CAD/CAM
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Lecture No. # 5
Parametric Cubic Curve

Last time we started the subject of geometric modeling and we had our first lecture discussing particularly about why parametric representation. So from today onwards, we will discuss parametric curves and then that will be followed by a parametric surfaces and then finally we go into solid modeling which is the subject of this. Now in this particular thing, first lecture in this particular series is tilted parametric cubic curve which is also called as Hermite curve. So we will study about this, staring with first initial introduction to curves which will be followed what exactly a parametric cubic curve is. Now when it comes to parametric curves also there are number of ways they can be classified.

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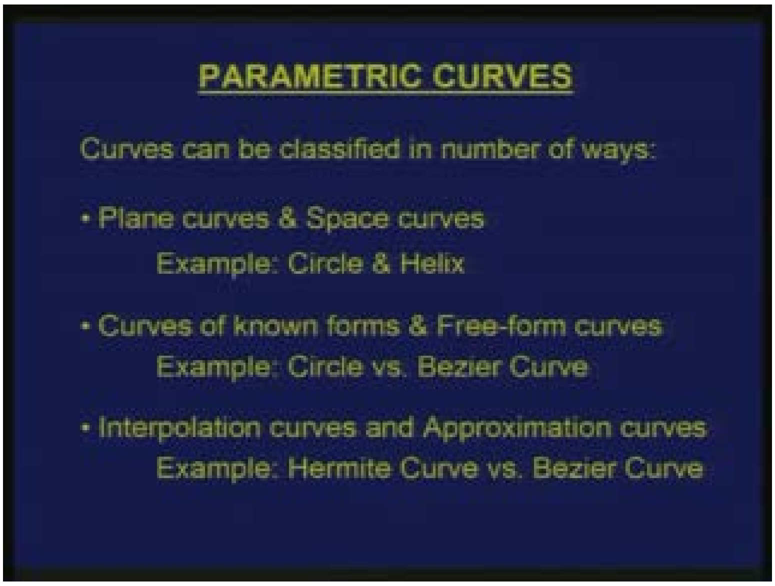

This classification is also applicable to curves which are implicit and explicit forms, not necessarily parametric like we differentiate between plane curves and space curves. So the definition is like if all the points of a curve can be made to lie in a single plane then you call as a plane curve. Otherwise if it is not possible to make all the points on a curve or a curve segment to make to lie in a single plane then you call it as a space curve. An example is for example a circle or a circular arc is an example of a plane or curve whereas helix is a classic example of a space curve which we are aware of.

Now one thing when we discuss about plane curve, when we differentiate between plane curve and space curve it is not necessary that the curve should lie completely in one of the standard planes. Suppose if I am working in let’s say three dimensional Cartesian geometry, you differentiate as xy plane, yz plane and zx plane. So the curve need not lie necessarily in one of these planes or it need not lie in a plane parallel to these planes, so even if the curve lies let’s say in any plane which is arbitrarily oriented to any of these three standard planes, still I call it as a plane curve.

A space curve is the one where it cannot be made to lie in a single plane. Similarly there is, one can classify curves as known forms and free form curves like there are certain forms with which we are quite familiar. We study that these forms and geometries extensively in our schools and colleges, for example everybody is familiar with line which is also a curve or a circle or let's say a conics like ellipse, parabola and hyperbola. So these are usually called as curves with known form.

Now the word form comes because with every geometry like line or circle there is a associated form. For example if there is something called as, a curve called as a line then you have a form called as straightness. Straightness is a form which refers to the geometry line. Similarly you may say circularity. Circularity is a form which is associated with something like a circle. Since these curves are quite common and these forms which are used are quite common, so we call them as a curves with known form. Compared to this we also have what is called as a free form curves where the form cannot be classified into one of the known forms.

For example, an example is given here is a Bezier curve which we will study later so that is nothing like a, just like a straightness or a circularity. There is a form a standard form which is associated with the Bezier curve, so this is another way of classifying. Now, most of the known forms like we are already familiar, so most of the study in this particular course or lectures consists of studying a free form curves with which we are not aware and another very interesting aspect is many of the free form curves which we are studying have been developed only in last a few decades. Particularly after the computers have come, these representations are developed in order to arrive at let's say mathematical representations which are also computer compatible.

So we can say a geometric modeling primarily consists of studying those curves which are free form curves as we are already familiar with the curves which are of known form. Then one can also classify a curve as interpolation curves and approximation curves. Now the definition here again is somewhat generic. I will call interpolation curves as all those curve where in order to define a curve I give certain conditions or let's say certain input. If these conditions and inputs are completely satisfied then I call it as a interpolation but if the given input and conditions are used they are not necessarily satisfied then you can call it as let's say an approximation curve.

For example I may give let's say a set of points and I want a curve which necessarily passes though these points. Then I can call it as an interpolation curve. Contrary to that, I may have a curve where there are a set of points which are given. These points are used to define the curve, curve may not necessarily pass through these points then I can call it as an approximation curve. And both are used extensively and curves can be converted from one form to another also as a part of representation. So we will study some interpolation curves and also a few approximation curves as a part of this lectures and curves.

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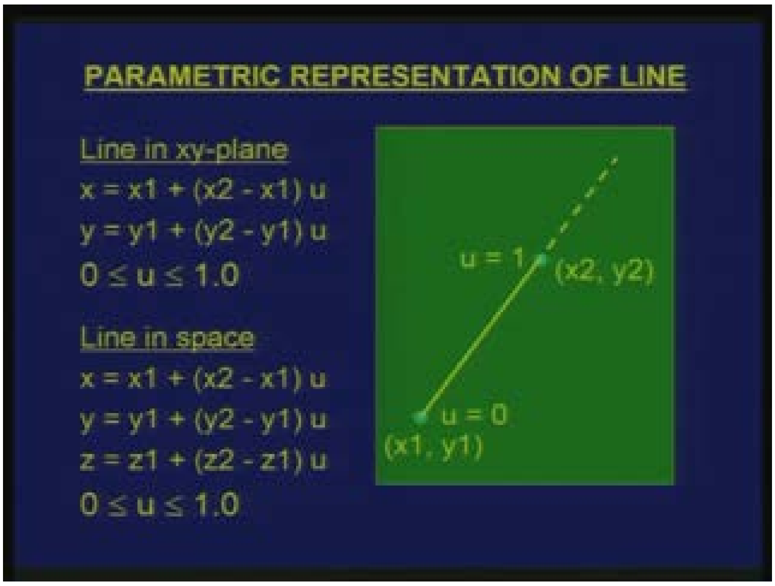

I will start with representations like parameter representation for those curves with which we already familiar to be comfortable and then we will move on to those free form curve. The simplest curve of course is known as a straight line. Now somebody may wonder whether a straight line can be called as a curve whether because it has no like it is not actually it has no curvature. So still one can, we can say a subset that is straight line is a subset of a standard curve where the curvature is zero at all the points, so or you can say radius of curvature is infinity at all the points. So it is a special curve and simplest of course is a straight line.

Now one way to define let's say a parametric representation of line is to take a start point and an end point. So there is a straight line which is joining these two points. So you are not referring to a line which extends infinitely in both the directions just like as we define in explicit form like most common representation for a straight line is y is equal mx plus c where m presents the slope and c represents the intercept which is this but in a parametric terms, you can easily define as a straight line which is passing through two points or a line segment which starts at a specific point and ends at another point. So here is a starting point x_1 y_1 and here is an end point which is x_2 y_2.

So I have a parametric representation which is x is a function which is written in terms of x_1 and x_2 and it is also a function of parameter u. And last time we studied that any functions of xy and z which are functions of single parameter basically represents a curve. In parametric representation, you also try to impose bounds it is like saying that u the parameter has a range starting from 0 and it goes all the way to 1. Now these bonds are also helpful in many ways. Firstly if I put u is equal to 0, I will get the starting point. If I put u is equal to 1, I will get like the end point that is a line segment and whenever I use a parametric representation like this I also have a direction for this particular line. It's like, there is always a fixed staring point and end point. Starting point is represented by u is equal to 0 which is x_1 y_1, u is equal to 1 gives me x_2 y_2. So there is a, straight line is going from you can say starting point to end point. I can also reverse it so the equation also changes accordingly when I do that.

The same equation can also be used to represent let say a line which extends infinitely. So in that case what you do is you just change the parameter range for u. You say that u varies from minus infinity to plus infinity. So I have a same form as y is equal to mx plus c which is commonly used. And this kind of representation is useful particularly in many ways also in terms of finding intersection of line with various entities.

Intersection of a line with a line, line with a circle or line with a surface or intersection of a line with a solid we use a parametric representation quite often and this form is very useful. This is basically an example of a line in x y plane which is shown here. I can extend this same definition for line in a space where I have two points x_1 y_1 z_1 and x_2 y_2 z_2 and these points need not, the two points need not lie let's say in a specific plane. They need not be in a x y plane or parallel to x y plane or any of the standard plane. So there is any arbitrary point in space and I can always extend the definition of line in a plane to a line in a space basically by adding one more equation that is for z as an extension of x and y. So this is a simplest known curve in a parametric representation.

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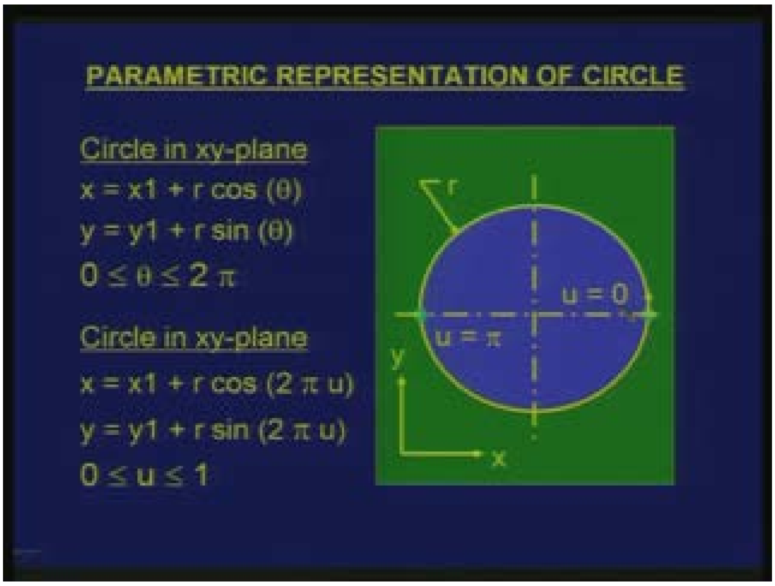

Second of course is a most common curve here is a circle and we also known that a circle is represented in a parametric form which is shown here, x is equal to x_1 plus r cos theta and y is equal to y_1 plus r sin theta and where theta has a range which goes from 0 to 2 pi. Now whenever I have a representation like this which is shown here, so x and y are the Cartesian system. And the center of the curve is represented here as x_1 and y_1 which is shown here. And the curve starts from a parameter u which corresponds to a point, u is equal to 0 and then it has also a direction. The direction is counter clockwise here and comes back all the way to the same point when u is equal 2 pi. This representation can also be used for example to represent let's say an arc also which we will see later.

Now generally what happens is in a parametric representation, I have a parameter which has a range like in the case of a circle it is going from 0 to 2 pi but it is more common to normalize this such that u is equal to 0 always represents the starting point and u is equal to 1 always represents an end point, so you try to normalize. So the same equations of circle in x y plane are written here where theta has been converted into a new parameter u where the u varies from 0 to 1 which has the same representation. This process is usually called as a normalization process in representation of curves and surfaces.

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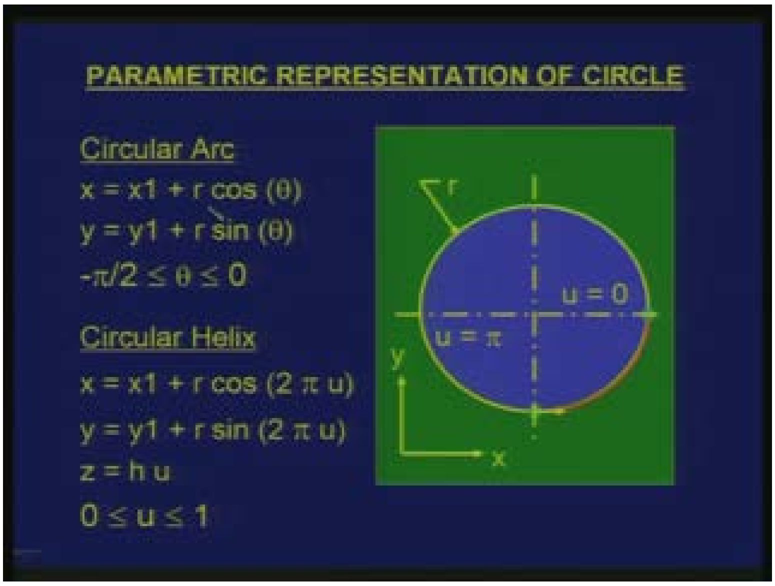

I can extend the definition of circle for a circular arc also. Now the only definition, only difference between a circle and circular arc having the same radius, same staring point and other aspects is that only thing the range changes. When the range is between 0 to 2 pi, you call it as a circle. When I have a smaller range for example I may have a range from minus pi by 2 to 0, so which is shown here in different colour. So I have a starting point which is this. Then I have an endpoint which is this, so it is going from let's say minus pi to 0. So that is another way of let's say representing and this is a representation for a circular arc.

So one can say that circle is a special case of a circular arc when the parameter has a fixed set then it becomes a circle, so arc is a common representation. I can extend the definition of a circle to a circular helix also which is shown here. This is a basically a helix which has an axis passing through the center which is x_1 and y_1. Then it has the same radius as that of a circle which was shown earlier. And now the only difference here is that as you have x and y which is changing with the parameter u, the z value is also changing that means the curve which is starts with initially lying in a x y plane and slowly moves out of plane. And it keeps traveling towards the planes which are like z is equal to 0 and z is equal to other positive values. And the curve ends like when u is equal to 1, when the z is value is equal to h. So this can also be used to represent like for example a circular helix.

I can also get different type of helix, if I have a z value which is different from that. For example it is not necessary that you always use a circular helix as a curve. Sometimes I may use a helix where as the u changes you have the radius which is continuously decreasing or increasing. So I can also represent that simply by modifying this particular equation. So this is an example of a space curve which is a known standard space curve which is shown here.

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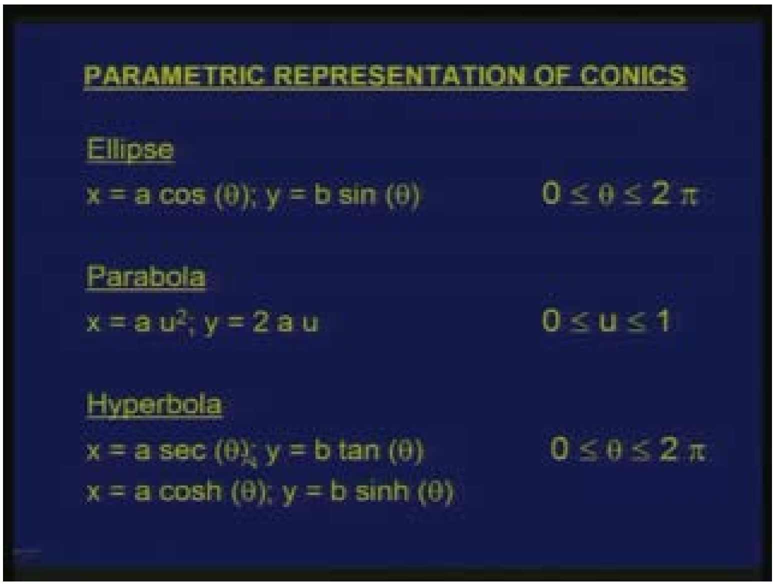

Of course we are also familiar with the parametric representation for conics. Both explicit and parametric representations are studied in colleges. So we have a representation for ellipse which is a cos theta and b sin theta, I can normalize this which is not normalized here. I can have a parametric representation for parabola, I can have a parametric representation for hyperbola either a sec theta and b tan theta. And it is also true that there is no unique representation for many of the curves.

For example circle which is represented as r cos theta and r sin theta also has another parametric representation which doesn't use cos and sin functions. So similarly we have an example which is shown here is a hyperbola which can be represented in a parametric way in two different ways, either I can use a hyperbolic function or I can use purely trigonometric function as it is shown here. These are all like examples of curves which are of known form.

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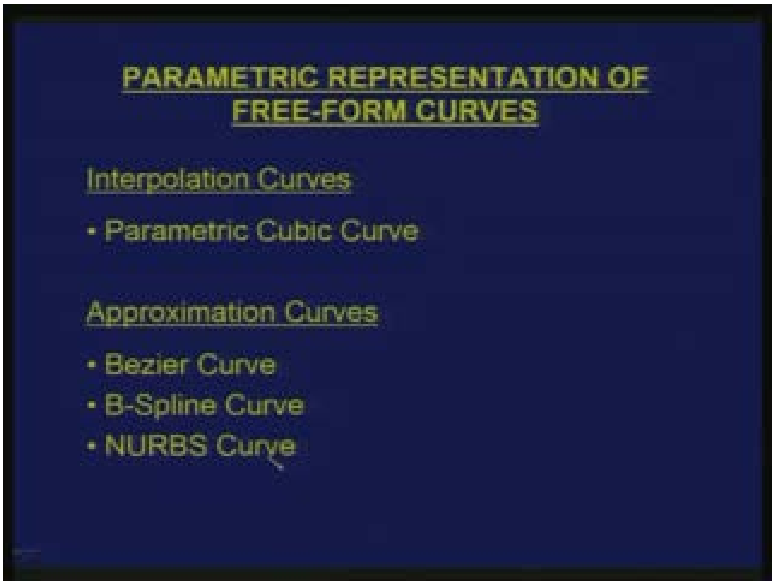

Now I will come back to our major like subject of study here which is concerned with free form curves. And in free form curves as I said we will be studying under two categories, one is interpolation curves and a second approximation curves. So we will start with first interpolation curve and one of the very common curves in this category is called as a Hermite curve or it also called as a parametric cubic curve. So we will start with that and subsequently in our other classes, we will be looking at approximation curves which are Bezier B-spline and of course the most important curve is NURBS which stands for non-uniform rational B-splines.

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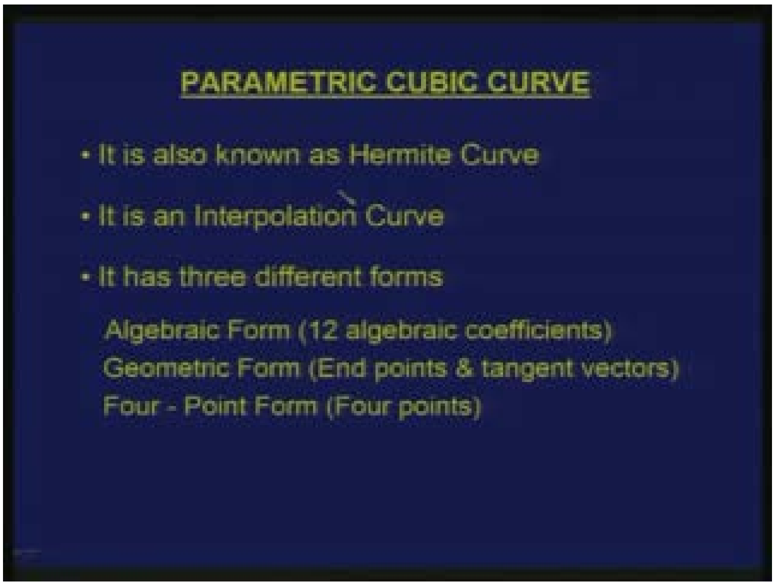

This is one you can say curve form which has become almost an industry standard and is in some way a super set of most of the curves which we are going to study. Parametric cubic curve is also known as a Hermite curve. It is an interpolation curve which I have already said. There are three different ways a parametric cubic curve can be represented. One is I can have an algebraic form, second is a geometric form and then there is a four point form. Now in order to know what are these forms, we will move ahead and look at these forms one by one.

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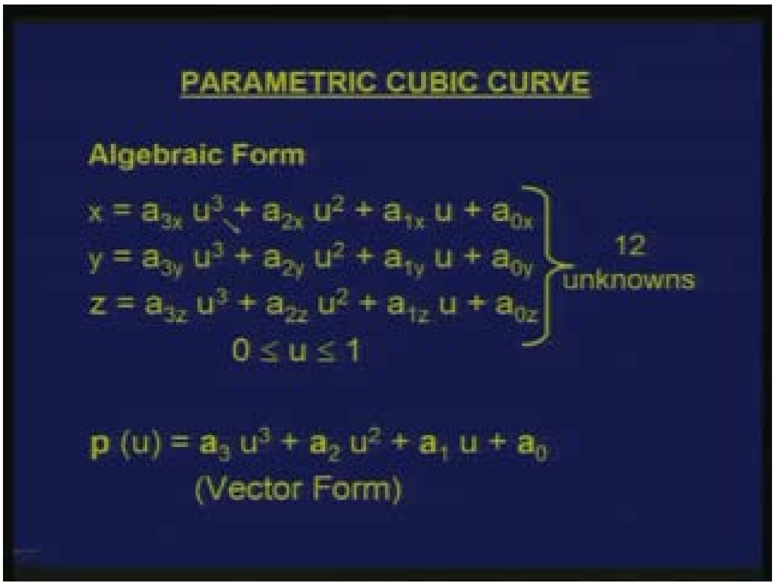

First is an algebraic form. As the name itself indicates that is parametric cubic, this curve is represented where xyz has a functions which are like cubic functions of parameter u. So x is a function of a cubic parameter of u, so which can be represented by 4 unknowns or you can say 4 variables which are shown here as a_{3x} a_{2x} a_{1x} and a_{0x}. Similarly I have y and z which are also cubic functions of parameter u that means in order to represent or in order to define a parametric cubic curve in algebraic form, one has to give 12 variable value that means if I supply 12 values which are known as 12 unknowns here which basically corresponds to these a values which are a_{3x} a_{2x} etc. If I supply these two, these 12 values I will get a different variety of curves.

Now this is, this is one convenient way in a sense you have too many variables to play around. I can vary the variables related to x like I define a curve and would like to modify. In order to modify the curve I can play around with any of these 12 variables and get whatever curve which I want, I can get it. So you have many variables to manipulate. The same is shown here in a vector form. It's very common to represent parametric curves in a vector form like what is shown here is a p (u). So what is p (u)? p (u) is nothing but a point vector which is xyz which are functions of u. So p (u) has 3 component xyz then a_3 is basically a vector which represents a_{3x} a_{3y} and a_{3z} and u cube is common to all of them. Then a_2 basically represents a_{2x} a_{2y} and a_{2z} and similarly other parameters. So the same set of algebraic equation which are shown three here, I can also represent in a vectorial form and a vector form as it is shown here. This is one of the very common way to represent a parameter.

Now, algebraic form of parametric cubic curve though it is very convenient but it is not very convenient to model in many situations. Imagine, I have to define a curve and giving 12 unknowns and playing with these 12 unknowns is somewhat very difficult. You really know which parameter effects the curve shape in which way. So sometimes it is very difficult to model the curve by giving 12 variables as an input. The more common form of parametric cubic curve is called as a geometric form.

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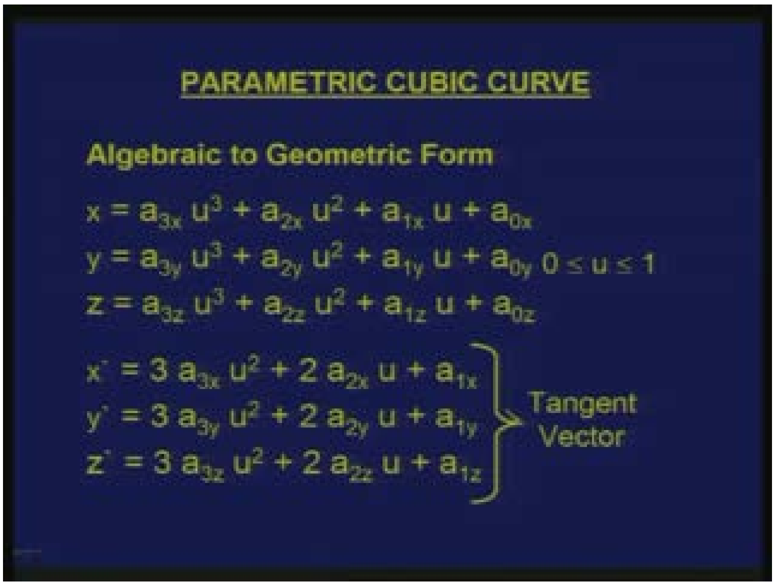

In geometric form what is done is like this is a way to convert, go from algebraic to geometric form. So this is algebraic form which we just saw in our previous slide. Now I will differentiate this particular curve with respect to the parameter u. Now if I differentiate the three curves then I am going to get these three equations that is which are basically the differential form. These are called as a tangent vectors. The x prime, y prime, z prime which are shown here basically are called as a tangent vector and tangent vectors are also function of parameter u. They vary like you have a different tangent vectors at different values of u.

As I substitute u is equal to 0 to 1, I get different values of this. Now in geometric form of parametric cubic curve instead of giving let's say a 12 unknowns as we do in algebraic form, what is done is you specify two points through which the curve is passing through, starting point and endpoint. So if I specify starting point that means I know the xyz values corresponding to u is equal to 0. Then I also specify the endpoint where the curve is ending. You are supplying xyz which corresponds to u is equal to 1. So you have already given 6 inputs start point and endpoint.

Similarly I also provide tangent vector at the starting point that is what is the x prime, y prime, z prime value at u is equal to 0 and also the tangent vector value at u is equal to 1. So you are giving total 12 inputs, earlier also you had a 12 inputs. Now only thing is you are instead of giving let's say 12 algebraic coefficients, I am giving only the start and endpoints and the tangent vectors at start and endpoints. Now this is a more convenient form of you can say a parametric cubic representation because if you really look at most of our design and manufacturing applications, more often we may have to define a curve which is passing through certain points like it has certain specific start point, endpoint and curve should have a slope along certain directions because it should have a continuity, it has to have a continuity with another curve with which it is blending. So this form that is a geometric form is a more convenient and most commonly used form than the algebraic form.

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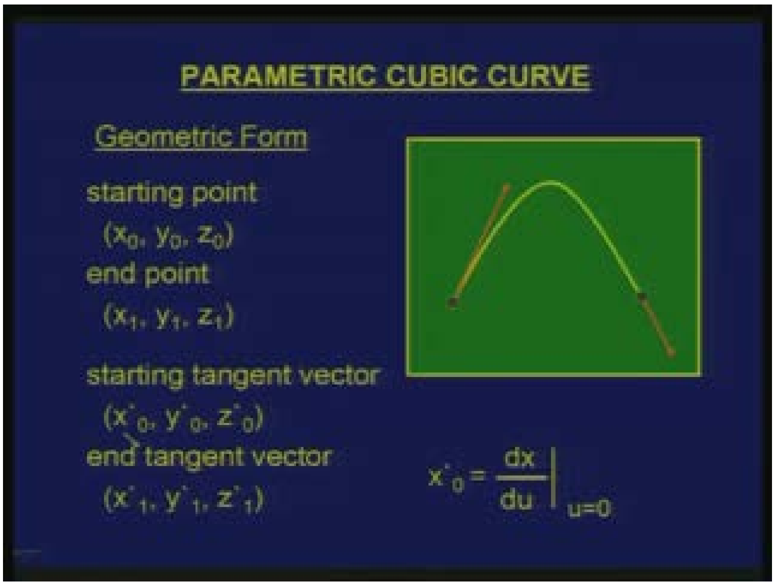

Now coming back to geometric form, what we have seen is that input which is given is the starting point x_0 y_0 z_0 which is shown as here. Then I also have an endpoint which is x_1 y_1 z_1 which are shown as another coordinate here. Then you have a starting tangent vector which is shown in red color here which basically refers to the x prime y prime z prime equations which we have written and if I substitute u is equal to 0 that becomes my tangent vector and n tangent vector which are nothing but x prime, y prime, z prime at u is equal to 1 which is the n tangent vector. So this basically, if somebody has to evaluate what is x prime 0, so you basically refers to I think it is written wrongly, this is du by dx this is not dx by du, this is du by dx sorry. I think it is correct, dx by du at u is equal to zero which is shown here which basically refers to x prime 0. And so if I am able to give 12 values, earlier you had 12 unknowns, now you are supplying 12 inputs in a geometric form, I can solve for the algebraic form.

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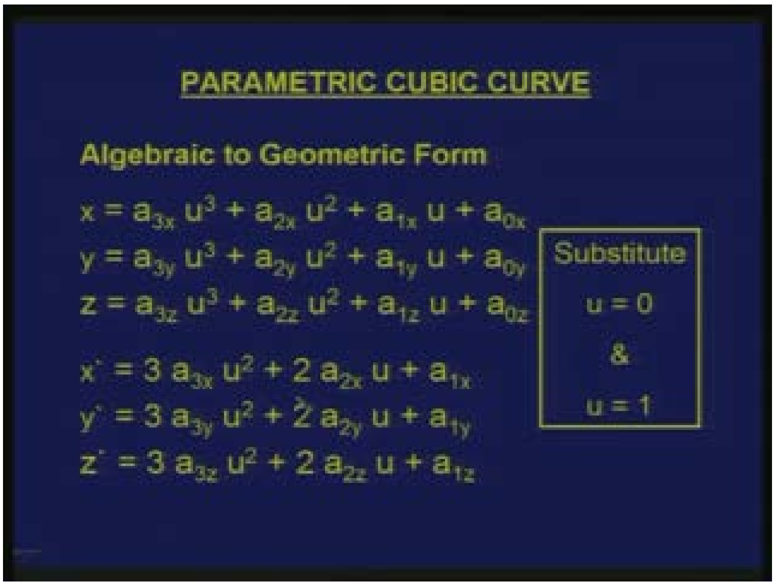

For example this was my algebraic form and these are the tangent vectors equations. Now I will substitute u is equal to 0 and 1. I can solve for these unknowns which are a_{3x} a_{2x} a_{1x} a_{0x}. And if I do that like if I solve these equations, in fact you solve independently for x y and z. For example x has 4 unknowns and now I am giving 4 inputs which are related to x. What are those? Starting coordinate of x, end coordinate of x then starting tangent vector x component and end tangent vector x component. If I substitute that, I can solve this 4 by 4 equation and get a complete representation for x purely in terms of geometric co-efficient not the algebraic coefficients as we discussed earlier.

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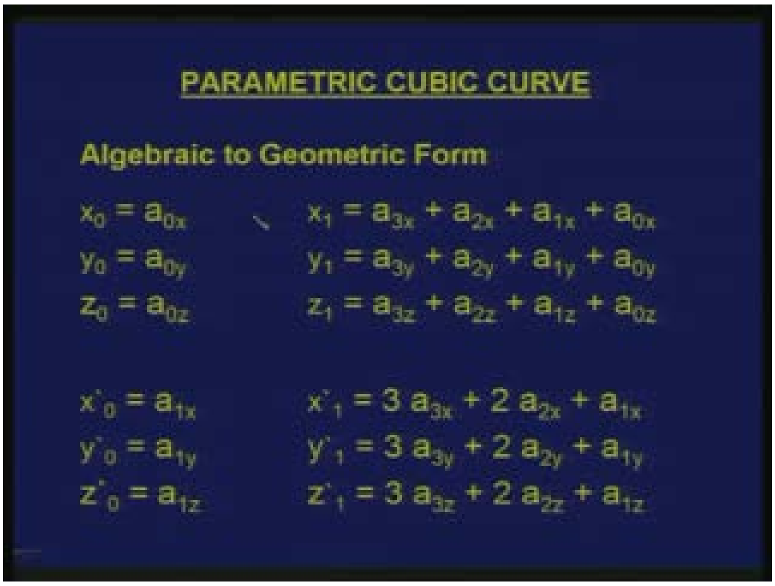

Now if I do that then these are the 12 equations which are shown here which you get. What has been done is you take this particular thing and substitute for example u is equal to 0. So then I may end up at an equation which is like x is equal to a_{0x} that is what is shown here, this is one of the equation. Next I will take this equation and substitute for example u is equal to 1 then I will get an equation which is a_{3x} plus a_{2x} plus a_{1x} plus a_{0x}. That is what is shown in the second equation.

Similarly if I substitute for example for the other set like here if I substitute u is equal to 0 then x prime becomes a_{1x} at u is equal to 0. That is what is shown here and similarly for the other value. So corresponding to x, I have 4 values. Now I will solve to get read of these unknowns a_{3x} a_{2x} etc and get a parametric cubic equation, purely in terms of my input this is x_1 y_1 z_1, x_0 y_0 z_0 and do that.

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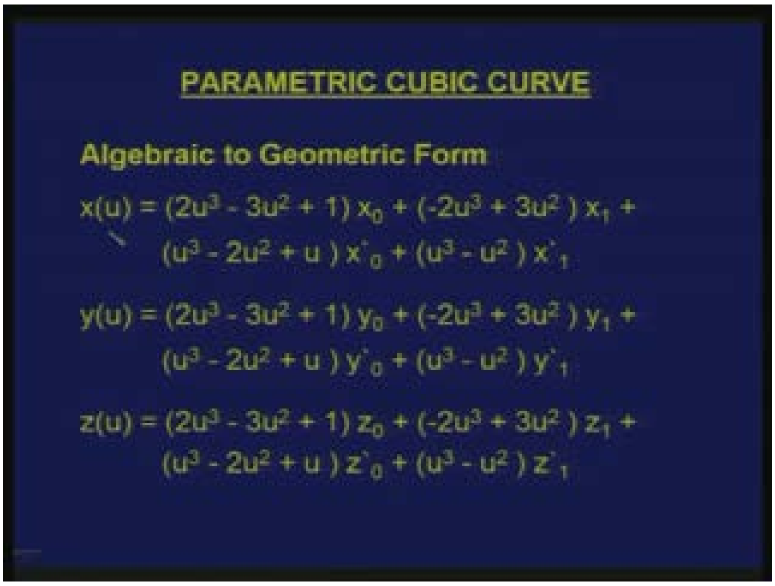

I am not going to solve here, I will just show you what is the final form which we will get it. Now once I solve and then try to rearrange the terms, I will get an equation for x. Now you see x is a function of u, it was earlier also and this is defined now it is a function of cubic equation in u, it is still cubic equation in u. Only thing is it is rearranged to show that what are the inputs which you are providing. What are the inputs? x_0, x_1, x_0 prime and x_1 prime. These are my input in a geometric form which are called as a geometric coefficients. So you are still supplying 12 values but those are geometric coefficients, not the algebraic coefficients as it was done which is inconvenient, this is more convenient. The same form can be extended to represent the y and also I can represent for the z.

Now, this is you can say a most common form of parametric cubic curve which is used. This cubic curve like if I want to go let's say one order lower, let's say quadratic equation or let's say if I want to go for a linear form they are like subset of this. Only thing is you have a few unknowns or coefficients or few coefficients become 0. So when they become 0 then you have like a quadratic curve or something but it can be used to represent any curve, any parametric curve which is algebraic in nature, up to an order which is cubic. Beyond that you have to like it is not called as a parametric cubic curve, you can go for what is called as a parametric quadratic curve or parametric quintic curve by taking more unknowns and solving them similarly one can go for that. But I think we have discussed earlier also, cubic form is enough for most of our design and manufacturing applications like because this gives you a curvature continuity at all the points.

You have a slope continuity and you also have a curvature continuity that means you have a continuity up to a second derivative which is enough for most of, you can say a design and manufacturing application. It's very rare that you go for a derivative which is beyond like a second derivative except in some applications like design of cams and others which are usually done where sometimes you need a curve forms which are beyond cubic. But for most of the applications this is enough and this is the most common form of representation.

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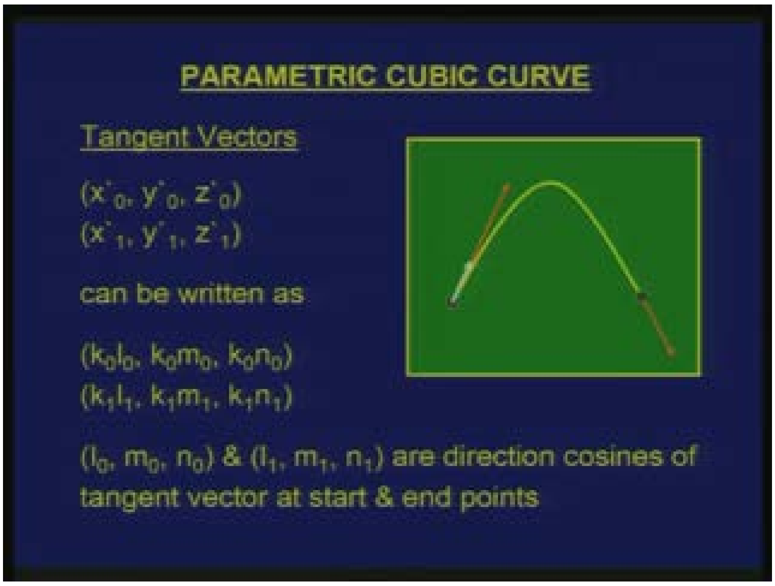

Now coming back to tangent vectors, I think let's try to understand what do we mean by tangent vector little more in detail. So we said that in geometric form, apart from the staring and end point I also provide these derivative values at a starting and endpoints. Now somebody can say that a tangent can also be represented by direction cosines. Isn't it? Suppose, if I want to represent how the tangent of a curve is happening, it is also possible to represent with direction cosines usually we use a terminology l m and n to represent the direction cosines. Are these tangent vectors different from direction cosines or they are same as tangent vectors.

So in fact they are related, for example the direction which you have a tangent vector is nothing but the same direction as those of a direction cosines which are represented but only thing is they differ in terms of magnitude. We can say that tangent vector has l_0 m_0 n_0 which represents the direction cosines and they are multiplied by a scalar value which is k_0 which basically represent what is the vector which is trying to define the curve. Now this k_0 is important if, like is it possible to define a curve where I give let's say a start point and the endpoint and only the directing cosines, like I don't give the magnitude or I don't give the tangent vector. I have a small difficulty there because whenever a direction cosines are given they are not independent, they are related. So if somebody gives me two direction cosines, I can calculate the third one because there is a related. So that means if I am giving let's say the starting and endpoint and only the direction cosines, I do not have all the 12 values to solve for those 12 algebraic equation. So that is why you also associate magnitude here that means these, like these direction cosines are multiplied by a scalar value to get k_0, that is a k_0 is scalar value.

Similarly I can use a different scalar value to multiply the direction cosines at the other end of, endpoint of this particular curve. This also implies that by changing the values of k_0 and k_1, I should be able to get different curves. So it is not the direction cosines which will define the curve it is actually the tangent vector that means as the magnitude increases, as k_0 and k_1 value has increased or decreased I am going to get a different curve.

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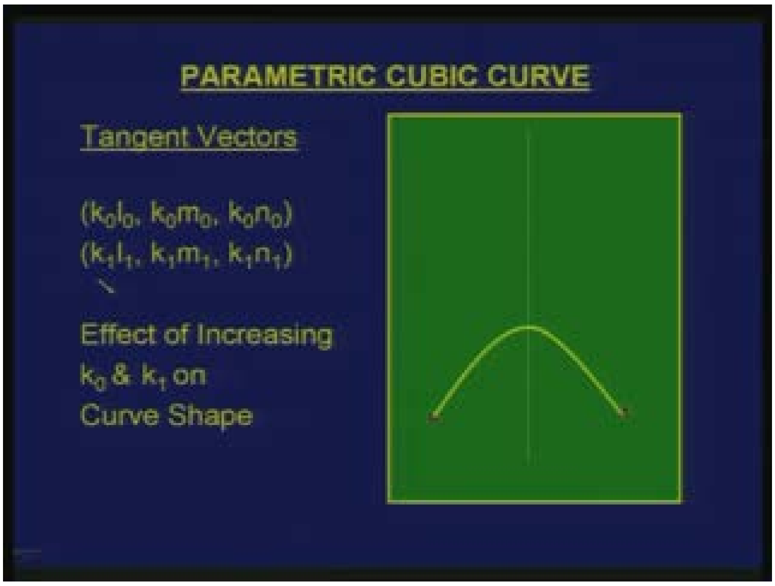

That is true with a parametric cubic curve that like I can just show you, like here is a simple example I have this curve is defined by two tangent vector let's say k_0 as one scalar value and k_1 as another. Now what happens if I increase let's say k_0 and k_1. What you are trying to do is like I have a certain magnitude which is continuously increased. As I try to increase the k_0 and k_1 values, I am getting I should get different curves. So suppose if I increase k_0 and k_1, the curve moves like I get a different curve. This curve also has the same like if I look at the tangent, it has the same direction as the previous one but the curve is different because it is a non-unique for a given direction cosines like this was if I increase further then it also starts forming loops which is a undesirable characteristic as far as the curve definitions are concerned.

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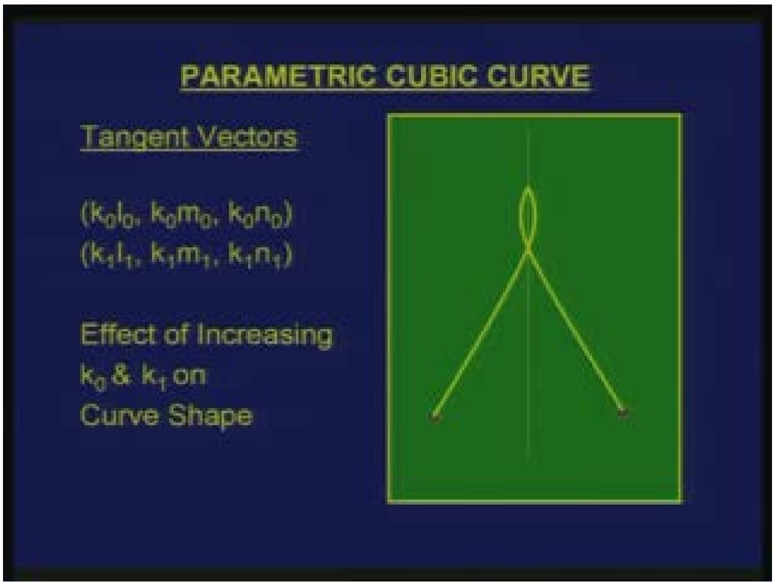

So, one has to be careful in using the tangent vectors to define curves that I don't use a very high values or use those values which are undesirable.

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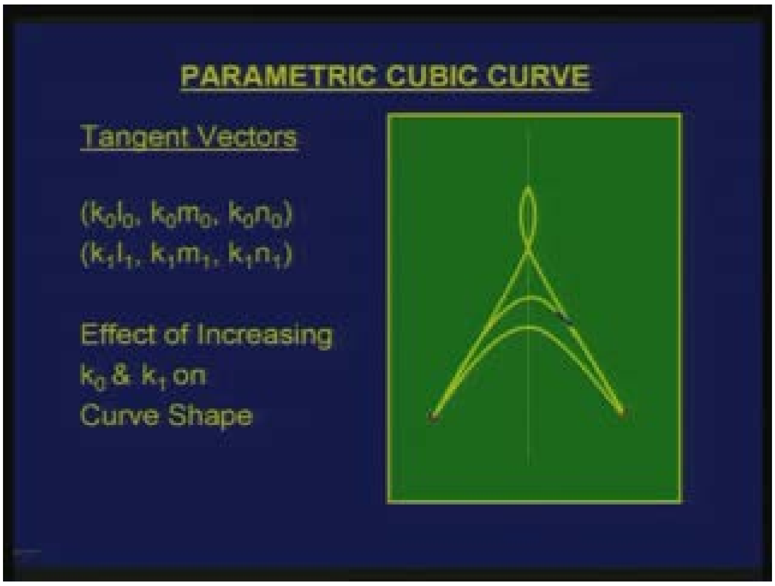

So if I look at this, these are the three different curves which are obtained by changing the values of k_0 and k_1, as it increases it changes it shape and at some stage like it starts forming loops also as it is shown here. So you never go to this point as a part of any curve modeling or surface modeling, this can happen in surfaces too in some situations. And this particular example which I have taken we try to put k_0 and k_1 as constant. That means they are increased proportionately. I can also have a situation where I try to increase let's say one of the tangent vector very high relative to the other. So that can also be used to give, change the shape of a curve to give some kind of a skew in the curve.

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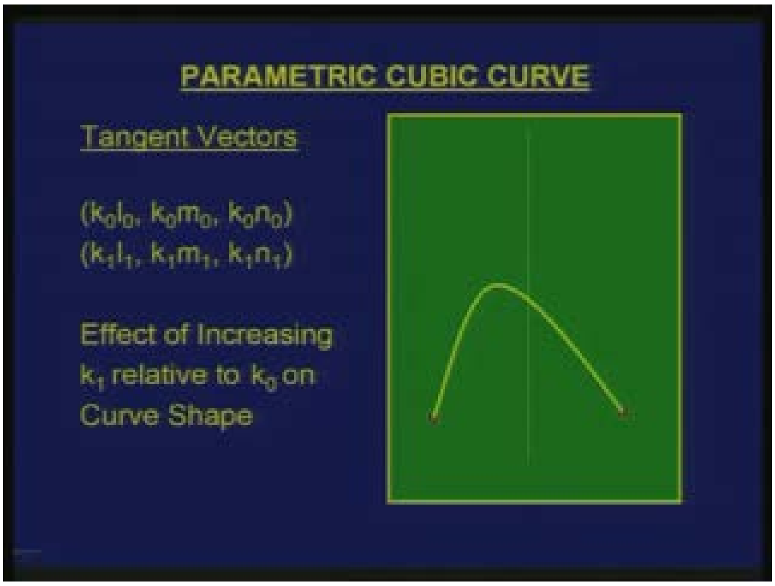

For example here is the curve. Now suppose if I try to increase let's say k_1, one of the parameters relative to the other I get another curve which is skewed in one of the directions whereas the same, you can say direction cosines are maintained in both the cases. So, one can say that in a geometric form the input which one has to provide is the start point, endpoint either I can say tangent vector with the magnitude or direction cosines which are multiplied with a scalar value which is k_0 and k_1.

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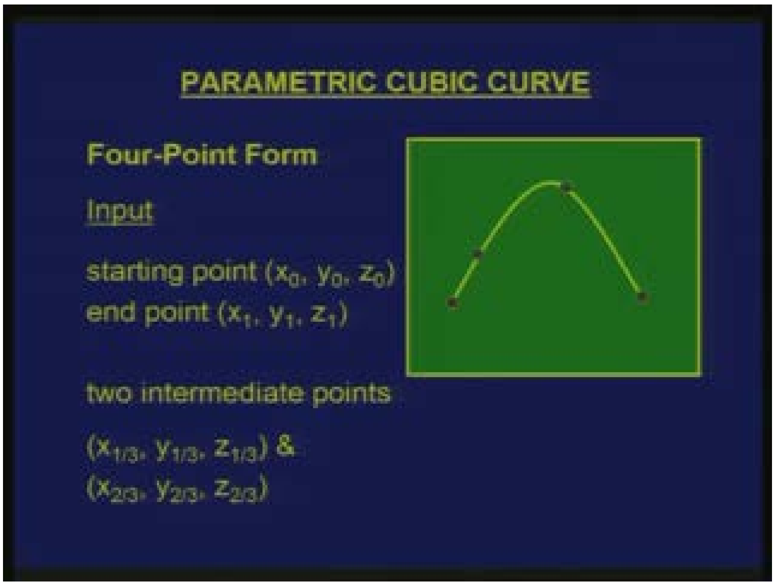

Now, so first form which we studied was algebraic form which was defined by 12 algebraic coefficients. Then we studied the geometric form which was also defined by 12 inputs but more convenient input in terms of start point and endpoint. The third important form of a parametric cubic curve is that given a four points through which the curve is passing, so I can probably define a cubic curve which is passing through these 4 points. So here are the four points which are shown here like I have point one, point two, three and four. The order of points is also important.

If I change the order, I get a different curve because the curve is like in a way traveling from one point to another in whatever is this thing. So order of points is important, so you have given start point which is a x_0 y_0 z_0 which is shown here. Then you have an endpoint which is shown as the endpoint then I give two intermediate points, in order to define this particular curve. Now let's say given a set of 4 points, do I get a unique cubic curve. Is the curve unique is let's say a question or do you have a multiple curves which are possible for let's say a given set of 4 points. It is unique when we define in terms of let's say a sense which is explicit sense. In a explicit sense when I have let's say y as a function, y which is a function of a cubic equation in x, I get a unique curve but it is not in the parametric sense. In a parametric sense that means when I am using a parametric cubic curve for a given set of 4 points I can get different curves which are possible through this.

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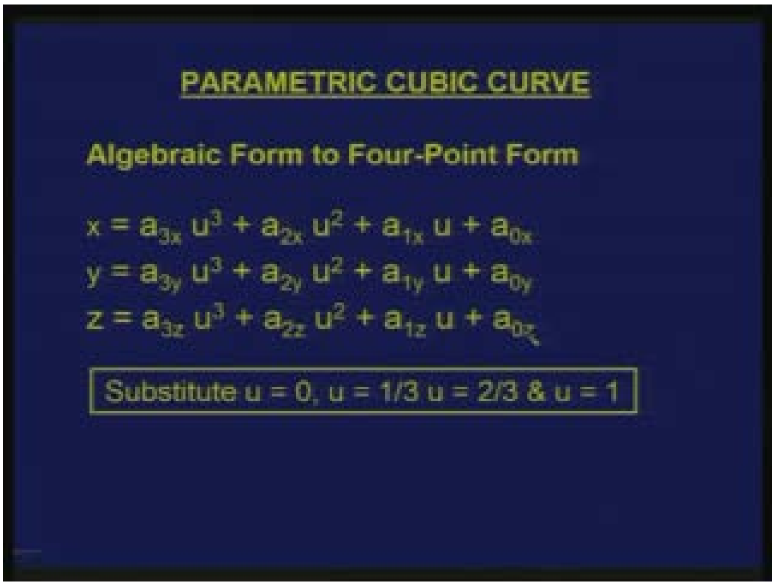

How is that is like if I look at the algebraic coefficients, this is my starting point for this. I have defined this as now I can substitute for those 4 points which are input now. So what are the input? x_0, y_0, z_0 which are shown here. So these are the 12 values which will form an input, so I should be able to solve this particular equation using those 12 inputs. So, I will substitute u is equal to 0, I will get this equation, one equation. I will substitute u is equal to 1, I will get another equation but what values of u which I will give for the intermediate points. It is like when the finally curve is defined, this is a curve which is staring from here which is u is equal to 0 and travels and goes to u is equal to 1. So I should be able to get different points on the curve by substituting a value between 0 and 1.

So for the two intermediate points, what values of u should these points correspond is a question. If I give different values, I get different curves like I can say that this is the starting point is 0 and this is 1. Here I will give as u is equal to 1 by 4 and I will give u is equal to 3 by 4 as another value or I say that know the parameter should be equally, it should be equal interval. In that case I will say this is 0. This corresponds to u is equal to 1 by 3 and this corresponds to u is equal to 2 by 3 and this is u is equal to 1. So if this is the situation which is you can say among various set of parameters, this is one you can say from which is commonly used then I should be able to solve this.

I will substitute for whenever I give a second point through which the curve is passing, I will substitute the value which is u is equal to 1 by 3. Whenever I give the third point through which the curve is passing, I will put the u value which is 2 by 3 and for the endpoint where the curve terminates, I give a value which is u is equal to 1 and then I will be able to solve. So there has to be, first you have to fix the u values for the intermediate points in a four point form in order to do that. So now again I have 4 simultaneous equations. For x, I have 4 equations which correspond to these 4 parametric values. Similarly for y and z, they can be independently solved. And I can again obtain a form which is something which is like similar to what we got for a geometric form and the form which you get is this.

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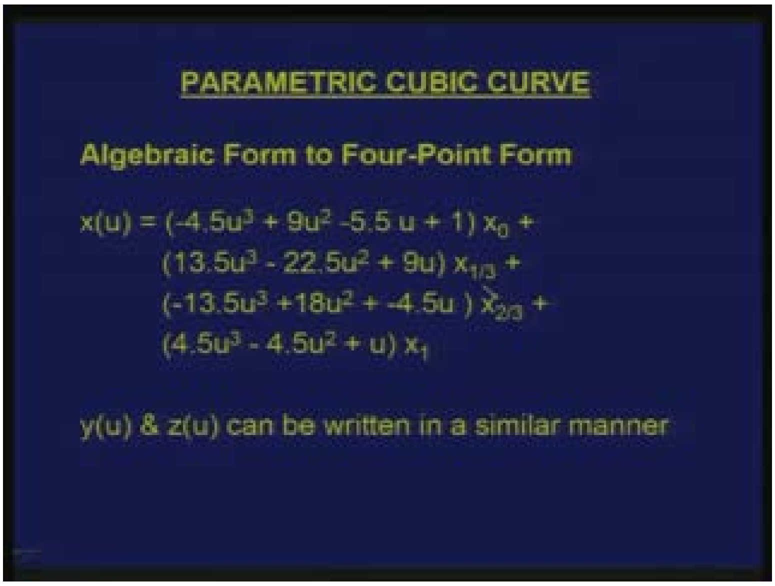

The curve looks now little more complicated like you have x which is a function of u. It is again a cubic function of u as it is shown here but the inputs which you are giving now are the x coordinates of the four point through which you are giving that is x_0 this corresponds to $x_{1/3}$ is shown to represent that x value has a parameter value which is u is equal to 1 by 3, $x_{2/3}$ and x is equal to 1. Now if I give the same thing but a different parameter value like as I said 1 by 4 and 3 by 4, I make at a different curve. So whenever you say that if given a four points whether you have a unique cubic curve, the answer is yes and no. It depends on whether it is a parametric or explicit representation which you are trying to use it.

Now coming back to this form for example where we used, it is always like how to basically choose these parameter values intermediate, for the intermediate points what parameter value should be chosen. Now suppose if I choose 1 by 3 and 2 by 3, that's one good way because I am having an equal parameter at the interval but the problem here is these curves are actually not located at a distance which are also uniform as it is 1 by 3 or 2 by 3. So, one of the ways to do is like I can calculate the distances of this. For example I can calculate a distance between these two points. I can calculate the distance between these two points and I also calculate the distance between these two points. So I have three distances, what are those. Let's say this is d_1 and the distance between these two points is d_2 and this is a third distance which is called as a d_3.

So this anyhow I will give u is equal to 0 but for this I will give a parameter range which is proportional to the distance. So what is that? D_1 divided by you have a d_1 plus d_2 plus d_3, for this I will give d_1 plus d_2 divided by d_1 plus d_2 plus d_3. And this is of course, u is equal to 1 which corresponds to d_1 plus d_2 plus d_3 divided by the same as a numerator. So that is in fact the most common way to represent a parametric cubic curve in a four point form but there is a disadvantage with that. What is the disadvantage? That means like I cannot have a standard form, I cannot have a standard equation like this which is used whenever I have a parameter which are fixed. If I have a standard form then these coefficients etc they are fixed when I am choosing u is equal to 1 by 3 and 2 by 3. I can just substitute and get it but there you have to solve the 4 simultaneous equations every time, for every different situation those have to be separately solved and then they have to be used. Now these are like in brief, three different you can say forms of a parametric cubic curve.

Now why are we calling this curve as an interpolation curve is in all the cases whatever input which I am giving, suppose if I give 12 algebraic coefficients, all of them have to be satisfied in order to like get a unique curve. Similarly in the geometric coefficients I give start point, endpoints, start tangent vector and end tangent vector. It has to be completely satisfied, you cannot violate or you cannot approximate them in order to get a curve.

Similarly when I said a curve is passing through 4 points, it has to pass through all the four points. It is not approximating these 4 points in anyway. So whenever the inputs are completely satisfied, you usually call as an interpolation curve, otherwise we will study later that we use geometric forms of curve, sorry the approximation curves where you specify a certain input. It is not necessary that those inputs should be satisfied in a complete manner. The input will be used to define the curve. The input will in some way modify or define the curve uniquely but it may not necessarily satisfy this. And when it comes to that, we will study three curves. One is a Bezier curve which is a simplest of that then B-spline curve which is an extension of a Bezier curve in some form and then there is a NURBS which is non-uniform rations B-spline which is a super set of a Bezier curve or a B-spline curve.

And if you are able to study up to B-spline curve that means if you are able to study up to NURBS that means you are able to understand or you know most about geometric modeling of curves. So that is what will be the subject of our lectures in subsequent classes. And also these forms we will be using when it comes to design and manufacturing applications which we study like one can also look at that I can use this parametric representation to generate NC tool paths or to find out like what is the curvature value at different places and how these curvature value should be used to let's say select a tool shape, some of you may have done in your earlier classes.

So the same thing like I can use this mathematical representation to also automate some of the design and manufacturing applications in like when we study about these topics subsequently. Otherwise the representation also is can be used to display the curve. In all the curves which you have seen, you have a parameter which is ranging from u is equal to 0 to 1 by giving different values between 0 and 1, I get different point. So I can plot the points or I can join them by straight lines to get a piecewise linear approximation to the curve and I can plot the curve on a computer screen for the display purpose. And a third important thing about study of curves is when you study about surfaces in many of the parametric representation of surfaces input itself is a curve like I can define a parametric surface whose boundaries, 4 boundaries are parametric curves. So study of curves is important from the study of surface point of view. It is not just the extension of that. In many of the surface forms, you basically get curve, you give a curve as an input. And later on we will also see that a surface, a parametric surface is the function of two parameters. And if I fix one of the parameters, I can get back the curve or it basically reduces to a curve which may be one of these forms.

It may be a parametric cubic curve or it may be a Bezier curve or it may be a B-spline curve or any one of these points. So thing I will just stop it here and I will take a few questions. If you have any questions, like is there anything which you did not understand when we discussed about parametric cubic curve. It looks like there are no questions, so we will stop it here and in the next lecture we will take up the Bezier curve that's another very important curve. In fact one of the interesting aspects of Bezier curve is it is not a very old curve. It is defined in 60's that is 1960's is the first time a definition to Bezier curve was given by a person by the name Bezier itself who was an automobile engineer, particularly to arrive at certain forms for automobile applications. And as I said it is an approximation curve and most common form of Bezier curve is a cubic Bezier curve which is defined by again given a four points as an input like you have seen the four point form of a Hermite curve or a parametric cubic curve.

In a Bezier curve also I give a cubic Bezier curve also I give four point to define but it doesn't pass through all the four points. It passes through only at the start and endpoints but in some way approximates or in some way it uses the other two points as an input to define the curve. So we look at Bezier curve more in detail when we meet here tomorrow or whenever when the next lecture is.
Thank you.