CAD/CAM
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Lecture No. # 4
An Overview of Geometric Modeling

Today's lecture is titled an overview of geometric modeling. Earlier we have seen in our introductory lectures that geometric modeling forms basis for the integration of many of the cad cam activities. So in the first three lectures, in this particular course we looked at geometric modeling applications in design in manufacturing and that it also forms a basis for integration of these activities. So from today onwards in next few lectures, we look at basics of geometric modeling like what you mean by geometric modeling, what are the various representation for representing geometric entities like curves, surfaces and solids. And how these representations are used particularly in design and manufacturing applications is the subject of this particular today's lecture.

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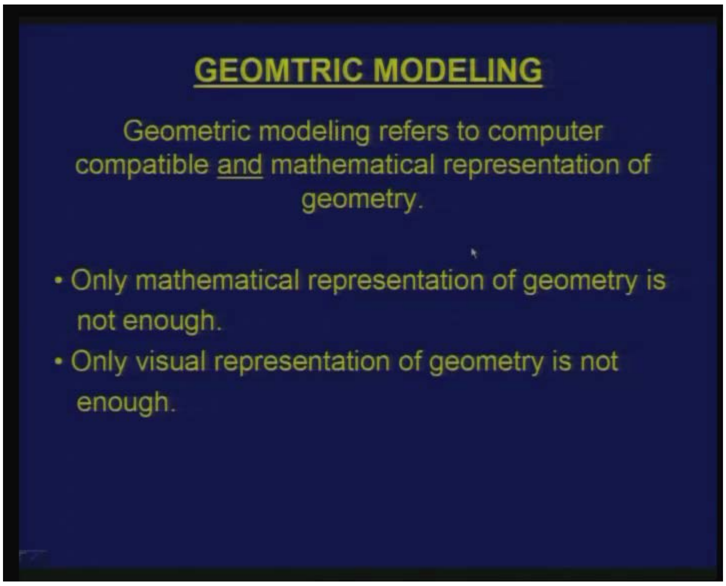

Now geometric modeling can be defined as computer compatible and mathematical representation of the geometry. If you really look at this particular definition, there are two aspects one is computer compatibility this is a must, second is a mathematical representation of geometry. If these two can be fulfilled in any definition I can call it is a geometric modeling. Now one can always give a mathematical definition which is not computer compatible or I can have a representation which is used purely for visual representation of the geometric object. So if I look at these representations which are like which fulfills only one of the requirements then that is not what we are looking for. We are looking for something which can fulfill both. For example if I look at a mathematical definition to geometry, we study in our schools and colleges, a complete course on solid geometry or coordinate geometry which is basically concerned with mathematical representation of geometric entities like line, circle, conics also surface entities like cylindrical surface, conical surface or a spherical surface.

So we study about how these are mathematically represented in our coordinate geometric course or three dimensional solid geometric course. So these representations are mathematical but they are not really computer friendly that means what I intend to do with this geometry cannot be fulfilled by these definitions directly. So I am looking for a better definition where I am able to use the mathematical representation also for doing some calculations which are related to cad cam or also to use this representation for visually displaying on let's say a computer screen. And if I go by other definition that is I can always give or I can always visually represent a three dimensional picture on computer screen without going into mathematical definition also, it's also possible. So, even that is not enough, so we are looking for something which is a combination of this thing.

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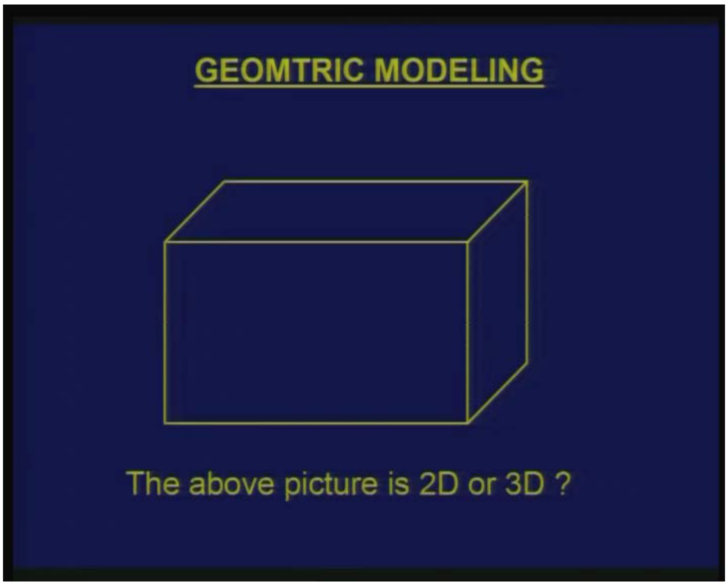

So if I just put this picture and say what does this particular picture represent? Is it a two dimensional figure or a three dimensional figure, what would be your answer? It's somebody, some people saying its three dimensional and some people are saying its two dimensional then there are few people who say like it can be both. Now purely looking at a picture you cannot really make out whether it is two dimensional or three dimensional. For example if I look at this particular figure, this figure can be constructed using a set of lines which are drawn in a plane which can give me an effect of three dimension. For example if you look at this particular thing you have 4 plus 3 that is 7 plus 2, 9 lines have been drawn to basically give you a feeling of a three dimensional object. So these 9 lines can be drawn in a plane and say that here is an object which looks like a three dimensional object or alternatively I may have really had a three dimensional representation for this that means I have actually constructed a geometric entity like a cuboid or a box and then I have removed the hidden lines and then are able to show this particular as three dimensional representation with hidden lines removed.

Now what basically implies here is that what you see is not what is correct. How it is represented internally? Let's say as a file or as an internal representation is what we are concerned. So when we say a mathematical representation there can be purely visual representation without mathematical also or like you may have seen that many of the pictures or sceneries or backgrounds you can always create three dimensional effect and there is no mathematical representations for that. It's purely a visual kind of effect. We are not interested in that either, what we are looking is something which can give me a geometric definition, at the same time the geometric definition should be computer friendly enough to do things which I need to do as part of cad cam applications. So that's what is basically the objective of the studying geometric modeling as part of this lecture. Geometric modeling is a vast subject and in terms of studying this particular subject, what we basically do is to look at how are various geometrical entities in terms of curves, surfaces and solids are represented.

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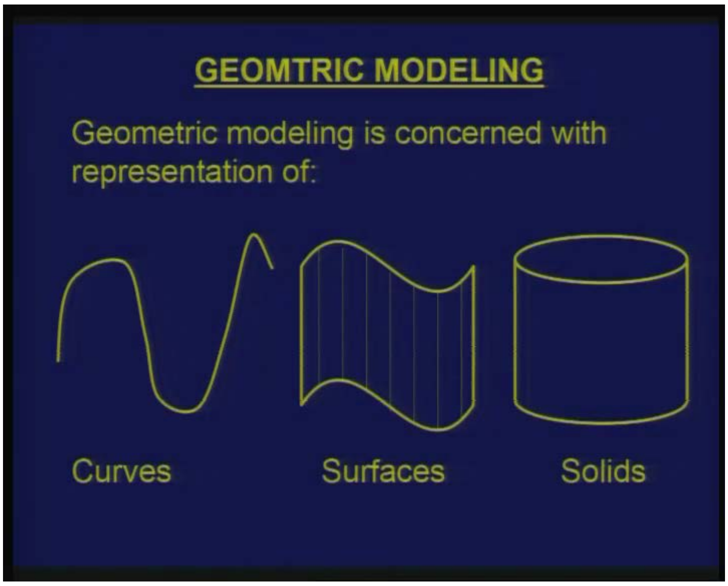

The representational aspects of curves and surfaces have some similarities like sometimes geometrical representation or geometric modeling of curves and surfaces together can be classified as surface modeling or modeling of you can say free formed curves and surface entities etc whereas representation for solids is very different from that of curves and surfaces in cad cam applications. Suppose if I am giving a definition for a curve, I can always extend it to surface one more one dimension higher and I can also extend it to solids but it is usually not done for certain basic reasons. So we will look at those aspects, why representation for curves and surfaces is slightly different or why there is a different representation which is used, which is not used for solids. So we will be studying about this. First we will take up study of curves and surfaces and then we will go for representational aspects of solid objects.

Now in terms of this study, you will come across for example if I am looking at curves and surfaces particularly, we may be looking at some of the geometric entities with which we are already aware of like starting from our school etc we study about many geometric entities like a line or circle or conic or if it is surface like it can be plane or a cylinder or sphere or a cone, so we are already familiar with some aspects of these geometries which are usually called as a standard geometries or more so what we call as a known forms. But cad cam also deals or when you have to use a geometric modeling for product design and manufacturing applications, one may have to deal with those surface features or those surface entities which are not one of the standard forms. Usually they are called as free formed curves and surfaces.

So what we would be looking as part of geometric modeling is start with known forms just quickly revise those forms and then go to what we call as free formed curves and surfaces.

And then once we have complete representation for all types of entities then we study about how these representations can be used to automate certain design and manufacturing applications so that will be the subject of this thing. And as I said curves and surfaces will be treated almost as one entity and solids will be treated as a separate entity in terms of this particular study. Now study of curves surfaces and solids is not necessarily like domain of a cad cam courses. This is also a subject of courses and computer graphics where one would be interested how to visually represent an object and also use a mathematical definition but not necessarily for design and manufacturing applications.

So the subject of geometric modeling has evolved along with computer graphics, so we will also see lot of graphic related terminology as well as concepts which are coming when we study about geometric modeling in this particular course. Now when it comes to representing curves and surfaces, we know that like we are aware of different types of representations which one can use.

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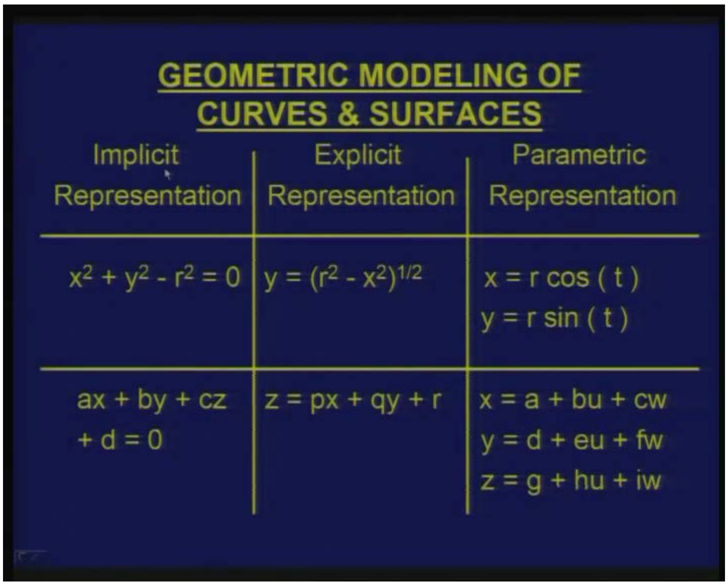

For example we have what is called as a implicit representation which we study in our geometry courses. We also study about explicit representation which is also a subject of courses on geometry and also parametric representation. We are familiar with parametric presentations for some of the geometric entities. Now one has to basically choose one of these three or let's say combination of these representations for cad cam applications. Now for example if I take an example of this, so what you see here is an entity like a circle which is represented in all three representations.

So if it is an implicit representation I can have an equation like x square plus y square minus r square is equal to zero where a curve is represented as a function of like f of xy is equal to zero is what is the representation or I can have an explicit representation where you try to model a curve as y as y equal to a function of an x. So the same circle can also be represented as an explicit representation which is shown here. Then you are also familiar with parametric representation of circle in terms of let's say r cos theta or r cos t or r sin t where t is the parameter which is varying for the variable for a curve.

Now among these three representation, now each one may have an advantage or disadvantage and may be one more suitable for cad cam application than the other. So we will look into that aspect as we go along. Similarly you have another example here like you have an equation of a plane which is given here. so it's a surface entity either i can represent it in a implicit form as ax plus by plus cz plus d equal to zero where you are using the function like f of xyz equal to zero or I can also say z as function of x and y which is shown here as explicit representation or I can also have a parametric representation where every point on a plane like every coordinate point on a plane xyz can be represented as a function of two variables which is u and w.

It is true that like whenever we represent a curve or a surface in a parametric fashion like whenever we have let's say x y or z which is basically has one parameter, it can be classified as a curve. Whenever you have two parameters like u and w as it is shown here, so it becomes a surface and we can extend it to higher dimension. I can have xyz which are functions of three parameters which can be used to represent a solid object. So we would be looking at these three representations and to look at which is more appropriate for cad cam application.

Now it is almost universally believed that among the three representations, parametric is the one which is most widely used for certain strong reasons which we will look into is why parametric representation has an edge over implicit and explicit representation for cad cam applications. But when we are studying about parametric representation and its advantages, we should be aware that implicit and explicit representations to have certain advantages. So it is not necessary to say that parametric representation is the only representation. I may use a combination of these for a given application also but more so, a parametric representation for most of the applications.

So let's look at why parametric representation, what are the advantages over let's say other two representations which are shown for curves and surfaces in our next part of the lecture.

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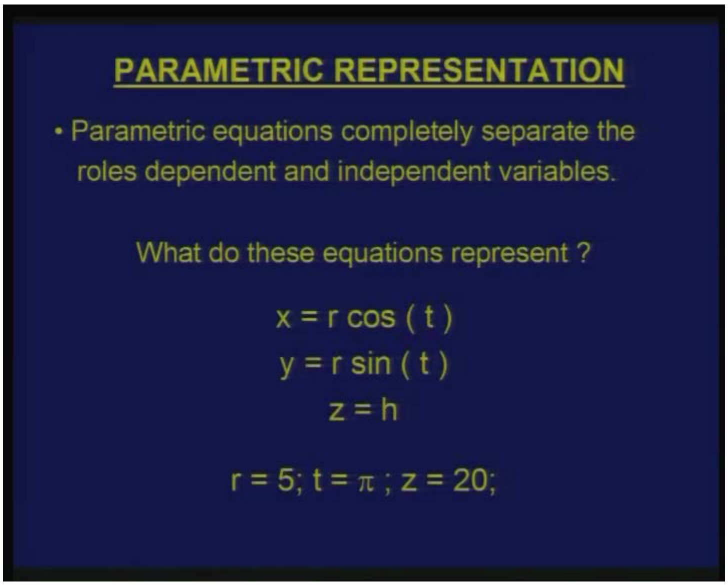

The first advantage of a parametric representation is that it completely separate the roles of dependent and independent variables like when you, for example when I write implicit or explicit representation which involves x y or z, you have one which is like dependent on the other, whereas when I am giving let's an parametric representation like xyz which are functions you are basically looking at them independently. So how x varies as parameter of t is of not much concern to you like how y varies with let's say a parameter which is there. So the three coordinates can be independently written and they can be changed if I want to change let's say a curve or a surface. So this is one of the major advantages you can say in terms of parametric. So, instead of looking at equation as a whole, you are looking at the independent aspects of xyz, how they vary as a part of this particular thing.

And another of course a very interesting aspect of this kind of independent variables is the equation which can be given for let's say geometric entity can be extended to higher dimensions very easily like if I say like given let's say x equal to r cos t, y equal r sin t, z equal to h what do they represent? What geometric entity? Somebody may say it's a cylinder, no, it's a circle or no, no it's a solid cylinder, I would say it can be anything. For example if you look at the values which are given at the bottom for r t and h, r is fixed which is 5 units, t is equal to pi and z is equal to 20. If I substitute these in the equation, I get a unique value for xyz so it's a point, it's a point, it's not a curve, it's not a surface or it's not a solid. So you have three parameters but how you vary, which are the parameters which actually vary and which are the ones which are actually fixed will decide whether these equations, the three equations for xyz which are given here will represent whether it's a curve, surface or let's say a solid entity.

Now I can always vary one of the parameters keeping the other two parameters constant. Now if I do that then I am going for one dimension higher than let's say a single point, a specific point which is mentioned. So you will be representing let's say a curve. Now if I go to that, I have the same equations but if you look at the parameters r is fixed which is till 5 units, z is fixed which is 20 units whereas t is a variable, t varies from let's say minus pi to pi. So you have three equations which are functions of a single variable. We know what is the entity which it represents. It's a circle, so because t is varying from minus pi to pi, so I can call it as a circle.

Now I can extend these two higher dimensions too and you get different representations by fixing a different entities. For example in these equations instead of fixing let's say a variable t, instead of varying let's say variable t, I may have varied let's say for example either r or h. If I do that what will I get? I will get a straight line instead of a circle. So parametric directions like what basically it's says that you have one parametric direction which is more like a circle, other two parametric direction which is like straight lines either by varying r alone or h alone, I will get a straight line which is basically a straight line entity. So you can like it's basically a three parametric directions which are a combination of two lines and one circle which is it.

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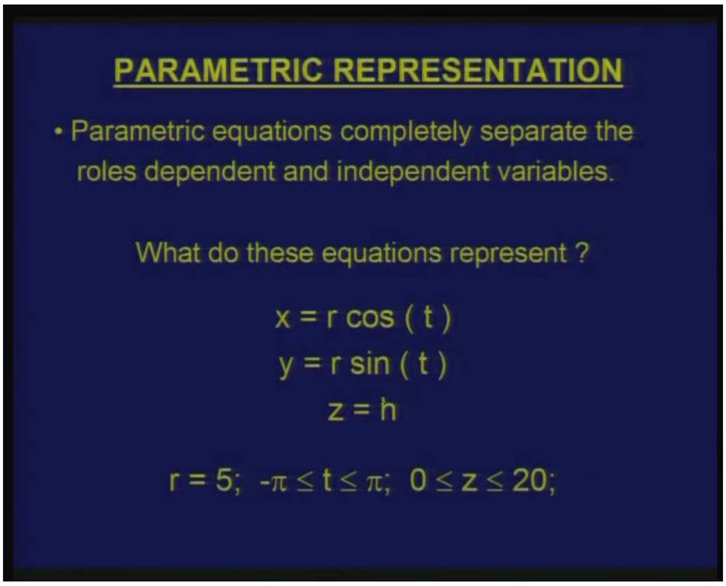

I can go to one more dimension higher, what you see here is r equal to 5 which is fixed whereas two parametric variables that is t and z are varying within certain range, t varies from minus pi to pi and z varies from 0 to 20. So what does this represent? It's a cylindrical surface, so it's not a solid cylinder. It's a cylindrical surface where the radius of this cylinder is fixed as 5 units, instead of these two I would have chosen some other combination also like what happens if I vary let's say other two variable, instead of t and z suppose if I vary let's say t and r, I keep z as a fixed. So you get a circular disk where the z is constant, so it's again like a planar entity, it's a surface entity with this or I may have gone for varying r and z keeping the t as a constant. It is a rectangle, so that's again a plane entity. So you are basically choosing a combination of these two.

We have two parametric directions which are lines, one is the circle. So if I take two lines, I will get a rectangle one line and a circle either I will get a cylindrical surface or let's say circular disc. So one can have a combination of any of these two entities and in our each case you get a surface as a result of this. So it's very clear that whenever we have a parametric representation if xyz are functions of a single variable as we have seen in our earlier representation. If I vary any one of them, I get always a curve entity like either a straight line or a circle has we have seen earlier.

Now if I vary any two of those, I always get a surface entity which may be a rectangle or circular disc or a cylindrical surface. I can go to one more higher dimension where you are varying all three parameters, r is also is varied between 0 and 5. Then I have let's say t which is varied between minus pi and pi and z which is varied between 0 and 20. So what does this represent? It's solid cylinder.

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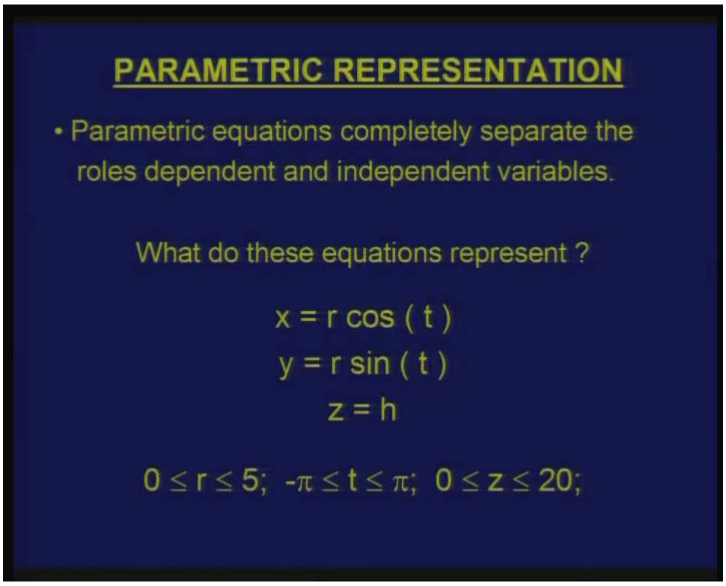

Now instead of let's say r which is varied from 0 to 5, let's say I vary the r value which is let's say from 3 to 5. So you still get a solid object but this time it's not a solid cylinder but it's a hollow cylinder. So since these parametric equations are functions of three variables where all the three variables are like, all the three parametric entities are being varied so you get a solid object which is that. The same concept can be extended to higher dimensions too. Now one can also say that for example I have a parametric representation which is a function of 4 variables or more than 4 variables too that may not have really a meaningful solid object as per as let's say a geometric modeling is concerned but in some cases it is used.

For example I would like to represent let's say a solid cylinder as it is shown here. And this solid is moving in space and when it is moving in space, I also have a function which is like a time, so I can bring one more variable which is r t z and as well as let's say another variable which is time. So I can have let's say a solid or I can represent a motion which are functions of 4 variables. So one can extend this to higher dimensions too but in most of the cases, this kind of representation are used for study of curves and surfaces that's what we are actually looking at.

Now in explaining this I have taken a very simple example of let's say a parametric equation. The same thing can be extended to other entities too. It is not necessarily for let's say a circle, cylindrical surface or a solid cylinder but it's true with any parametric entity where if I go for parametric equations which are functions of three parametric variables it's always a solid. It may or may not really give me a meaningful solid but it is a solid.

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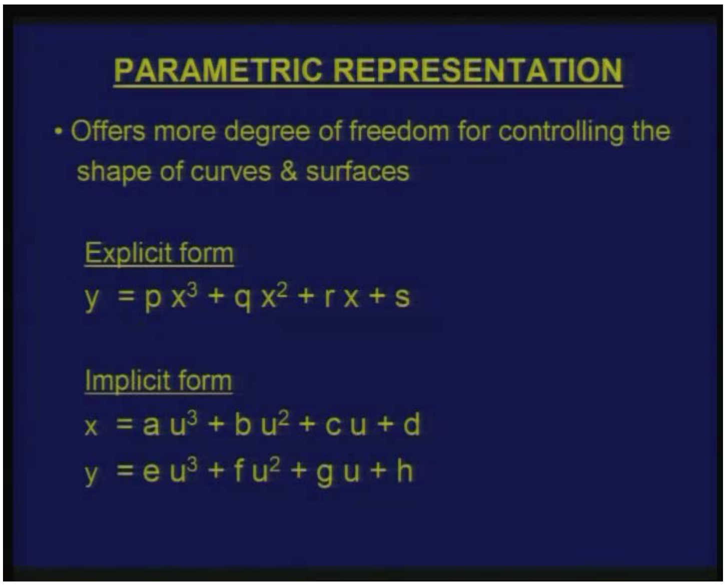

Another major advantage of parametric representation is that it offers more degrees of freedom for controlling shape of curves and surfaces like I have given two, you can say curves here which are y as a function of x and xy as function of parameter. So what you see is two different types of representation, one which you call as a explicit and second I think sorry this is not implicit, what is shown here is it's a parametric form. It is wrongly written as implicit, so this is a parametric form.

Now if I look from explicit and parametric form both these things basically represent a polynomial which are cubic in nature. A cubic equation in explicit form and again a cubic equation which is in parametric form where x and y which is there. Now what is the basic advantage in these two representations is like here you have 4 variables. By giving different values to p q r and s, I get different cubic curves. Same thing is true by giving different values to abcd efg and h, I get different parametric cubic curves but a designer has more variables to play with in the case of parametric than in the case of an explicit form. Here you have only a 4, whereas here you have 8 variables. So usually somebody who is in the business or who is concerned with designing a curve for a specific application or designing a surface for a specific application would be interested to have as many degrees of freedom as possible or as many variables as possible so that one can play around.

And secondly you know that when you are actually playing with abcd, you are only concerned with x coordinate and when you are doing with efgh, you are only concerned with y coordinate. So I can also have their roles which are independent in this particular case. So same degree of polynomial which is cubic in nature because we know that cubic is one of the most commonly used representation also like one can go one degree below that is quadratic or I can go one level higher than cubic that is quartic or one level higher which is fifth which we call as a quintic but cubic is the most commonly used representation for representing many of the curves and surfaces. The reason is very clear.

Usually, whenever we have a curve or a surface you look at the continuity requirement. that means the curve should be like, you should be able to differentiate it at least two times because whenever we have first differentiation of x and y, we are looking at more like how the slope varies. And when I am going for a second differentiation, I am looking at curvatures. So curvatures also, curvature information is also very very important for many of the cad cam applications, so cubic is one. If I go for one order higher that is quartic where you have polynomial where you have terms like u to the power of 4 or x to the power of 4, it really doesn't serve much purpose because your mathematical calculations become more and more complex. At the same time it also gives you a curvature continuity even one level higher but that is not really needed for most of the applications. So in most of the cases, we represent cubic form.

Another reason for using a cubic form for polynomial more commonly is that up to cubic, we can also get closed formed solutions. I can get analytical solutions. If I want to know what are the roots of let's say a cubic equation, it should be possible in many cases to know what are the roots in a purely analytical form or by giving a formulae whereas once I go for a higher order than cubic then you have to use numerical methods. And we know that whenever the numerical methods are used, you have associated mathematical problems and also there are other issues. So cubic has advantage that it serves the, we can say this is the minimum representation which serves most of the applications and also I have analytical solutions which are possible. So this is most widely used representation for this. So I have just chosen cubic as an example to represent this. And of course you can always, other representations likes quadratic and linear are subset of this by making one or the other variable zero, I get the other representations.

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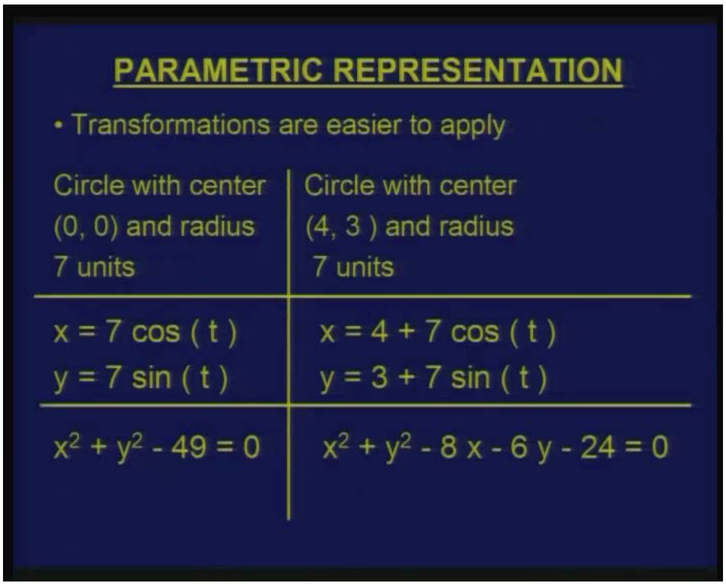

And coming back to the parametric representation, there is one more advantage which is very evident is that transformations are easier to apply. In a cad cam work or let's say when you are dealing with computer graphics, you are not only dealing with geometric entities but also their relative positions. How one entity is placed with respect to the other or how let's say an entity is defined with reference to some Cartesian or other coordinates systems or an object is initially constructed at some convenient position and then it is moved within the Cartesian space to some other position. So when we move it, its parametric and other whatever may be the mathematical representation it changes. So this is usually called as a transformation.

So transformations are usually, the most common transformation which we use in a cad cam applications are translation where you translate an object from one place to another place or rotation, it can be about, rotation about an axis which may be parallel to xyz axis or it may an arbitrary axis which is inclined to xyz axis. You also deal with the, in fact translation and rotation are rigid transformations. They do not change the shape of an object but I can also have transformations where I am going to scale an object. There are also transformations called shear where you are again changing the shape of an object. So these are non-rigid transformations. So these are very commonly used in many of the cad cam and graphics applications and we should also look at that particular mathematical representation for geometry which gives convenience in terms of applying transformations.

Now this I am basically demonstrating giving a simple example like let's take a circle and this is a circle with radius of 7 units. Now what is shown here is basically like this is a parametric representation which is shown here and what is shown here is basically an implicit representation for the same circle. Suppose if the circle has the center 0 0, I can write down it's parametric representation as x as 7 cos t and y as 7 sin t where t varies from, for the complete 2 pi as the parameter range. The same equation in an implicit form can be written as x square plus y square minus 49 is equal to 0.

Now this circle is moved such that center now is at 4 and 3 instead of 0 and 0. Now when you do a transformation that means the circle has been moved to another place where the shape is not changed, size is not changed, shape and size has not changed only thing is now it is at a different position with reference to a Cartesian coordinates system which I have chosen. What happens to the equation? A parametric equation which was this thing, so you just simply add the translational variables that it is moving 4 units in the x direction, 3 units in the y direction directly to the parametric representation. So if I look at this particular aspect, this right hand side basically denotes the shape and size aspect of the entity which you are representing whereas 4 and 3 or translational variable or transformation variables which are used.

Now let's look at that same for when I am having an implicit representation. The same equation which is same circle with radius of 7 units with at a center has a different equation now. Now if I look at these two equations, it's very difficult to make out, first thing is it's the same entity because the transformation variables have mixed up with the actual representation of shape and size. So it's very difficult to separate it out, in fact circle is very simple entity, so you can still make out by let's say or one can still dig out what are the transformation variables and what the actual shape and size. But when it comes to more complex geometric entities, this becomes more and more complex.

Since I am able to do that this kind of transformation variables are I am able to separate it out from shape and size aspects. So this parametric representation has a great advantage in this case. Now what is shown here is only a translation. Same thing is true for rotation also like in fact rotation is you will see slightly a different formulae but basically what one does is that you are actually representing a point as a vector in a parametric representation. Suppose if it is a entity in a plane, you have a vector has two components which is x and y. Suppose if it is in a space then you are looking at xyz. So you are basically transforming a vector in a parametric representation. So, the vector has three components x component, y component and z component and whereas you do not have any such representation here implicitly. And we know that the transformations are also I think you are going to study later. Transformations can be represented in a matrix form like when an object is translated from one place to another, what is the actual translation can be represented as a matrix form. Similarly a rotation can be represented in a matrix form and you are representing a point in vector form. So, all the transformations can be represented as some kind of a multiplication of vectors and matrices. And we know that when it comes to particularly programming geometric entities or when I have to write a program where transformations and geometric manipulation of geometric entities are involved, vectors and matrices are very convenient to handle in programming rather than like solving with many of the algebraic entities which are sometimes difficult to do. So you can say that parametric representation gives you a programmer friendly environment because you are able to represent the geometric entity as a vector and transformations as in a matrix form.

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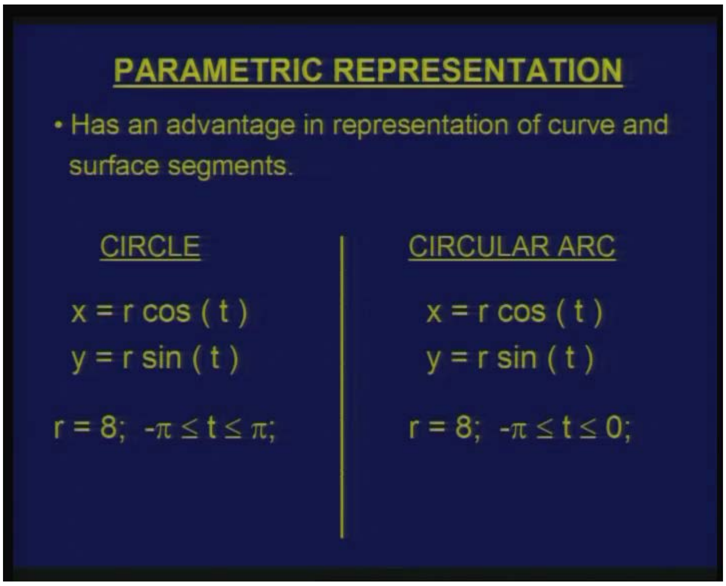

The other advantage which you have particularly with parametric representation is that you are able to represent the objects that is curves and surfaces which are inherently bounded. Now whenever we study about let's say a line or give a representation, we really do not look at a line, there is a starting point and then there is an end point for the line because in most of the cad cam applications one has to deal with let say a line which has a fixed starting point and fixed end point. A line which extends infinitely in both the directions has not much meaning, you cannot have an object where there is a straight line which extends infinitely in both the directions. So I want to put bounds on let's say any entity which I choose.

So since whenever you have a parametric representation, you also try to represent the bounds which is the parameter which is being varied and what is the range of variation is also specified. So you are primarily representing the bound entities like if I take this particular example, for example I have a circle which is r cos t and r sin t and if I take let's say the range as minus pi to pi, I get a complete circle. Now the same representation can be used to represent a circular arc, you don't have to have a different representation like imagine representing a circular arc in implicit or explicit form. It's very difficult to do it because you do not have a representation or you do not have a provision for representing the bound entities whereas by simply changing the parameter range to a different let's say minus pi to 0, now equation has not changed but I am able to represent let's say a circular arc instead of a complete circle and this is true with many of the curves and situations, curves and surface situations where you would be doing it.

Now somebody you can say it's also possible to put bounds in implicit and explicit representations by putting either range in x or y but it's not always true like firstly when I am having a range in terms of x and y, I may have a multi valued functions, so I may not be able to represent this uniquely that's also one of the problem. I think I will refer to that sometime later in our lectures. So why we have a difficulty in representing let's say segments in implicit and explicit representation, whereas it is much convenient to do in terms of (Refer Slide Time: 00:39:45 min). Now this kind of representation also helps you when I am trying to find the intersection of curves for example.

Let's say if I am trying to find out let's say whether two arcs intersect or not, you are actually not trying to find out whether the two corresponding circles intersect or not, you are interested in only the arcs segments. So I can always make use of this parameter range to know because corresponding circles may be intersecting but the arcs may not be intersecting. So such a situation if I want to distinguish then probably I may have to, I have an advantage in using a parametric representation here.

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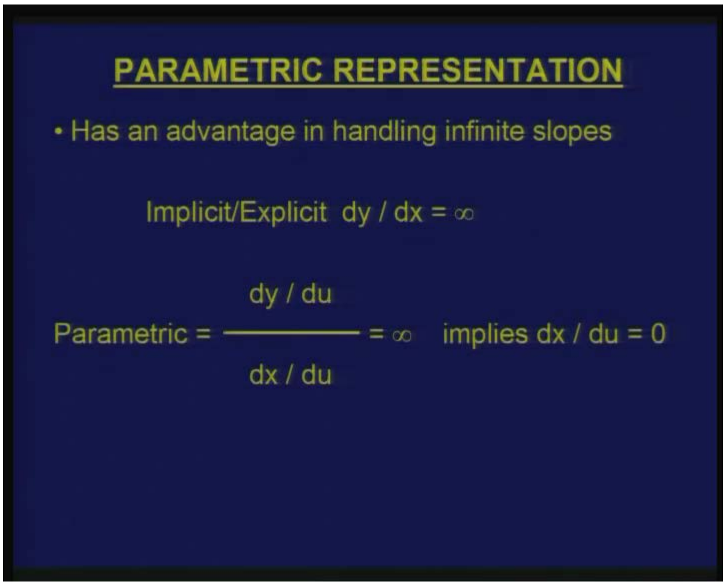

One more advantage which you see here which parameter representation has in terms of calculation of slopes are some of the geometric properties. If I represent let's say a curve in an implicit or explicit form and you are trying to find out what is the slope of a curve at different places or suppose if I take a circle and I am trying to find out what is the slope at different points. I may have a situation where the slope becomes infinity or I will have a points where the slope becomes infinity. Now suppose if I am writing a program, I am writing a computer program to calculate slope at various places, the moment you land up at a point where the slope is infinity your program fails because the number infinity is something which cannot be handled by a programming languages that is something which cannot be easily defined.

So in order to evaluate a slope like if I use a formula like dy by dx which is a very most common form, I have a problem. So I cannot have a check like even to implement a check whether the slope is becoming infinity or not is also difficult because you cannot say that if dy by dx is equal to infinity kind of logical statements are not possible. If I am using a parameter representation, you are actually trying to take two components separately that is whatever is the dy by dx which you are trying to represent, you are splitting as dy by du divided by dx by du. Whenever the slope becomes infinity, dx by du become zero. Zero is a number which can be handled easily in a programming languages.

So I can always put a check, if dx by du become 0 or equal to 0 then you have something to do that means this is the case of infinite slope and this case has to be handled separately. So the advantage that zero has an advantage over infinity in terms of handling in programming languages that is possible with parameter representation that is one of the advantage. It's not necessarily restricted to slopes, we may have a many situations where you may end up with values like infinity in many cases whereas those can be very easily handled when it comes to a parameter representation.

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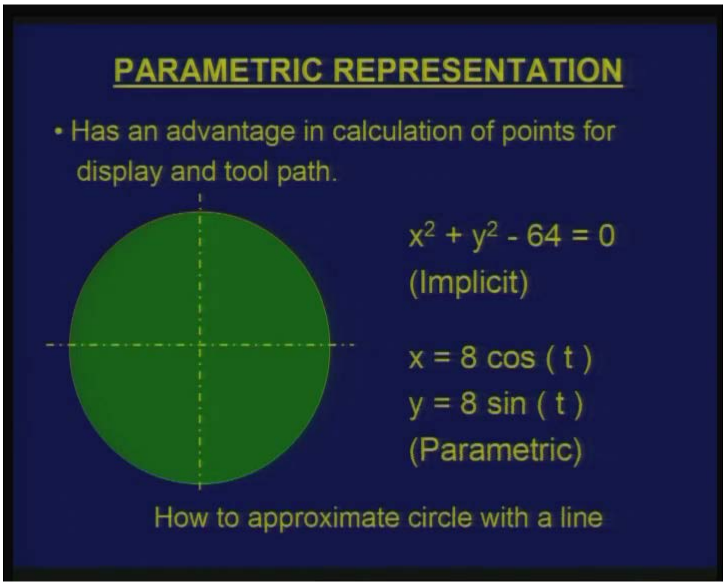

Then of course one of the, a very clear advantages, a major advantage as far as cad cam applications is concerned is in terms of a discretizing an entity. I have again taken an example of a circle. In most of these cases I have taken a circle because this is one entity with which we are already familiar and most of the advantages can be demonstrated using this single example. Here is a circle which is shown and you have an implicit representation for this circle. The radius of this circle is given as 8 units, center of the circle is 0 0 which is shown here. Now you have both implicit representation and explicit representation. Now this circular entity has to be approximated with let's say straight line. This we do quite often in cad cam applications.

For example, I have a curve which may be a circle or which may be any free form curve and in order to display this particular curve or in order to approximate this particular curve, I take number of points on the curve and join them by straight line and say that you have a piecewise linear approximation per a curved entity. Now whenever for example, I have let's say an algorithm for which works only for polygons not for curved entities. I can always take a circle and convert into a polygon with number of edges and still apply the same algorithm.

So there are many situations where I can basically go and we need to do this kind of discretization of a curved entity in terms of straight line. Same thing is done for surfaces also. I may have a free form surface and this free form surface has curved geometry. So instead of dealing with a curved geometry, I would like to approximate it as let's say facets which are basically plane or a polygonal faces. So you try to approximate a surface entity with a piecewise polygonal approximation. Now whenever we do that, there are problems with implicit and explicit representations which is clearly shown here as a simple example.

Now one thing is suppose if I am trying to take number of, I want to display this particular circle. So in order to display this particular circle, I use a piecewise linear approximation by calculating, computing number of points on this circle and joining them by straight lines. If I use a representation like this how do I compute the, how do I go about let's say computing points on a circle, maybe I will vary one of the variables. Let's say I vary the x, I know the range of x for example in this case that is it goes from minus 8 to plus 8, so I can take this particular range and try to calculate y. So I will get number of points and join them by straight lines to do that, but I have a problem firstly when I give a value of x in some places I have two values for y. So which one to choose? That means I have to keep tracking in both the directions or I may have a cubic curve where you have an implicit and explicit equation which is cubic in nature.

So if I give x then I have three roots and I have to evaluate all the three roots. Some may have a meaningful roots like that means I may have a real roots or I may have imaginary roots too. So I have to keep track of like how the curve is moving and if there is a multivalued functions, how to track this particular curve is also a big issue whereas in a parametric representation, whenever you have a curve when I move from let's say one point on the curve which I call as a starting point to end point. One thing which is continuously varying is a parameter, so just by varying one parameter I am able to get as many points as possible, you should do. So if I want to draw let's say do an approximation of this circle as a piecewise line, so what I will do is I will vary the t. Here I know what is the range of t, t has a range from let's say 0 to 2 pi or minus pi to pi depending on how you represent. And I take small intervals of t and compute x and y values between 0 and 2 pi and I get number of points and I can join them to get let's say a linear approximation to this particular circle whereas you have a difficulty when I go for implicit and explicit.

Moreover you also have some kind of a uniformity when I am using a parametric representation like even if I am able to track let's say both the directions, for a given value of x if I have two values of y, if I take equal intervals of x values I have highly unequal intervals for the y values. That means a small change in x value here results in a large difference in y value whereas here if I take a x value large difference in x value somewhere towards the center results in only a small difference in y value. So you have a non-uniform kind of variation for x and y, if I vary one variable and try to calculate the other variable.

When it comes to parameter representation, you have an advantage. It's not true for all the entities. For circle it's okay but for others it is not necessary that by varying a t or by varying a variable I will always get equal size line segments but it would be much more uniform than implicit or explicit representation in many cases. So by varying a t as let's say certain value taking let's say if either a interval of let's say 5 degrees, 10 degrees or 1 degree depending on my requirement, I am able to get the straight line approximation which is more uniform and which has a unique value.

So computing becomes much easier like for example if I have to write a program to display a line as a series of circles, naturally I would prefer this rather than using that as a mathematical equation for writing a program. Now, approximation of a curved entity in terms of lines and polygons is not only for the display purpose. It displays one aspect like as I said, circle can be approximated as lines in fact surface is also can be approximated as polygons. This is one of the extensively used, you can say tool in computer graphics. Whenever you see let's say an object which is represented let's say a good seen a computer graphic seen which you see, many a times the object looks like a curve but actually internally it is basically approximated as a series of triangles or series of polygons because in most of the computer graphics algorithms whenever I do operations like rendering etc, I have an advantage in dealing with triangles and polygons than with curved entities.

So one of the most common you can say aspects in computer graphics for display purpose is approximate for let's say a curved surface with a series of triangles or series of polygons that's it. Many a times this approximation is so smooth that you can't even notice when you see an object on the computer screen like for human eye, it almost looks like a curved surface but internally it is approximated. Now this kind of approximation is not necessarily for purely visual purpose or displaying or for computer graphics applications, you also need this kind of approximation for applications like manufacturing. For example if I have to generate let's say NC tool path, for example I have a curved geometry and I have to take my tool along a curved geometry let's say for machining purpose.

So what you do is in a typical CNC machine, machining programming that means a part programming either you use a linear interpolation or let's say a circular interpolation like a circular interpolation basically uses codes like G_{02} or G_{03} whereas a linear uses $G_{0\ 1}$. Now, any entity which is not line or a circle is again approximated as piecewise linear approximation. So, if I have a free formed curve, I take number of points on the curved entity and then join them by series of straight line tool paths to get let's say curved path, so that's a very common.

So approximating a curve or a surface as let's say piecewise linear or polygonal approximation is not necessarily for display. It is also true for an NC tool path calculation. So here also we see that a parametric representation has an advantage. So what I basically try to do particularly in this lecture is to give you enough inputs to convince that parametric representation is the way to go for cad cam applications. But at the same time, we should not really like undermine the other representation. There are situations where implicit or explicit representations have an advantage. For example I can give you taking the same example of a circle.

Suppose if I have a point, I want to know whether this point is inside the circle or outside the circle. How do I do that or I have a straight line and then there is a point, I want to know whether the straight line is on which side of the straight line whether it's on the left or right. So in those situations, you have other representations which may be useful. For example if I want to know whether a point is inside or outside a circle, I can use an explicit representation very conveniently. Substitute the coordinates of the point in the equation of a circle and check whether the value which you are getting is 0, negative or positive. So, depending on that you can say the point is either on the curve or outside the curve or inside the curve, whereas if I want to use the parametric representation for that, I may not be able to use it that conveniently as I do it.

So, summary is that parametric representation has major advantages but a combination may be useful in many situations. So I may try and opt for a combination of these representations if the other representation also has an advantage. So, we basically try to look at the representation and their advantages. Now, what will do is carry forward this like we will take up start with representation for curves, how some of the curves are represented parametrically and then we move on to surfaces. Once we have finished with a curves and surface representation both for the known forms as well as free form, then will take up some applications where these curve and surface representations are used to automate or to take certain decisions in a cad cam environment. So that would be like sequence of lectures which we would following starting from today. So if you have any questions please raise them. So any questions or is this clear? Then I will, yeah somebody. Yeah, any questions? Then I will stop it here and we will take up with a parametric representation of curves in our next lecture.