CAD / CAM
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Lecture No. # 9
Parametric Surfaces II

So these days we are discussing the subject of geometric modeling of surfaces and we will continue with our subject. Last time we looked at surface definition in two different ways. One is surfaces of known form like cylindrical surface or planar surface and we also looked at sweep representation of surfaces. Particularly the linearly swept surface and circularly swept surface where surface can be defined by defining two curves. One is a generatrix curve which is swept and the directrix curve which is the direction in which we are sweeping. We will take up two other forms of a definition in this class.

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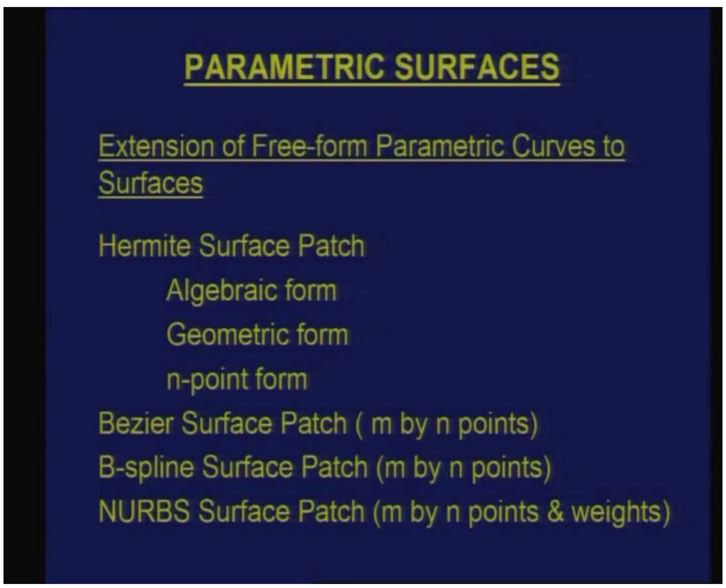

So first is we can extend free from parametric curves which we discussed earlier, the same definitions can be extended to surfaces also. If you recall we discussed parametric cubic curve or Hermite curve, Bezier B_Spline and NURBS curve. So in a similar fashion we can extend the definitions to surfaces. So what we will see is we look at Hermite surface patch. As I said the word patch here basically refers to a surface with fixed boundaries. And just like we had Hermite curve, here too we have three different forms of Hermite patch definition algebraic form, geometric form and n point form or defining a Hermite surface patch which is interpolating a given set of points. So that is another form, similarly a Bezier surface patch.

If you recall like a Bezier curve is defined by n plus 1 control point. So here or you can say a given fixed control points. Now since it is by parametric, you have to give a number of points in both the directions. It need not be m need not be equal to n. I can have for example five control points in one direction and four control points in another direction. Same is true with B-Spline surface patch where you define m by n points but only thing is the order is defined by separately by giving a value of k as we used in B-Spline curves, here we use two values k and l. And similarly the NURBS surface patch where you not only give control points but also associate weights with this particular thing. So let's look at these forms today.

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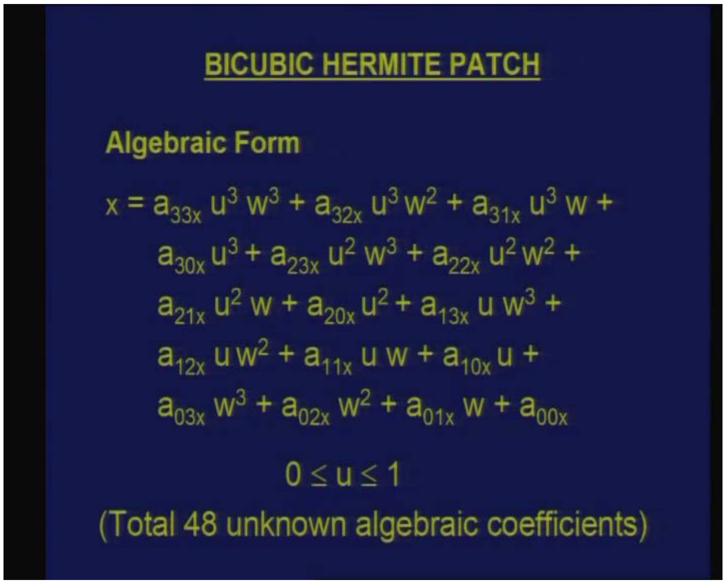

First is algebraic form of a Hermite patch. Now whenever you say Hermite patch like one of the most standard form here is a bicubic. Since surfaces or you have two parameters, u and w which are standard notations. So if the surface is cubic in both u as well as w then you call it as a bicubic, this is the most common. One can also think of having cubic quartic patch or quartic quartic patch or quatic quintic patch so or cubic quadratic patch. So there can be different versions which are possible and most common version of course is bicubic Hermite patch. If you recall in the algebraic form of parametric cubic curve or Hermite curve, we had x which is a function of u. there were 4 unknowns. Isn't it? There were 4 algebraic coefficients which could define in the case of xyz, 4 into 3 there were 12 algebraic coefficients.

Now if I want to represent x as a function which is a cubic function in u and w, there are 16 possible terms like I have constant term u term, u square and u cube term. Similarly I have constant w, w square and w cube, all the combination of this leads to 16 unknown coefficients. So algebraic form x value is represented by these values and we are using coefficients are represented by value with two values which basically represents the power of u and w which is used. x represents that, coefficient belongs to the x similarly I may have a_{33} y when I am actually representing let's say a y equation for y. So how many algebraic coefficients which one has to provide, in order to completely define a bicubic Hermite patch is...

No, I have a 16 here for x 16 into 3. So there are total 48 unknown algebraic coefficients which are shown here and you also have a parameter which is u is equal to 1. What is missing here is w also. So you have zero w which is also ranging between 0 and 1 which makes it complete definition of this thing. Now it may be difficult to visualize or somebody to design let's say or represent or model a bicubic patch by manipulating these 48 unknowns would be rather difficult because how the shape changes like suppose if I vary one of the 48 algebraic coefficients and if I want to see how the shape changes, I do not have a direct, we can say perception of which coefficient will give me the desired changes etc. So this is not a very standard form of, this is not very you can say commonly used form of a bicubic patch just like we had a Hermite curve where we preferred to use geometric form. So here too you have a bicubic Hermite patch which is in geometric form.

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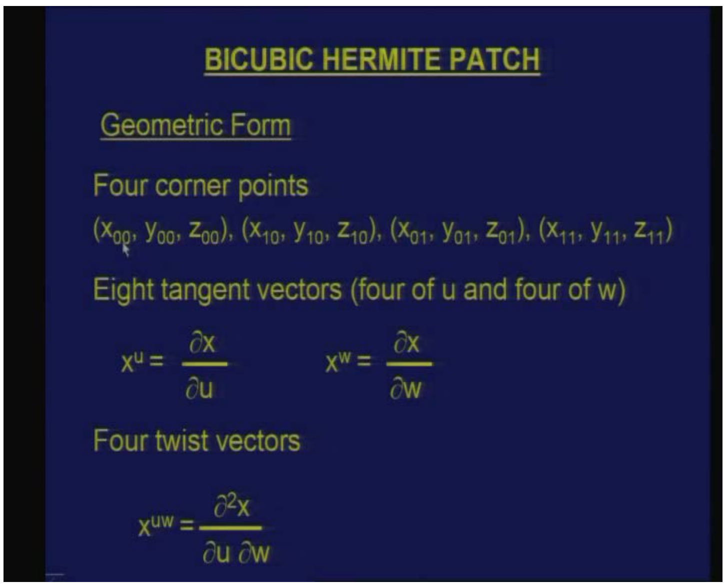

So coming to geometric form, going back to Hermite curves, the curve could be defined by specifying two endpoints. That is start point and end point and the tangent vectors at the two end points which could completely defined the geometric form of a cubic curve. Here just like a curve has a start point and the end point, a parametric surface will have a four corner points, it will have four boundary curves wherever these boundary curves are intersecting you have four corner points. So, one can give these 4 corner points as input. So they are also shown here, 0 0 basically says that this refers to a point where u is 0 and w is 0. And accordingly you have other four corner points which are given. Then you have 8 tangent vectors because there are 4 tangent vector at 4 corner points which are derivative of u, derivative with respect to u. So this is like if I have x of u this is del x by del u. So at any point I have three components that is del x by del u, del y by del u and del z by del u.

Similarly I have 4 tangent vectors which represent for the w which are del x by del w, del y by del w and del z by del w. So you have at any particular point, you have 3 x y and z coordinates, three of these values and three of the tangent vector with reference to w. and we also use twist vectors which is a second derivative with respect to u and w. These are called as a twist vector. So at every point you have this is also a vector, so I have 3 values of this. Now if I look at how many inputs which i am giving at different places is at every point I have 3 of the coordinates. This vector consists of three components, three of this and three of this. So at every point you are actually giving 12 values and since there are 4 corner points, I have 48 values. So if I give this 48 input values, I can solve the algebraic form and I can get a bicubic Hermite patch which is in the form of a geometric form.

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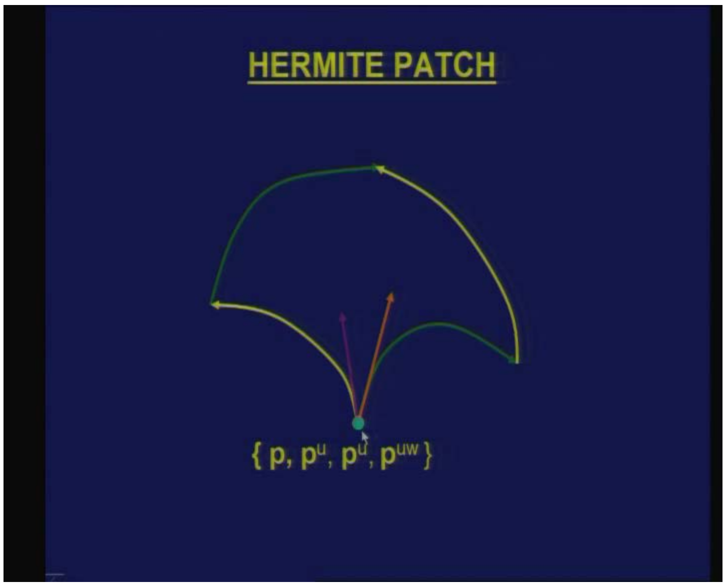

So if you really look at here like here you have 4 boundary curves which are shown here, the opposite curves are shown in one color so you have 1 2 3 and 4 curve. At any given point I am specifying 4 vectors. One is the point vector which is x_0 y_0 z_0 or whatever xyz coordinates at that point. This is the tangent vector with reference to u, del x by del u del y by del u del z by del u I think this is w. This is p of w, so this is like del x by del w del y del w and del z by del w and the twist vector. So by giving these values at 4 corner points, I am completely defining the various surfaces define which is bicubic in nature.

So this is you can say a better form than the algebraic form because if I want let's say surface to be have a certain tangential direction at any particular point, I can make it happen or I can also specify twist vectors at corner points. Then the third version of Hermite patch is analogous to the four point form which we use for the Hermite curve that is a parametric cubic curve. What we did is like we took four points in a space and through which I can have a Hermite curve which is interpolating all the four points. One of them is a starting point, other one is an end point but it passes through other points. But only other requirement which we need to fix here is what will be the parametric values for the intermediate points. So one of the forms which we used is suppose if I have 4 points which I call as 1 2 3 4.

The first is the starting point and fourth is an endpoint in between there points are given parametric values which are like equi spaced in parameter space. That is u is equal to 1 by 3 and u is equal to 2 by 3. If I follow the same, if I extend the same definition to bicubic Hermite patch, there are a 16 points given in a space which is these are defined in a order because this is like 4 by 4 points.

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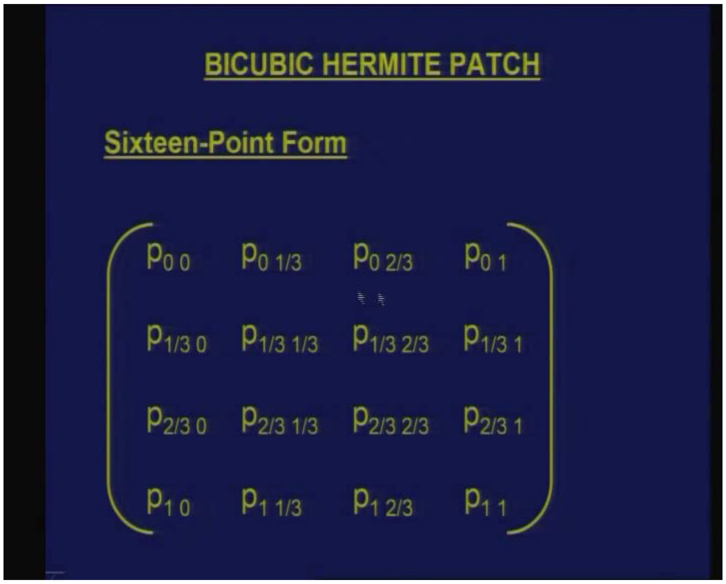

This is a matrix, so I want a curve which passes through all the 16 points. Now this basically can be given, if I specify 16 coordinates that means I have 16 x y and z values total 48 input values. So I have 48 algebraic coefficients, I can solve it and I can also choose 1 by 3 and 2 by 3 as one of the options to define the surface. So you are defining a point which is like p_{00} and then there is a point which is p_0 and 1 by 3 where the coordinate values are u is 0 and w is 1 by 3 and accordingly, if I give this matrix I can again solve and I get the 16 point form of this.

One of the interesting things about curves and extending them to surfaces is parametric curves can be represented in matrix form. We saw that when we discussed in curve like since it is cubic in u, on one side I have u cube u square u one terms and then there is a coefficient matrix. Now when it comes to surfaces the same coefficient matrix is valid. Only thing is we have to take once as it is, second time a transpose of that and I can represent extend the same matrix definition nothing changes, coefficient matrix remains same. Only thing is now you have instead of having let’s say a three matrices which are multiplied to get a curve, you have 5 matrices which will be multiplied to get a surface.

So since extension is very straight forward, many of the routines which I develop for curves like when we are writing programs or when I am developing the software for geometric modeling, I can make use of most the concepts for definition of surfaces too. So these are the three you can say more standard forms of a bicubic patch. Coming to Bezier curve we were discussed about Bezier patch.

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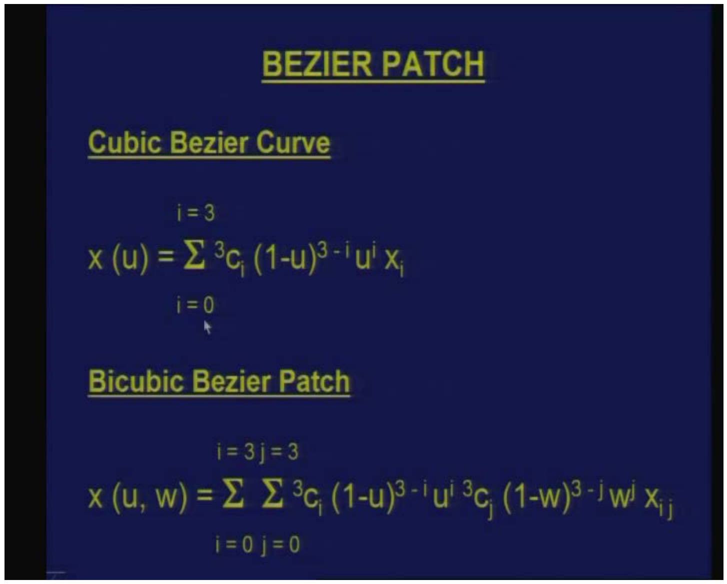

We discussed about Bezier curves and we also put down equation of cubic, quartic and quintic Bezier curves. If I recall the definition of cubic Bezier curve, I have 4 control points, so which goes from let's say p_0 to p_0 p_1 p_2 and p_3. And if I want to define let's say xyz coordinates x coordinate has been equation for that has been reproduced here. So this is defined by a binomial coefficient then i have a term which is 1 minus u to the power of 3 minus i u to the power of i and x_i, similarly for y and z.

Now if I am let's say moving to surfaces, surface patches, I can define like cubic, cubic patch which is called as a bicubic patch or cubic quartic patch or cubic quintic patch or any combination of that. So if I am going for let's say bicubic Bezier patch that means I have cubic in both then you have x coordinate which is now function of u and w. Now you have, here you had a summation of how many terms? 4 terms. Here you have a basically a summation of 4 into 4 that is 16 terms. So there are total 16 terms which will come in definition of bicubic patch like I have the same binomial coefficient which was used here like 3 c_i, 1 minus u is to the power of this remains same u also remains same but now I also have an additional term which is for other parametric 3 c_j 1 minus w to the power of 3 minus j and w to the power of j. The number of inputs which you are giving here are the 16 that is i goes from 0 to 3, j goes from 0 to 3 so 4 into 4 there are total 16.

So how convenient it is to basically extend the definition from let's say curves to surfaces is clearly evident by an example like this and there are many of the properties which are, which we discussed for Bezier curves is also valid for Bezier surfaces like in the case of Bezeir curve, the curve passes through first and last point. In this case the curve passes through four corner points, it doesn't' pass through other this thing. And similarly the tangent vectors etc are defined by the points in those subsequent that is adjacent points. Same property is valid here too and other property which we discussed is convex hull property, convex hull property is also valid in the case of Bezier surfaces.

And we also discussed that if I take summation of all the coefficients which was unity, here too you have the same concept that the summation of all these things is unity. So most of the properties of Bezier curve which we discussed are also valid for surface only thing is you are giving a matrix of points instead of series of points as it was done.

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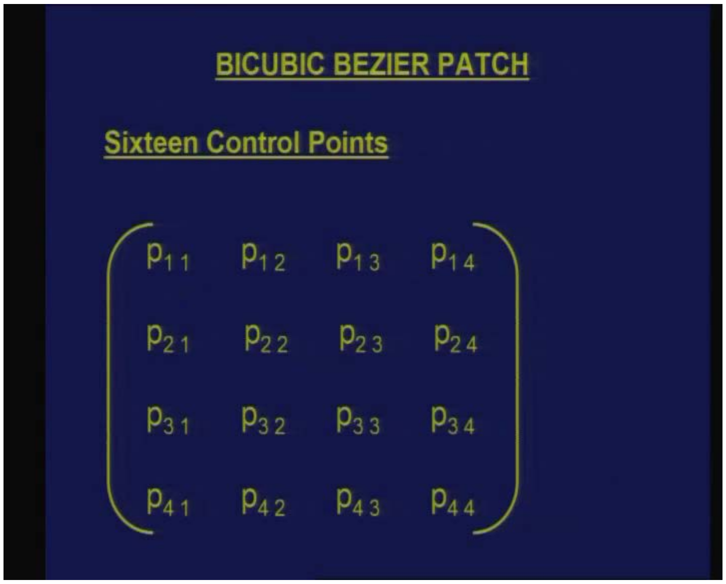

So these are the 16 control points which form input, we can say 1 1, 1 2, 1 3. order is important I cannot if I change let's say a point from if I put p_{12} here and p_{23} here I will get totally a different surface or that is important that was true in the case of curves to the point order was, like here is a simple example of a bicubic Bezier patch which is shown here.

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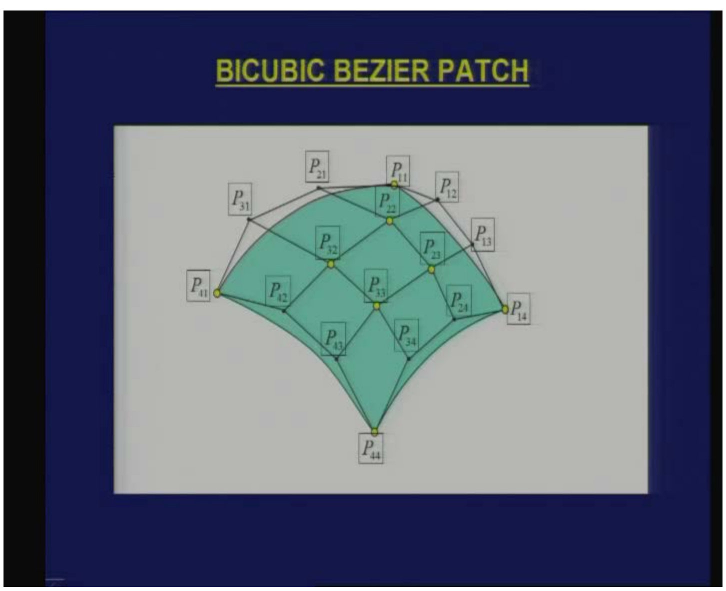

So you can see the 16 points which are defined and the curve is only interpolating 4 corner points that is which are there in the 4 corner whereas all the other control points will influence the shape of this particular surface or patch by moving any control points, I can always change the shape of this surface and whereas it doesn't interpolate the points which are given in this case. So this is a typical, a bicubic Bezier patch which is shown here, like I can extend the definition to a Bezier patch which is in m and n. I may have m m control points along the parametric direction u and n in the case of w direction.

So the extension is equation changes accordingly and the disadvantages of Bezier curve which we discussed are also valid here is suppose if I define a Bezier patch which is defined by m by n control points. If I change any control point, any of this the shape changes. So you call it as a global propagation of change which is also valid here. Now to overcome this, a better way to do is to use a B-Spline patch which is an extension of a Bezier patch.

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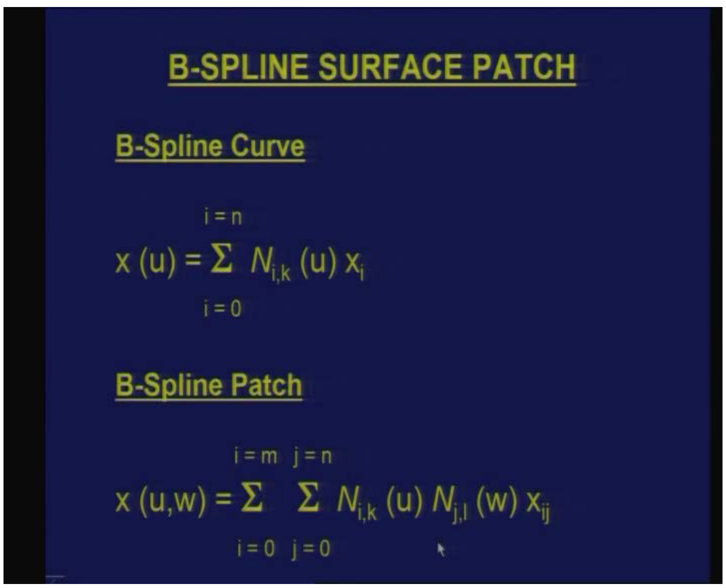

So a B-Spline surface patch let's recall the definition. A B-Spline curve is defined by a basically what you call as the B-Spline basis function. We looked at some of these functions and these functions are different for different values of k and i is the total number of control points here is n plus 1 which goes from i is equal to 0 to n. And when they are multiplied by x_i I get this function. we have seen that this is what we call as a non uniform B-Spline function because these basis functions are uniform in the middle but towards the end they are modified in such a manner that the curve is made to pass through the first and last points which is there and this basis functions can be evaluated using a recursive function which we have also seen. Having known what a B-Spline curve is, knowing a B-Spline surface patch is a trivial.

Now you have a surface which is a function of u and w. Now you have a summation of basically a term which are both along u and w. Now I can, it need not be m need not be equal to n, I can take like 10 points along u direction and let's say 15 points along w direction, this is completely our choice. And now you have same basis functions that is $N_{i,k}$ and $N_{j,l}$. This shows that the value of k which we use to select, whether you want a cubic equation or quadratic can be different in both the direction like you can say that it's cubic in u direction whereas I want one order higher or lower in w direction. So that is why you associate another input which is l.

So what is the input which I give in order to define a B-Spline surface patch is m plus 1 into n plus 1 control points, there xyz values. Other than that I have to supply two values k and l. k and l which will define what is this thing which you are using and I have $x_{i,j}$ which is basically a matrix of points which are defined in this manner. And if I take leave the $x_{i,j}$ and take summation of all these thing, it is still unity. So that property is valid as far as B-Spline surface patches to in this case. But given let's say B-Spline surface patch let's say I choose k and l both to be equal to 4 and let's say I have a large number of m and n points let's say 15 and 20. Then if I change one of the control points, the shape of the surface will be changed locally that means every control point will have some influence locally but it will not change the entire surface. There is no global propagation as it happens in the case of a Bezier surface patch. So because of this B-Spline patches, you can say more common version which is used compared to a Bezier surface patch.

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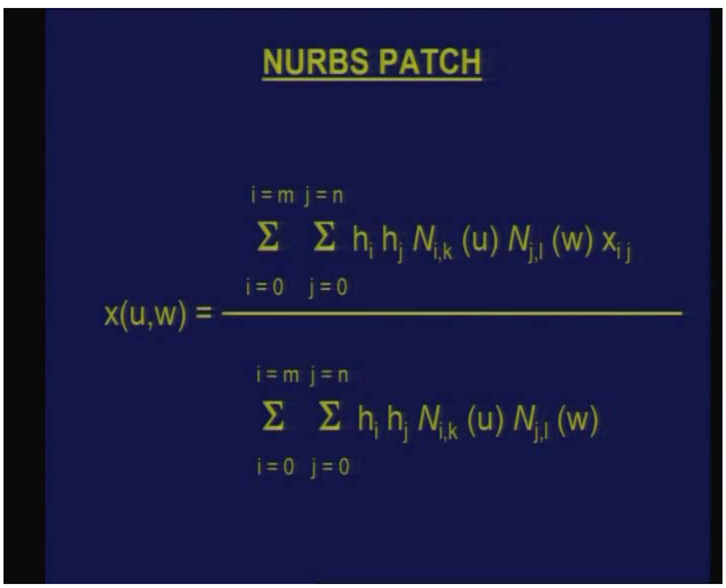

So extending it to the last in this category that is NURBS patch is also straight forward. If we look at what we have is we had only a summation which is a one direction. If you recall the NURBS patch like what is shown here is x as a function of u and w. Now in order to define a NURBS surface patch whatever input which we supply for let's say a corresponding B-Spline patch has to be supplied. That is m plus 1 into n plus 1 control points, also specify what are the like k and l which is the direction but you also specify m plus 1 into n plus 1 weights which we will define like how the weights also influence the shape of curve not only the control points which are this thing. And, since you take a rational version of this so this is called as a rational B-Spline. And since B-Spline is non uniform because it is made to pass through points, so you call as a non uniform rational B-Spline. So we can say NURBS patch is something which is more like a standard one can think of it as a standard representation for surfaces when it comes to this. So this is like a typical extension from curves to surfaces. Mathematically there is no change as far as definitions are concerned except that you are adding one more dimension to this particular thing.

And as I discussed earlier, instead of extending dimension from let's say 1 to 2, I can do it for three and I can have a Hermite hyper patch which is basically a piece of solid. It's basically a piece of solid which is bounded by 6 free form surfaces. Every point inside a solid can be obtained by giving values of u v w between 0 and 1. And I can extend the definitions to like now suppose if I want specify let's say a tri cubic hyper patch that is what it is called one of the standard form is suppose if I want let's say if I want to specify number of points, so then I will give 64 points.

If I want let's say, if I want to specify in a geometric form, you give the 8 corner points. You also give the tangent vectors for this particular thing along u v as well as w because now you have one more dimension. Then you also give, at every point you have to give the 8 values so you give the point. First derivative with respective u v w then you also go for second derivative which is with respective u v w. How many combinations are possible like I can go for del square x by del u square del square x by del w square and del square x by del v square. So those are the three, how many are over? So I have 7.3 tangent vectors which are the first derivative three for second derivative. And the eighth value which I am going to choose is del cube x by del u del v del w.

So if I give these values, 8 values at every corner point I can also define a geometric form of tri cubic Hermite hyper patch. That is an extension of that. Same is to like I can have Bezier hyper patch, B-Spline hyper patch and NURBS hyper patch. So a very complex solid can be easily represented by like giving definitions like a NURBS hyper patch. But as we discussed earlier, it is not a very convenient way to do that because you have too many inputs and this thing.

So a best way to define let's say a complex solid like hyper patch is to just define 6 boundary surfaces and say that it meet certain continuity requirements. So that is the more common version because inside points do not have much this thing like suppose if I want to change the shape of the solid object, inside control points they do participate but they do not have much influence whereas it's the boundary points which are more influential. So you try to use surface form even to define a solid that in order to define let's say a hyper patch, I say here are the 6 boundary surfaces which is an input to define the hyper patch but the definitions can be extended. In fact people try to extend it to a dimension higher than three also.

So it's like you can go for one more dimension now that visually it may not be make any meaningful this thing but mathematically it may make some sense. For example I have a solid which is changing its shape continuously. So I have one more variable which is time, so I can have let's say a hyper patch in n dimensions to mathematically represent a solid which is continuously changing its shape that could be one of this thing. In fact there is one version of NURBS surface which is used is just like I associate the weights with a NURBS surface.

I can associate many other variables also like suppose I am representing a surface where I have a temperature is varying on this surface like I want to basically input the temperatures also. If I am using a NURBS representation, you not only give the control points, you not only give let's say the points which are like the weights, I have also say that these are the temperature and pressure values. people have try to extend for representing some of the mechanical quantities measurement quantities to be associated with patches but that's not very standard, only a few attempts have been made in that direction.

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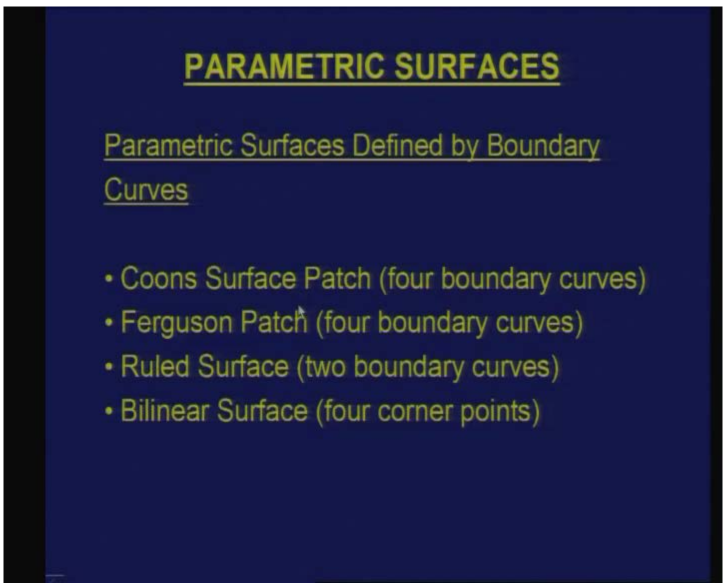

Now let's come to the last category of defining parametric surfaces like in last class, we have seen that one of the convenient ways to define surfaces is not to give mathematical equations or interpolating points but I give purely boundary curves and by giving the boundary curves also, I can define completely the surfaces. Now among these there are four common, four important versions which we are going to look at. One is a Coons surface patch and the Ferguson patch is another. And most of these definitions have come only in last 50 years like it is only in 60's when Coons and Ferguson patch was first time defined. And they were use just like Bezier patch then ruled and bilinear surfaces are of special interest, we will look at them also.

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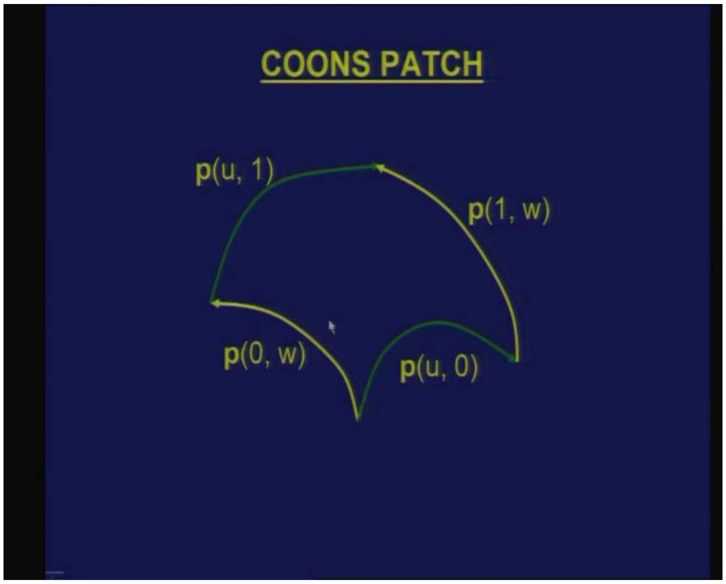

Now what is the input which one gives in order to define let's say a Coons patch like I would like to, there are 4 boundary curves of a surface are known. Now I would like to define a surface which interpolates these boundary curves in certain manner. So an input to Coons patch is four boundary curves, I think you may not have seen this kind of definition in any other forms which we discussed earlier. So I am giving one parametric curve which I call as p_{u0} this means w is equal to constant whereas u is varying from 0 to 1, here to w is equal to constant which is 1 u is varying from 0 to 1. These curves are u is equal to constant curve, u is 0 and w is varying from 0 to 1. Here u is constant that is u is equal to 1 and w is varying from 0 to 1. So if I specify these four boundary curves because sometimes it is easier to define curves rather than surfaces because of the complexity of equations.

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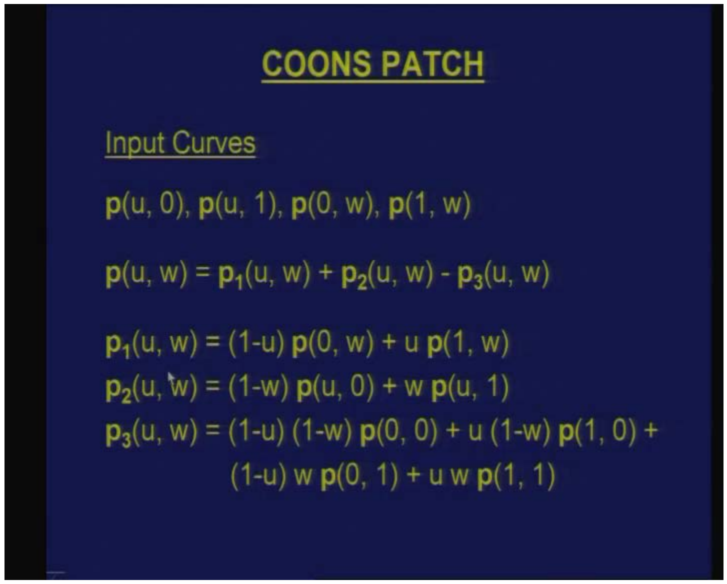

If I input these then a Coons patch can defined using a definition like this. So these are the four boundary curves which I have reproduced here. This is basically defined by 3 terms which is called as p_1 p_2 p_3. Let's look at what are these terms. First is a p_1, you are taking the two curves which are opposite curves and linearly interpolating. So these are actually u is equal to constant terms, constant curves and you are linearly interpolating these two curves. I take two other curves and linearly interpolate along that and basically add these two terms.

So if I add these two, I will get what is called as p_1 by p_2 but p_1 by p_2 is not sufficient because if I just add p_1 and p_2 it actually doesn't meet certain boundary conditions. For example if I try to put u is equal to 0 or w is equal to 0 and try to get back my original equation, I don't get. So there is some extra information additional information which comes into this which has to be subtracted. That is called as a p_3 term, so p_3 time is basically a linear interpolation of the corner points which are like p_{00} p_{10} because if you really see here the corner points are coming in both the curves. So it's basically it is coming twice.

So they has to be, they have to subtracted in order to get that. If I look at this, these are the four boundary point's p_{00} p_{10} p_0 p_1 and they are, this is basically a linear function this is called as a bilinear function. So if this bilinear function is subtracted from let's say the linear interpolation of curves, the surface which is obtained is usually called as a Coons patch.

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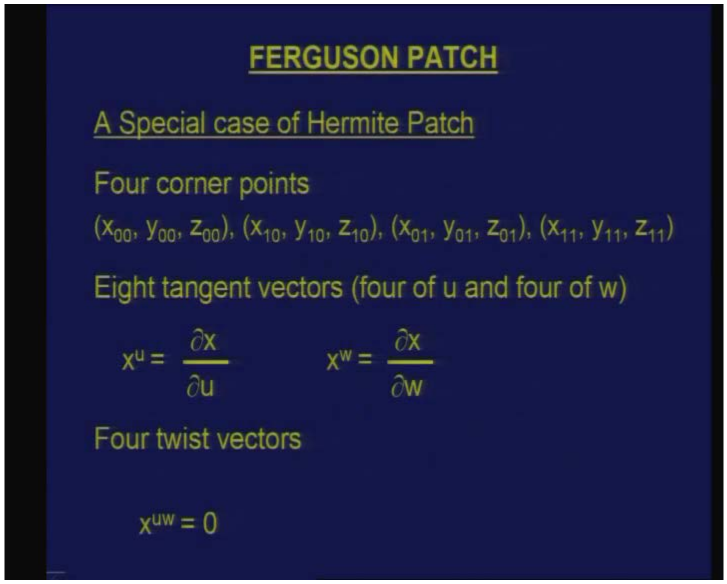

Let’s look at, so that is one of the ways like given a four curves how to do an interpolation first time a proposal was given by person Coons so that’s why it is called as Coons patch. There is another variation which was proposed by Ferguson patch, this is first time proposed by a person by name Ferguson. Ferguson patch can be very easily understood if we have, if you know what a geometric form of bicubic Hermite patch. We have already discussed that. What are the inputs which we give in bicubic Hermite patch is four corner points. Tangent vectors with respect to u and w and also twist vectors which are second derivative. If I put twist vector as 0 and keeping all the other inputs are same then this basically gives you a Ferguson patch. So the input here to define let’s say the same definition which we had for bicubic Hermite patch is valid here to all that is how much in like the input which you are giving has been reduced. The input which you use to give was like for every point you use to give a 16 values, now you are only giving 12 values because others are 0. And the surface which is obtained is called as a Ferguson patch.

Now what is the difference between these two? Suppose is if I take for example a bicubic Hermite patch, it has what you call as c_2 continuity. That means you have a continuity which is one order higher than the Ferguson patch. Here the continuity as far as c_2 is not maintained, you sacrifice the continuity requirement basically satisfying the c_1 continuity. But you also try to simplify the definition because you are not looking about something like second partial derivatives. Even if I have partial derivatives with respect to u and w, I am able to define equation mathematically.

So it is basically a simplicity and continuity compromise which gives a difference between a Hermite patch and the Ferguson patch which is shown here. So one of the ways to define a Ferguson patch is take the equation of Hermite patch and substitute 0, I have a new surface which is a Ferguson patch.

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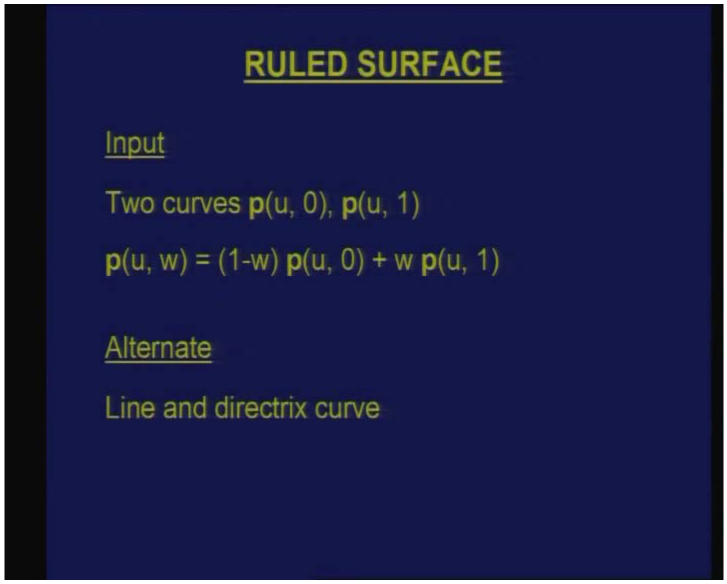

Then the third form of defining a surface using boundary curves is using what is called as a ruled surface. Ruled surface can be defined in two different ways like in earlier cases whenever we had Coons and Ferguson patches, you are giving all the four boundary curves which were this thing. I think one thing which I forgot to mention is like for example if i take 16 point form of let's say bicubic patch. Isn't it?

So if I take 16 point form then there are 16 points which are defined in order to do that but if I take Ferguson patch, I don't require all 16 points. 12 are enough because you are actually not going for that. What are those 12? Basically, whatever are the intermediate points like which are not there on the boundary, you are not trying to define. So it's, you can say that coming back to a ruled surface you have a similar situation here. Instead of let's say defining all the four boundary curves, I am just defining only two boundary curves which are opposite. That means which are either u is equal to constant terms or w is equal to constant curves. And if I have a linear interpolation between the two curves, this linear interpolation gives me a surface which is called as a ruled surface like what is the input for a ruled surface is I give two curves which is w is equal to constant that is w is equal to 0 and w is equal to 1. So this is a parametric in u this is another parametric in u and these two u parametric curves are linearly interpolated along the direction which is w. If I see this equation, if I substitute let's say value like suppose if I put w is equal to 0 I will get this curve. If I put w is equal to 1, I get the other curve.

So any curve is basically some kind of a mixture of these two with linearly interpolated between the two. There is alternate way of defining a ruled surface. We looked at when we discussed about sweeping swept surface, you have a generatrix and a directrix. So if I take a very generic curve and sweep it along another curve then I get let's say a generic swept surface. If I reduce one of them to straight line that is generate let's say if I reduce the directrix, if I take a directrix curve and the generatrix curves is a straight line. So if a straight line is swept in a space along certain special curve direction then the area swept is also the ruled surface which is obtained in this. If I just change the other way I get linearly swept surface. I can take any curve and if I choose directrix as a straight line. So that’s a linearly swept surface. If I just reverse it, I get a ruled surface which is that.

Now ruled surfaces are very very popular in many of the design and manufacturing applications. The reason is very simple. The definition is very very simple. Firstly you are defining just two boundary curves and you are linearly interpolating. So mathematical definition is very very simple, this is just like an extending a mathematical parametric representation of a straight line which basically interpolates if I put this as a point this becomes a straight line whereas if I to a curves it becomes a ruled surface. Mathematically very simple, easier to program and easier to compute many of the values.

Secondly it also gives you a linear direction. One of the parametric directions is a straight line which is also an advantage you may be aware that this has an advantage in terms of manufacturing applications. When I have to mission this particular surface then I can always take let’s say my tool along the linear direction because in any CNC programming you have linear and circular interpolation. If one of the parametric directions becomes line or a circle, generation of CNC programs is easier to write it. So that is one reason why but at the same time it also gives you a very complex geometric form with one direction being a linear direction. Because of this, it’s also used in construction work. Suppose if I have to let’s say a fabricate or let’ say construct let’s say a roof which has a very complex form, if I choose ruled surface as one of the forms, it’s convenient because I can always have a reinforcing elements along the directions which are which you call as a rules, all the ruled directions. So because of this there are many applications which they use ruled surface. In fact these were also used extensively in an aerospace application. When people were looking for free from surfaces, instead of going for let’s say a bicubic patch or a Bezier patch where mathematical definitions are little more complex than this.

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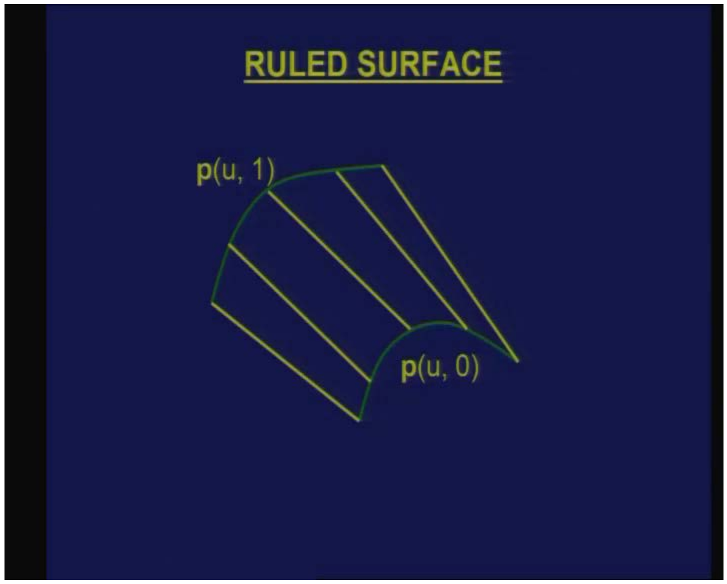

One can restrict the definitions to a simple like ruled surface. If I choose the form in this way like if I look at this ruled surface can be defined, I have one curve which is called as w is equal to zero curve and here which is w is equal to one curve. Parameter u is changing along the curve 0 to 1, parameter is changing. If I just take the corresponding points, for example I take u is equal to 0.5 on this curve and u is equal to 0.5 on the other curve and just joint them by straight line corresponding parameter values, the resulting surface is nothing but ruled surface which is shown here and a special form of ruled surface is what is called as a bilinear surface.

Suppose if these curves which are being interpolated are also straight line like I have w is equal to 0 curve which is a straight line, w is equal to 1 is also straight line. And you have a linear interpolation of two lines in a space then also you obtain a very complex form of surface which is called as a bilinear surface. As long as these 4 points which are defined are not coplanar, suppose if 1 2 3 and 4 points which are use to define a bilinear surface are coplanar then you get basically a plane which is a simple.

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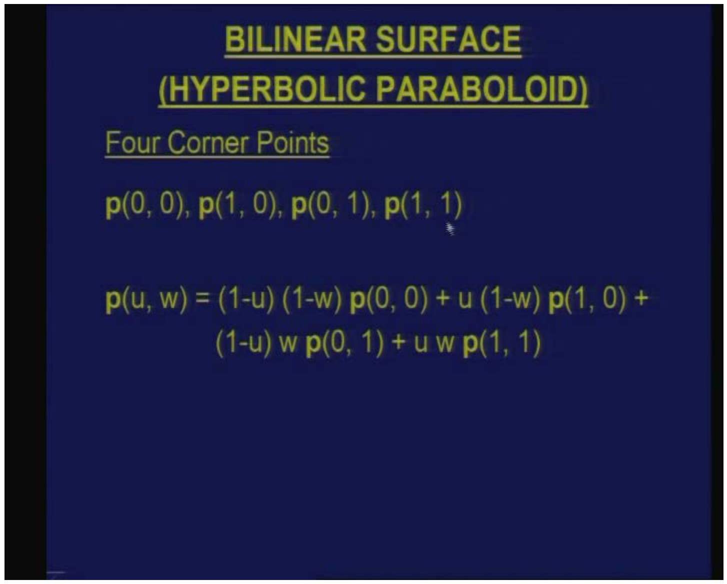

So a bilinear surface is also called as more common name is hyperbolic paraboloid and the definition is even more simpler than the ruled surface. all that which you need to do in order to define a bilinear surface is to define just four control point sorry, four corner points like u is equal to 0 and w is equal to 0 then u is 1, w is equal to 0 and this is u is 1 and w is equal to 1. If I give 4 corner points, 4 corner points means just 4 into 3 12 values. Given a 12 values, I can define a bilinear surface and a bilinear surface is linear in both u and w.

So you will take terms like 1 minus u 1 minus w, we can very clearly see if I substitute u is equal to 0 and w is equal to 0, I end up at one of the corner points. If I substitute u is equal to 1 and w is equal to 1, I end up at another corner points. So this is basically probably simplest form of defining a surface mathematically after let's say a definition of a plane. plane may be probably simplest but after that you have, this is one of the simplest form of defining a free form surface is just linearly interpolate four corner points which is called as a bilinear surface.

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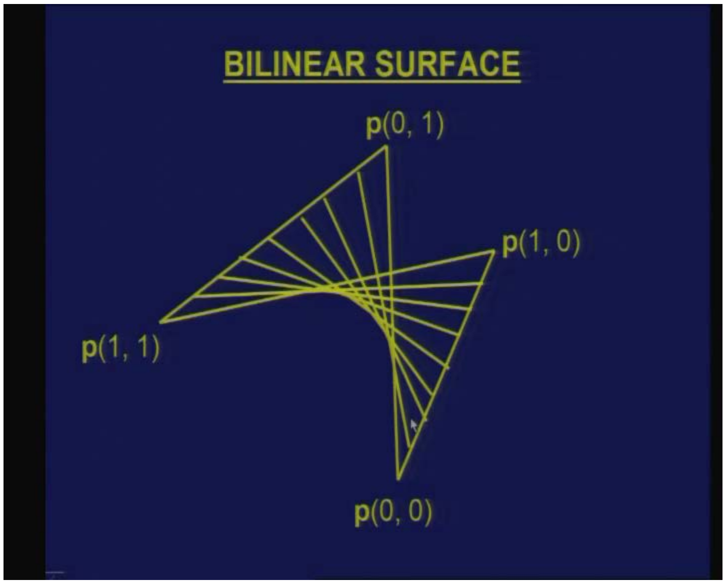

Like, this shows a bilinear surface a typical. So I have 4 corner points which are shown here and what is done is you take the corresponding value of parameter on both the curves and then join them by straight line. though surface looks like a complex but mathematical definition is quite simple like many of the, one of the bilinear surface is one of the common forms chosen for a like big roofs or many of the buildings like the roof which you have an IIT Delhi convocation hall is also a bilinear surface because it's easier to construct. At the same time the curve is really curved in, it's a free form curve it also gives an allegiants because curvature is continuously changing from point to point that is principle curvatures. So this is one of the simplest and allegiant forms of defining a surface.

So this brings basically we can say a completion of our study of surfaces. We looked at four categories, two in our previous class and two in this. Given a particular situation, I can, depending on the situation I can choose any one of these forms we have also seen that one can be converted to another form also. That means many of the standard forms can be represented as swept surface, ruled surface has commonality with sweeping. So there are lots of common things between these surface definitions. And we have also seen that the definitions of which we have given to curves and surfaces can also be extended to solids but we will study mathematical that is geometric modeling of solids in a slightly different way because most of the solids which we use in our day to day life have simple geometries. And we would like to give what we call as a solid modeling techniques. There are three major categories which we will take up when we meet next which will be the subject that is a solid modeling.

So one can say that a geometric modeling constitutes study of geometric modeling, two different we can say two different major branches. One is study of curves and surfaces what we did so far and another is study of solids using a solid modeling techniques like boundary representation, constructive solid geometry or special enumeration techniques is another this thing. So that would be the subject for our next class. So this is, I will stop it here and if there are questions please raise them out. Is that clear? Then if there are no questions we will stop it here and we will meet next for our lecture on solid model.