Computer Aided Design
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Lecture No. # 40
Solid Modeling Using Octrees

In the last class we had talked of special enumeration, special occupancy enumeration. And we said that one of simplest ways of enumerating space, let's say you have two dimensional space using the set of squares is by dividing the space into an array of cells and they are having an x by y array of zeros and ones which tells us whether a particular cell is occupied or not.

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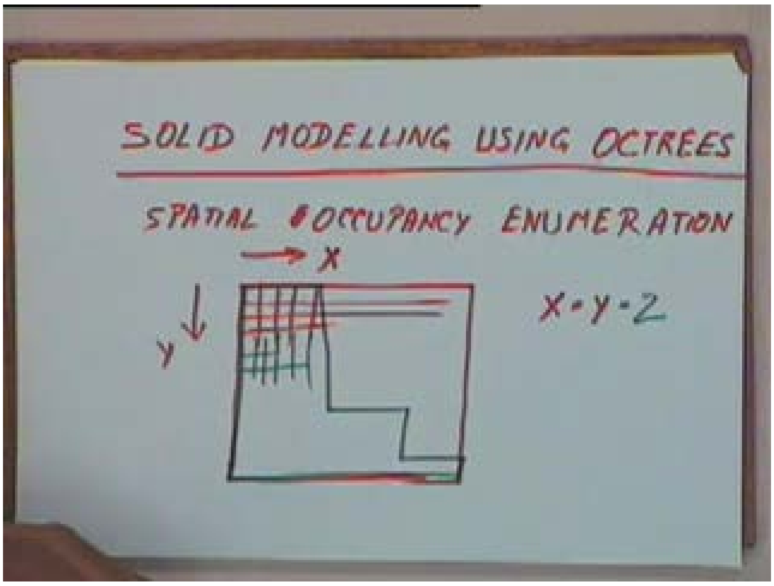

If we use a mechanism like this then, let's say if we have resolution of x in the x direction and y in the y direction, we need space which is equivalent x times y. And even if let's say our objective as something like this then a portion which is let's say a large portion which is totally outside the region that you are trying to model or a large portion which is totally inside even that portion will get enumerated or will get divided into cells. This is quite inefficient and especially if we extend it to three dimensions then you need an array of a size of x into y into z then it can become very inefficient.

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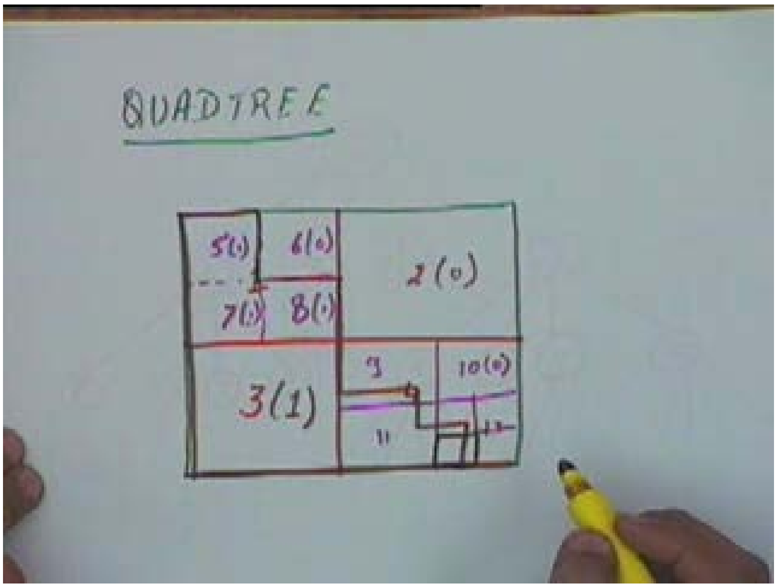

So in order of take care of this, one method that is used what is called a quadtree. This is a technique which is used to model let’s say a two dimensional surfaces. For three dimensional we will go on to octree, we will come to that. So, for a two dimensional surface if we have any area like this which has to be modeled, we will first divide this into 4 regions. Let’s say this is our region number 1 2 3 and 4 and this region if it is let’s say if I have figure to be modeled is something like this and this is a region am trying to model. Then we see whether each region is inside or outside. If you look at this region, we cannot decide whether it is totally inside or totally outside. If you look at region number two that is totally outside.

Similarly region number 3 is totally inside and region number 4 is again partially inside and partially outside. So for region number 2 we will store a 0. For region number 3 will store a 1 and when you consider region number 4, we will divide it further into 4 parts. So if I divide region number 4 we will get let’s say parts like this. Now this region number 1 is now divided again into 4 parts let’s number them 5 6 7 and 8, 5 is totally inside that gets a 1, 6 gets a 0 because it’s totally outside. 7 gets a 1 and 8 gets a 1.

Similarly if I consider region number 4, I will again divide this region number 4 into 4 parts because region 4 is partly inside and partly outside. So let’s say this becomes something like this, after 8 let’s say this is 9 10 11 and 12. Again a region number 9 I cannot decide but 10 is totally outside, so 10 I will give it as 0. 11 again I can’t decide, 12 also I can’t decide. So again I will take 9 and divide it into 4 parts and we continue this way. And the way we will represent this is in the form of a tree. I will come to that, I will come to that please.

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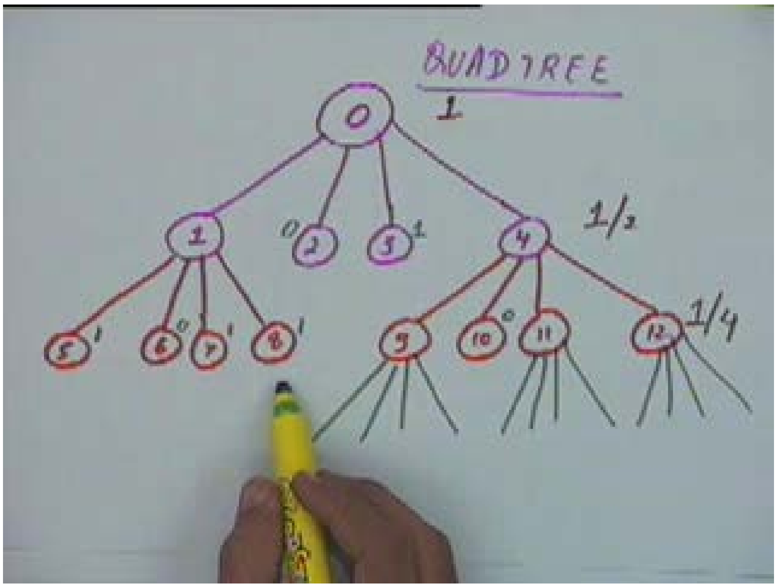

So let's say this complete region is represented by one root level nodes. This has been divided into 4 parts, this is region 1 2 3 and 4. Now for region 1, we haven't been able to decide but region 2 has been given a 0 because totally outside the region that we are trying to model. So for region 2, I will give it as 0. Similarly region 3 I have given it a 1 and then region 1, I am again dividing into 4 parts and region 4 also has been divided again into parts. So region 1 is divided into 5 6 7 and 8, 5 6 7 and 8 and 4 is divided into 9 10 11 and 12. Now again 5 has been given let's say 1 and 6 has given a 0 and 7 and 8 have been given a 1 each. Between 9 10 11 and 12, 10 is given a 0, the other 4 are again divided further and so on. So at every node let's say this is my node number 0, at every node we are dividing a space into 4 parts and each part we are trying to say whether it is inside or outside.

And this way we will continue till the whole of a solid or the whole of the area has been marked with zeros and ones or till let's say some portion, let's say if I divide this 12 further. Now this portion is totally also, this is also totally outside. Let's say this portion is again partly outside and partly inside but now I might decide that this is too smaller portion to be considered. Again so I will neglect this portion. So we decide upon a criteria as to how much area has been lost, how small is the portion that is being lost or gained. For example in this I might decide, I will consider totally to be inside. So this area I will give it a code of 1, this one. We will decide upon an accuracy level and according to that we will give zeros and ones to every region. So our area that we trying to represent will not be modeled accurately, it will model only up to the accuracy that we desire.

And larger the accuracy that we want, more the number of times we will have to divide a, subdivide the area. If you want a very accurate model, let's say if you have want to go up to microns then we will have to keep subdividing up to a large extent which means that the depths of the tree will become very large and the space involved will also become very large. And if you consider square rotating represented at this level let's say if this has a size of one, unit size, the size here will be half, the size here will be one fourth because you are dividing a square into 4 parts into 2 equal parts in either direction. These two parts are going to be equal. So the size of the square here will be one fourth and the next square we will one eight and so on.

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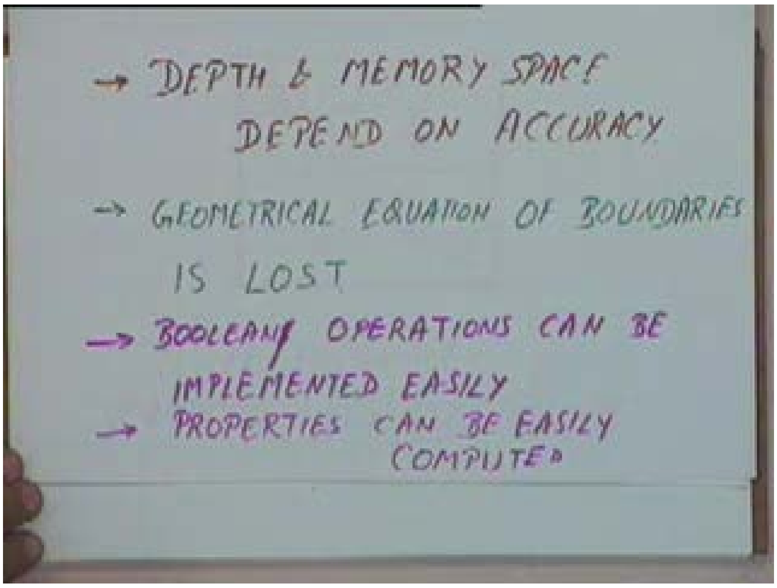

So depending on the accuracy that we want to go up to the depths of the tree will depend on that. So the depth and memory space, they will both depend on accuracy. The other thing that is important in this is that let's say in this region, I want to model a curve which looks like this. That means I want to model let's say this area, I will start by subdividing this into 4 parts.

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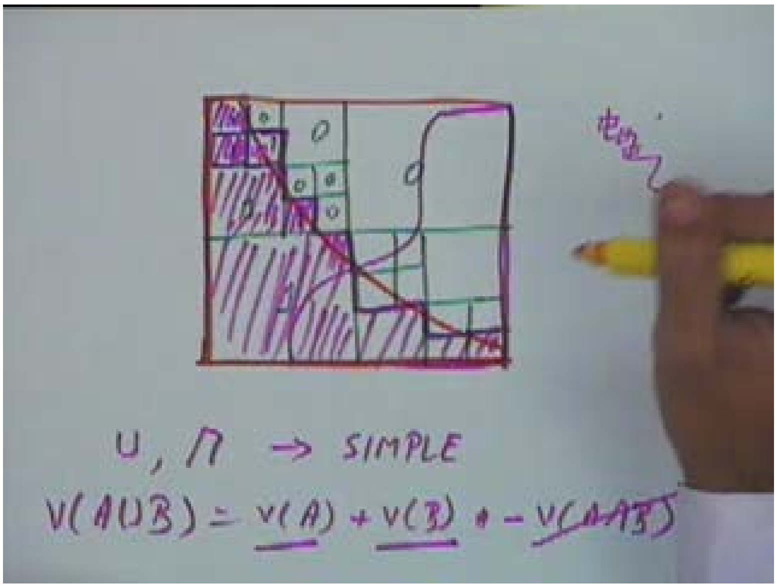

Again I will start, I will subdivide this, subdivide this, subdivide this and then if possible subdivide this, this way, this way and so on. And finally I will get a representation which is something like this. Every node is divided into 4 parts and so on but now the mathematical equation of this is lost. Again let's say this is may be an arc of a circle, from this representation this is very difficult to say that this is an arc of a circle. Now we only have sets of a zeros and ones which define the different cells. The whole thing is represented by set of zeros and ones like this and so on. This is let's say 1, this is 0 0 1. Now from this data it is very difficult find out what is the equation of the curve.

So in a two quadtree, we say that the geometrical equation or equation is of boundaries is lost. That is one big disadvantage of this. We don't known what would be the equation, what will be the equation governing the curve, the bounding curve. Some of the advantage of this model let's say if I have an area which is like this and I have another area which is let's say something like this. And we want to find out let's say the union or the intersection that can be found out very easily because every cell I can see whether it is a 0 or a 1 and decide it about whether it is going to be included in the final area or not. So the algorithms for union and intersection will be very simple. But again the time that these algorithms will take, that will depend on the resolution that we have chosen because we have chosen a very fine resolution.

My algorithm might still take a very long time because I have divided into very small small cells. So time involved might be large but the algorithms are going to be very simplistic. So we say Boolean operations can be implemented easily. If we want to compute the geometric properties that is let's say the area of this region, I only have to take the area of each of the individual cells and keep adding. If I look at this tree, the area at this point is 1, the area of all over this level is going to be half into half which is one fourth, area of all over that this level is going to be one sixteenth.

Similarly, if I want to compute the moment of an inertia that can also be done because I know each cell whether the area exist there or not. So the properties of this area can be computed easily. Of course, we can't say anything about the time, we can say anything over the time only if we know the resolution. The resolution we are looking for is very high then the time involved will again go up but the algorithm will be simple because no two areas have any portion in common. If I take this cell and this cell, they are disjoint. If you had remembered earlier I mentioned that if you say the volume of A union B is equal to volume of A plus volume of B plus sorry minus volume of A intersection B. If I consider any two cells in a quadtree representation, the area of A intersection B will always be 0. So we only have the area of A plus area of B or moment of inertia of A plus moment of inertia of B. So properties can be easily computed. So these are some of the advantage and disadvantage of this method.

The biggest disadvantage is that if the accuracy is large, we need a lot of space and that the geometrical definition of the boundaries is lost. These are the biggest disadvantages of this method. Any questions regarding this? The other thing is let's say if we want to display an area which has been represented like this, for a two dimensional object display can be done very easily. I just have to start drawing each of the squares when they are one. If you look at this tree, whenever a leaf node is 1 I can draw the square, if it is 0 then I can omit it. If you do that we will get or display which would, for curve like this my display might look something like this. It won't be accurate, even portion which are outside the curve they will be shown as inside because my curve instead of looking like the smooth curve like this, it will look stepped. It looks something like this, it won't look like a smooth curve, it will look stepped like this because you have divided it into a set of squares.

And if you look at the boundary, the region around the boundary is divided into a smaller size cells and the region which is away from the boundary, they are divided into a larger size cells. But if we have a very complex boundary which has lot of details, in that case we need to go into a very small size cells only then we can represent the boundary accurately because the accuracy at the boundaries is always lost in these methods. Now if we want to model three dimensional solid, so far we are talking of modeling of areas, there are two dimensional areas.

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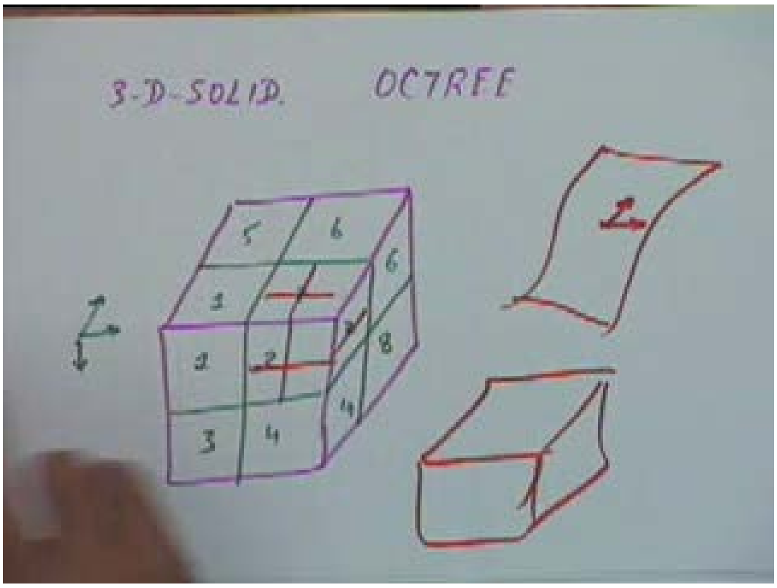

If you are talking about 3 D solid then we extend the same thing into an octree. This tree was called a quadtree because every node had four children, every node had four children, so it is called a quadtree. Now if we go into three dimensions, instead of talking of a unit square, we will now talk about the unit cube. And this unit cube we will divide it into 8 portions whereas let say 1 2 3 4, this one will be 5, this is 6, 7 will be at the bottom corner and this one will be 8, the faces of 2, this is 4, this is 6 and this is 1. So every cube will be divided into 8 octants and the tree that we have, every node will have 8 children.

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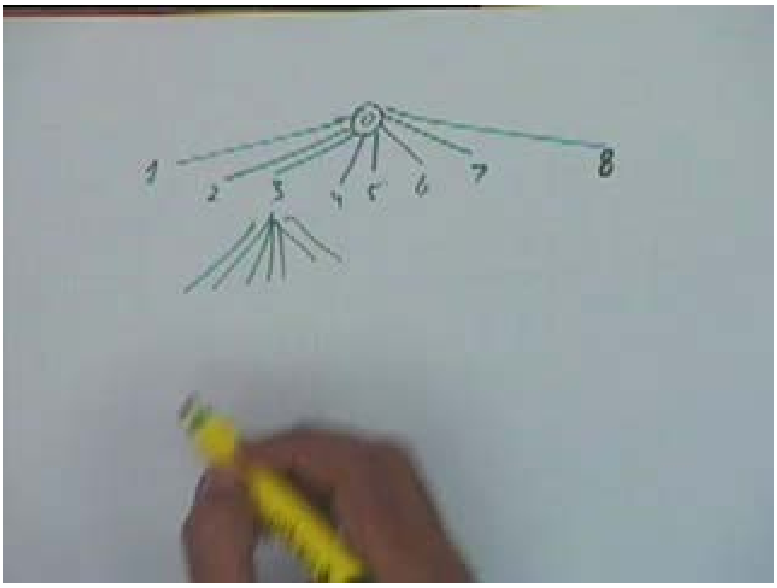

So if this is my root node 0, it's like 8 children 1 2 3 4 6 7 and 8 and each of these nodes will be either a 0 or a 1 or it will have again 8 children after that. So this way, this is a simple extension of quadtrees into a three dimensional space. In two dimension, we had 4 children for every node, in three dimension will have 8 children.

Essentially what we are doing is in every dimension in the x direction let's say this is a y direction and z direction, we are dividing the unit length into two portions. This is divided into two portions here, two portions here, two portions here, so we get two cube that is 8 parts of a cube. And similarly in two dimensional space we got 4 parts. For a three D object, we get an octree and this octree will have the same property as quadtree. Here also the depth and the memory will depend on the accuracy; here also the geometrical equation of the boundaries will be lost. The Boolean operations can be implemented easily because every cube, for every cubic cell that will either a 0 or 1, so if we have computing the union that can be done easily. And properties can be easily computed. So if I want to find out the volume, I only have to traverse octree and I will get the volume.

Similarly if I want to get the moment of inertia, I only have to traverse the tree and I will get the volume. For a tree like this, for all the leaf nodes will compute the property either the volume or the moment of inertia and narrator so that can be done quite easily. Any question about quadtrees and octrees? Any question about solid modeling? There is no question, I will wind up now and if you have any questions we can discuss that right now, otherwise I will wind up the... Unigraphics is using basically a combination of CSG with or B-REP. It has graph based model for the primitives plus its using set of Boolean operations. So actually if a modeling using an octree representation then that can be seen very easily, I mean the display that you will get will have edges and you will see small cube and so on because one normally doesn't like to go into a very high resolution in the case of octree representation. One thing the display using octree models that can be quite simplified depending on the of course technique one uses.

If one wants a good rendered image with the nice shading then it is not going to be very good because let's say I think it should mention that. If you have a keyboard like this which shall be divided into very high resolution and so on and you want to find out a surface normal at any point that is not going to be easy because the geometric equation is lost. Normally, if you have let's say a three dimensional surface and you want to find out its surface normal at any point, you can compute the derivative in one direction, derivative in a second direction and take the cross product. So you will get a surface normal but if you have a model represented in octree representation, getting a surface modeling is not very easy. The best that can be done is you draw each of the cubes independently.

So the surfaces of the cube which are visible you keep drawing those surfaces one after another. Again once you have a good enough of representation you stop. That is the way display is normally generated. Yes, yes yes near the commonly used alternatives yes. You have some variations of B-REP but there are essentially all graphs there. If you have some kind of information being represented, strictly speaking of the B-REP the complete model is represented as a union or intersection of half spaces, so let's say if you want the represent a cube, we will represent the equation of each of the faces and then say which half space is inside the solid and the intersection of all of those will give us the final solid. But some kind of graph based representation is always used in which adjacency information is being stored. That is most commonly used form of thing.

Each of these representations whether it is B-REP or csg or octree, in some situations one is helpful and in some other situation other is helpful. Quite often we keep switching from one to the other. You have the algorithms available for getting the octree representation of a B-REP model or octree representation from a CSG model of simple primitives and the other way around. So such, in fact some algorithms are also available for getting the equation of a model represented by an octree model but that is quite commonly needed. So algorithms are now available but they are complex.

They will do some kind of fitting between the cuboids. Each cuboid being a 0 or a 1, they will do some kind of resolution technique and get equation of a curve but it is complex. Between these three techniques, one can switch from one to the other but at the cost of time. Patron, Pattern is essentially a plane prosperous for a finite element analysis model. Say it doesn't have extensive solid modeling capabilities.

In octrees, see the pending on the accuracy you are storing the model depending on some accuracy. The display is depending on some other accuracy. So, if you want to get a display which was finite and you have stored a model that is not going to be possible. Sometimes what is also done is that in the equation I have stored, the equation of all the surfaces that is stored and then you wanted to use that and you need the octree model, you generate the octree model immediately so that you don't do the geometrical information also. In that case you are using the combination of two. Some kind of boundary representation plus an octree. After use, only when you need the octree representation. Any other questions? In that case I will call it a day now.