Computer Aided Design
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Lecture No. # 36
Surface Modelling (Contd.)

In the last class we are talking of modelling bicubic surface patches and for that we had derived this expression that p of u v will be given as UMB multiplied by M transpose into V transpose or it can be given as U into A into V transpose where A is the algebraic matrix and B the geometric input matrix.

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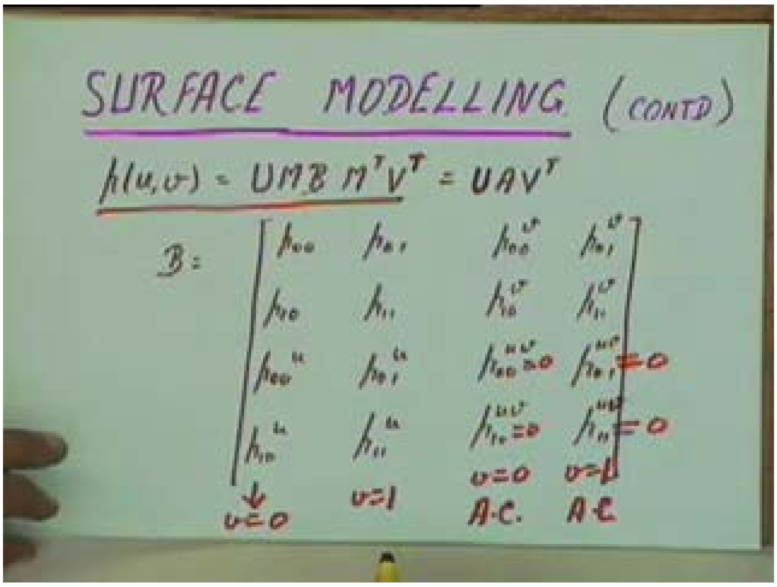

The matrix B is given by this set of 16 vectors p_{00} p_{01} p_{10} p_{11} p u at 0 0 p u at 01 and so on. And in this matrix if you look at this first column we said that this, these 4 terms together will specify a pc curve which is the v equal to zero curve. Similarly these 4 terms will specify the pc curve for v equal to 1. These 4 terms will specify the auxiliary curve for v equal to 0, this is the auxiliary curve and similarly this is the auxiliary curve for v equal to 1.

And similarly these 4 rows will specify another 4 set of pc curves, first two will be pc curve, the next two will be auxiliary curves and this is how we can formulate this 4 by 4 matrix of the input vectors. And we also mentioned that if these 4 twist vectors are put equal to zero, in that case this is called Ferguson patch or a F patch.

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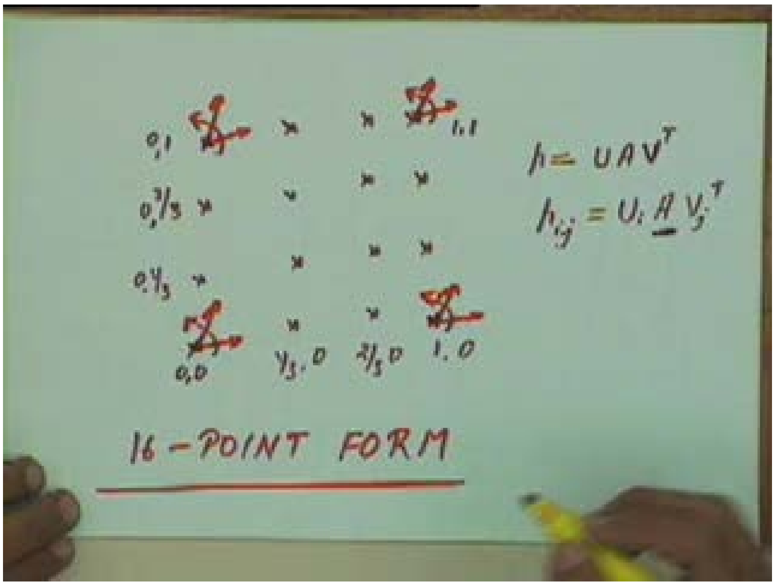

Now what we will see is that instead of inputting a surface through these 16 vectors, these 16 vectors are the 4 corner points. Tangent vectors at the four corner points and the twist vectors which let's say if I represent them by 4 curved arrows like this. So these are the 16 inputs we are giving right now but generally it is difficult to give input in the form of twist vectors or in the form tangents instead of that we sometimes prefer to give input as points. So if you want to give the input as let's say 16 points on the surface then we can give an array of 16 points at specified values of the parameter u and v. Let's say this is at 0 0, maybe this is at 1 by 3 0, this is at 2 by 3 0 this is at 1 0. Similarly this maybe it is at 0 1 by 3, this is at 0 2 by 3 and this is at 0 1 and this is at 1 1, others you can fill up. If you give the input as these 16 points and we say that this bicubic surface patch should pass through these 16 points. In that case again we can write down, we know the equation of the curve as p is equal to UA into V transpose.

At each of these points we can write down p_{ij} as U_i into A into V_j transpose that means I am essentially putting U equal to i and V equal to j in this expression and this will give me a set of 16 equations. I can solve those 16 equations simultaneously to get the 16 parameters or 16 variables required for the A vector, for the A matrix.

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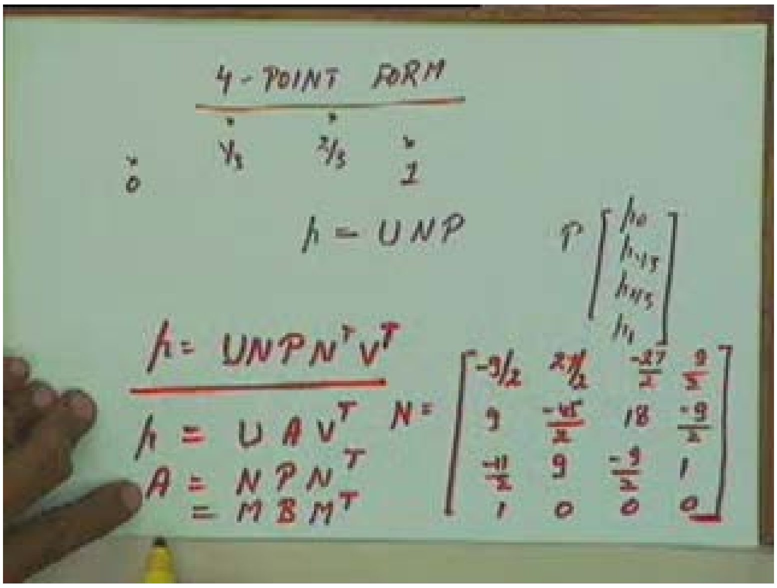

If you remember when we carried out the same exercise for pc curves, for pc curves when we gave the input in the four point form 0, 1 by 3, 2 by 3 and 1. For the pc curve we had mentioned or rather we had derived that p will be given by UN multiplied by P where this matrix P consisted of these 4 points that is p_0, p_1 by 3, p_2 by 3 and p_1. This was what we referred to as the four point form of the pc curve. Similarly if you are given the input in this form, we can derive an expression for this 16 point form of the surface and for this 16 point form we can shown that p will be given by UNP multiplied by N transpose multiplied by V transpose. That this matrix N is the same as this matrix N and just to recapitulate this matrix N, I had given it last time also it is given by these terms. So if the input is given in a 16 point form like this, that means if these 16 points are given as input at these location 0 0, 1 by 3 0, 2 by 3 0 and so on.

Then for this 16 point form, we can get this relationship where N is given by this matrix and P consists of the matrix of these 16 points. The detailed derivation for this, you can look at from the book but this result is it can be shown to be this. And we have already shown that p is equal to U into A into V transpose. So the set of algebraic coefficients will be equal to N into P into N transpose, just by equating these 2 right hand sides. That A will be NPN transpose and we have already shown A to be MBM transpose. So this is also equal to MBM transpose. If we compare it with the formulation for the pc curve, the formulation will be very similar. Their A was equal to NP and it was equal to M times B. So you have a similar formulation in this case also.

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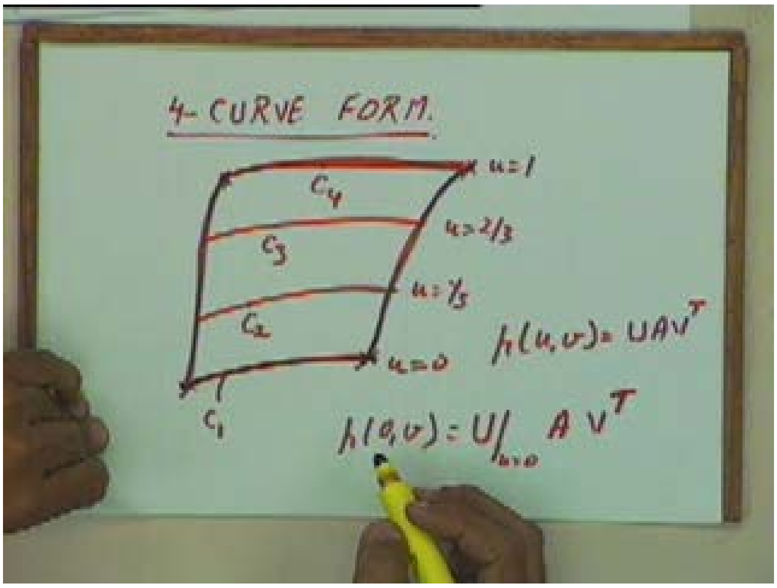

Now the other form of that we can have for the bicubic surface patch is what is referred to as a 4- curve form. Again I just give the outline and let's if this is the surface that I want to create, this surface can also be created by inputting 4 curves in the same direction. If I consider this let's say this is my curve 1, this is my curve 2, this is my curve 3 and this is the curve 4. These 4 curves can be input let's say for standard values u equal to 0, u equal to 1 by 3, u equal to 2 by 3 and u equal to 1. So for these 4 standard values let's say if we input these curves and again we can derive the expressions for the algebraic matrix. We can write down the equation for each of these curves as say p of u v is equal to UAV transpose. In that if you put u equal to 0, we will get p of 0 v will be equal to U at u equal to 0 multiplied by A into V transpose. If this curve is given, similarly if the other 4 curves are given we will get a set of simultaneously equations and we can get expression for A using that. So if we are given 4 curves then an expression for A can be derived from those 4 curves also.

The only thing one has to be careful about is that normally you would prefer to give these 4 curves in the same direction. If I give two curves in this direction and the other two curves in this direction, that might not be correct because at the common point we might not be specifying independent variables, we will have to be careful about that because in the beginning we had seen that if we give this as one curve, this as a second and this is as a third curve and this as a fourth curve and we did not get a unique bicubic surface. But that was because the 4 curves were not in the same direction and we had common points that have been given. But if you give four curves in the same direction then we don't have problem of this type and we can specify a bicubic surface patch in this manner also. Any question up to this point?

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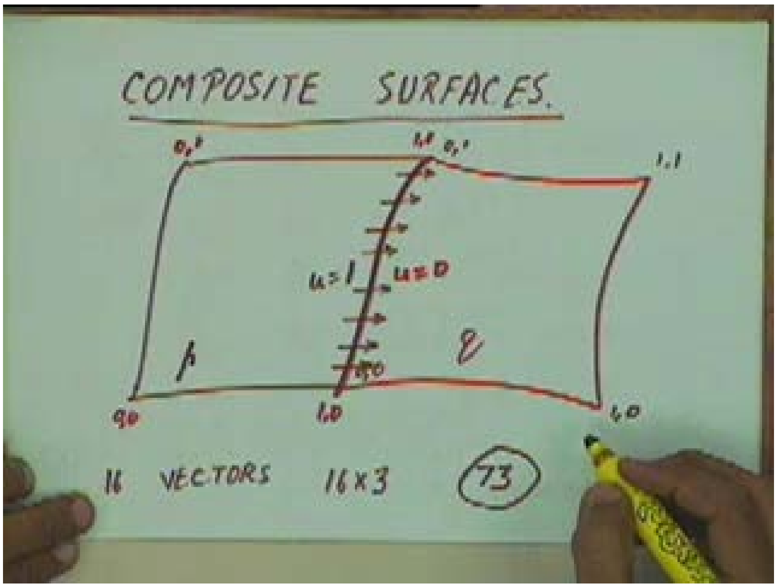

The next thing let's see is how to formulate composite surfaces. Let's say if we are considering two surfaces. Let's say this is a surface patch q and this is a surface patch p and let's say this is 0 0 point for this surface patch. This let's say the 1 0 point, this is the 1 1 point and this is the 0 1 point. Similarly for this surface or the q surface, let's say this is the 0 0 point, this will be 01 point, 1 1 point and this is the 1 0 point. Now what we are bothered about is that at this common edge, we should have a certain kind of continuity. We have already mentioned that for specifying any bicubic surface patch, we need 16 vectors which mean that for any surface, for any bicubic surface we have 16 into 3 that is 48 degrees of freedom.

So if these two surfaces or these two surface patches are totally independent, we would have had 48 into 2 that is 96 degrees of freedom but since we want to have let's say c_1 continuity at this edge. The two surface patches will not have 96 degrees of freedom. The number of degrees of freedom will be much low. So let's just see what, how we can ensure that there will be c_1 continuity between these two surface patches. Let's take the standard geometric form for the input for both of them.

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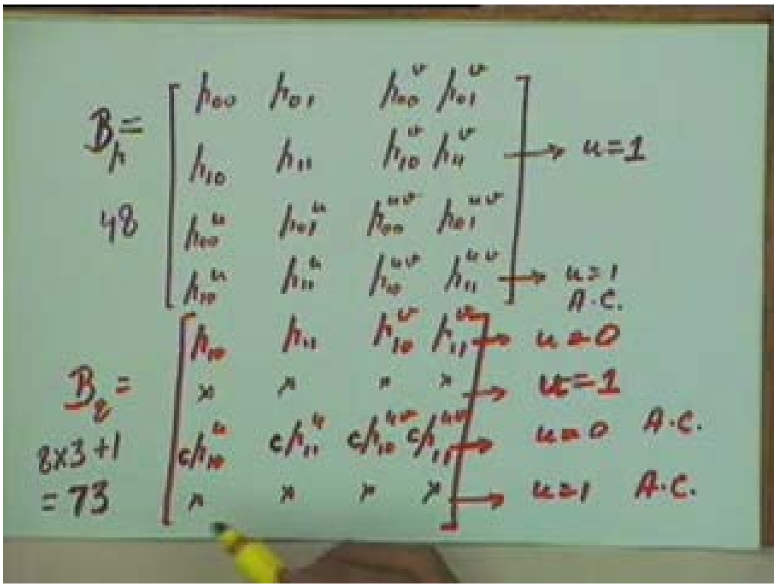

Let's say for this surface if we write down the standard geometric form, we will get B that is the input, the geometric input matrix as p_{00} p_{01} p_{10} p_{11}. This is the input matrix for this p surface, for the first surface that I have made. In this surface this edge is the edge given by the curve u equal to 1. So the edge given by the curve u equal to 1 that in this is given by this row. Similarly if I want to ensure c_1 continuity then along this edge, the tangent vector in this direction should be proportional for both the surfaces, only then we will get c_1 continuity. And the tangent vector in this direction along this curve is given by the auxiliary curve corresponding to this curve by the u equal to 1 auxiliary curve. So the u equal to 1 auxiliary curve is given by this row. So if we want to ensure c_1 continuity between these two then the u equal to 1 curve for this patch and the u equal to 0 curve for this patch should be identical that will ensure a c_0 continuity that means these two edges will be common.

Secondly the auxiliary curve along this, for the q patch should be the same as the auxiliary curve along this for the p patch. Now if we want to write the B matrix for the q patch, let's say that this is B for p and this is B for q. Again this is a 4 by 4 matrix and the first row is the u equal to 0, u equal to 0 curve and the third row will be the u equal to 0 auxiliary curve. The second row will be v equal to sorry u equal to 1 and the fourth row will be u equal to 1 auxiliary curve. And now to ensure a c_1 continuity here, the u equal to 1 curve for p and the u equal to 0 curve for q have to be identical which means that this first row here has to be the same as this row.

So the first row here would look like p_{10} p_{11} p v at 1 0 and p v at 1 1. Similarly the third row here should be proportional to the fourth row over here. Only then we can ensure that the tangents are in the same direction everywhere. So if that is to be ensured, this fourth row here will get repeated here in the third row with a constant attached to it. So, p u at 1 0, p u at 1 1, p u v at 1 0 and p u v at 1 1, so these four terms will come here with a constant attached. Only then the tangent vectors along this edge will be in the same direction. The magnitudes can be different. So further B input matrix for the q patch out of the 16 terms, 8 terms will get fixed like this and the remaining 8 terms can be anything. Whatever the remaining 8 terms be, the two patches will always have a c_1 continuity at the common edge. Again, 72 degrees of freedom, that I don't think so. I will be just calculate the degrees of freedom.

Let's come to that. The number of degrees of freedom for these two surfaces put together, the first surface has 48. The second surface we can input 8 vectors that is 8 into 3 plus this constant. That is 48 plus this will give us 73 degrees of freedom because this auxiliary curve and this auxiliary curve they can be proportional. So we have introduced a constant c and the magnitude of this constant c can be changed, so that gives us an additional degree of freedom. Either direction should be same, the magnitude can be different. So this way if you are having two adjacent composite surfaces instead of having 96 degrees of freedom, we will get 73 degrees of freedom.

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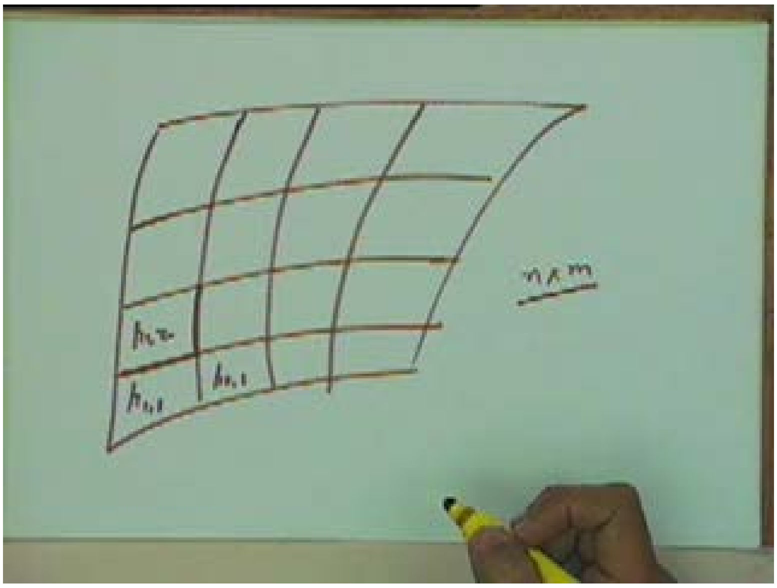

And this concept can be extended further to having an array of composite surface patches where each segment that I have made here either surface patch having a c_1 continuity along this edge for the next one. So let's say if I call this as a p_{11} surface patch, this is a p_{21} surface patch and so on. This is a p_{12} surface patch and between these two we will have a c_1 continuity over here, between these two we will have c_1 continuity along this edge and so on. So a formulation for having a let's say n cross m array of composite patches can also be made on similar lines. In fact you will find such a formulation given in the book. If you want to look at the details you can look into that. Any questions regarding bicubic surface patches?

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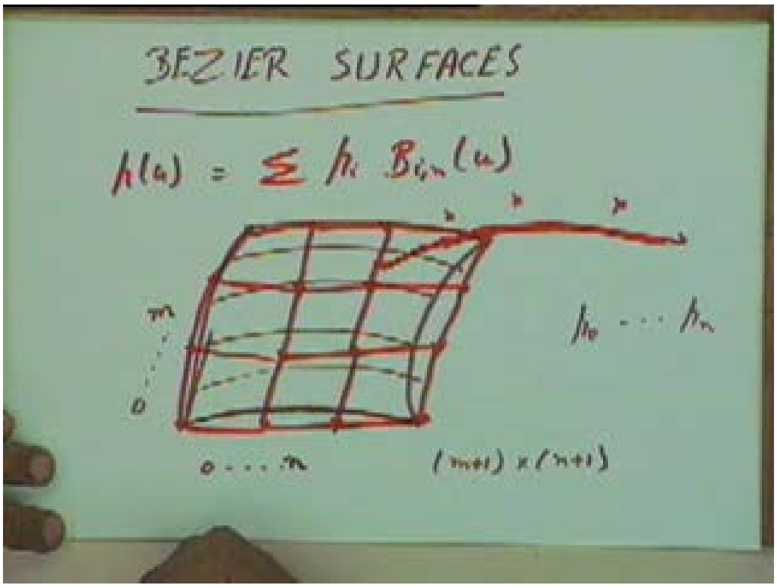

Then let's take up the next type of surfaces, those are the Bezier surfaces. We have earlier defined Bezier curves and for Bezier curves, we have mentioned p of u will be sigma p_i multiplied by $B_{i,n}$ of u, this was for Bezier curves. And in this case we are giving a set of control points as the input and a curve was being defined between, within the convex hull of these points. Now instead of having a single one dimensional array of points, we will talk of a two dimensional array of points that means if let's say this is the array of points that is input, I am joining them right now by simple straight lines. Within these we can define a Bezier surface which might look let's say something like this and let's say it will have a family of Bezier surface, Bezier curves in-between.

So we can define a Bezier surface in this manner where let's say in one direction we can have n plus 1 points, in the other direction we can have m plus 1 points. Earlier we are defining a points from p_0 to p_n. Now in one direction we can have points going from 0 to n, in the other direction from 0 to m, so this way we will get an array of n plus on cross n plus 1 points.

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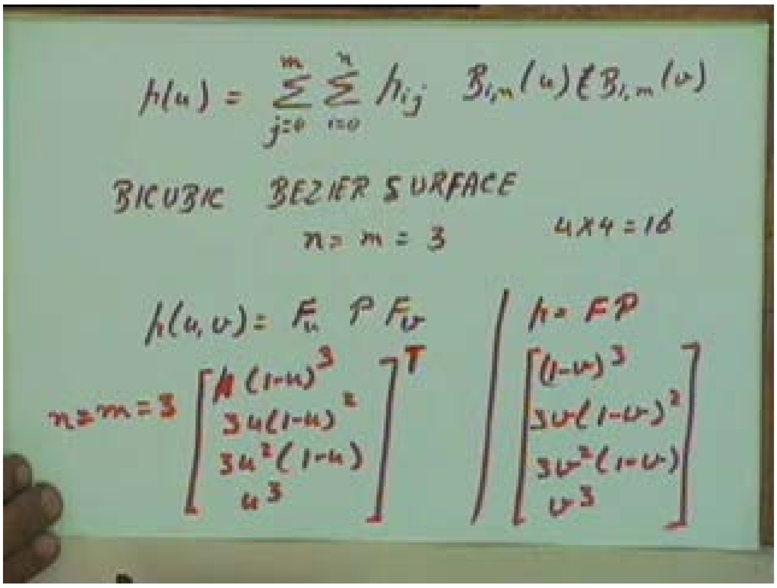

And the mathematical formulation for Bezier surfaces would be where the summation from i varying from 0 to n and j will be from 0 to let's say m. So in the case of this Bezier surfaces, you will find that a degree in one direction can be n, the degree in other direction it can be m. So if you are talking of bicubic Bezier surface, for that we will say n will be equal to m will be equal to 3. The total number of points will be 4 by 4 which will be 16. And then for a bicubic Bezier surface, we can say that, we can show that p of u v will be equal to let's say F of u multiplied by P multiplied by F of v.

If we compare it with what we had for Bezier surfaces, you are saying p was equal to F multiplied by p. A p was the array of control points. In this case we can say F of u multiplied by P multiplied by F of v where F of u that can be let's say for, if you are talking of a bicubic surface patch we can say that this will be and similarly F of v will be having similar terms containing v. So this will be the transpose of this and the F of v will contain terms like and this is what I have written is only for the case of n is equal to m is equal to 3.

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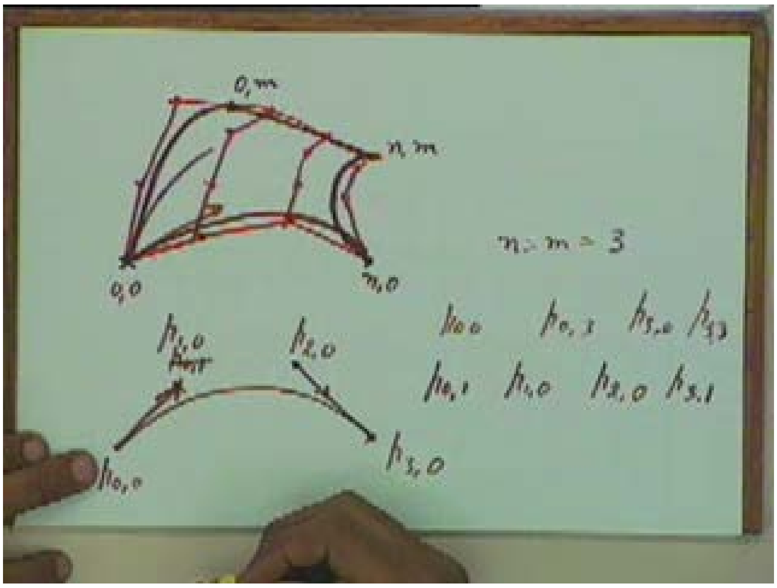

For a bicubic Bezier surface, if you have a surface which is given like this, this surface has to pass through the 4 corner points because let's say this is my point which is the point 0 0 in the array. Let's say this is the point which is n 0, this is the point which is n m and this is the point which is let's say 0 m. This is the corner point I have taken over here. This surface will pass through these 4 corners points. But it will not pass through any of the other control points just like we had in the case of Bezier curves because this surface is an approximating surface and not a interpolating surface. These four bounding curves that I have drawn right now, this curve, this curve and this curve, these four bounding curves they will be the Bezier curves. This can be seen easily from this formulation p_{ij} into $B_{i,n}$ $B_{i,m}$ and suppose u equal to 0. I will still get of the form of the Bezier curve.

Similarly if I put u equal to 1, I will still get a form of the Bezier curve and so on. So in this surface these four bounding curves they will be Bezier curves. So what that mean is if I consider this curve, this curve is defined using these 4 points. So if I draw the view here that means that on this curve, the tangent in this direction will be from here to here in this direction. That means this will be the direction of tangent vector because this is a Bezier curve, so direction of tangent vector has to be from the first control point to the second control point.

Similarly the tangent vectors at the end will be in this direction and the same thing can be said about the other three surfaces or the three curves also. That means for a bicubic Bezier surface when n is equal to m is equal to 3, the points p_{00} p_{03} p_{30} and p_{33} these points will define the 4 corner points but if I consider the points p_{01} and p_{10} they will control the two tangent vectors at the corner, at the point 0 0. This point is p_{00} and this point that I have made this is a point p_{01} sorry p_{10}. This is a point which is p_{20} and this is the point which is p_{30}. So 1 0 and 0 0 will define the tangent vector here. So in additional to these points p_{01} and p_{10} will be controlling the tangent vectors at this corner point. Similarly if I consider this point, you will get p_{20} and p_{31} these two will control the tangent vectors at this point, these two in conjunction with point 3 0.

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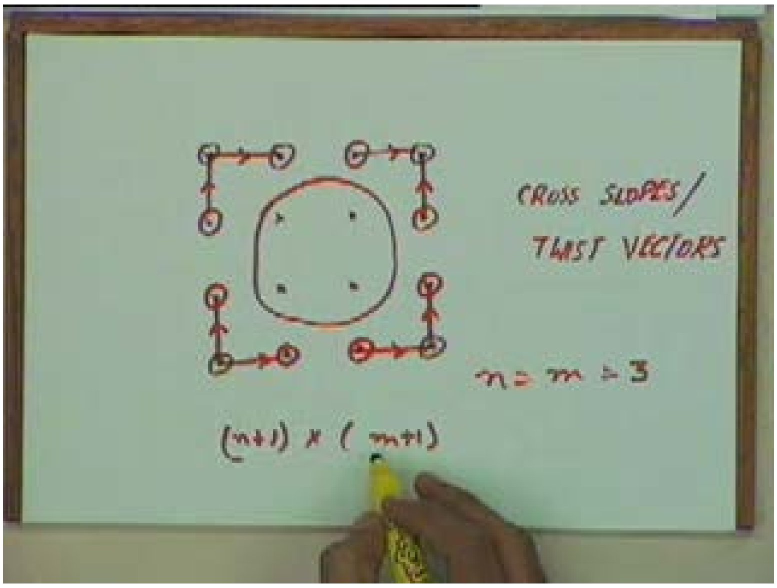

So if consider, I will just draw a rectangular array, these four points control the four corner points. These two will control the tangent vector in these two directions, these two will control the tangent vectors in these two directions, these two will control the tangent vectors in these directions, these two will control the tangent vectors in these directions. So out of these 16 points, 4 are controlling the corner points and 8 are controlling the tangent vectors and these 4 they will be controlling the twist vectors at the 4 points. So if I specify these 12, I can specify the 4 bounding Bezier curves.

But these 4 will help us control the twist at the corner points, the twist vectors or the cross slopes or twist vectors. And again we can say, something we said earlier for Bezier curves that if you are given this array of input points and I change any of these points, my tangent vectors at this point will not get effected where at this point the tangent vector in this direction will be controlled only by these two points and the tangent vector in this direction will be controlled only by these two points.

Similarly at this point the tangent vectors are controlled only by three points and we can extend that to a general, a surface of degree n. If we have a surface of degree n, if I consider the k, if I consider k points in this direction, the k th derivatives will be controlled by only those k points. Similarly k points in this direction will control the k th derivative in that direction and so on. That is a property it is specific of Bezier curves and Bezier surfaces. So we can define bicubic or we can define Bezier surfaces using this kind of formulation. Any question with respect to the Bezier surfaces?

16 is the number of points for as to give for a bicubic Bezier surface that is when the degree in both directions is equal to 3. If you have a higher degree curve, we can have higher number of points. If you want to have a quadratic Bezier surface then you need to give only 3 into 3, 9 points but normally cubic is the lowest degree one goes down to. Anything else with respect to Bezier surfaces?

Similarly if I have let's say, if I have a higher degree Bezier surface that means I have n plus 1 cross n plus 1 points. If I change any of these intermediate points, my continuity of the corners will never get affected. The continuity will be affected only by k points in this direction and k points in this direction. So if I want c_1 continuity only one point here and one point here that will only affect by continuity. So if n and m is large that can be important in controlling the continuity in the case of composite surfaces. Anything you want to know about Bezier surfaces? In that case let's quickly see how we can define B-spline surfaces in a similar manner.

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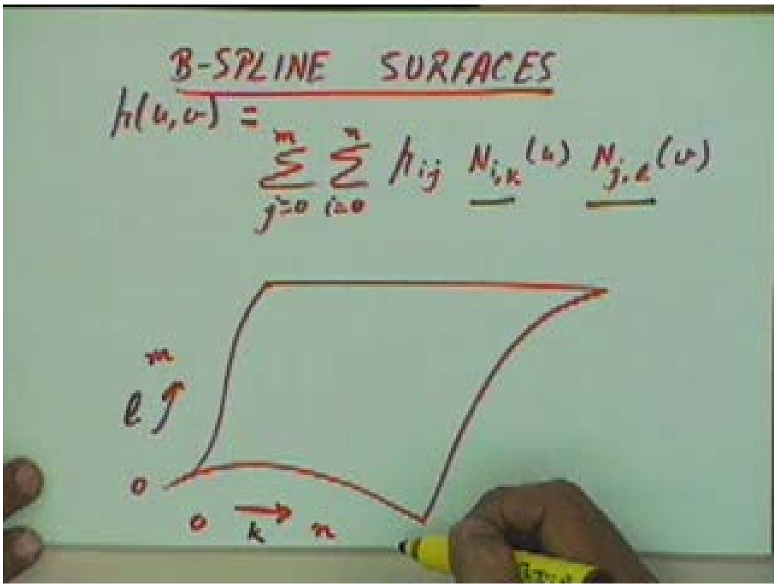

So if you want to define B-spline surfaces, we will extend an expression of B-spine curves and will say p of u v will be given as summation in both direction p_{ij} N_{ik} of u multiplied by N_{jl} of v where i goes from 0 to n and j goes from 0 to m, just as we have defined B-spline curves. Now if let's say this is the general surface that we want to define, we have number of points in this direction going from 0 to n. The number of points in this direction going from 0 to m and in addition to that we define the degree of this curve in this direction as k and the degree in this direction as l. So this k and this l will control the degree of local control in the two directions. And the definition of N_{ik} and N_{jl} will be there in a manner similar to what we had done for B-spline curves. These definitions will remain the same.

So if you are defining a bicubic B-spline surface, we will put k equal to l equal to 3 sorry equal to 4 that will define a bicubic B-spline surface and the number of points in either direction can be anything. So we will be able to define an array of surface segments or surface patches as you are defining curves segments for B-spline curves. And all the property that we had for B-spline curves will also be extended to B-spline surfaces. That means the degree of local control will be there and in addition to that we can define a closed surfaces, we can define what we had periodic and non-periodic surfaces.

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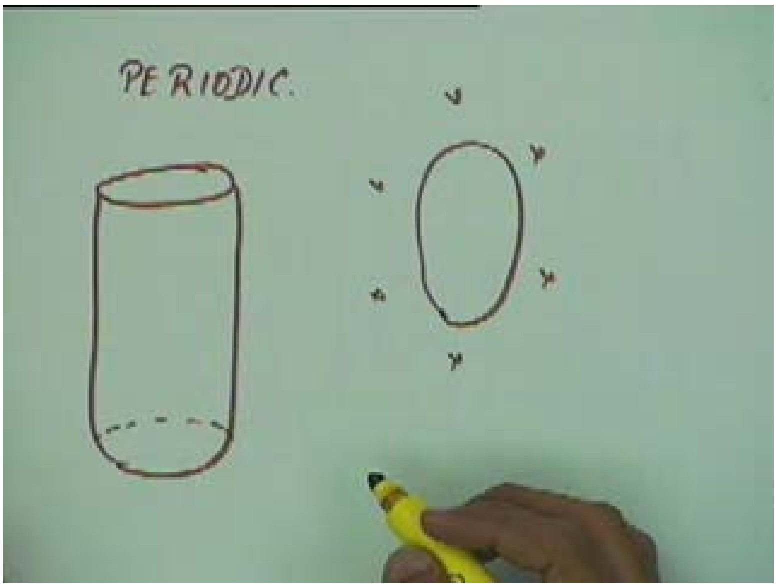

If we remember we had defined periodic curves and for a closed periodic curves, if we had an array of points like this, a closed curve or the smooth curve like this. Similarly we can define periodic surfaces and we can define closed periodic surfaces which can be totally closed surfaces. For example we can also have surfaces like this, this is closed in one direction open in the other direction, periodic in one direction and non-periodic in the other direction and so on.

So the basic definition of the B-spline surfaces will be given by this and on top of this basic definition we can define periodicity and we can define close and open surfaces in either direction. In one direction it can be open, the other direction it can be close. In one direction it can periodic, in the other direction it can be non-periodic. So we won't go into details of these surfaces.

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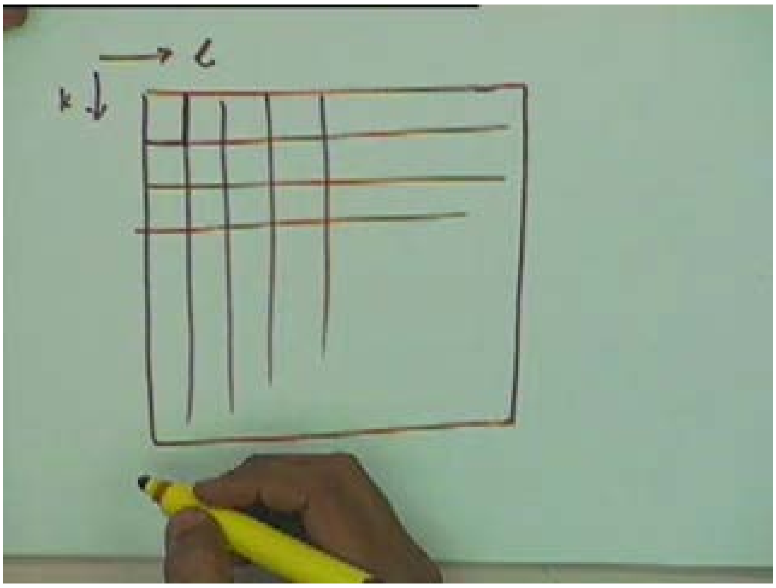

The basic idea behind these surfaces is that these surfaces are essentially giving us an array of surface patches. We get an array of surface patches like this and in each patch, we will have a degree depending on the degree of the B-spline surface. So let’s say if you are taking a degree k here and degree l in this direction and each of these patches will have a degree k and degree l and the number of points controlling each patch will be governed by these parameters k and l or the total number of parameters might remain anything. So this is how we can define B-spline surfaces. Any question on B-spline surfaces? So with that I will close this topic of surface modelling. From the next class we will see how we can model solids and how we can represent solids for different applications. Thank you.