Computer Aided Design
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Lecture No. # 28
3 D Tetrahedral and 2 D Quadrilateral Elements (Contd.)

In the last class we are talking of the 3 D tetrahedral elements and we had seen the formulation for a general tetrahedral element consisting of 4 nodes and then we have seen that if you have a cube, how we can split this cube in to a set of tetrahedral elements.

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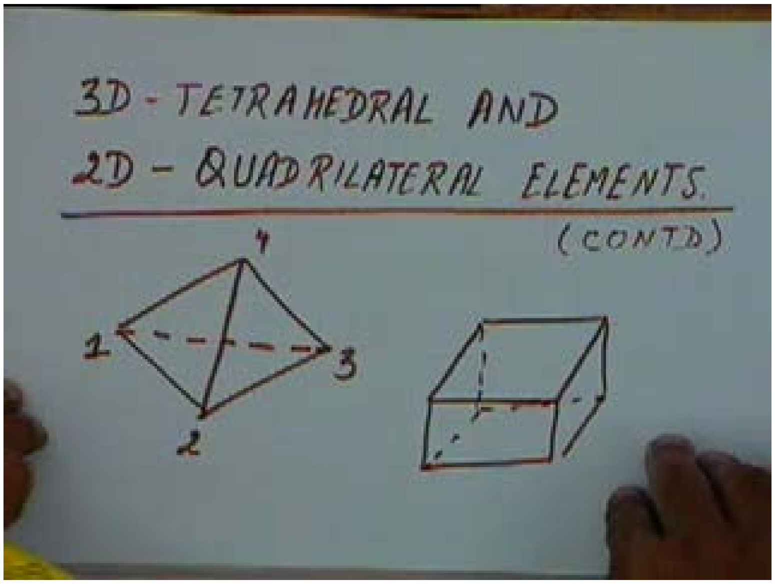

If we are splitting a cube into tetrahedral elements, we had considered the splitting mechanism like this. We said that if you have this cube given by 1 2 3 4 5 6 7 8 these 8 nodes and you want to split it into tetrahedral elements, we can take elements like 1 2 4 5, 2 3 7 4, 4 5 7 8 that is 4 5 7 and 8 and 2 5 6 7, it's 2 5 6 and 7 and the fifth one is 2 4 5 7 which is inside all these 4 elements. These 4 elements are basically at corners 1 6 8 and 3.

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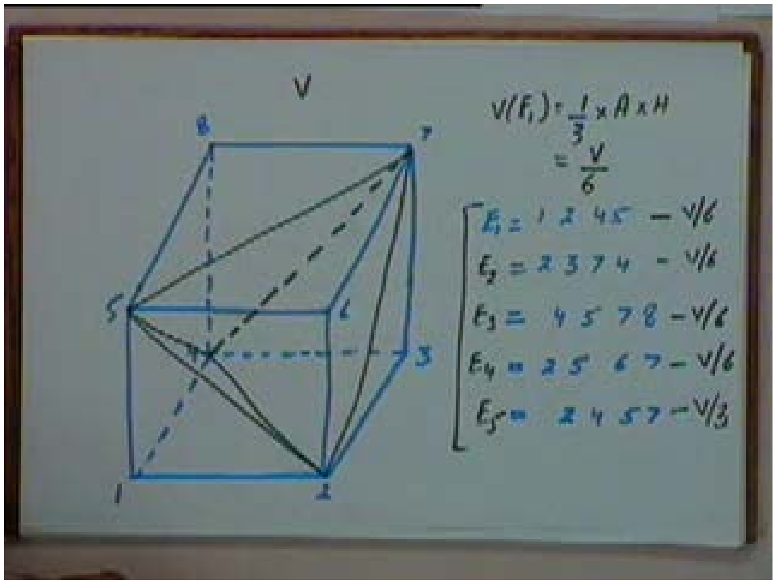

If we cut off the corners, we will have a tetrahedral element in the center that is this element number 5. And we said that if you look at the volumes of these 5 elements, the first 4 have a volume of v by 6 and the fifth one is volume a v by 3. And if you want to get elements of the same size that is you want to have the same volume for all the elements, we can divide it in other way and that is like this.

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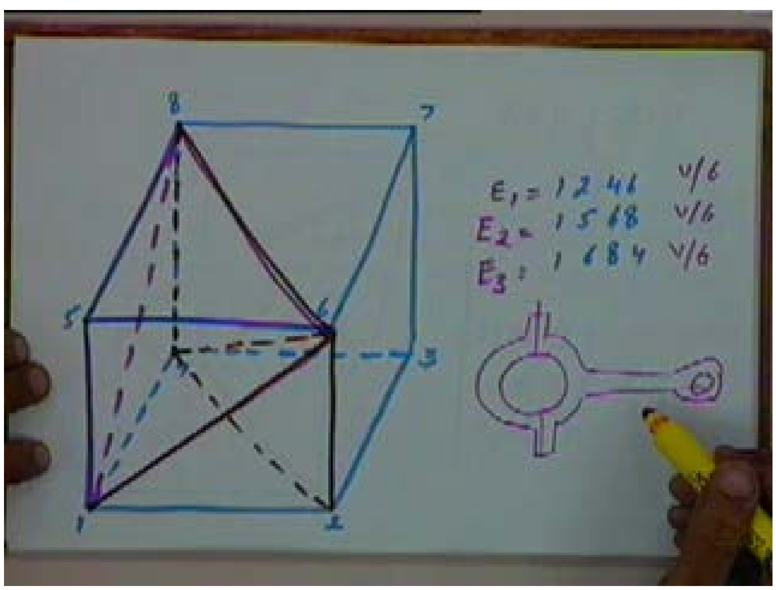

If you this 1 2 3 4 5 6 7 8 as the cube, we can take vertical diagonal plane. In case that means if you join 8 and 6, you take this vertical diagonal plane and divide this cube into two parts and then each of these half’s again divide them into 3 tetrahedrons each.

So if I take 1 2 4 6 so 1 2 4 and 6, so 1 2 4 and 6. So this tetrahedron 1 2 4 and 6 is 1 tetrahedron, 1 5 6 and 8 so 1 5 6 and 8, so you join this. This is second tetrahedron and 1 6 8 4, so 1 6 8 and 4 this is the third tetrahedron which has obtained if I remove this 1 2 4 6 and 1 5 6 8 from this half. So, this half will be divided into 3 tetrahedrons. So this is let's say my element 1, this is my element 2 and this is my element 3. From these 3 elements if I look at the volume of each of them 1 2 4 and 6, I can look upon this triangle 1 2 6 as the base and 1 4 as height of the pyramid. So the volume of this will be v by 6. So v is the total volume of the cube. Similarly 1 5 6 8, 1 5 6 can be looked upon as the base and 5 8 as its height.

So volume of this will also be v by 6 and 1 6 8 4 will be one half of this cube that is v by 2 minus v by 6 minus v by 6, so this will also be v by 6. So we will get 3 tetrahedrons each with the volume of v by 6 whereas similarly in the other half, we will get another 3 tetrahedrons each with the volume of v by 6. So this way we can split this cube into 6 tetrahedrons, each having a volume of v by 6. So if I have any general 3 D object, that's' last time we are talking of a simple connecting rod, I can split this connecting rod into a set of cubical elements and each cube I can split into a set of tetrahedrons. And for each tetrahedron, we know how to get the local stiffness matrix and the local force matrices and then we can use it in the global matrices. Any question on this part? In that case we will now go on to some other elements, typically two dimensional quadrilateral elements. Earlier we have seen a simple quadrilateral element consisting of 4 nodes and we said that for this 4 noded element, let's say we define a zeta coordinate as zeta equal to minus 1 here and zeta equal to plus 1 here and eta equal to minus 1 here and eta equal to plus 1 here.

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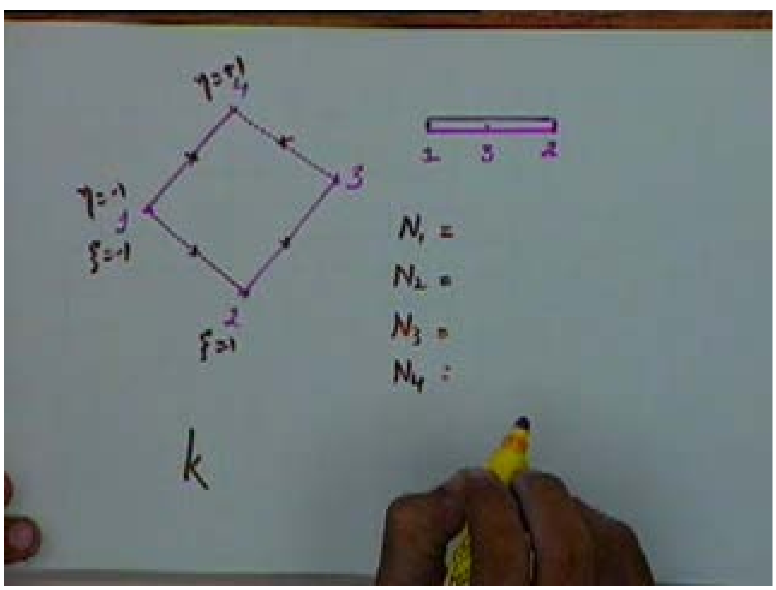

So we converted the xy coordinates into zeta eta coordinates and mainly define the 4 shape functions N_1 N_2 N_3 and N_4 using this zeta and eta coordinates. And then you also mention that in a element like this, if you have to find out the local stiffness matrix k, we have to carry out integration using numerical techniques because integration would involve terms of zeta and eta which have to be integrated over the element. This was for the simple 4 noded element. Instead of the 4 noded element, we can have 8 noded element, we have 4 additional nodes like this. The advantage that we will have in this that if you remember earlier when we are talking of a 1 dimensional problem, we had a 2 noded element and you are doing a linear interpolation between these 2 nodes. Then we went for a 3 noded element and we said we will do quadratic interpolation between these nodes. Similarly, now we will go for an 8 noded element and we will try to do a quadratic interpolation. The way we will define an element will be something like this.

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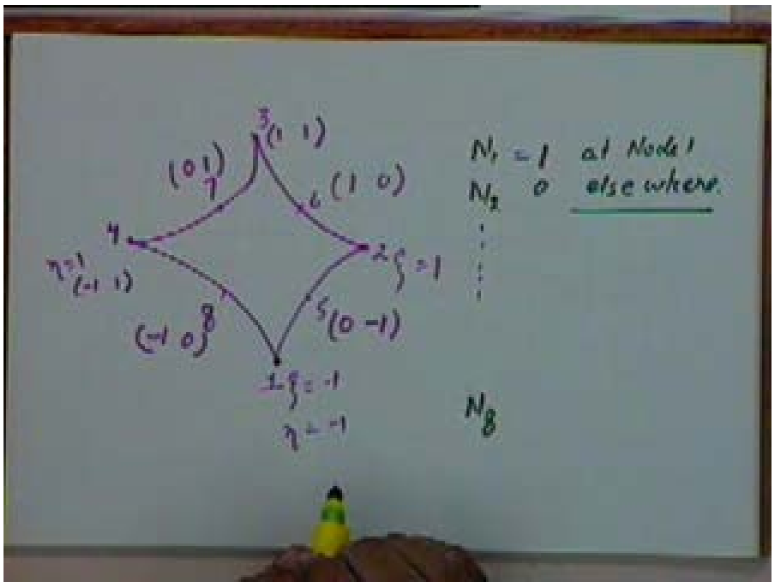

You consider 8 nodes like this and we said node 1 will define zeta equal to minus 1 and eta equal to minus 1 and at this point we will get zeta equal to 1 that means zeta is varying in this direction and eta will vary in this direction that means along this curve, we will get theta equal to 1. So this point will be the point minus 1 1, this point will be the point 1 1 and typically if zeta is varying from minus 1, eta is varying from minus 1 to plus 1 here we will get eta equal to 0 so this point will be minus 1 0. This point zeta will be 0 and eta will be minus 1, so this point is 0 minus 1 and at this point zeta is 1 and eta is 0, so we will get at this point to be 1 0 and this point to be 0 1. 1 0 and 0 1 all these are in terms of zeta and eta. So now if we have the 8 nodes, we have to define 8 shape functions, so the 8 shape functions. Again we will have the same constraints that is N_1 will say, will be 1 at node 1 will be 0 at all the other nodes. So N_1 will be equal to 1 at node 1 and 0 elsewhere and similarly for N_2 and so on.

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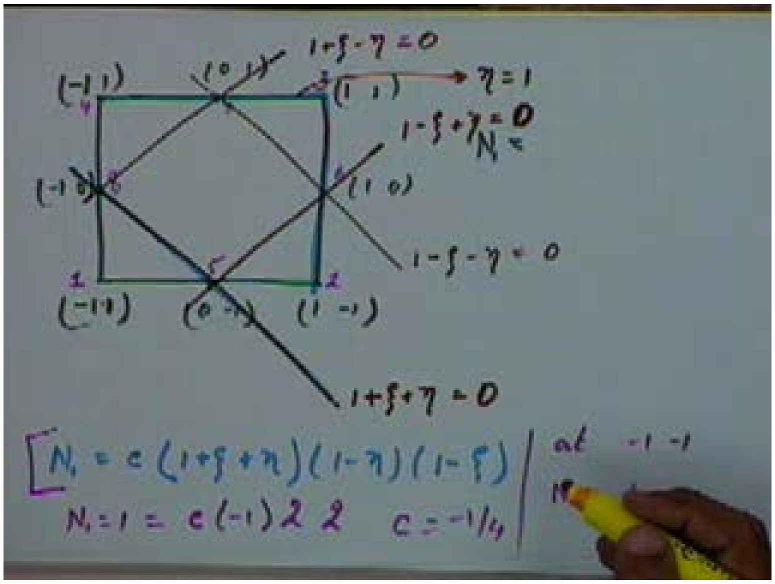

So let's read out this element in terms of the zeta and eta coordinates. So these are 8 nodes, this is 1 2 and if you have to define a shape function let's say N_1 which is 1 at this node and is 0 at all the other nodes. We will first draw straight lines. The equation of this straight line would be 1 minus zeta minus eta equal to 0 that means along this straight line, you find zeta plus eta will be equal to 1. You can verify at these 2 points, zeta plus eta will be equal to 1. Similarly if you consider this straight line, this will have the equation 1 minus zeta plus eta will be equal to 1. Similarly this line will have the equation of 1 plus zeta plus eta equal to equal to 0 equal to 0 sorry.

Now zeta plus eta will be minus 1 along this and similarly along this edge, this 1 we will get 1 plus zeta minus eta equal to 0. Basically for all these 4 lines, we have 2 points and I am writing down the equation between the two between the equations of the lines between these 2 points. Now if I consider node 1, for node 1 I want N_1 to be 0 at all the other nodes. So I will consider let's say this straight line, this straight line and this straight line. So if I consider these 3 straight lines and I say that N_1 will be 0 at all these 3 straight lines that will ensure that N_1 will be 0 at all the other nodes. So that would mean that N_1, we will say N_1 will be of the form some constant multiplied by the equation of this straight line is 1 plus zeta plus eta. So this is 1 plus zeta plus eta. The equation of this straight, this straight line the top one is eta equal to 1. Again? The shape function can be defined in any way, the choice is totally up to the user. You can take shape functions according to any formulation as long as the particular set of conditions.

So we will we can take it this way or we can take it any other way, there is no problem on that. Now only thing that will change would be the stiffness matrix that you will get. So this N_1 we will say some constant multiplied by 1 plus zeta plus eta multiplied by the equation of this line that will give us 1 minus eta and similarly this vertical line will give us 1 minus eta. So I take my N_1 to be of this form then we can ensure that N_1 will be 0 at all these nodes except at node 1. And then we have the condition that at minus 1 minus 1, N_1 is equal to 1. So if I put zeta equal to minus 1, eta equal to minus 1 in this equation, we will get N_1 which is equal to 1 will be equal to c times, I am putting minus 1 here we will get minus 1 into 2 into 2. So that means c will be equal to minus 1 by 4. So N_1 we will get it in this form. Similarly you can take N_2 and for N_2 we will say that we will consider this vertical line, for N_2 we will consider this vertical line, this horizontal line and this inclined line. For N_3 we will consider this vertical line, this horizontal line and this inclined line and so on. For N_5 I will consider this vertical line, this horizontal line and this vertical line. N_6, I will consider these 3 lines and this way I can get all the 8 shape functions. And once I have the shape functions, the process of getting the stiffness matrices and the force matrices would remain the same. We will again carry out the same integrals, we need the numerical techniques for evaluating the integrals and we will get the stiffness matrices. Any questions up to this point?

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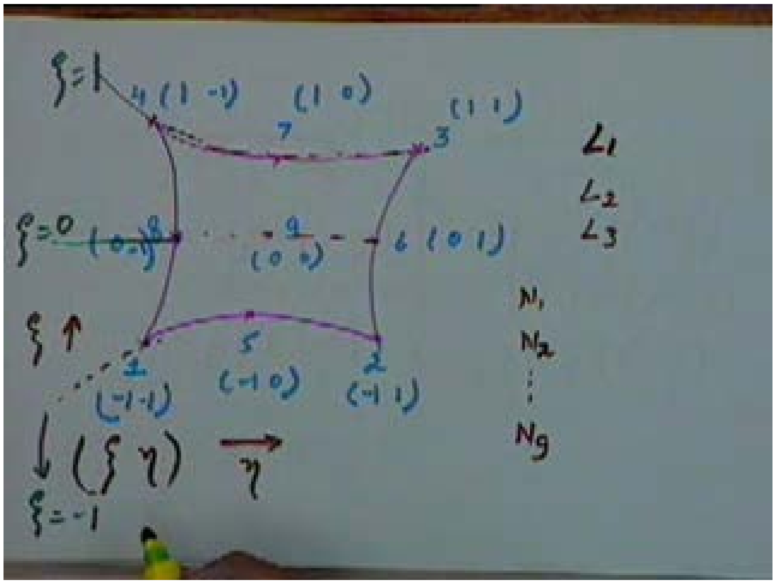

Next let's see here, if you take an element which is a 9 noded element now and we will take an additional node in the center. So let's say, so if you look at this, this point has coordinates of minus 1 1 minus 1 minus 1. This has minus 1 1, this has 1 1, this will have 0 1, this has 1 0, this will have 0 0, this has minus 1 0 and this will have 1 minus 1 and this will have 0 minus 1 (Refer Slide Time: 19:25). And now we have we need to define 9 shape functions for this. Again these are the zeta and eta values. So these values of zeta and eta, I am re-defining my x and y in terms of zeta and eta. So for an element like this, what we will do is we have to define N_1 N_2 so on till N_9. The way we can do it is we will define one line like this and other line like this and the third line like this. In this direction what is changing is zeta and in this direction my eta is changing. So we can define 3 functions L_1, L_2 and L_3. If you remember in a simple quadrilateral 3 noded element, in this element we have said N_1 N_2 and N_3. We have taken quadratic variations for this and for N_1 we had mentioned that this will be minus half of zeta into 1 minus zeta, N_2 we said would be 1 plus zeta into 1 minus zeta and N_3 we said would be half of zeta into 1 plus zeta.

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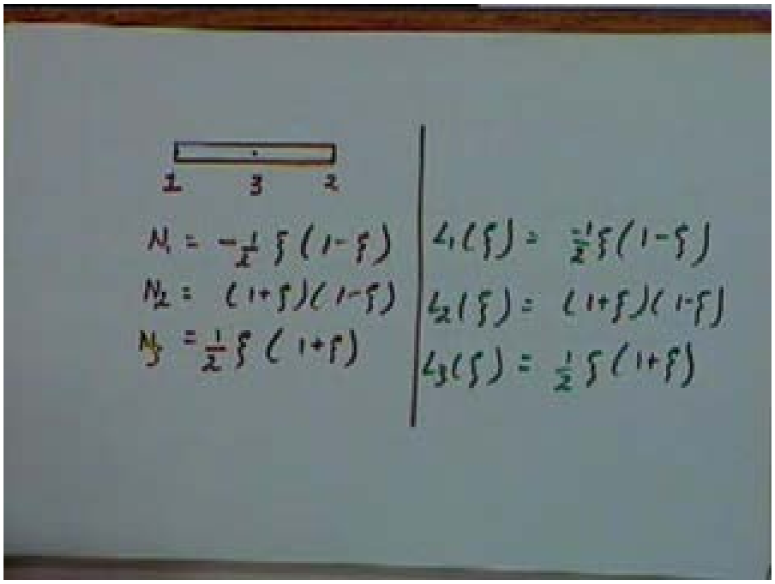

Now what we will do is we can look upon this element, adjust some extension of a 1 dimensional 3 noded element in the second direction. That means we will consider this line to correspond to a function L_1, this line to correspond to a function L_2 and this line to correspond to a function L_3. And similarly we will consider a function along this line, along this line and along this line. So what we will do is analogous to this, we will define L_1 of zeta to be minus half of zeta into 1 minus zeta, L_2 of zeta to be 1 plus zeta into 1 minus zeta and L_3 of zeta to be half of zeta into 1 plus zeta. That means L_1 is defined along this like, so L_1 this is 0 at zeta equal to 0 and zeta equal to plus 1. So if I consider this line, this line is zeta equal to minus 1, this line is zeta equal to 0 and this line is zeta equal to 1.

So if I consider L_1, this is 0 at zeta equal to 0 and zeta equal to plus 1. So L_1 will be 0 at this line as well as at this line, while at this line L_1 may be equal to plus 1. So L_1 will be equal to plus 1 at this line. Similarly L_2 will be 1 at this line, will be 0 at this line and at this line and L_3 will be 1 at this line and 0 at both these lines. Similarly what we will do is we will define in other 3 functions L_1 L_2 and L_3 along the in a vertical direction.

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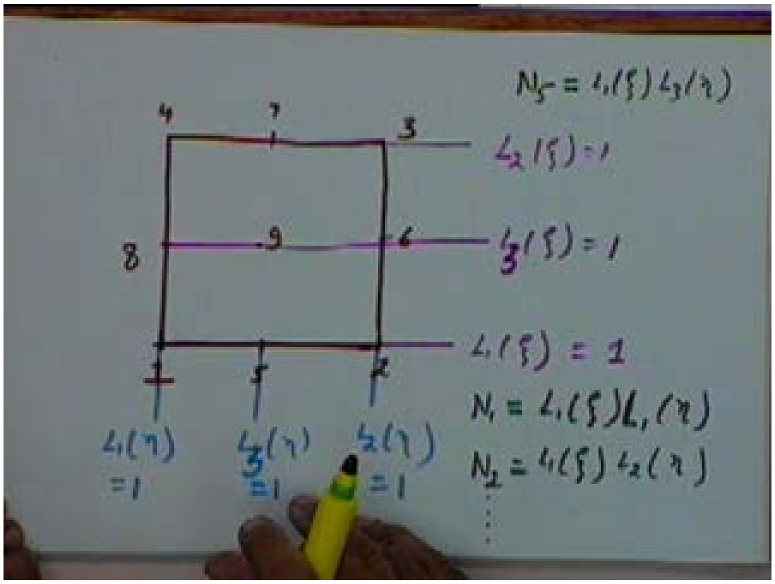

So consider vertical direction. We will draw a separate figure. I will again draw the element as a rectangular element. So we have defined, along this edge we defined L_1 of zeta which is equal to 1 along this edge and 0 at both these edges. Along this edge we have defined L_1 of L_2 of zeta to be equal to 1 and along this edge we have defined sorry… here you have defined L_2 of zeta and here you have defined L_3 of zeta. Similarly we will define along the vertical in the vertical direction L_1 of eta, L_2 of eta sorry L_3 of eta and L_2 of eta. This is equal to 1 here, this is equal to 1 here and this is equal to 1 here. Now the shape functions for the point 1 which is N_1 will define N_1 to be equal to the product of L_1 of zeta and L_1 of eta, L_1 of zeta multiplied by L_1 of eta. This way I will ensure that N_1 will be 1 at this point, will be 0 at all the other points.

If I take N_2, I will take N_2 to be a product of L_1 of zeta and L_2 of eta and so on. For N_5, I will write it here. N_5 will be product of L_3 of eta and L_1 of zeta. So this way for all the 9 points, I can write the shape functions as a product of L_1 L_2 L_3 of zeta and L_1 L_2 L_3 of eta. So for point 9, I will take a product of L_3 of zeta and L_3 of eta that will ensure that it is 0 at all the other points. And again once I defined the shape functions, the process of finding on the stiffness matrices and the force matrices remains the same. So in any in any element, the basic thing is to find out the, this is to specify the shape functions for the element and define the element accordingly. The stiffness matrices, force matrices etcetera can be followed by a process of integration which we have already seen a number of times. Is that okay? Because the basic advantage that one gets of taking elements which are already 8 noded or 9 noded. The basic advantage is that if I have 3 nodes along an edge, I can do the quadratic interpolation along that edge. That means earlier when I was having the simple 4 noded element, my 4 noded element was like this. So the edges have to be defined in a linear manner, I cannot get curved edges.

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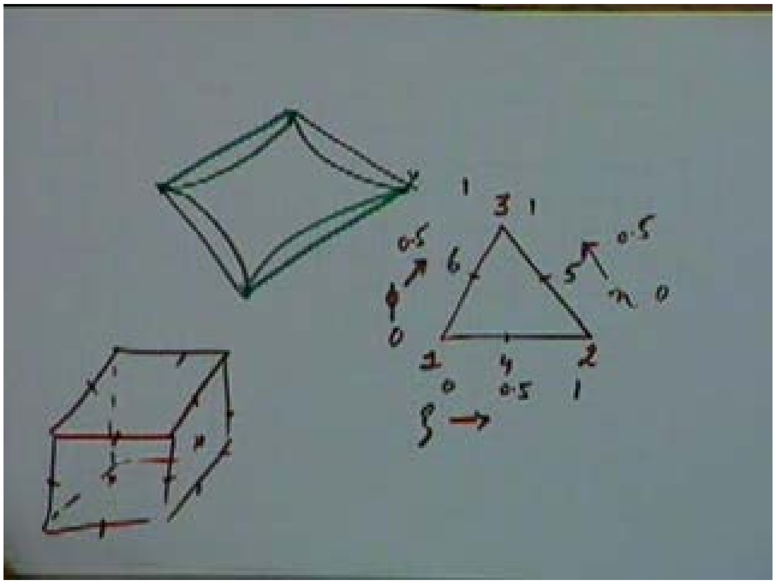

Similarly my stress and strain patterns will also be equally respected. I am doing linear interpolation everywhere. So for 4 noded elements, I have to have straight edges, for a 8 noded element I can have curves like this and so on. So, extra nodes on the same element will give you more flexibility in defining the element. Similarly we have also seen a triangular element and we had defined a triangular triangular element lines 1 2 and 3, 3 nodes. We can also define a 6 noded triangular element. And again we can define 6 shape functions for that. The one way of defining the shape functions is that along each edge, you would have a parameter like zeta, eta and phi. So zeta is varying in this direction, eta is varying in this direction and phi lets say varies in this direction.

Let's say this is 0, 0.5 and 1, again eta it's a 0, may be 0.5 and 1 and again phi is changing as 0.5 and 1 and again 6 shape functions can be defined in a similar manner. For node 1, I will consider this straight line and this straight line. For node 2, I will consider this straight line and this straight line and so on. Now this way we can define a 6 noded triangular element also. In the same ideas can also be extended to 3 dimensions.

Right now we have talked of an 8 noded quadrilateral element, we can define a cubical element. First we can define a 8 noded cubical element. Then we can define a 20 noded cubical element, at the center of every edge we can have an extra node and so on. So, 12 edges that give us extra 12 nodes. You can have 20 noded cubical element, we can also have a 27 noded element. In the center of every face you have an extra node, 6 faces per the center of the cube, we have a 27 noded cubical element. Then you do define a 27 shape functions accordingly. The basic process can be the same. Any questions up to this point? Then I will stop here. In the next class we will consider how to generate a mesh from a given object, how do you generate a mesh automatically for any kind of element.