CAD / CAM
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Lecture No. # 8
Parametric Surfaces

Earlier we spent about 4 lectures discussing about parametric curves. So we looked at interpolation curves like Hermite curves and also we looked at approximation curves which include Bezier curve, B-Spline curve and also we extended that to the definition of Nurb's. Now from today onwards, we will be discussing surfaces that are parametric surfaces. After having studied curves, understanding surfaces is not really difficult because most of the definitions which we studied as a part of curves can be directly extended to surfaces also. So that we will see as we go along. What I will do is in the beginning of this course, we looked at parametric representation of curves and surfaces.

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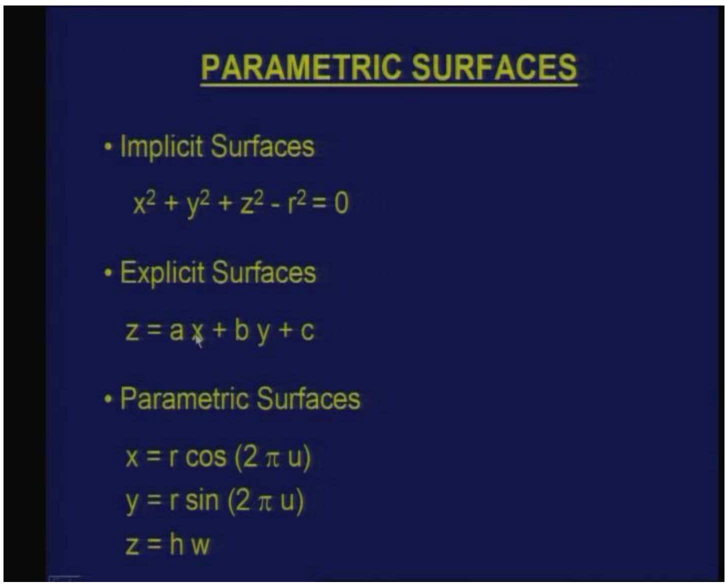

So even with surfaces you have all the three representations which are available like I can define a surface an implicit surface. So here is a simple implicit form of a surface, you know what this surface represents. So it's a sphere with a center at origin and radius as r units. Here is another example of z is equal to function of x y which is an explicit form of surface. So this is a simple plane equation which is there or I can also have like a parametric representation of the surface which is shown here. We have earlier discussed that if xyz are function of a single parameter it's a curve, if they are functions of two parameters then it's a surface. So here you have an example of xyz which are functions of two parameters u and w and you also know what this particular equation represent. So this presents if I have a variation of u and w, I get a cylindrical surface. Again with this thing like axis of the cylinder is along the z axis and whereas radius of the cylinder is r units which are shown there and parametric surfaces are also bound like you try to give bound for u and w. So if I say, if I put bound for u as 0 to 1 then I get a complete cylindrical surface one complete circle. If I put let's say value of w also as 0 to 1 then you also know what the height of the cylinder is, so this goes from 0 to h. So h is the height of cylinder which is this but we will be discussing primarily parametric surfaces in this particular case but before I do that there is one specific, you can say an implicit form of surface equation which is of interest is called as quadric surface.

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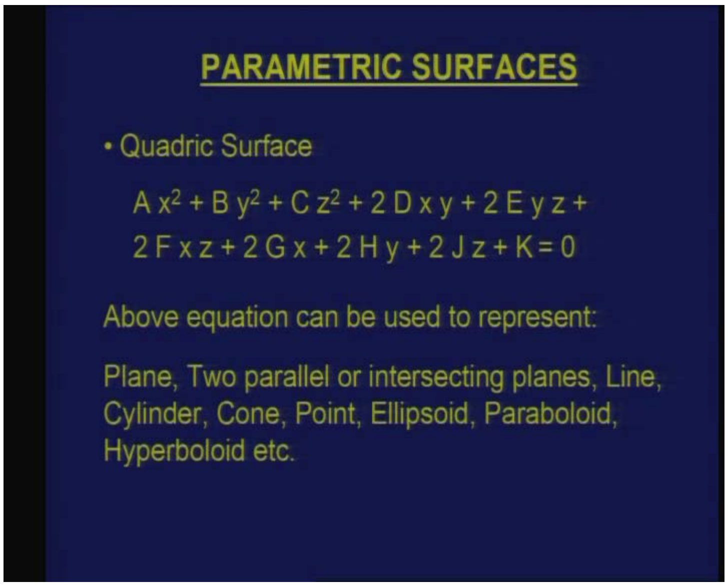

Now what is a quadric surface like if I have a equation like this what are you saying in this equation is that you have all the terms which up to like a degree of 2. So if I have xyz are the, you can say the Cartesian system then you have terms like x square y square and z square. I have terms like xy, yz and zx, I also have x term y term and z term and constant term. Now this type of equation is you can say an extension of having only a linear equation. If I look at for example only xyz and constant term, it represents a simple plane equation. If I extend it to one more higher degree then this is called as a quadric surface in fact it represents a family of surfaces, it is not a single depending on whether these coefficients AB CD up to K if they meet certain conditions if depending on the conditions, it can be used to represent many of the surfaces not only surfaces even one can look at other entities.

For example it can be used to represent a plane or it can be used to represent two planes which are either parallels or intersecting. I can use this to represent a line, it can be a cylinder or cone, it can be just a point or it can be ellipsoid, paraboloid or hyperboloid. So variety of surfaces can be represented. This is also used extensively in many of the CAD applications where I use a quadric surface to represent a family of surfaces in many cases. But as I said this is just one of the implicit form of surfaces which we are using and you can also extend it to cubic but that is of not much interest in at least as far as I know. Now coming to the parametric surfaces, now how do you define let's say a parametric surfaces of our interest here. It can be done in number of ways like how do you construct a surface or how do you model a surface, how do you mathematically represent a surface is what is of interest here.

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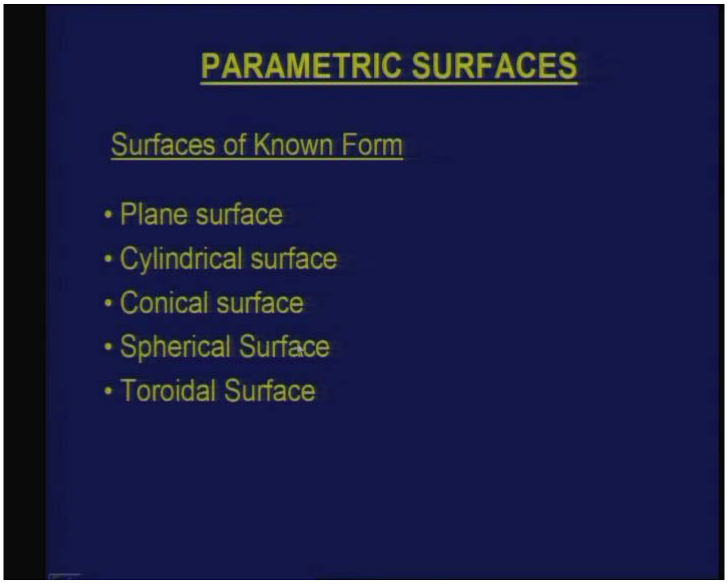

Let’s start with surfaces of known form. We are familiar with these surfaces; we know what a plane surface is. This also like when you say a known form, the form is also known for example a plane surface has a known form called flatness. Then you have a cylindrical surface which has a form known form called cylindricity then you have a conical form which is called as sometimes called as conicity and you also have a spherical surface which represents a form sphericity and there is a toricity which is a toroidal surface.

So these are like some of the very standard known forms of surface which are also studied in explicit and implicit form in courses like coordinate geometry or 3 D geometry or solid geometry, when they are taken up in schools and colleges. Now I think you are familiar with all of them, I don’t know whether you are familiar with toroidal surface.

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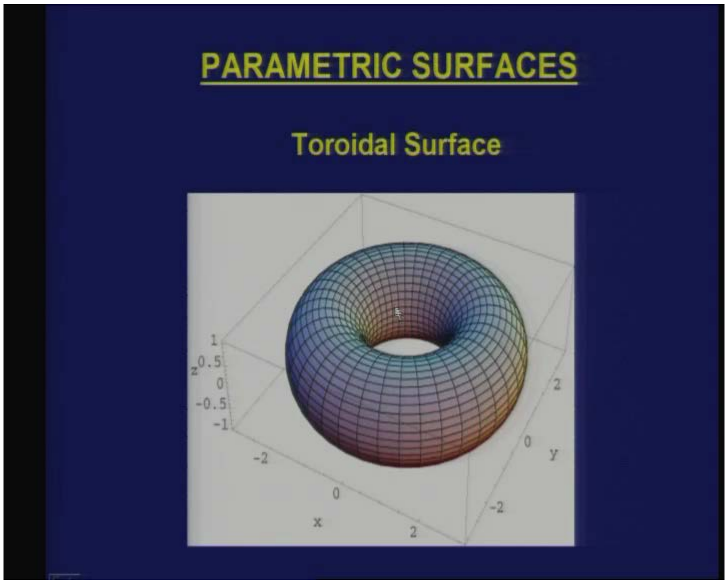

A toroidal surface looks like this. So this is torus, it looks like as if you have a rubber tube which is filled with air. So whatever is the surface which is representing is a torus. This also has two parametric directions, one parametric direction is this, another parametric direction is shown. So the parametric lines are also shown for a torus. Now these five surfaces standard known form in fact you will find in many of the component geometries. Usually whenever we try to design component geometry, we see that it has features which are combination of these known forms only because they have a direct relation with manufacturing. They can be very easily realized, they can be easily produced by any of the manufacturing processes.

So that is why they are also known form or when one goes for let’s say an inspection things like cmm using a coordinate measuring machine, you too have these known forms as features which can be measured like I can measure cylindricity, I can measure toricity, I can measure flatness which are known forms of this. So one of the ways to define parametric surfaces is what we know is the known forms of surface.

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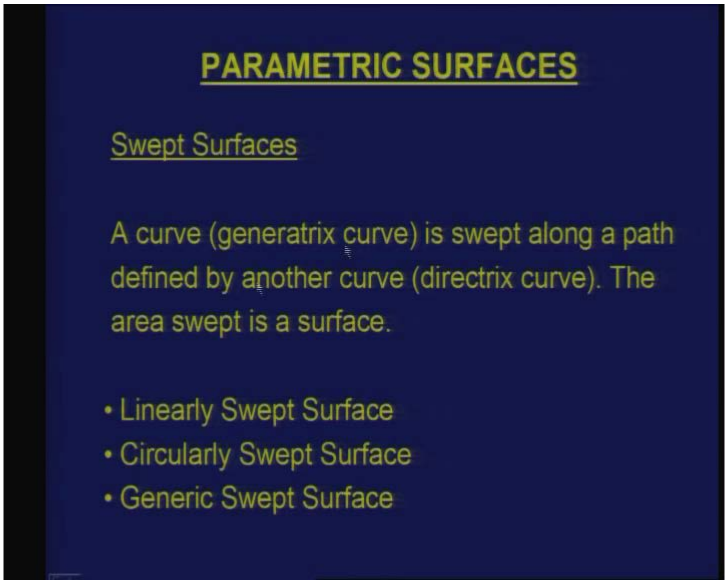

Then probably one of the most important ways to define a parametric surface is what is called as a swept surface or a concept of sweeping. So in swept surface, what is done is you take a curve, it can be a straight line, it can be arc, it can be any free form curve, this is called as a generatrix curve. Now this is swept along a path which is defined by another curve which is called as a directrix curve. Then the area which will be swept by the curve is a surface that I call as a swept surface. Now sweeping is a concept which is not restricted only to surfaces, one can sweep a point along in fact whenever we define a curve it is nothing but you have a point which is swept along a certain path.

So the area swept sorry the distance which is swept by the point, the path which is swept by the point is what you call as a curve like when we represent a parametric equation xyz as a functions of u, you can say that u is a parametric this thing which is a sweeping direction. Then among the swept surfaces there are a few which maybe of which are commonly used like most common is of course a linearly swept surface where the directrix curve is a straight line. So the direction along which you are sweeping a curve is basically when it is a straight line, you call it as a linearly swept surface or if the directrix curve is let's say arc or a circle, it's a circularly swept surface but we need not restrict to these two directrix directions only I can take any arbitrary curve.

I can take one arbitrary curve and sweep it along another arbitrary curve to obtain a surface so that you call as a generic that means you are extending the sweeping concepts to direction rather than line and circle which is there. But of the three the special interest is of line and circle. Again, as I said line and circle are the known forms. If I would, if I can have a combination of let's say a free form and a known form then I can use let's say I can take a free form curve and sweep it along line or a circle to obtain let's say a mixed form of surface.

One direction is a known form one parametric direction another the parametric direction is a freeform. So this is one of the very commonly used this thing. You may have seen like when you go to a CAD package and try to model let's say a curve or a surface even solid for example sweeping can be extended to solids. If I take let's say a cross sectional area or let's say a surface itself and sweep it along certain direction then you get the volume which is swept is a solid. When you use a CAD package to model the component what you are, most of the time which you use is like sweeping.

You say extrude or revolve, they are all basically extrude and revolve are nothing but linear and circular sweep which we are using. Then you also see a sweep as a option in many of the packages which basically represents the generic sweep which is shown here. So this is one of the very easy, sweeping is one of the very easy convenient way to build objects and also convenient way to represent mathematically.

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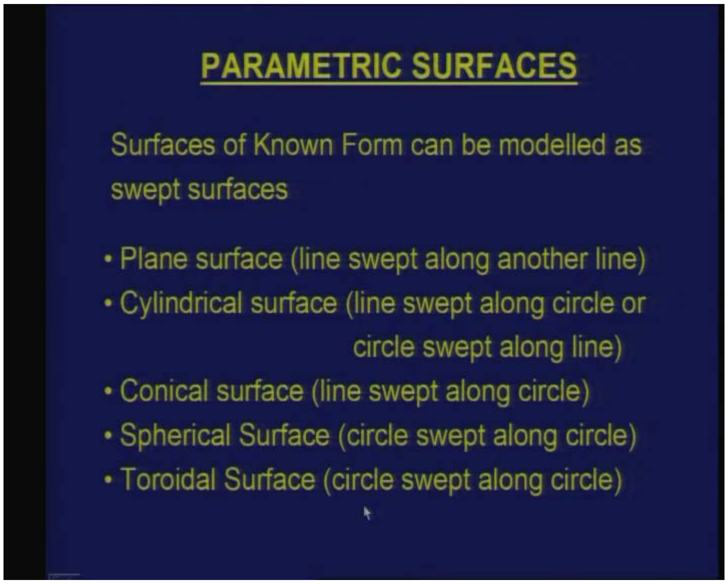

So we will look at swept surfaces little more in detail. So this is a third form. First form was the known form of surfaces second is using sweeping to define and of course before I go to the third form here is like surfaces of known form can also be modeled as swept surfaces like we have looked at 5 known forms they are all nothing but you can say they can also be modeled as swept surface. A plane is nothing but a line is swept along another line that means both directrix and generatrix curve are straight lines. Cylindrical surfaces have two combinations that is a line is swept along a circle which is there. So in this case line is basically a generatrix and circle is a directrix or it can be other way round I can take circle as generatrix and line is a directrix. So there are two combinations by which I can represent a cylindrical surface as a swept surface.

Conical surface, a line is swept along a circle which is there. Spherical surface is a circle swept along another circle. Then toroidal surface is also a circle which is swept along another circle. Only thing is both are, in both the cases you have circle as generatrix and directrix but only thing is there is a difference in terms of where these circles are situated and what is the radii of the circle that we will define. So, one can also say that maybe though topology is different, a toroidal surface reduces to spherical surfaces for certain conditions. When the radius and position of the circle matches so I can, toroidal surface basically converges becomes a spherical surface. So all these surfaces can be in fact when we try to write mathematical representation that is parametric representation of known forms, we make use of sweeping as a concept to do that because it's easier to write in this case.

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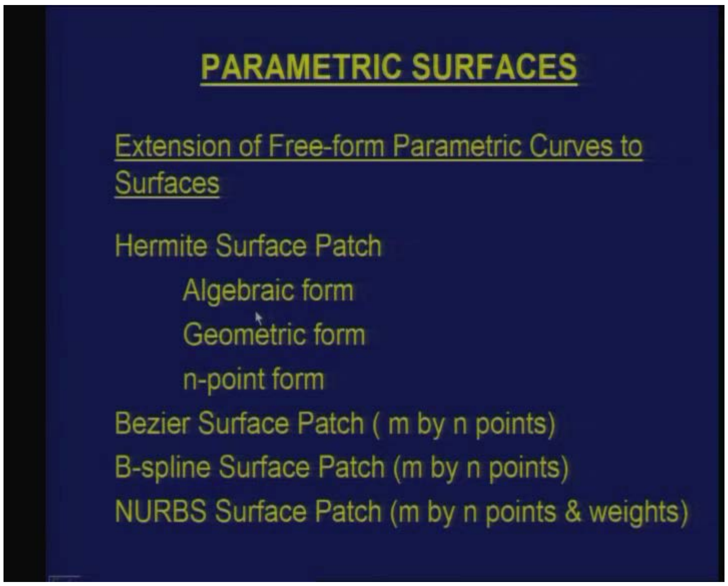

Coming to the third form like we have seen the known form of surfaces, we have seen sweeping can be another method which can be used to represent surfaces. Then whatever we discussed in our previous classes for free form curves like Hermite curves, Bezier curves, B-Spline curves and Nurb's curve. The same definition can be extended to surfaces like I can call it as a Hermite surface patch. Now I am bringing the word patch, patch is basically represents a surface which has fixed boundaries. That means unlike like, just like we differentiate between a line and a line segment or a plane and a polygon, so you also differentiate between a surface which extends infinitely and a surface which has definite boundaries, you call as a patch, a surface patch.

So one can define a Hermite surface patch, I can use algebraic form to represent. If you recall when we discuss the parametric curves, how many unknowns were there? So we had 12 unknowns. Now here I have xy and z is parametric equations that are cubic in both u and w. So if how many terms are possible for like if I had like 12 unknowns in the case of parametric Hermite curve that is a cubic curve. So if I take let's say a Hermite surface patch which is both cubic in u and cubic in w then for x itself, I have 16. So there are 48 algebraic coefficients which can be used to represent in algebraic form of a surface. Now that is only for what you call as a bicubic. It is not necessary that I should chose the same you can say a degree of polynomial for u and w. I can for example go up to cubic in u and I can restrict to only let's say a square terms as far as w is concerned. So you get a different combination. But again the one of the most combination is what you call as a bicubic which is cubic in both u and cubic in w. So if I am taking let's say a bicubic this thing, algebraic will have 48 unknowns that is 16 unknowns for the x 16 for y and 16 for z so, y 16 that we have seen. You have terms like constant, you have u and w terms, you have u square w square terms, u cube w cube terms then you have all the other combination u square w u w square. If I take all combinations then I have total 16 terms which comes and for xyz total 48 algebraic coefficients which come, which can be used.

So in fact when I write it in a matrix form an algebraic form, I can simply use the same matrix which is used for parametric curves and just extend the surface. So it's a very simple way to represent, we will see that later as we go along. Then there is a geometric form. When we discussed about PC curve parametric cubic curve geometric form, we gave again like 16 inputs. That is start point, end point and then the tangent vectors which are given, which also supply additional 6 values. Now here you have to basically give 48 this thing like if I am defining let's say a surface patch then I will give 4 corner points.

So the 4 corner points will give me xyz values. So 4 into 3 how many unknowns? So I have 12, 12 which are already covered as a part of it. Then whenever there is a surface which is a function of u and w, I can take a partial differentiation with respect to u and w, so I have del x by del u and del y by del u. And this value for example at four corner points, If i take for example del x by del u as one of them or del y by del u and del z by del u all the points so I get how many more. Additional, 24. 12 for the points that is 12 for first differentiation that is partial differentiation with respect to u and another 12 for partial differential with respect to w but that is enough.

So I have to go for one more differentiation, so you also use the values for del square x by del u del w. These are called as just like we had a points and tangent vectors, the same thing del x by del u is called as a tangent vectors here too but when I go for a second differentiation those are called as a twist vectors. So I can define a bicubic patch by defining four corner points then tangent vectors, you have two tangent vectors one along the u direction and another along the w direction then the twist vectors those combination will give me enough inputs to solve algebraically. So this we will see but I am just trying to give you an overview of how to define this. Then we also looked at four point form, there are four points through which a parametric cubic curve passes.

Here I define 4 by 4 that is 16 points, so this is two dimensional matrixes which will you be defining 4 by 4 and each consists of 3 points, 3 coordinate's xyz. So 16 into 3 again, I have 48 inputs and I can this thing. But you have to again choose parametric values for the intermediate points. Just like we did it in the case parametric cubic curves, so here also one has to choose. I can have n-point form. Why did I put it as n-point is for example I can have a matrix of 5 by 4, so that the curve is quantic in one direction not quantic, quartic and cubic in other direction which is possible?

So one can choose any of these combinations as far as surfaces is concerned, since you have a parametric direction. Same thing can be extended to other points like you have a

Bezier surface patch common is bicubic Bezier surface patch. in this case m is 4 and n is also 4, so you have 4 by 4 points that is 16 points which will define the Bezier curves or I can have B-Spline surface patch, you define by m and n. and you also define like in the case of B-Spline curve, we used additional variables. What is that? Other is order like for example when I choose use the value of k suppose; if this k is equal to 4 it represents some of equation.

Now when it comes to B-Spline surfaces, k can be same for u and w or it can be different. I can choose that it's cubic in both the directions or I can also choose a different this thing, so this option also exists. When it comes to NURBS, you have the same definition as B-Spline but for every control point, you also supply additional scalar value which is called as a weightage which will define this. So once we, like once we know what are the parametric curves extending them to parametric surfaces is not really difficult, it's quite trivalent. And we can also use these for example combination with sweeping. For example I can take two Bezier curves, one is a generatrix curve and another is a directrix curve and I can get a very generic swept surface which is a part of this particular.

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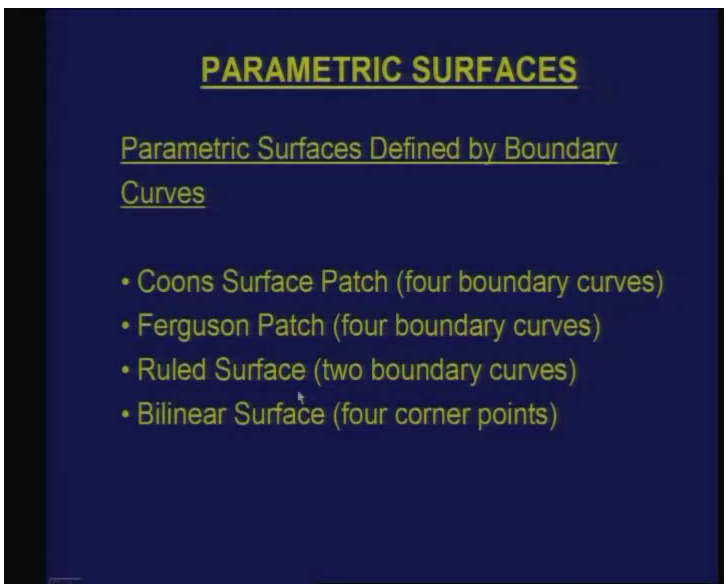

Then when it comes to surfaces, you also have a few important surfaces which can be defined by boundary curves. Let's say if I have, I am given 4 boundary curves let's say for every surface patch, the boundary is a curve. Now, these boundaries basically refer to fixing one of the parameter like for example if I say u is equal to 0 that means that becomes one of the boundary curves. When I say u is equal to 1 that is a secondary boundary curve, w is equal to 0 and w is equal to 1 are the two other boundary curves. So if I just give input as my 4 boundary curves, I can define a number of ways the surfaces can be defined that means the surface basically interpolates the boundary in some way.

The two such interpolations which have being, which have come up in the last century are Coons patch and Ferguson patch. These are surface patches which take four boundary curves to define, so we will look at the equation of this. This is also a very simple equation. Then I can also have what is called as a ruled surface which takes only two boundary curves, two opposite boundary curves and these are joined by straight lines. That means you have a linear interpolation between two opposite curves. If I carry out a linear interpolation then this is becomes a rule surface this is also of special interest in many applications. Then bilinear surface is when I have basically a ruled surface where the two boundary curves are straight lines.

It is like a best way to define a bilinear surface is just by giving four points. Whenever I give a four points, they may or may not be coplanar. If they are coplanar then you get a plane that is that's a simple equation. suppose if they are not coplanar then you need to have a curved surface which interpolates all the four points and one of the simplest way to do is to use linear equation in u and linear equation in w. So if I am using u and w as a linear then I have terms like constant term, u term, w term and u into w u w term also comes into picture.

So a bilinear surface is another special form, I think you may be aware of that ruled and bilinear surfaces are of special interest particularly in NC machining for machining advantages which is. So this is a fourth way of defining like first is a known form, second is using the sweeping, third is extending the whatever we studied for curves to surfaces, fourth is I defined boundary curves and try to interpolate in one of these ways. So if I am using let's say any package which offers modeling of surfaces, it gives almost all these options. Now you are free to choose what combination one of the combinations of this either I can use a standard form or sweep or this in order to build more complex geometries.

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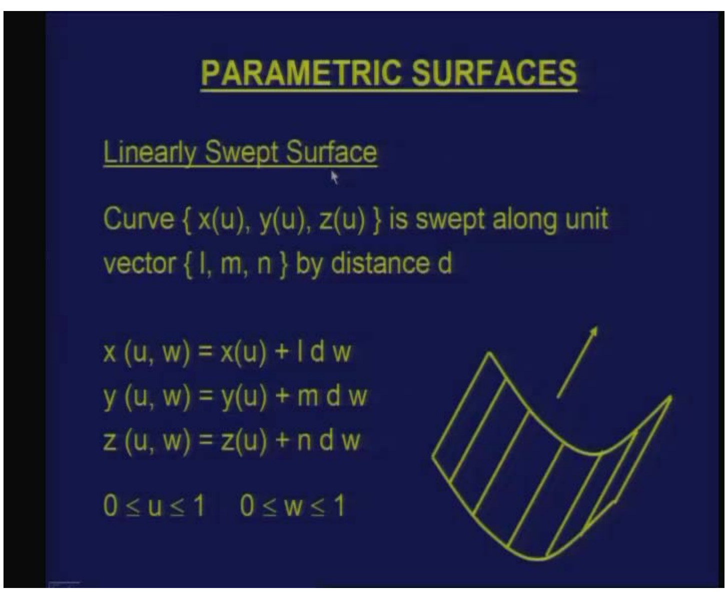

Let's look at since we are already familiar with known form of surfaces, I can take up as a first sweeping, sweep representation of surfaces as the first topic. And we have also seen that sweeping can also be used to represent known forms, so it covers two of the category of surfaces if we study about this. Now in a linearly swept surface, now we are trying to give mathematical definition. So far we were seeing only conceptually like I have a curve. So input to a linearly swept surface is a curve, what is this curve. This is a generatrix curve which is swept and directrix curve is a straight line. So I can directly specify what the direction cosines along which it is swept are and what is the distance by which the sweeping is done by specifying these two, I can define completely this thing.

So what is the input curve xy and z which are parameters of u. So the resulting surface now is parametric in both u and w, so this is the curve plus l is basically the x component of direction cosine multiplied by d and w is the parametric direction which specifies this. So this is now a surface equation with range between u and w ranging between 0 and 1 which is shown here. So I can take let's say a curve and then sweep it along a direction to get a surface which is shown here. What is the parametric direction u which is that of a curve which is already known? The w represents the direction along which you are sweeping this particular thing.

Another way to define this linearly swept surface is I take a curve which is xy and z, instead of specifying let's say a unit vector lmn and the distance by which it is swept I can just specify two points. Whenever I specify two points in a space, I can always have define a vector completely that is start point of a vector and end point of vector. This vector includes two things, I can get lmn which are direction cosines, I can also get magnitudes. So suppose if this point is x_1 y_1 z_1 and here is a point which is x_2 y_2 z_2 then I have x_2 minus x_1, y_2 minus y_1 and z_2 minus z_1. What do they represent? That represents nothing but ld, ld is x_2 minus x_1, md is y_2 minus y_1 and nd is z_2 minus z_1.

So in this case the equation becomes x of u plus x_2 minus x_1 w y_2 minus y_1 w and z_2 minus z_1 w. One very important thing is these points which are x_1 y_1 z_1, x_2 y_2 z_2 need not lie anywhere on the curve or anything, it can be anywhere in the space. As long as they when once you say a vector, it can be anywhere in this thing, it can have any starting point any end point. As long as magnitude and directions are the same, so it really doesn't matter, it's a same vector which I am trying to represent. So a linearly swept surface can be defined by a vector which has magnitude and direction that is the direction along which you are sweeping together with a generatrix curve. This curve can be simple curve; it can be a Nurb's curve or any of the curves which one can think of. It can be closed or open curve; all options are allowed as far as linearly swept surface.

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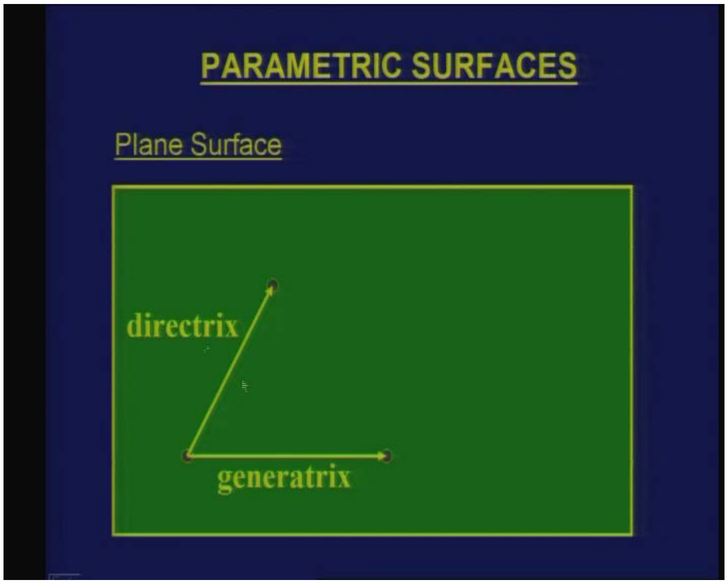

Now here is a simple, probably a simplest example of linearly swept surface is a plane. I have a straight line which is call it as a generatrix then I have another straight line along which it is swept which is directrix. So if I sweep this particular thing along line, I should get a surface which is a planar surface.

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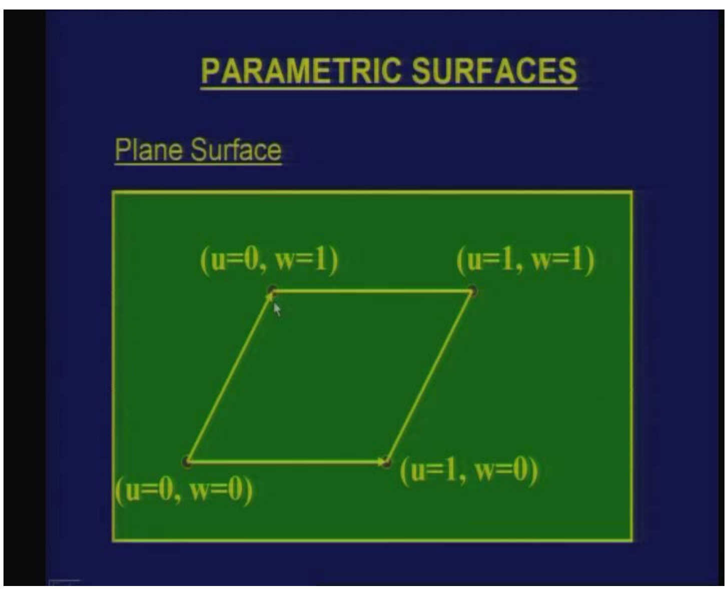

So this is a planar surface which will be obtained and you also, one should be very clear of the parametric directions like the generatrix curve which we used has a parametric u, so this is u is equal to 0, it goes all the way up to u is equal to 1. Parametric direction is a w, so it starts from w is equal to 0 and goes to w is equal to 1 and other corner represents u is equal to 1 and w is equal to 1. Now one of the, this form can also be used to represent a parametric equation of a triangle. See for example what are you specifying is I can define this entire plane by defining three points. A plane surface can be easily defined by specifying three points through which it is passing like this maybe a starting point, I have a second point let me in fact I can go to this.

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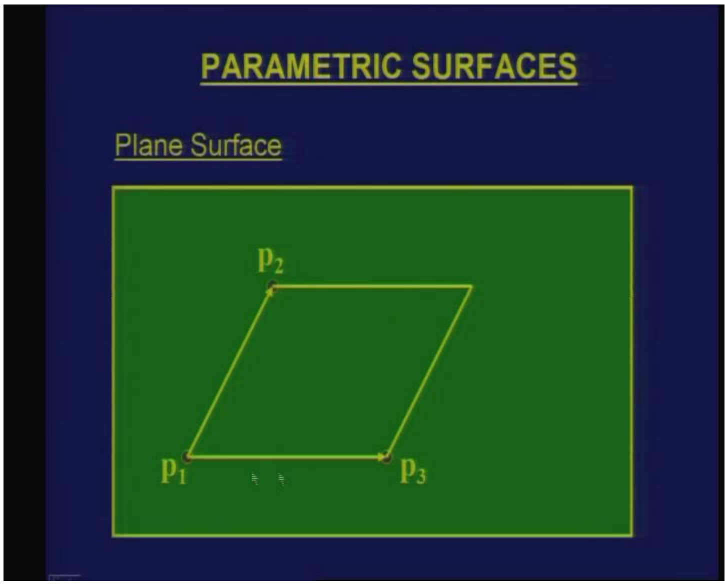

Like here is a starting point I have another point which is let's said P_3 or P_2 whatever it may be. So this gives me one vectorial direction, I have another vectorial direction which is P_2 minus P_1. By giving the three coordinates that is x_1 y_1 z_1, x_2 y_2 z_2 and x_3 y_3 z_3 I am able to represent completely the plane equation which we do. How do you do that? I know the two points, so I can get the parametric equation of line. Once I know the parametric equation of line I can apply the linear sweep equation because I know the other vector which is this thing and do this.

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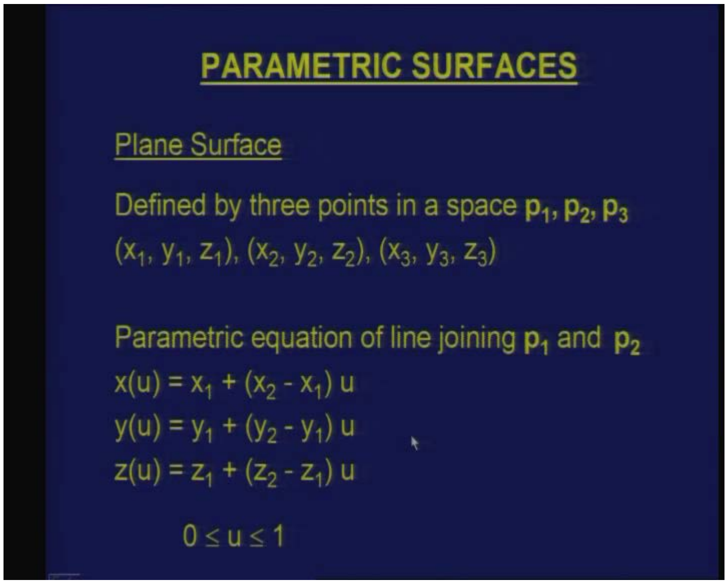

So, one of the ways to define is like given a three points P_1 P_2 P_3 which are x_1 y_1 z_1. So parametric equations which are joining P_1 and P_2, we know that it can be written in the form of x_1 plus x_2 minus x_1 u, y_1 plus y_2 minus y_1 u and this is. So this is the parametric equation representing two of the points which goes from P_2 to P_1. Now I can sweep this along a direction. What is the direction? That is P_3 minus P_1 which is another direction which is given.

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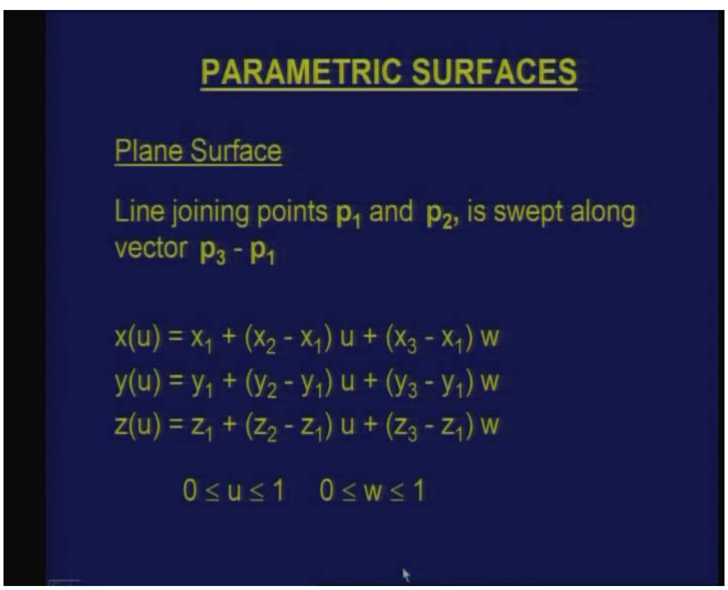

So this was, this portion was the earlier equation. Now I have to add ldw. Isn't it? mdw and ndw. ld can be directly replaced by x_3 minus x_1, y_3 minus y_1 and z_3 minus z_1. So if somebody says what is the parametric equation of a plane then this is one of the ways to define. There are three points which are given and here is I think this is not clearly represented. It's not just function of u, you should also have u and w and then this is complete equation of the plane. Now, as I said, this can also be used to represent a triangle.

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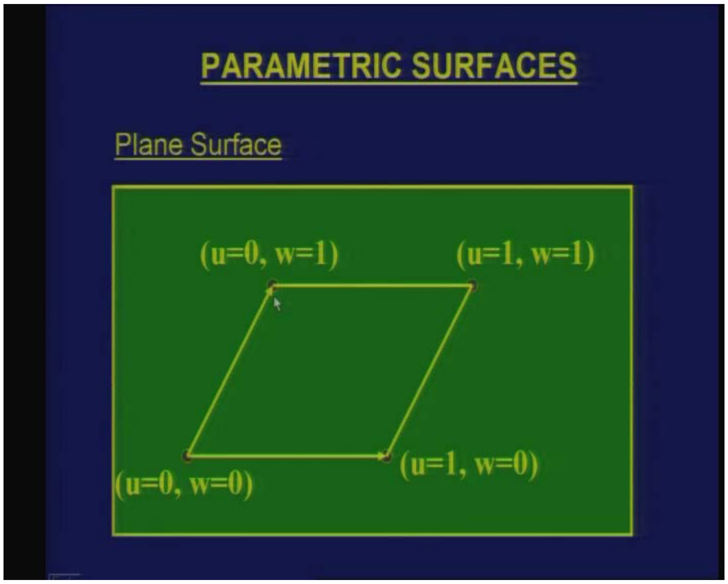

If you have seen this like given three points, what should be the parametric equation of a triangle. If I use the equation as we discussed, I am getting actually two triangles. That is this one and this two. So I can also differentiate like saying that there is one portion of this parametric equation represents a triangle. That means you can put additional condition in u and w, in order to represent that. So it can be very easily used to represent a triangle also. We will see what that condition is how you basically split a parametric surface so that you are concentrating on only part of the surface etc. So this is a definition of a plane surface, a simplest of the linear.

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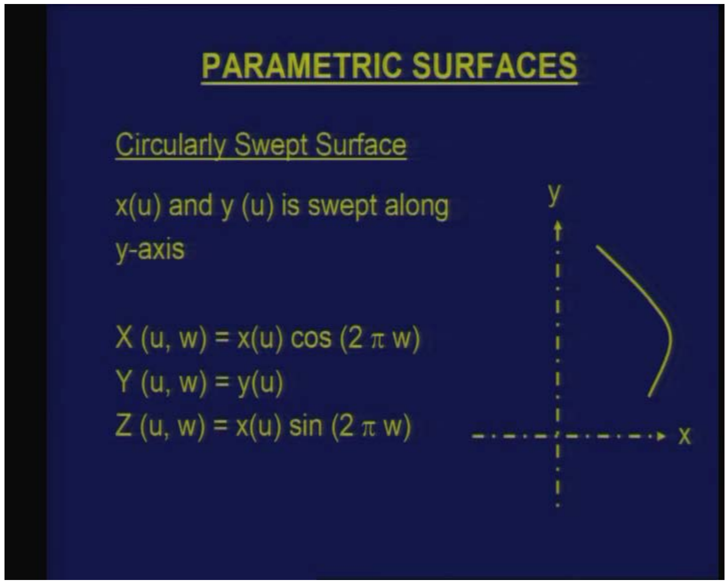

Circularly swept surface can be also be defined by sweeping let's say a generatrix curve along a circle. Now there can be a many combinations of this like I can choose that the curve is swept along one of the known axis, xy or z which is simpler to represent. Other option is I have a curve which is you are sweeping along an axis which is lying in xy plane but it is not one of the standard x or y axis or I may have a situation where the axis is arbitrarily oriented with xy and z axis, all combinations are possible. But often you choose one of the standard axes because it's easier to represent mathematically. If I want to sweep let's say a curve along an arbitrarily axis, what I do is first I transform it such that the axis coincides with xy or z axis using transformation. Apply the sweeping and then do the reverse transformation, so that I get back the original equation.

So transformations together with sweeping can be used to represent a parametric equation of let's say a circularly swept surface along any arbitrary axis. Now let's look at I have a curve let's say this is a parametric curve which is a function of u, this is a curve which is lying in xy plane. So you have x and y which is a parametric of u. now this is swept along y axis. When I sweep it along y axis, I have XYZ. Now this capital XY and Z represent the coordinates of the surface which is obtained which is a function of u and w.

What happens to x coordinate of any point? As I am sweeping, the x coordinate actually moves towards the center then goes in the other direction and comes back. So the variation is more in the form of a cos of a function. So this is the x coordinate of the point of the curve multiplied by cos of this particular function. If I have any point which is a y coordinate on this particular curve and if I am sweeping along the y axis, then y coordinate doesn't change because height of this particular point, any point from the x axis remains same. So y coordinate here will be same as that of y coordinate of the curve. Now as I am doing that initially this curve was in the xy plane. Now it goes out of xy plane and at some stage this is in yz plane, as it is sweeping then it comes back to again xy plane.

So as x coordinate is changing as a function of cos, you have the other. So this is like a circle. So if I have a cos as function, other function should be a sin which is the x coordinates of the curve. So just by using x of u and y of u and by adding additional terms, multiplying with additional terms you are able to represent a simple circularly swept surface.

Now I can extend this to like sweeping let's say circularly swept surface, when the curve is swept along x axis or let's say z axis. It's a simple, one has to basically change these equations a slightly and one can do that as a part of this. But one thing which one has to keep in mind is also that one has to consider a positive sense of rotation and a negative sense of rotation otherwise your parametric direction may change if I don't use properly the cos and sin functions in this manner. So, this is a very simple definition of you can say a circularly swept surface.

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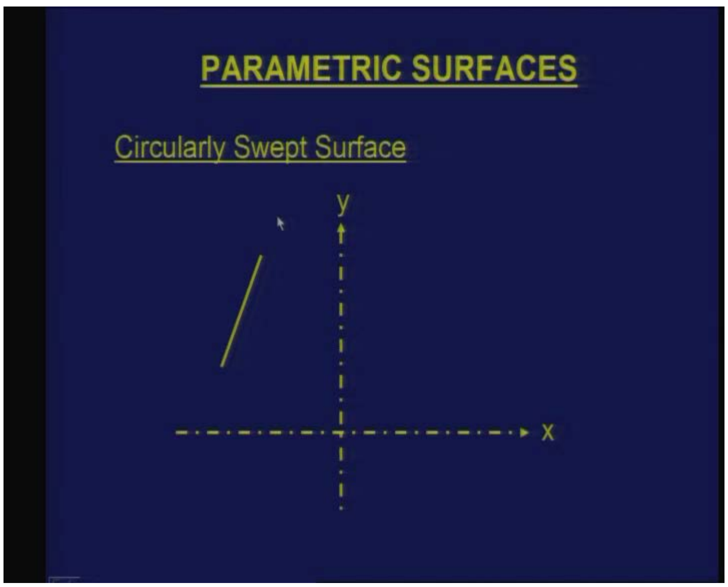

One of the of course simple examples of sweeping is let's say I have a straight line, this straight is defined by two points. I can write down the parametric equation, so I know x of u y of u on this. Now I substitute this in this to get basically a surface which is nothing but a conical surface where u represent this direction and w represent the circle which is basically obtained as a part of this.

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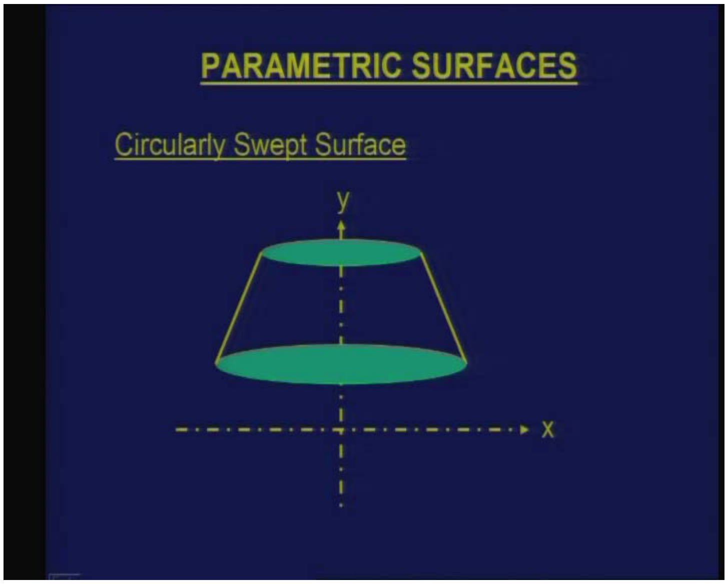

So like when I sweep it, basically I get the swept surface which is a conical surface.

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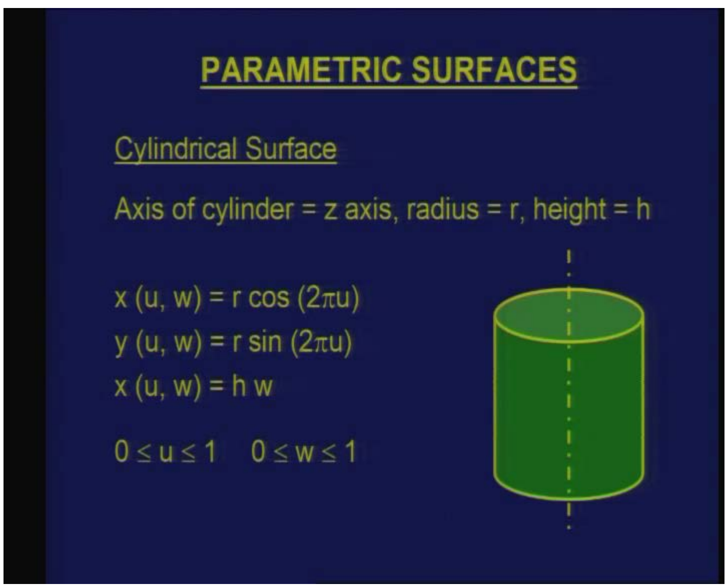

So this can be used to represent some of the known form of surfaces. For example a cylinder can be very easily represented as a circularly swept surface. I can take a straight line or another way is that I know the equation directly in a parametric form, I can put down. If I am given that axis of cylinder is z axis and radius is r and height is h, then I can directly write down the equation of this particular thing. But do you think this is like given this kind of a thing, is it a unique kind of a thing. If this is the input, axis of cylinder is z axis, radius is r and height is h. Is this equation unique or there can be some changes like in this particular input what is not shown is what is the starting point of let's say a cylinder or where does the cylinder lie along z axis.

Now this is represented here. Suppose if I put w is equal to 0 then I get, sorry this is z, I think this coordinate is z. So if I put w is equal to 0 then I get z as 0, if I put w is equal to 1 then I get z as h. So, it is actually going from 0 to h but cylinder may lie somewhere else. Now suppose it may, I can have minus hw. So if I have minus hw that means it is the same cylinder which meets all these conditions but lies somewhere else, so this equation is not unique as far as this is concerned. I may have to give additional input in order to make it unique along the z axis where it should be situated in the space, I have to give certain additional input. So this is one of the simplest, I think circularly swept surface which one can think off.

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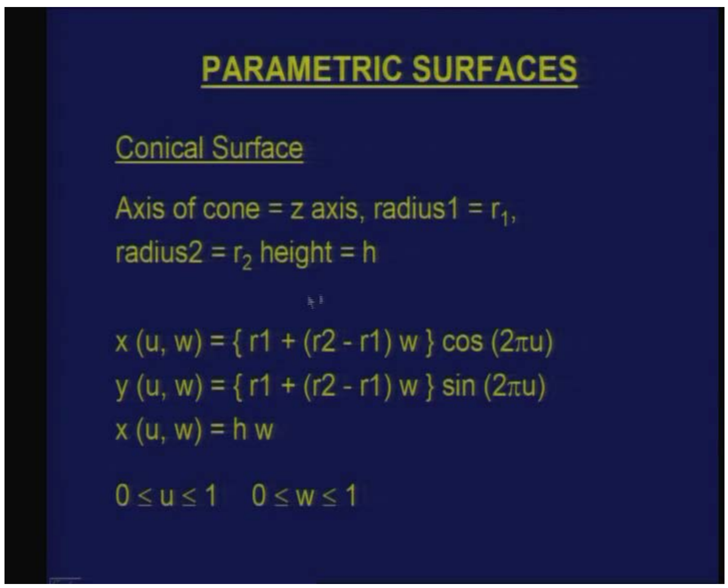

Similarly we can define a conical surface. Axis of the cone is given which is z axis and radius is given there are two radii which can be given either it can be a truncated cone or it can be a complete. So in one case it becomes probably a 0. So I have radius 1 as r1 radius two as r2 and then there is a height which is given. Now I can write down the equation in this form, so if I put let's say w is equal to 0 then I am getting one circle. If I put w is equal to 1 then I am getting another circle which is corresponds to r 2 of that and the height varies height of the cone is given as h. so this can be completely defined except for the again like the position which is not specified in this case. So this is another example of circularly swept surface which is of a known form. Same thing can be done for spherical surface and a torus surface. If I take the generatrix curve and the directrix curve both are circles, I can represent an equation of a sphere of a torus very easily using this particular form.

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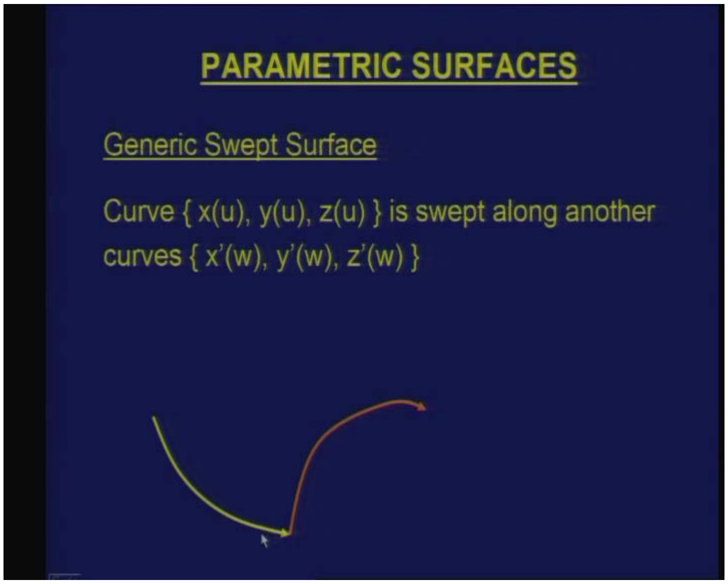

Now coming back to a generic swept surface, the input here is two curves. What are those curves? One is xy, yu, zu which is one of the curves which is given. Then you have another curve which has x prime y prime z prime which is a function of w. So this is one curve let's say this I call as a generatrix curve and here is a directrix curve which is given. Now when I sweep it, what happens is basically you have to take for example you know two parametric directions, so this is basically the area which will be swept can also be represented.

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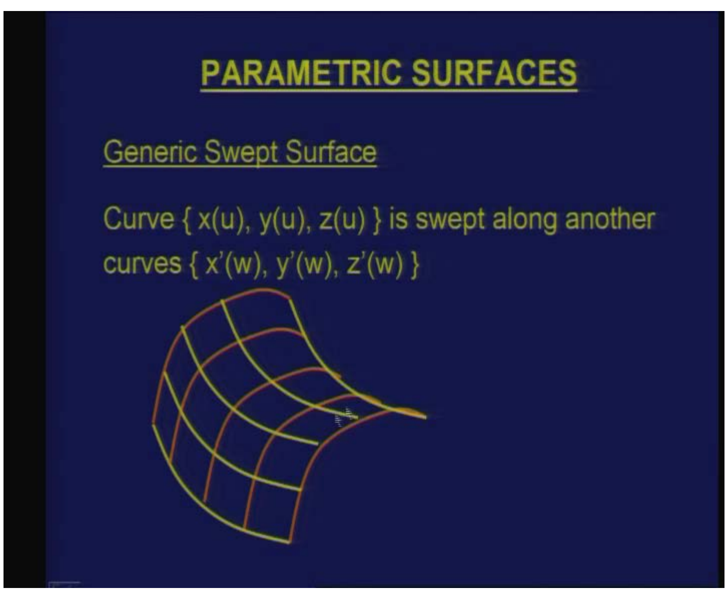

The surface looks something like this. So here is a curve, now this is swept along that. So I have two parametric directions which is given to me which basically represents area which is swept by the surface. Now in order to define this this may look like a complex surface but it is defined by just two curves, one is a generatrix curve. Curves are easier to represent than surface. So complex surfaces can be easily modeled using curves which is even this. Many a times this is used for what you call as a sweeping is used to define what is called a mixed form where one of the forms is either a line or a circle whereas the other is something which is a free form type of the curve. This is a very common in many of the CAD CAM applications.

So I think I will just stop it here because what we covered today is primarily an introduction to surfaces. What is left now is to look at two other forms like how do we extend surfaces like whatever we discussed for parametric curves to surfaces. We won't go that much in detail because we have understood the concepts. So we will put down the equation and try to look at a few common surfaces. Then we will also take up the surfaces which are defined by boundary curves which is of special interest which includes ruled surface or Ferguson patch etc which is there and once that is over then we will take up the solids. But overall the subject of parametric curves and surfaces together forms you can say one entity which one important aspect of geometric modeling. Sometimes you can call it as a surface modeling as a broader area of a surface modeling which is used.

Concepts can be extended to solids also like I can have like just like I have a segment curve segment or a line segment, I have a surface patch then I can have a piece of solid which is bounded it is called as a hyperpatch. So you have a Bezier hyperpatch, I can have like tricubic Bezier hyperpatch this is defined by 4 into 4 into 4, 64 points. If I give 64 points as an input, I can define a Hermite hyperpatch or Bezier hyperpatch or B-Spline hyperpatch or even Nurb's hyperpatch can be.

So same concepts can be extended but usually those are not very common like representing a surfaces solids as a hyperpatches is not really this thing because that kind of definitions are basically little too complex to deal with in terms of programming and others. And also since a solid can be easily represented by a boundary surfaces, so I would like to deal with those complex solids also as consisting of 6 boundary surfaces which are easier to deal in one of the solid modeling representations which we will see later is this thing. but there are special ways to represent solids which we will also see as we look at like constructive solid geometry or boundary representation or there is a third method which is called as spatial enumeration methods which are extensively used for certain applications. So we will look at those aspects when we come to solids but this is just an introduction to parametric surfaces which we had. Now I think it's a time for questions. What are the conditions that a toroidal surface reduces to a sphere? What are the conditions that a toroidal surface reduces to this? I think let me go back to toroidal surface.

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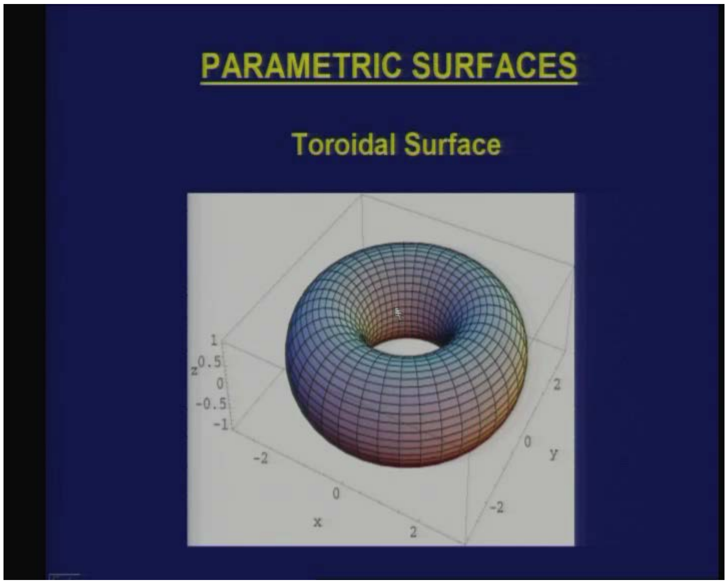

So how are you representing let's say a toroidal surface? I have an axis and like how do I basically obtain let's say toroidal surface. What is the generatrix and what is the directrix let's look at this. For example, these parametric directions which you are seeing that I can call it as a generatrix. Then this is a directrix kind of a thing. now first thing is radius of both should be same generatrix and directrix that is one thing and another thing is generatrix curve like if I take this, this particular curve the generatrix curve which I have has the same center which is the center of the torus. So suppose imagine that this circle moves towards the center as it starts moving. In fact when it comes to a sphere you need not actually take the complete circle, I can just take semicircle like how do I represent a spherical surface.

I take a semicircle which is swept 360 degrees along this. Here you are taking a complete circle and do it. When I do that when it comes to this thing, I can take only a semicircle even if I take circle still I will get spherical surface only that is new thing but though mathematically it reduces but topologically they are very different. What do you mean, what do I mean by topology is that like a torus has a hole. An object with a hole is very different from an object which is completely a solid like a cube can be easily deformed to make let's say a sphere. So they are in a way topologically they have the same. Those objects which do not have holes in terms of geometric modeling terminology, you call that they have a genus which is 0. This is a genus which has one like you know one hole which is there so that is usually represented as a genus. But it's not possible to basically deform these two, a sphere without changing the topology and other.

So mathematically though it reduces but topologically they are very different solids surfaces or solids you can say. I think that answers your question that. Yeah, any other? Yeah, you can also say that. Your question is in a bicubic Bezier patch, 16 points are there so is it like every four combination represents a Bezier curve. Yeah, it is in fact it reduces to that like you take 16 points. So this is actually 4 by 4 that is very important, not arbitrarily 16 points. It is a two dimensional matrix of three dimensional points in all the surfaces what you give is a 2 D matrix of 3 D points.

Yeah for example if I take choose 4 points at a time like 16 points are let's say represented as a matrix, if I take only one row any specific row or column it represents a Bezier curve, it definitely represents a Bezier curve. Yeah, the only thing advantage of using let's say a cones patch kind of thing is suppose I want to represent a surface, one boundary curve is a straight line another boundary is a circular arc, third is a Bezier curve fourth is let's say a B-Spline curve still I can get a very good surface which interpolates this. So you can use combination it doesn't prevent you to use a specific format but we will come back later that one form can be converted to another. That is that can be very easily done like if for example if I take let's say a Bezier curve with four boundary curves, I can probably comeback to one of the other forms we will see that later. So none of these are unique, one can be converted to another multiple ways of representing the same surface is possible like somebody can say that cylinder is a ruled surface, it is ruled surface.

If I take two curves which are circles and if I join them by let's say a straight line then I get a cylindrical surface which is also a ruled surface so one can be converted to another.
So, any other question? In the case of cone, you said the line is a generatrix and the circle is directrix so in whether we need the x is also. See once you say that directrix is a circle, circle has an axis circle cannot be defined without axis. So the axis is automatically defined. The circle can be anywhere in the space needing not be xy yz or zx plane or parallel to these planes. So once I say that there is a circle anywhere lying in the space, it has an axis, so you know the axis. So you don't have to give specifically additional input as an axis. I think we will stop it here.