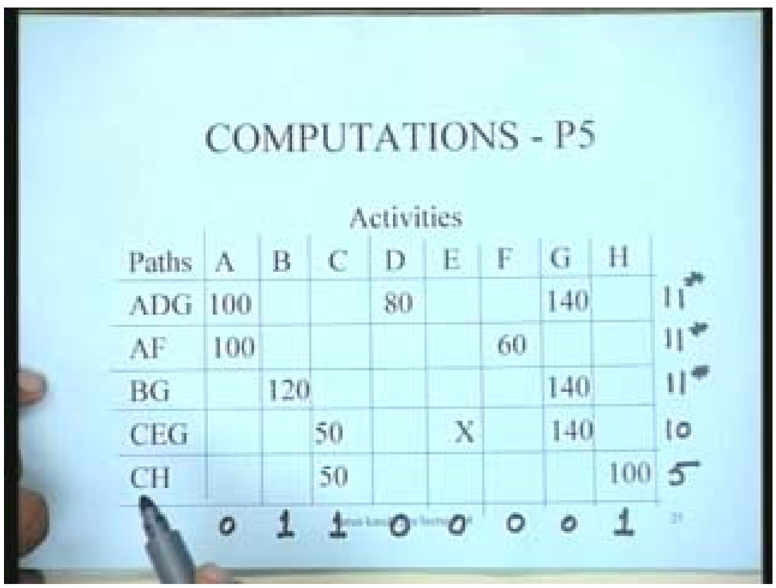
What we see is crash D F and B. What happens is we move from point 4 to point 5 in the network, I mean in the project cost curve. We are actually generating the direct cost curve in this process and from P4 to P5 our decision is crash B D and F by 1 day. This would result in an increase in the cost by 260 rupees per day and this is only 1 day reduction and what we find is that the total cost becomes 780 above the normal cost which was simply zero which is the datum that we had taken for particular value of generating this curve.

(Refer Slide Time: 43:13)



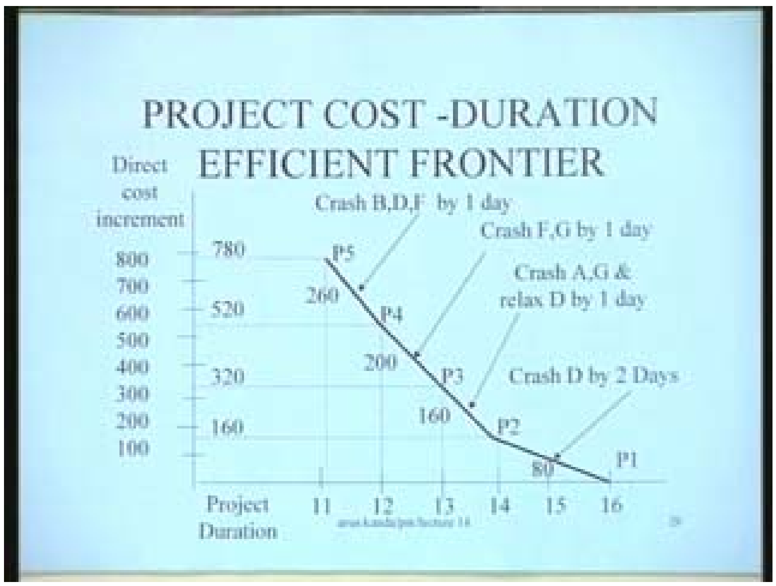
Let us now try to see what is the status at P5? When you see P5 what we had done was D F and B are the various values that we had tried to reduce and let's see here. At point P4 we had the durations of these parts as 12 but after crashing D F and B all these 3 paths will become of duration 11; 11 each and as far as CEG was concerned no activity on this particular path has been crashed in this particular iteration. This will remain at 10 and this will come to 5 and as far as the activity's slacks are concerned what we have crashed is D has been crashed. From 1 it's now totally crashed. F has been crashed. F does not have any slack anymore and B had a slack of 2 days. So it still has a slack of 1 day, others are the same. This is 0 1 1 and then as far as E is concerned it could never have been crashed and this activity is 0 and then this is 1. At this stage what do we find? You find for instance that this particular path A, D, G none of the activity is possible. Is it possible to crash the path ADG? This has a slack of zero, this has a slack of zero. These are fully compressed activities. This path cannot be reduced. We can rest assured that the project cannot be reduced any further and you need not check all the paths. Even if one path becomes critical that will ensure that you cannot reduce the duration of the project any further. But in this case even this path cannot be reduced and however B G can be reduced. It can be reduced by 1 day but it's pointless to do so. That's the point.

(Refer Slide Time: 45:33)



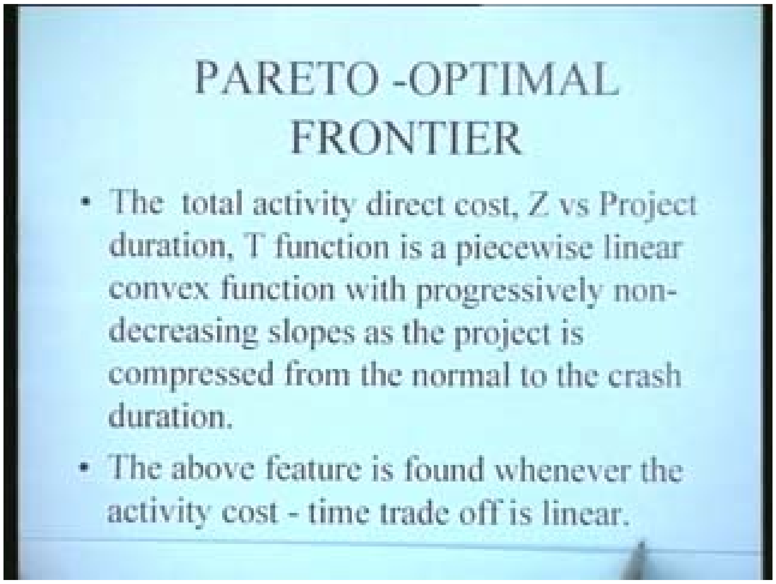
What we have seen here is that we cannot reduce the duration of the project in this case below 11 days. What we have observed is we have defined this curve. This is a very important curve from the managerial point of view. It's known as the project cost duration efficient frontier. This curve that we have generated is known as the project cost duration efficient frontier or it's often also known as the Pareto optimal frontier or some people just call it the project cost curve. We have this curve and it has certain interesting properties. If you have assumed linear costs for the various activities this cost curve is piecewise linear with a progressively increasing slope. That means it's a convex function. It's never a concave function. This is a property which can be proved also from the theory of linear programming and what you see is that you have this curve and what we have here is the managerial decisions. If you want to reduce the duration of the project from 16 to 14 days it's best to crash D by 2 days. From this to this it's best to crash A G and relax D by 1 day. That's what we have discovered. These are the managerial decisions and it tells you a number of things. It tells you for instance that the project can be done in a normal duration of 16 days to a crash duration of 11 days in this region and if I want to do it in any other point of time or any other duration then this is going to the minimum cost that is required for achieving that particular activity and in order to achieve this what activities have to be crashed and relaxed. That's the specific schedule that you will know from this particular curve.

(Refer Slide Time: 47:35)



We can see that the total activity direct cost versus project duration T function, Z-T function we can call it the total activity direct cost Z versus project duration T function is a piecewise linear convex function with progressively non-decreasing slopes as the project is compressed from the normal to the crash duration. This is a feature of all the Z-T curves with linear costs and the above feature is found whenever the activity cost-time tradeoff is linear.

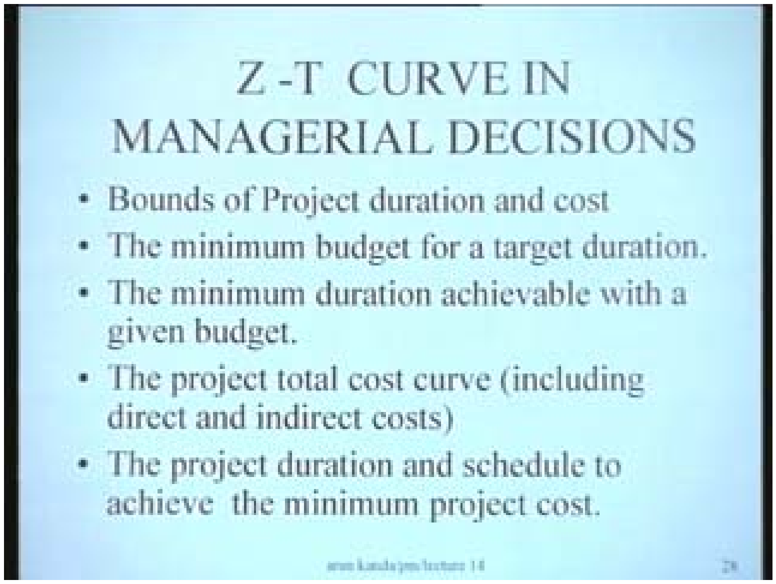
(Refer Slide Time: 48:17)



Whenever the time-cost trade off function is linear, the Z-T function is in fact piecewise linear and progressively increasing cost slope. How is the Z-T function useful to the manager? The Z-T function is useful for a variety of managerial decisions. For instance it tells the manager what are the bounds on the project duration and the project cost? For instance you know during the time we had the ASIAD games here, the flyovers which are normally constructed in India in about 3 years were constructed in something like 1 year and even less than that; 10 months. What is the possible normal duration and the crash duration comes from this kind of exercise and the kind of cost which you are willing to pay the cost increase that also is available.

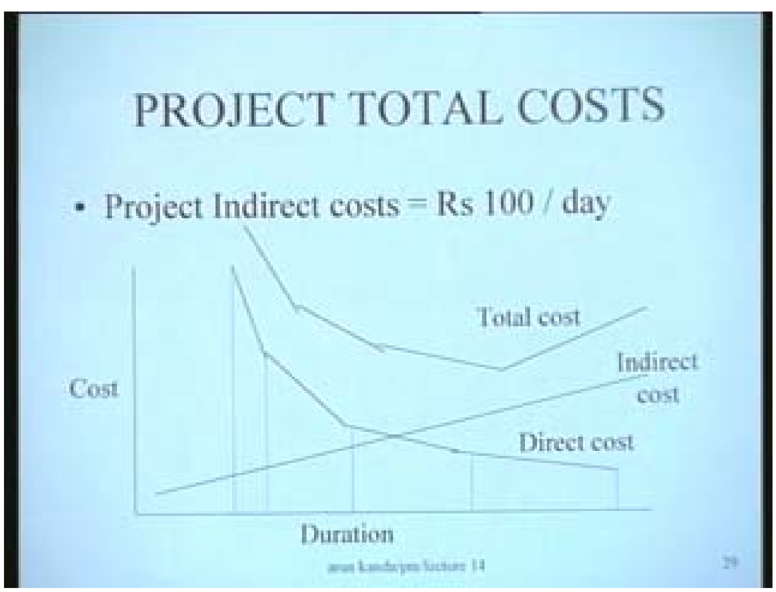
You get bounds of both project duration and cost from this particular analysis. You know what is the minimum budget for a target duration? That’s another thing. It helps you in cost planning and it also tells you the minimum duration achievable with a given budget. It depends on whether you look at it this way or whether you look at it this way. The total project cost curve that is including the direct and indirect costs that can be easily derived from this the Z-T frontier because all we have to do is super impose the indirect costs and get the optimum. The project duration and schedule to achieve the minimum project cost are easily obtained from this particular curve.

(Refer Slide Time: 50:09)



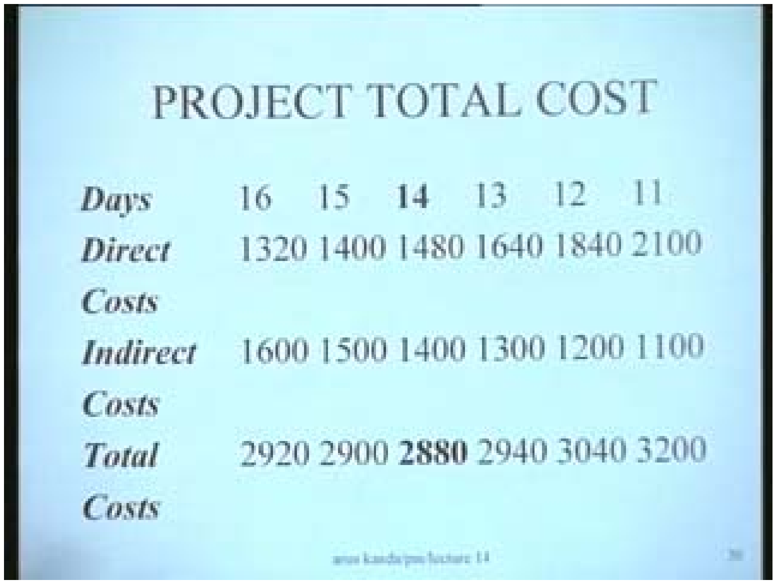
To continue the example what you see let's suppose that the project indirect costs for the example that we were talking is about 100 rupees a day. We have actually worked out this curve. That's the direct cost curve which is a progressively increasing function. The indirect cost of the project is 100 rupees a day which is the cost of maintaining the site is a cost of this nature. With this cost and the indirect cost if you add the two together you would get the total cost and wherever the total cost exhibits a minimum that should be the targeted duration of the project when you are preparing a plan because that would minimize the total direct and indirect costs.

(Refer Slide Time: 50:56)



For the example that we were doing you can work this out very easily.

(Refer Slide Time: 51:01)

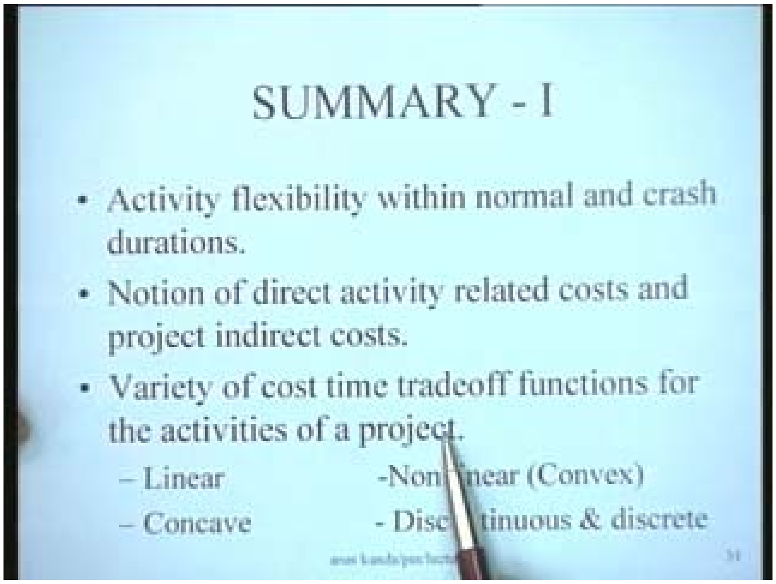


Our project duration was from 11, 12, 13, 14, 15 and 16 days we have calculated the minimum direct costs which we have obtained from the Z-T curve which we just did the analysis. If the indirect costs are 100 rupees a day, for 16 days then it will be 1600, for 15 days it will be so much, you have this so that the total cost is something like this and therefore in this particular situation the optimum would be to target for a 14 day duration which would give me a total minimum cost of 2880 rupees including this much from the direct cost component and this much from the indirect cost component and since it's a 14 day schedule I can easily work out from the Z-T curve as to which activities have to crashed and by how much.

For instance for the 14 day schedule you know that activity D has been crashed by 2 days. You know what are the durations of each of the activity? Which are the activities to be crashed and I think another important thing that emerges from this analysis is that indiscriminate crashing of activities is really not desirable. You notice that even when we reached the last point we had not crashed all the activities. Some of the activities still had some amount of slack left. However if you indiscriminately crash all the activities you would still get a duration of 11 days but you would be paying a much higher cost. So selectively crashing the activities is what is really required when you want to reduce the project cost to that particular value. This is another interesting thing in this particular analysis.

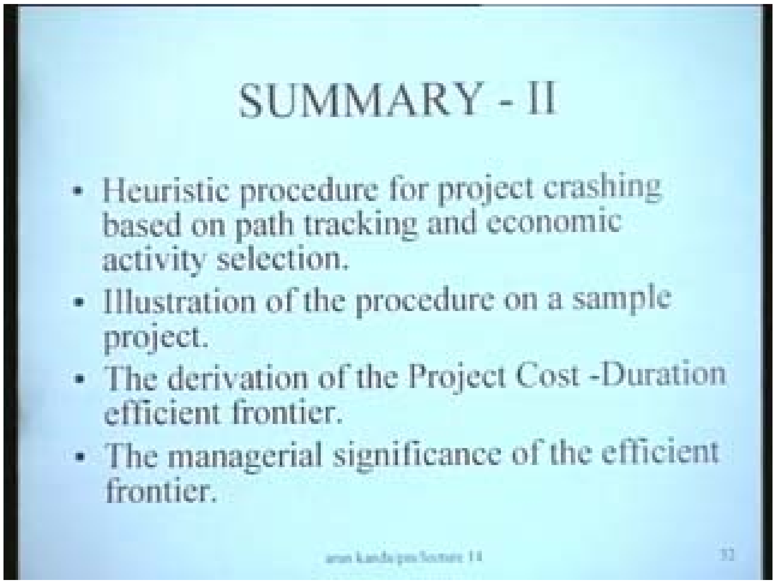
Let us now summarize what we have tried to do in this particular lecture. We have seen first of all that most activities have certain flexibility in their durations and this flexibility is expressed within the normal and the crash durations. For each activity there is a normal duration and a crash duration and in between there could be a variety of different cost-time relationships for that particular activity. Then we saw the notion of direct activity related costs and the project indirect costs. In the direct activity related costs when you reduce the duration of the activity the cost goes up and it's vice versa for the project indirect costs and we saw what their compositions were like. Then there can be a variety of cost-time trade off functions for the activities of a project.

(Refer Slide Time: 53:52)



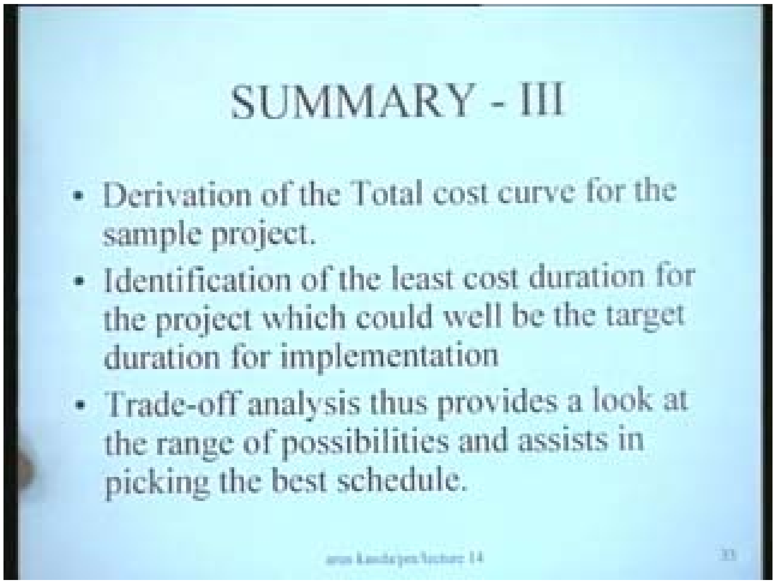
You might talk of linear, non-linear convex or concave or discontinuous and discrete time-cost trade off functions to represent different situations.

(Refer Slide Time: 54:06)



Then we looked at a heuristic procedure for project crashing based on path tracking and economic activity selection. If you keep track of the various paths and economic activity selection if you do but there was a trap involved here because identifying the best possible set of activities was not always easy; illustration of the procedure on a sample network that we have seen for this particular case. The derivation of the project cost-duration efficient frontier we did for this particular network and the managerial significance of the efficient frontier in terms of the kinds of decision that can be obtained from this and finally we did the derivation of the total cost curve for the sample problem which is the sum of the direct and the indirect costs. We did the identification of the least cost duration for the project which could well be the target duration for implementation and we found in general that the trade off analysis thus provides a look at the range of possibilities and assists in picking the best schedule that we have in a particular network.

(Refer Slide Time: 55:19)



As we saw there were difficulties in doing this kind of analysis in a heuristic way. We look at some optimal procedures for doing the whole exercise in our next lecture. Thank you!