Computer Aided Design
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Lecture No. # 35
Surface Modelling

Last time I had given this definition of a bicubic surface patch and we said that if we take 2 parameters u and v, in any point on this surface patch can be given by this double summation where i varies from 0 to 3 and j also varies from 0 to 3 and this summation can be expanded out in this form. That is a_{33} u cube v cube so on. Basically a_{ij} into u to the power i v to the power j.

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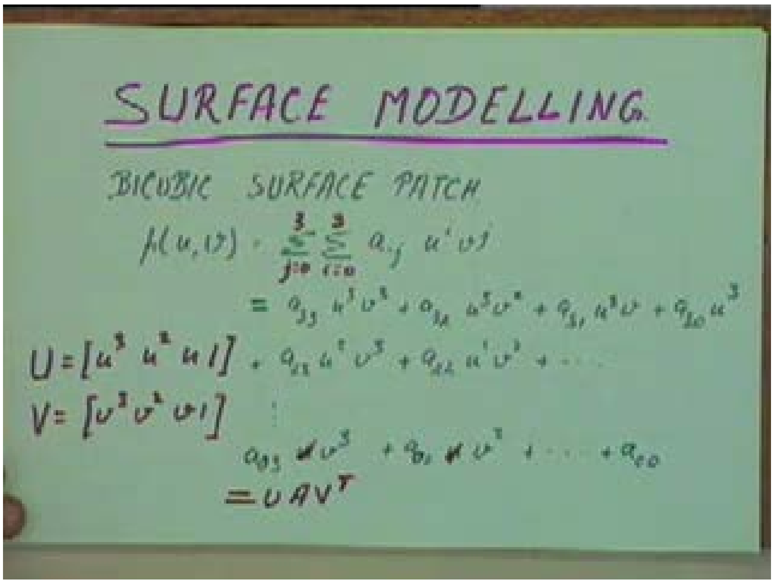

And this again we can write it in a matrix form where u is the power vector for u and v is the power vector for v for the parameter v. So this is the definition of a bicubic surface patch.

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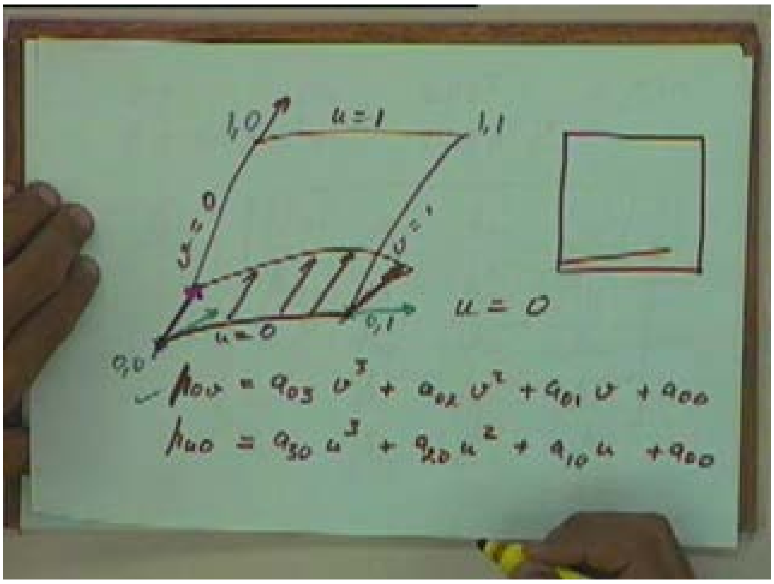

Now if you take this definition, we are trying to define a surface patch which is let's say something like this. In this surface patch if you put, let's say u equal to 0 that is in this definition if you put v equal to 0, we will get this last row of terms. And this we can write as p of 0v because u is equal to 0 in this case, p of 0 v will be equal to a_{03} multiplied by v cube plus a_{02} multiplied by v square plus a_{01} multiplied by v plus a_{00}. This is for u equal to 0 that is why I have written as p of 0 v. And this is an equation of a pc curve and let's say if we consider this to be the u equal to 0 line, this mean that for a bicubic surface patch this bounding curve will be a cubic curve or a pc curve.

Similarly if I consider p of u 0 that means I put v is equal to 0 now, in that case I will get a_{30} u cube plus a_{20} u square plus a_{10} u plus a_{00}. And this will be the v equal to 0 curve which is this curve that means these two bounding curves will be cubic curves. Similarly, I can also write down the equation for these bounding curves by putting this is v equal to 1 and u equal to 1. So if I put let's say u equal to 1 in this expression, none of the terms will vanish but it's the powers of u will all become 1. So my coefficient of v cube will be sum of these four.

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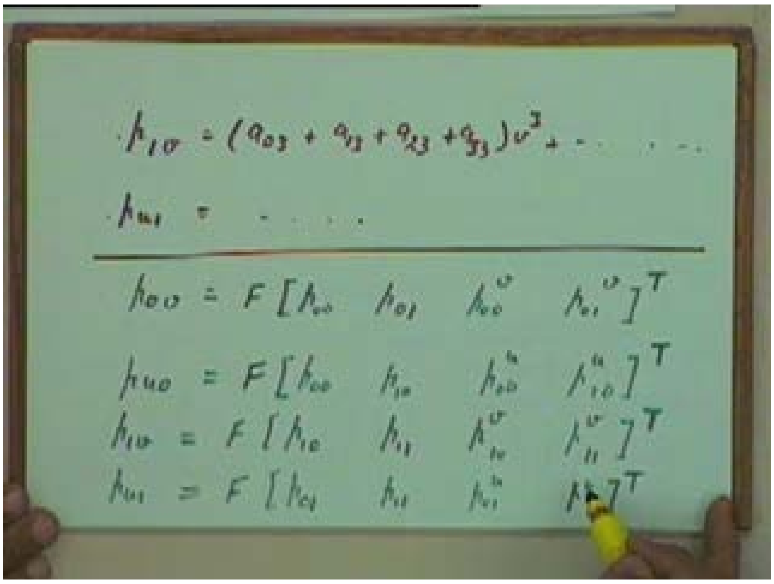

So what we will get will be p of, I am putting u equal to 1, so 1 v will be equal to a_{03} plus a_{13} plus a_{23} plus a_{33} multiplied by v cube that is sum of these four terms and so on. Similarly sum of these four terms will give us a coefficient of v square, sum of these four terms will give us a coefficient of v and sum of these four terms will give us the constant term. So this we can continue that way. And I can similarly write p of u 1 by putting v equal to 1, that will also be your pc curve in that manner.

So what that essentially means is these four bounding curves will be pc curves or parametric cubic curves. Now I can write these four curves using a standard geometric curve form. So if we do that we can write them as p of 0 v, that will be equal to let's say F times p_{00} p_{01} p_{00} v and p_{01} of v transpose. And v is the varying parameter, so I have put v equal to 0 right now. This is the u equal to 0 curve. I am considering starting point of this curve and the end point of this curve. In term of u and v this starting point is 0, 0, this point is in the point 0, 1 and similarly we will use that later on. This is 1, 0 and this is 1, 1, this is in terms of the parameter u and v. So this point is p_{00}, this point is p_{01}.

If I consider tangent in this direction that is a tangent in the v direction, that is the tangent in the v direction at 0 0. And similarly this tangent is a tangent in the v direction at 01. So this pc curve, this one we can write that in a geometric form in this manner. Similarly the other pc curve that is p of u 0, again we can write down in a geometric form as F and for this the input will be p_{00} p_{10} p_{00} at u, I mean p u at 0 0 and p u at 1 0. Similarly the other two curves that is these two curves, I can write them also in a parametric, in a standard geometric form and we can say p_{1v}, this is a curve for u equal to 1 so that is this curve, the starting point is p_{10}. So we will say F into p 1 0, ending point is p 1 1 and the tangent is in the direction of u, in a direction of v sorry. P v at 1 0, so this will be p v at 1 1 and the last curve that will be p_{u1} that will be defined using p_{01} p_{11} p_{01}, p u at 0 1 and p u at 1 1. So these are the four equations or the bounding curves of this surface patch. We consider this surface patch, the bounding curves 1 2 3 and 4, for these four bounding curves we can write down the equations in a standard geometric form like this.

Now if we look up the definition of the surface patch, this definition this has got 16 variables from a_{33} to a_{00} or 16 vector variables. That is a total of 48 parameters in this formulation or 16 vector parameters. And if you look at these 4 equations, there were 16 geometric inputs. So if we consider these four curves for inputting this surface patch, if somebody inputs these four curves one might expect that would be sufficient when we input the surface patch when it is not. Because these four curves are including only 12 parameters, they are not 16 different parameters, p_{00} and p_{00} are common and similarly p_{11} are common, p_{01} p_{01} are common and so on. So these four curves are defining only 12 distinct parameters. They are 12 because we have 4 corner points and at each point there were 2 tangent vectors. Let's say one in this direction and the other in this direction, so far we are using only 12 parameters. So for inputting a surface patch geometrically, we will need at least 4 more geometric vectors to be input, 2 4 8 10 sorry 2 4 6 and 8 and plus 4 corner points. These 12 are not sufficient to input the surface patch. So what we will do is in addition to these, in addition to the vectors we are using in this formulation, these vectors.

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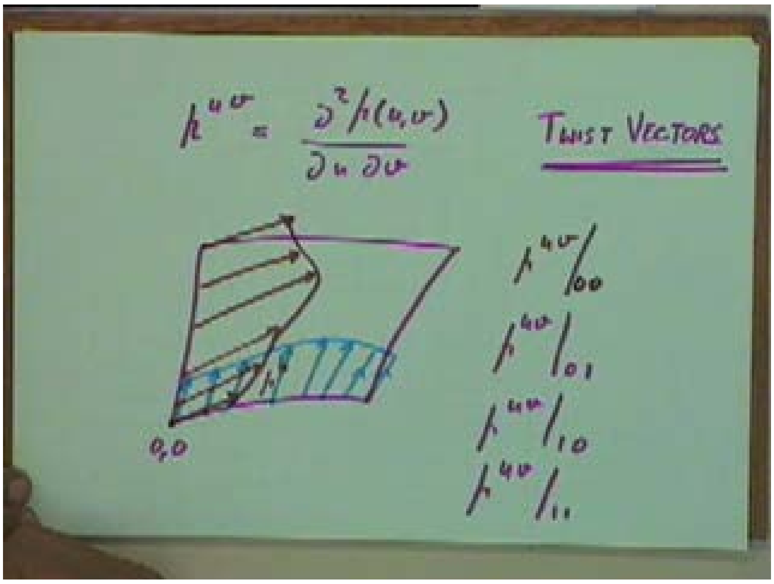

We will define 4 more vectors and those vectors will be a second derivatives which are defined as here p is a function of u and v. So these second derivatives, these are called twist vectors and they basically tell us that if you consider any point on this surface, let's say for the sake of I would let's start with this point. A second derivative at this point will tell us how the slope in one direction changes as you move in a second direction. For example let me draw the, this is a point 0 0, at this point my tangent is in this direction, this is the p u tangent at 0 0. As I move further to this point but tangent vector may be change its magnitude like this. As we move further, its value may be changes further.

And let's say at the last point here, at this point may be the tangent vector is like this. And so the tangent vector may be taking a shape like this as I move along this curve. So this is a variation in the tangent vector as I move along the other direction. The tangent vector in the u direction, how that changes as I move in the v direction. And that rate of change at this point is a twist vector at this point. So if I say p u v at 0 0 that tells me the magnitude of this second derivative at this point 0 0.

And that gives an idea of how the tangent in the u direction will change as we move in the v direction or how a tangent in the v direction will change as we move in the u direction. And we will soon write an equation for the variation of this tangent vector in the other direction. So this twist vector gives us the magnitude of the variation at a particular point. So now for defining this bicubic surface patch, in addition to these 12 inputs we will take 4 more inputs which are the twist vectors at the 4 corner points, so that means p u v at 0 0, we will have p u v at 0 1, p u v at 1 0 and p u v at 1 1. So these 4 twist vectors will give us the 4 additional inputs that we need to define this surface patch geometrically. So if we consider these twist vectors, we will write down the equation for this variation in the slope and I will again read out that part of the figure.

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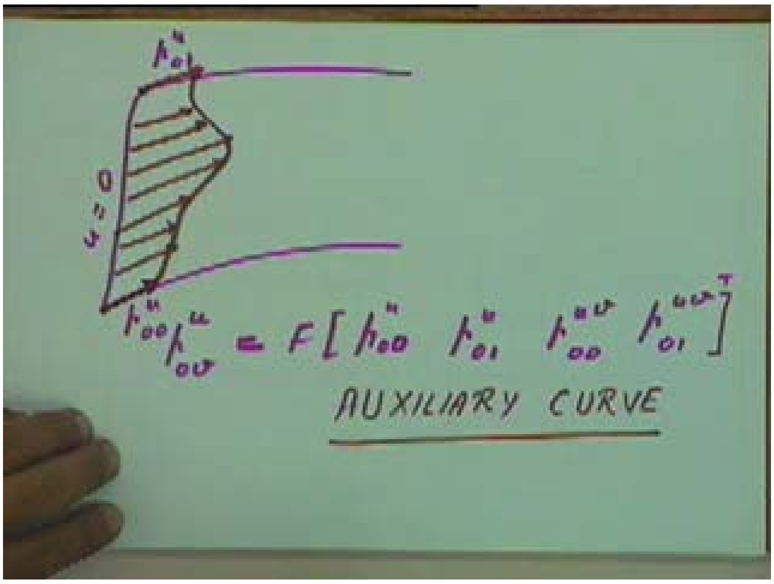

If we consider this variation in a tangent vector that is the tangent vector in the u direction, as we move in the along the vector p 0 v. This is a tangent vector in the u direction so that I have taken u to be varying in this direction which means that this is my curve u equal to 0 and my u is varying in this direction. So this is my tangent vector p u at 0 0 and this is a tangent vector which is p u at 0 1. So if I write down the expression for p u at 0 v that will be let's say F times p u at 0 0, p u at 0 1 then p u v at 0 0 and p u v at 0 1. So, p u at 0 v that means the u direction tangent vector at any point on this curve can be given by this formulation where F is again the set of blending functions we were using for a cubic surface sorry for pc curve and p u at 0 0 is this tangent vector, p u at 0 1 is this tangent vector, this is a twist vector at this point and this is a twist vector at this point and this formulation is similar in nature to the formulation of a pc curve except that instead of the end points, we now have the tangent vectors this is referred to as an auxiliary curve.

My original curve of p 0 v that is this curve u equal to 0 curve. An auxiliary curve is a curve which gives us the variation of the tangent vector in the other direction along that curve. So this auxiliary curve is giving us a variation in the u direction tangent vector as we move along the u equal to 0 curve. Similarly we can have an auxiliary curve at this end, an auxiliary curve at this end and an auxiliary curve. An auxiliary curve at any curve will give us the tangent vector or the variation in the tangent vector in the other direction. We will just use this, the concept of this auxiliary curve to get the standard geometric form for a surface patch. This is how we are defining an auxiliary curve to the u equal to 0 curve.

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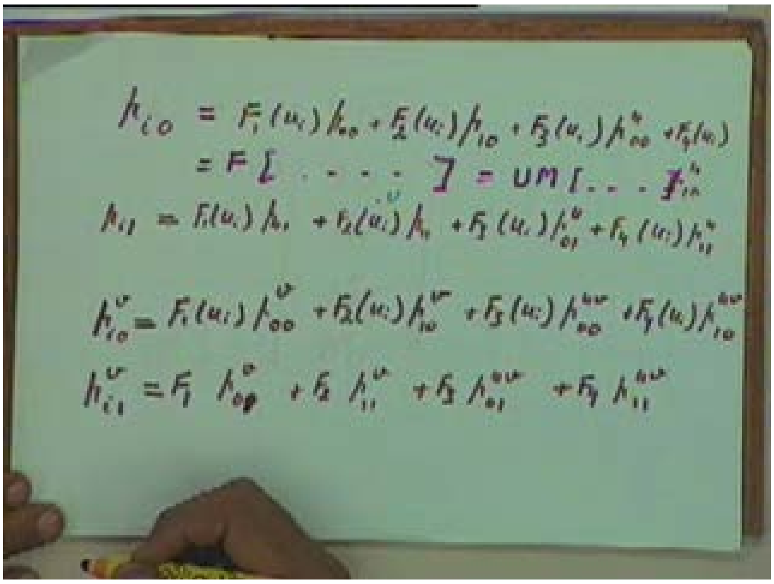

Now if we write down an expression for p_{i0}, basically I have put u equal to i. This will be obtained from the… from this curve by putting u equal to i. So this will give us F_1 at u_i multiplied by p_{00} plus F_2 at u_i multiplied by p_{10} plus F_3 at u_i multiplied by p u at 0 0 plus F_4 at u_i multiplied by p u at 1 0. From this p_{00} p_{10} p u at 0 0 and p u 1 0, this four terms I have put over here. Again? This is my polynomial. In fact I haven't used that terms so far. My polynomial is basically standard geometric form of a pc curve. So this is a p_{i0}. What am I basically trying to do now? That if I consider the surface patch like this, this is I am considering v equal to 0 curve that means I am considering any point over here.

Now if I consider curve here, I want to write down the equation of any arbitrary curve on this surface patch. This is a curve given by u equal to u_i. If I want to write down the equation of this curve, what I need is the starting point here, I need that end point over here. I'll need the tangent vector here plus I will need the tangent vector here. So I'll try to write down the expressions for each of these four vectors. So that I will get the equation for any arbitrary pc curve given by u equal to u_i.

Once I have the equation of this arbitrary pc curve, I will try to find the location of any arbitrary point in terms of the geometric input that we have given because all these expressions are being taken in terms of the geometric input p 0 0 p 1 0 p u 0 0 and p u 1 0, these are the geometric input that we have given. So in terms of this input, I will try to write down the expressions for this point, for this point, for this tangent vector and for this tangent vector.

Once I have expression for these four, I can write down an expression for this pc curve. So let's just see how we do that. We have an expression for p_{i0}, p_{i0} is this point. This point is p_{i1}, this point is p_{i1}, so let's and this curve will be p v at i 0 and this tangent will be p v sorry p v at i 1.

So let's now write an expression for p_{i1}. So p_{i1} that I can obtain from this equation, this will give me an expression for p_{i1}. I will put u equal to u_i in this that I will get as F_1 of u_i multiplied by p_{01} plus F_2 of u_i multiplied by p_{11} plus F_3 of u_i multiplied by p u at 0 1 plus F_4 of u_i multiplied by p u at 1 1. This will give me p_{i0} and p_{i1}. Now the next vector that I want is p v at i 0. So p v at i 0, for writing down the expression for p v or i 0, I will consider the auxiliary curve corresponding to this curve which will be the curve that will be p v of u 0. And let's first write down an expression for this auxiliary curve p v at u 0 that will be defined as F multiplied by, I want the tangent vector in this direction.

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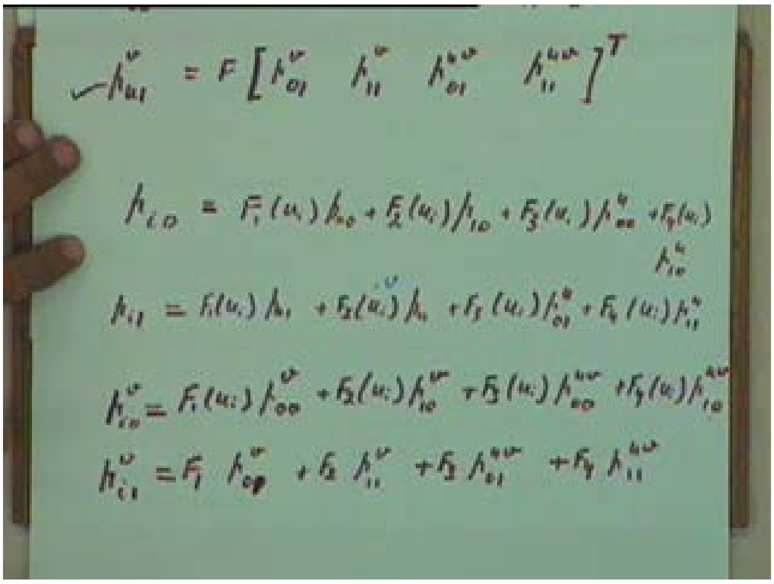

So, that will be p v at 0 0, the tangent vector at this point which will be p v at 1 0, the twist vector here and the twist vector here. So, that will be p u v at 0 0 and p u v at 1 0. This is the auxiliary curve corresponding to this curve. And similarly the equation an auxiliary curve corresponding to this curve can be written and that will be p u 1 v, the first term here will correspond to this vector which will be p v at 0 1, p v at 1 1, p u v at 0 1 and p u v at 1 1.

So, now if I take this expression and in order to get this term p v at i 0, I put u equal to u_i in this. This term or this expression will give us F_1 at gui of p v at 0 0 plus F_2 at gui of p v at 1 0 plus F_3 u_i of p u v at 0 0 plus F_4 u_i p u v at 1 0. This is what I am getting from this expression and for the last term what I want as an expression for p v at i_1 that I will obtain from this auxiliary curve and that will be equal to let's say F_1 times at p v at 0 1 plus F_2 times p v at 1 1 plus F_3 times p u v at 0 1 and plus F_4 times p u v at 1 1. This is what I will get from this auxiliary curve. So now we have got these four expressions for the starting point, end point, starting tangent vector and the ending tangent vector for this pc curve. So once I have these four vectors, I can write down the equation for this pc curve and an equation for this pc curve will be given by p equal to FB which I can write that as, in this as I move along this curve my v is a varying vector.

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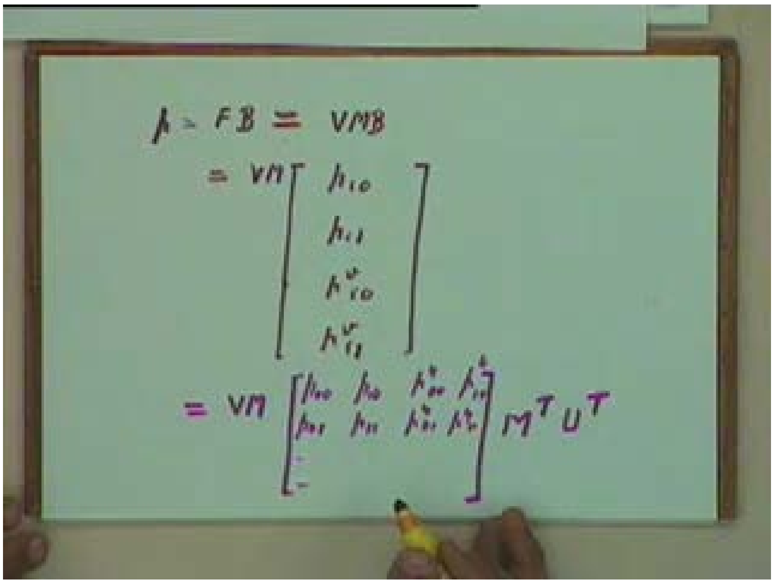

So the equation for this curve will be VMB. Because my variable parameter will be v and not u. So the equation for that curve will p equal to VMB or if I use the term that I have given, this B vector will consist of these four expressions. So this I can expand that out and what I will get will be VM. The first term is p_{i0} and second term will be p_{i1}, third term will be p v at i 0 and four term will be p v at i 1. In each of these four expressions would be used in this thing now.

Now if I consider p_{i0}, p_{i0} is nothing but F multiplied by p 0 0, p 1 0, p u 0 0 and p u 1 0. So in order to get this one term I will take F multiplied these four terms. Now this F is nothing but UM multiplied by these four terms. So I will write this whole thing as VM multiplied by a matrix which I will just explain what that matrix will be M transpose multiplied by U transpose. And this matrix will consist of the 16 terms, we will get from this 1 2 3 4, 1 2 3 4, 1 2 3 4 and 1 2 3 4. So these terms we will put them over here and what we will get will be p 0 0 p 1 0, p u at 0 0 and p u at 1 0. The second row will be p 0 1 p 1 1, this will be p u at 0 1 and p u at 1 1 and will be another two rows, I will just complete that in the next page. So this whole expression I can write down in the matrix form like this. And these 16 terms are the geometric input that you are given for this surface patch.

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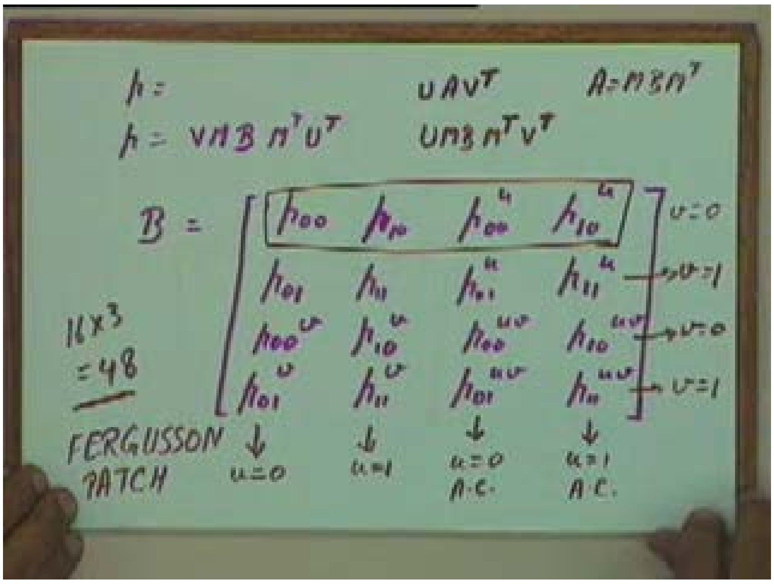

This matrix as a p is equal to VMB M transpose U transpose where B is p 0 0 p 1 0 p 0 0 u p 1 0 u p 0 1 p 1 1 p 0 1 u p 1 1 u. Here I will get p 0 0 v p 1 0 v p 0 1 v p 1 1 v. And similarly p 0 0 p 1 0 p 0 1 p 1 1 and this will be uv uv uv in this way. The 16 inputs that we are giving will come in this form p 0 0 p 1 0 01 11 00 10 01 11. This pattern of four will be the same throughout, these four terms are the tangent vector in the u direction, these four are the tangent vector in a v direction and these four are the twist vectors. And these four are the geometric inputs I mean the location input. This is how we can define the geometric matrix for inputting a bicubic surface patch. If you look at this matrix, in this matrix if we consider the first these four terms 0 0 1 0 p u at 0 0 and p u at 1 0. These four terms together will define the v equal to 0 bounding curve.

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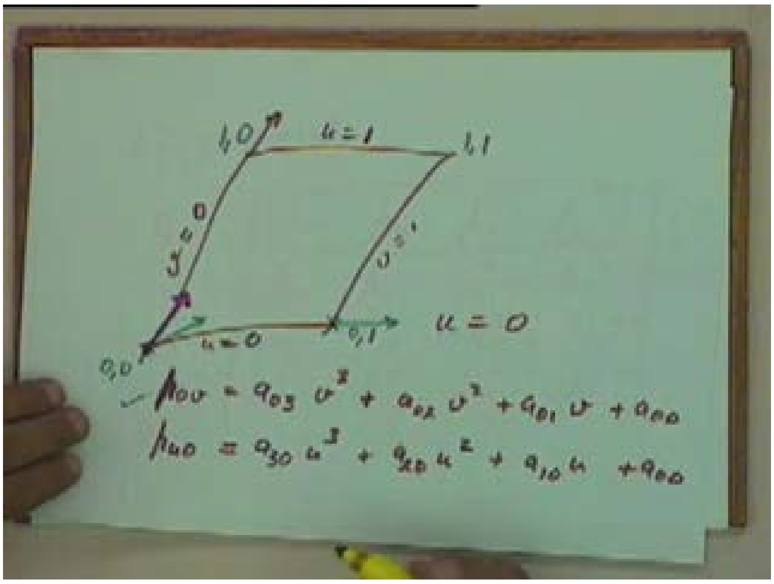

The v equal to zero curve, this curve, this is being input by p 0 0 p 1 0, a tangent vector in this direction which is p u at 0 0 and the tangent vector in this direction which is p u at 1 0. These four are these four inputs. If I consider this next four, I will get the v equal to 1 curve, p 0 1 p 1 1 p u at 0 1 and p u at 1 1, these inputs required for this curve. Similarly if I consider this column, this is the input required for the u equal to 0 curve. This is the input required for u equal to one curve. So the four bounding curves can be read off directly from this B matrix or this input matrix.

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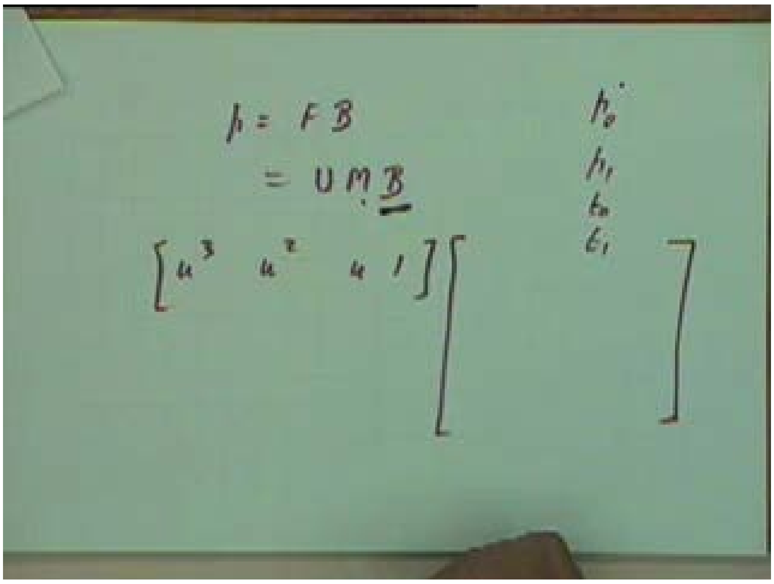

No, if you remember for a pc curves you mention p is equal to FB which is the same UMB. F vector is set of four blending functions. Now F can be written as U times M, so M is the matrix of coefficients, U is the power vector, U is u cube u square u 1. M is the set of coefficients, say multiply this row by this first column I will get my first blending function, this row by the second I will get second blending function and so on. So this matrix M is the same matrix as what you are using for pc curves. It is you know coefficients will be the same but this matrix B is not the same as what we are using here. Here this is a 4 by 4 matrix, while for the case of a pc curve we are just consisting of four vectors.

Now input required for a pc curve is only four geometric vectors. So, the B matrix of the pc curve consisting of p_0 p_1 t_0 and t_1. These are the four vectors required for the B curve, for the B matrix while here for the B matrix we need these 16 vectors. But this M and M transpose they are obtained from the same M matrix, matrix of coefficients. Now if you look at this column p u at 0 0 p u at 0 1 p u v at 0 0 p u v 0 1, these four terms they will give us an auxiliary curve. And that auxiliary curve will correspond to the u equal to 0 auxiliary curve.

Similarly this column will give us the u equal to 1 auxiliary curve, u equal to 0 auxiliary curve is essentially the curve, the auxiliary curve corresponding to u equal to 0. That means the variation of the tangent vector along this curve that means something like this. This auxiliary curve is given by this column, this column gives us the auxiliary curve at u equal to 1. Similarly the auxiliary curves at v equal to 0 and v equal to 1 can be read off from here v equal to 0 and this is v equal to 1. And in fact this matrix we can either write it like this or we can write it as UMB m transpose v transpose. Sometimes in the book you will find it written in the sequence.

The only difference is whether you the direction of parameterization which you are consisting is the u direction and which is the v direction, whether you consider this to be the B matrix or you consider transpose of this to be B matrix that doesn't make any difference. And if you compare this with the definition of the bicubic surface patch may be we said this is equal to let's say uAv transpose. The p is equal to uAvt transpose, so we will say A will be equal to MB m transpose.

So if we know the B matrix, we can find out the set of algebraic coefficients by looking at or by evaluating MB m transpose. Any question up to this point? So for defining a bicubic surface patch, we have seen that we needed 16 vectors. That means a total of 16 into 3, 48 inputs given to specify bicubic surface patch uniquely. If you do not specify let's say twist vectors, we will not get a unique a surface patch. And standard input for a bicubic surface patch can be given using this matrix and these 4 twist vectors in some sense will be controlling the curvature at the 4 corners.

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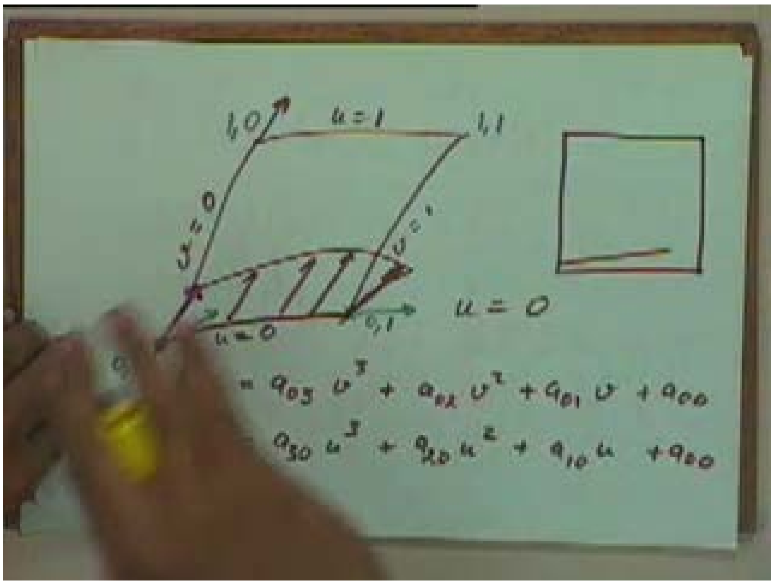

That mean let's say if I take a very simple case, I consider a simple rectangular surface patch and if I just control, if I just change the twist vectors, I can get surfaces with varying curvature at the corner. If my twist vector is 0 at all the 4 locations, I will get a flat plane surface but if I change my twist vectors, I will get a different bunch or a different shape of the surface in between. So these 4 curves are controlling the boundary while the 4 twist vectors are controlling the bunch or the curvature in the rest of the region. In fact if we consider these 4 twist vectors to be equal to 0, if we put each of these 4 twist vectors equal to 0, again well said we will get plane surface only when the 4 bonding surfaces, 4 bonding curves are plane. The 4 bonding curves are not plane then you won't get a plane surface.

If we have general cubic curve here, general cubic curve here, a general curve here and general curve here then you still get a three dimensional surface but your twist vectors will be 0 and that kind of surface is called a Ferguson patch, Ferguson patch or an F patch. If the 4 twist vectors are put equal to 0 and each of these 4 terms I will put as 0 then you get a Ferguson patch. Any question on this geometric form of a bicubic surface patch? In that case

I will wind up now. In the next class we will see some other formulation for the surfaces and then we will go into some other types of surfaces in this case. And that’s all for today.