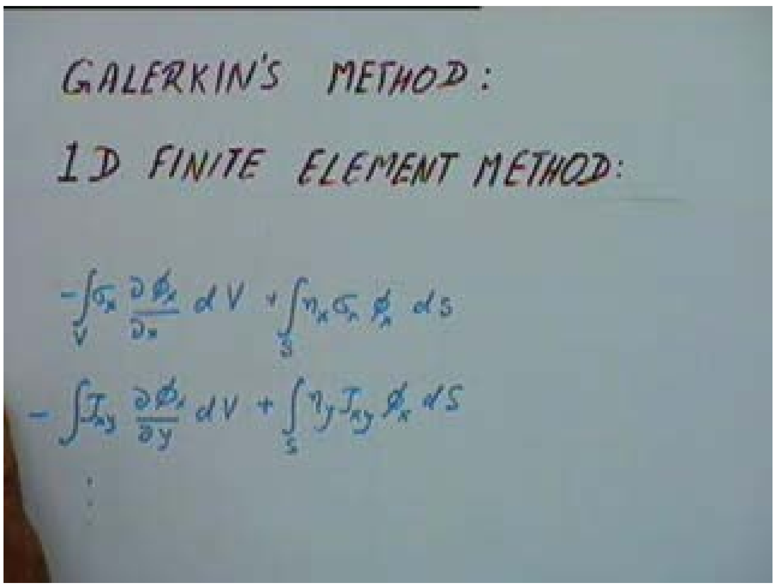
Computer Aided Design
Dr. Anoop Chawla
Department of Mechanical Engineering
Indian Institute of Technology, Delhi
Lecture No. # 17
Galerkins's Method: 1 D Finite Element Method

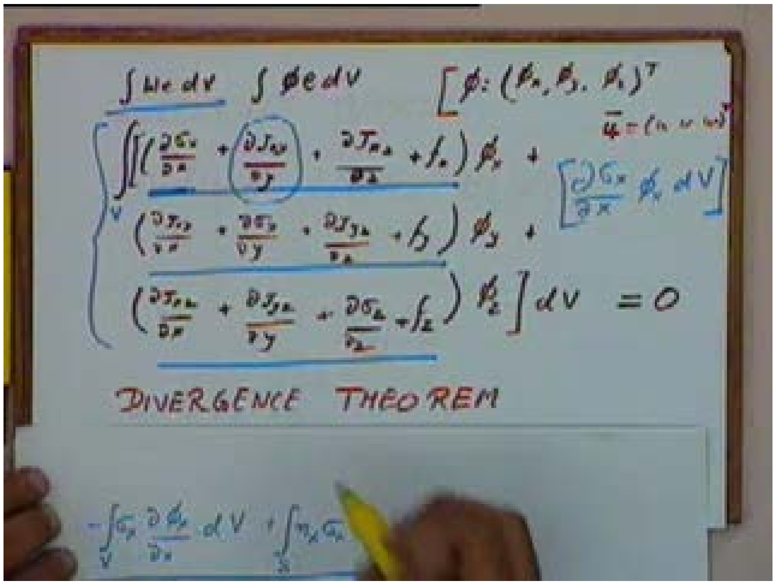
Today we will be continuing with the Galerkin's method and after that we will go on to one dimensional finite element problems.

(Refer Slide Time: 00:01:10 min)



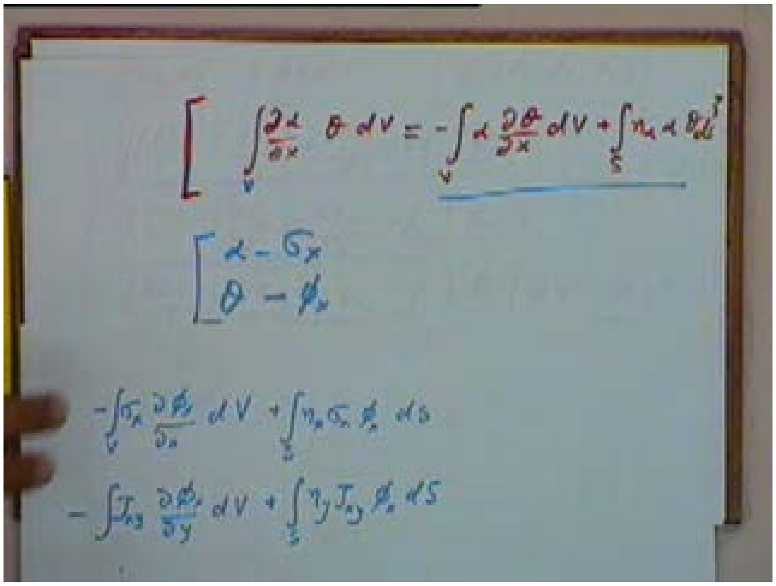
We will see how the one dimensional finite element problems are solved using both the Rayleigh-Ritz method and the Galerkins's approach.

(Refer Slide Time: 00:01:19 min)



If you remember yesterday I had introduced this expression and I said that when we are talking of this weighted integral, the integral of the or the weighted integral of the error term this in the case of a elastic three dimensional problem will come to this equation. That is this first part of the, this part is the equilibrium equation in the x direction, this is the equilibrium equation in the y direction and this is the equation in the z direction and each of these terms multiplied by the corresponding weights in the x y and z direction and then integrated over the volume and this is the volume integral and this we said it should be equal to zero.

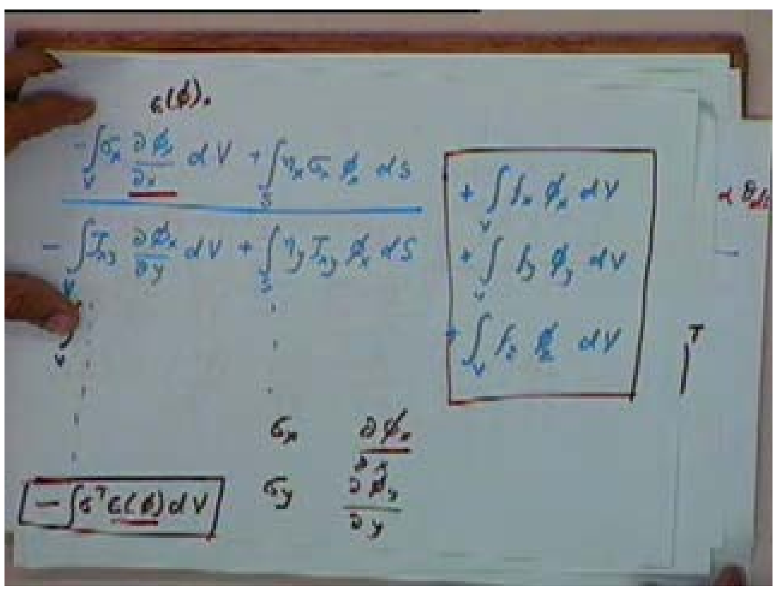
(Refer Slide Time: 00:02:05 min)



Now in order to simplify this integral we will be using what is call the Divergence theorem and I had mentioned the divergence theorem is this theorem. That is if you have a term alpha and a term theta and you want to evaluate the volume integral of del alpha by del x times theta dv, that will be equal minus of the volume integral of alpha times del theta by del x that is this term derivative of that with respect to x into dv. That is the volume integral of this term plus the normal in the direction of alpha or n alpha multiplied by alpha that is this term into theta into ds that is surface integral of this. So this is what we call as the Divergence theorem.

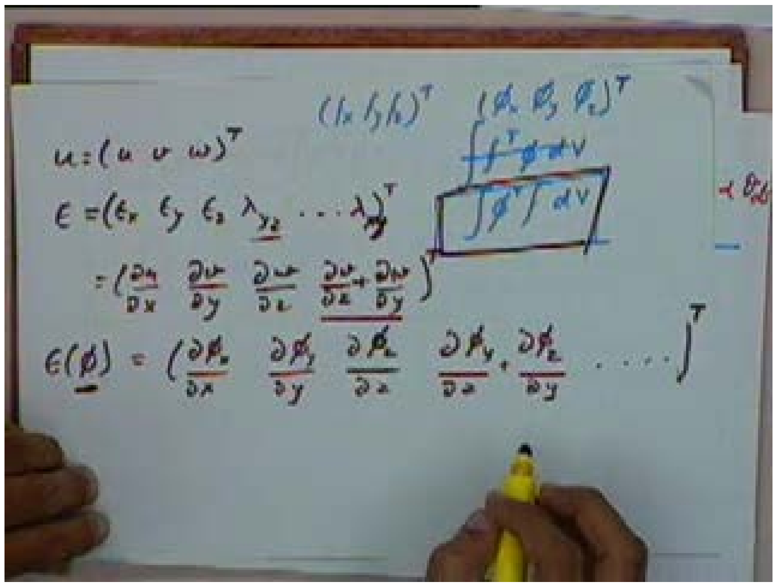
Now if we apply this Divergence theorem to the terms coming in this equation, this first term this one, del sigma$_x$ by del x multiplied by phi$_x$ multiplied by dv that is this term is I will just repeat it here, del sigma$_x$ by del x multiplied by phi$_x$ multiplied by dv. If you compare this with the term that you have here, your alpha is the same as sigma$_x$, the theta is the same as phi$_x$ and if I put these two over here, this first term would become something like this. In this expression instead of alpha I have put sigma$_x$, so I get integral of sigma$_x$ instead of theta I am putting phi$_x$. So del phi$_x$ by del x times dv, the volume integral of that with a minus sign plus the surface integral of n alpha alpha theta ds. So n alpha in this case will become n$_x$, alpha is the same as sigma$_x$, theta is the same as phi$_x$. So n$_x$ sigma$_x$ phi$_x$ multiplied by the surface term ds and the surface integral of that. So the first term in the expression that is this term or this term can be simplified to this term, this expression. Similarly the next term that is there that is this del tau$_{xy}$ by del y multiplied by phi$_x$ multiplied by dy.

(Refer Slide Time: 0:05:11 min)



The second term, the del tau$_{xy}$ by del y that term we will simplify to an expression of this type, tau$_{xy}$ will come here tau$_{xy}$ into del phi$_x$ by del y because here we have got the derivative with respect to y, so tau$_{xy}$ into del phi$_x$ by del$_y$ multiplied by dv and the volume integral of that and the second term that we will get will be n$_y$ tau$_{xy}$ phi$_x$ ds and the surface integral of that. This way for all these terms, for three into three for these nine terms we will be able to get 9 expressions of this type and plus we have got this term fx phi$_x$ so this I will write here separately plus integral of fx phi$_x$ dv volume integral plus volume integral of fy phi$_y$ dv and the third term is this that is plus the volume integral of fz phi_z dv. So I will have a set of like terms like this plus I will have three terms like this. Now if you look at, if you look at these terms, these three terms I can just write them as integral of f transpose into phi dv. Your phi or just I will write it as integral of phi transpose times fdv where phi is nothing but phi_x phi_y phi_z column vector and f is nothing but fx fy fz transpose, so these three terms can be written like this.

(Refer Slide Time: 00:07:46 min)



Now if I look at these terms if you remember the definition of epsilon, if you have any deformation u the corresponding epsilon this u is a vector u v w, the corresponding epsilon or the strain vector, this we said would be $epsilon_x$ $epsilon_y$ $epsilon_z$ $lambda_{yz}$ and so on transpose. Again $epsilon_x$ we said is del u by del x, $epsilon_y$ is del v by del y, $epsilon_z$ is del w del z. Similarly we had terms for $lambda_{yz}$ and so on.

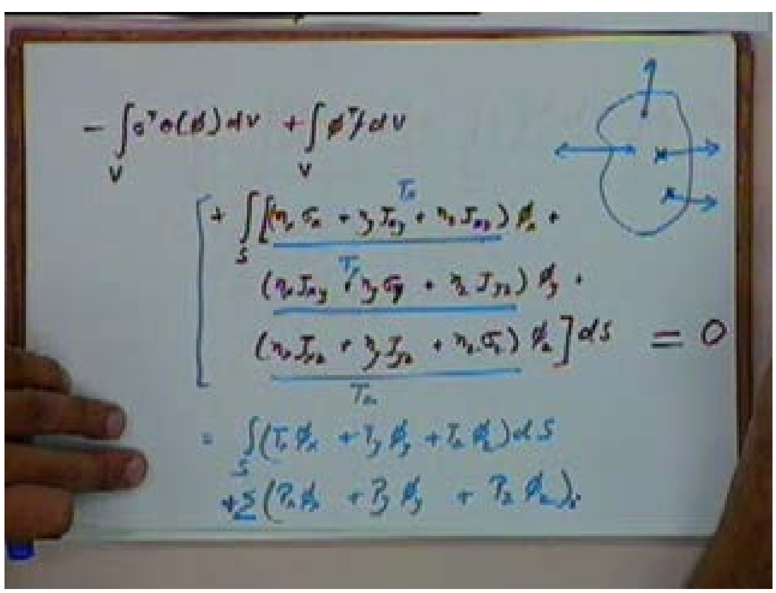
Now here we have got a term on phi_x that is del phi_x by del x and we also said that the phi that is the weights that we are giving are consistent with the boundary conditions that is one of the constraint that we had in the Galerkin's approach. So we can effectively say that we can take this phi to be equivalent to a displacement and del phi_x by del x is the strain in the x direction that will be there in the object, if this phi was given as a displacement. So if I write down expression for epsilon of phi, this would be equal to del phi_x by del x, $epsilon_y$ term that will become del phi_y by del y, $epsilon_z$ term will be del phi_z by del z. The $lambda_{yz}$ term here what we would have got would have been del v by del z plus del w by del y. This term would have been $lambda_{yz}$ corresponding to that we will get here del phi_y by del z plus del phi_z by del y and another two terms like this. So this vector is the strain that would be there in object, if phi was the displacement that was given or the phi was the displacement field that was there in the object. So epsilon of phi that is not the actual strain in the object but the strain that would be there if this displacement phi was given and if you look at these terms first second, this term is nothing but the component of epsilon phi, epsilon phi is the vector so the first component of that, that is the component in the x direction.

this is nothing but, this is nothing but a part of the term that we will have in the end that is corresponding to, the last term is lambda$_{xy}$ that a term that we will have in end that will have this term of del phi$_x$ by del y. So if you look at all these terms that we will have, we can collect all these terms together and we can say that this would be equal to minus of the integral of sigma transpose multiplied by epsilon of phi multiplied by dv. Now this epsilon of phi as I said is the strain that would be there if phi was the displacement that was given to the object and we have already said that this displacement has to be consistent with the boundary conditions. And this sigma transpose is the actual stress in the object, actual stress distribution in the body.

Now from this again, the first term of sigma transpose is sigma$_x$. The first term of epsilon phi is del phi$_x$ by del x. If I multiply these two, I get the first term here. The second term of sigma transpose will be sigma$_y$ and the second term of epsilon phi will be del phi$_y$ by del y and that will be the next term that we will have over here minus integral of the volume and so on. So all the term that one gets from this will be the term that we will be getting from this column. So this, all these terms can be compiled into one single term and that is this term. And as I said this phi, this is possibly a displacement field that we are giving to the object and if you familiar with the principle of virtual work, this phi is actually the virtual displacement that we are giving to the body. What we are calling as weights earlier that is actually a virtual displacement that we are giving to the body.

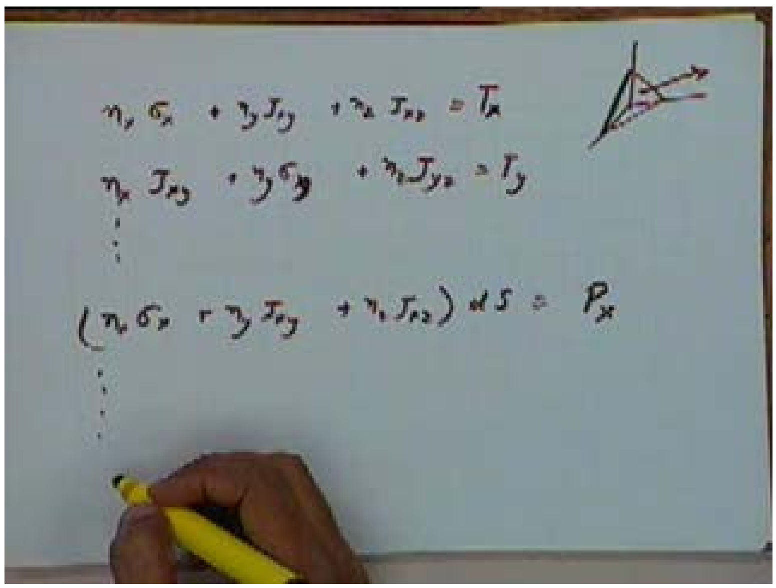
If you give it a virtual displacement then the strain due to that virtual displacement is the strain epsilon phi and this sigma transpose is the or this sigma is the actual stress in the body because of the existing loads. This sigma and epsilon are because of two different factors, epsilon of phi is because of the virtual displacement, sigma is because of the actual loads. This is how we get this one term and then we will collect all these terms together and what will finally get will be an expression of this type minus integral of sigma transpose epsilon of phi dv, the volume integral plus volume integral of phi transpose fdv.

(Refer Slide Time: 0:13:47 min)



This is the term that we just, I just mentioned, this is this term. This has been collected by taking these three terms together. If I take these three terms together, I will get this term and if I collect all these columns, the term in this column together I will get this term. The remaining terms I will just write them down them as they were and what I will get will be plus the surface integral of n_x sigma$_x$ plus n_y tau$_{xy}$ plus n_z tau$_{xz}$ multiplied by phi$_x$ plus n_x tau$_{xy}$ plus n_y sigma$_y$ plus n_z tau$_{yz}$ multiplied by phi$_y$ plus n_x tau$_{xz}$ plus n_y tau$_{yz}$ plus n_z sigma$_z$ multiplied by phi$_z$, this whole thing multiplied by ds. These are the terms I will get by collecting these terms and these terms are n_x sigma$_x$ phi$_x$, the other term are n_x sigma$_x$ phi$_x$. This term is n_y tau$_{xy}$ phi$_x$, this is n_y tau$_{xy}$ phi$_x$. This is I have collected all these terms together to give me the surface integral and then we are saying that all this will be equal to 0. Now let's again look at the equation that I had given in the beginning for the first lecture on finite elements that is we had given a set of equations like this n_x sigma$_x$ plus n_y tau$_{xy}$ plus n_z tau$_{xz}$ is equal to T_x. This is for any element on the surface.

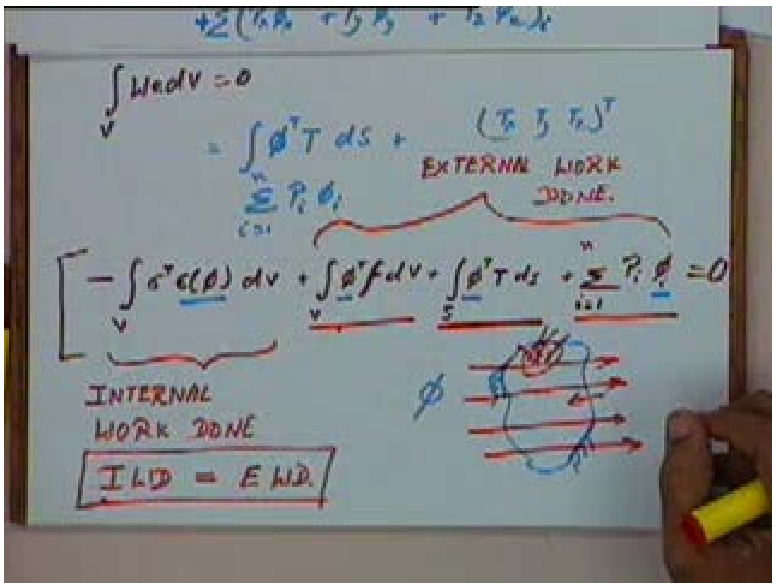
(Refer Slide Time: 00:16:31 min)



We had said that if we have an element like this and if we write down the equilibrium equation for this element, we will get this equation in the x direction, we will get a similar equation n_x tau$_{xy}$ plus n_y sigma$_x$ plus n_z tau$_{yz}$ is equal to T_y. So n_y sigma$_y$ plus n_z tau$_{yz}$ will be equal to T_y and a similar equation for the direction. This is for the tractive loads acting on the surface that is forces per unit area, these are the tractive forces acting on the surface but if there any point loads on the surface then our equations will be different. If in addition to the tractive force, we have point load acting like this. In addition to these equations, we will have to write n_x sigma$_x$ plus n_y tau$_{xy}$ plus n_z tau$_{xz}$ all this multiplied with ds will be equal to a point load in the x direction. If you don't give any point load then if I integrate over the complete surface over this surface, I will be getting a term like this, it will be equal to the total tractive force in the x direction. But if I have a point load then effectively I will have to multiply this by the surface area and that will be equal to the point load in x direction. Similarly I will get another two equation like this.

So now if I look at these terms, if I look at these terms for all the tractive forces, I can say that this term is nothing, this term and that will be equal to dx. This term is nothing but this term and that is equal to T_y and similarly this term is equal to T_z but in addition to that for the point loads since I am calculating the complete surface integral over the surface, the point loads will give me additional terms of P_x multiplied by phi_x and so on. So this surface integral will be equal to, I get surface integral of T_x phi_x plus T_y phi_y plus T_z phi_z multiplied by ds plus for the point loads I will get P_x multiplied by phi_x plus P_y multiplied by phi_y plus P_z multiplied by Phi_z and since the point loads are discrete I will have to carryout a summation over all the point loads. If I have a number of point loads over the complete surface, if I have a surface like this, a body like this I have a number of point loads. For each of these point loads I will get a term like this. So I will carry out a summation over all the point loads and these phi_i's will be the phi at each point.

(Refer Slide Time: 00:20:51 min)



So if I make these substitutions my integral this integral term, these two terms will become… Now this is nothing but the integral of, I can write this as phi transpose T ds because I am using, the T will be nothing but T_x T_y T_z transpose and this would be this plus sigma P_i phi_i i going from 1 to n if I have n point loads. So this expression will give us these two terms and this complete expression that will become minus of volume integral of sigma transpose epsilon of phi times dv plus volume integral of phi transpose fdv plus surface integral of phi transpose Tds plus sigma i going from 1 to n Pi phi_i and this term should be equal to 0. So this would be the expression that we will get by evaluating the integral that we had in the beginning that is integral of Wedv should be equal to 0. Any questions up to this point, this derivation?

Now if we look at this I said that if we give this object a virtual displacement as phi, if I have an object like this and this has some boundary conditions and let's say this part is fixed maybe this part is fixed or some such thing and we give it a virtual displacement consistent with these boundary conditions and that virtual displacement is phi. This strain is the strain caused by this virtual displacement. Again this is also the virtual displacement, this is also the virtual displacement and this is also the virtual displacement. So if I look at this term, the first term that is, this first term this is nothing but the internal work done by the virtual displacement.

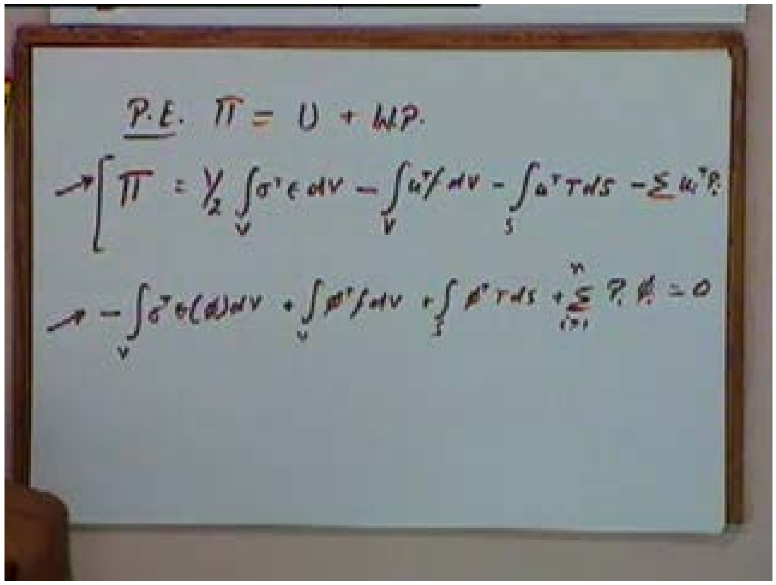
When you give the object some virtual displacement then this is the internal work done that is the work done against the internal stresses and these three terms give us the work done against the external forces. Let's say if I have some point load over here or if I have some body forces acting like this and I try to give this object a virtual displacement then the work done against these forces, against these body forces will be given by this, force multiplied by the distance. These are body forces multiplied by the virtual displacement we are giving and we will integrate that over the volume. So this is the work done against the body forces. If you look at this, you have a question there?

Student: Can we really separate the external work and the internal work because as soon as there is an external body force acting, it's really you know give rise to internal stresses in the body. So how can we really do? I will just comeback to your point in a minute.

Now if you look at this term, this is the work done by the virtual displacement against the tractive forces that means there are any forces are acting on a surface that is the pressure or force acting on per unit area basis then this term will give us the work done against that and this will be your work done against the point loads. So this complete, these three terms will give us the external work done, this one, this is for the body forces that means let's say gravity, work done against gravity or there might be some magnetic forces acting on the body which will again give on a per unit volume basis. So all those forces are the forces coming under this f that is the body forces. This T are the tractive forces that is forces are acting on the surface and these P's are the point loads, point loads acting on the body.

So these three terms will give us the external work done. If I look at this I get the internal work done or minus internal work done plus external work done is equal to 0 or the internal work done will be equal to the external work done and in fact this is basically a principle of virtual work, a principle of virtual work, whenever we give any virtual displacement the total work done will be equal to 0. So what we are saying is the total work done that is basically equal to 0. A part of that is what we are calling as the internal, done against the internal forces and a part of that is what we had, the work done against the external forces and that will give the answer to your question. The total work done by the virtual displacement will be equal to 0. So essentially this Galerkin's approach for elastic bodies boils down to the principle of virtual work for the elastic bodies. Any question on this Galerkin's approach? So now we have seen two methods, one is the Rayleigh-Ritz method and the second is the Galerkin's approach.

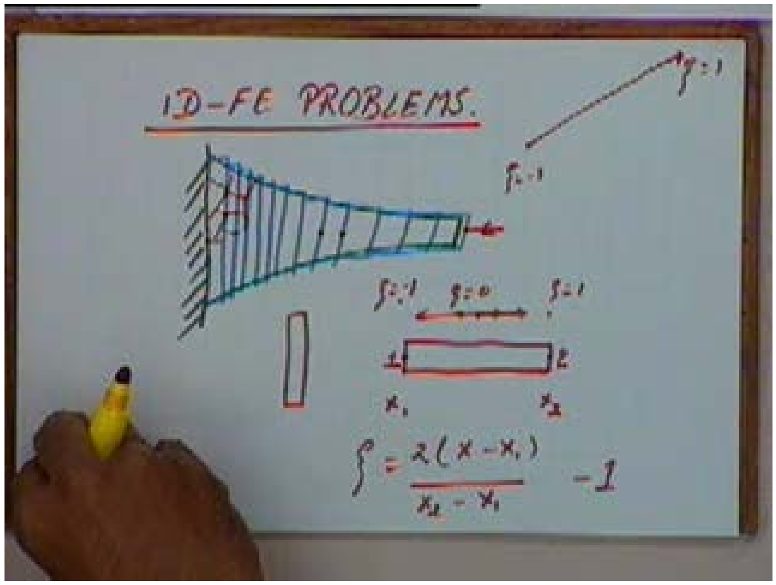
(Refer Slide Time: 00:27:57 min)



Just to recapitulate in the Rayleigh-Ritz method, the final equation that we got that was the equation for the potential energy term and we said that the potential energy of the object, the potential energy of the object P E or pi that is equal to U plus the work potential, in the term U is half of integral of sigma transpose epsilon dv volume of volume integral of that minus the work potential is given by the volume integral of u transpose fdv minus the surface integral of u transpose Tds minus sigma u_i transpose P_i. This is the expression for pi for potential energy and this we are using in the Rayleigh-Ritz method and in the Galerkin's approach, we just got that minus of the volume integral of sigma transpose epsilon of phi dv plus the volume integral of phi transpose fdv plus the surface integral of phi transpose Tds plus sigma i going from 1 to n P_i phi_i is equal to 0. This is the external work done sorry internal work done and this is the external work done and the two should be equal and the total work done should be equal to 0.

This equation we will be using when we use the Rayleigh-Ritz method, this we will be using when we use the Galerkin's approach. We will see that during both these approaches, we will be able to get the solution to the finite element formulation for different kinds of problems whether they be one dimensional, two dimensional or three dimensional. Now what we will see is how these methods can be used for simple one dimensional problems.

(Refer Slide Time: 00:30:03 min)

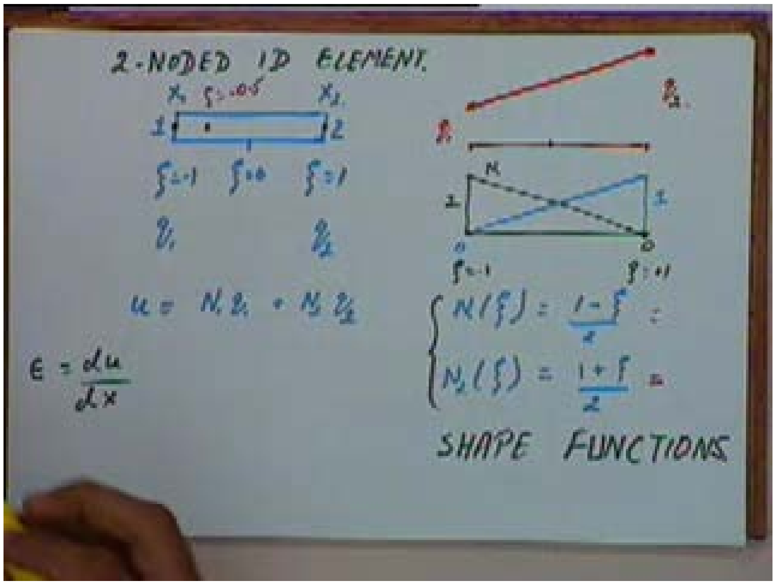


So for simple one dimensional problem, when we are talking of one dimensional problems what we do is that if we take any object, we are talking of one dimensional bodies, we will basically be talking of rods. So let's say if we take a rod like this, we want to divide this body into a set of small one dimensional elements. So while dividing into set of small one dimensional elements, we will approximate this by elements of this type. Each of these will be treated as one single element and we will assume that within this element, the area of cross section of the element remains the same that means let's say if I consider this element, this will be an element which will look like this that means there is no variation in the area of cross section. So along the length of this element, its thickness and height would remain unchanged that way it will vary only in one dimension that is why we call them as one dimensional element.

If we want to formulate the same thing as two dimensional problem then we can approximate it by triangles or quadrilaterals or something like that. Those could look maybe something like this but right now we are just trying a simple one dimensional formulation of this problem. So we will assume that there is no variation in the other two dimensions and all loads are also only acting in one direction, so all the loads will also be acting in the x direction only. So if I take any element, that element essentially looks like this. It is just a rod like element uniform cross section, on this element let's say this is a point 1 and this is my point 2. At this point let's say the x coordinate, if my element is here my x coordinate here and let's say x_1 and at this point that is let's say the x coordinate here is x_2.

What we will do is we will define a local coordinate system for the body, the local coordinate system would be that instead of talking of the x coordinate for this, we will talk of let's say the zeta coordinates. That means let's say in the center we will say we will have zeta equal to 0, sorry in center let's say here. At this extreme we will get zeta equal to 1 and at this extreme we will get zeta equal to minus 1 and zeta will vary linearly from one point to the second point. So our zeta will be equal to, again you will find two times x minus x_1 divided by x_2 minus x_1 minus 1. So add this point x is equal to x_1 so I put x equal to x_1, here this term will be 0 and zeta will be equal to minus 1. At this point x is equal to x_2, so x_2 minus x_1 this will cancel out 2 minus 1, we will get plus 1 over here. Similarly at the center that is x_1 plus x_2 by 2 we will get zeta to be equal to 0. So essentially zeta will be changing from minus 1 to plus 1 in a linear manner. Here we have zeta equal to minus 1, here we have zeta equal to plus 1.

(Refer Slide Time: 00:35:09 min)



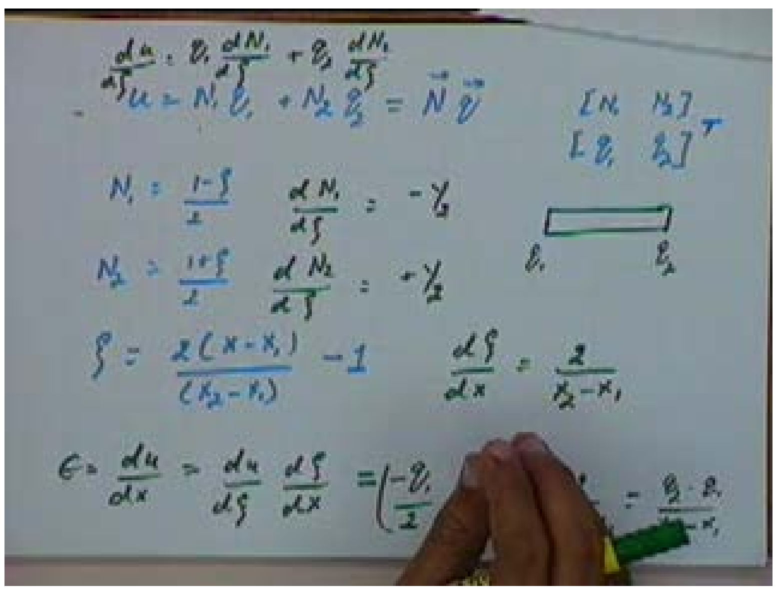
If we look at this element, the x value here is x_1, here it is x_2. Now let's say in the final displacement that is given to this body because of the loading that is acting, there is some displacement throughout this body, some deformation throughout this body. Let's say the deformation at this point is q_1 and the deformation at this point is q_2. So the deformation here is q_1 where deformation here is q_2. Now if we know the deformation here and the deformation here, we are interested in finding out deformation at any point inside this element.

The deformation at any point inside this let's say is given by u and we will assume that this deformation is a combination of q_1 and q_2, it depends only on q_1 and on q_2. So that's why we say it's a linear combination N_1 q_1 plus N_2 q_2. So the deformation at any point inside this element is given by this linear combination where N_1 is the function of zeta that is equal to 1 minus zeta by 2 and N_2 which is also a function of zeta is equal to 1 plus zeta by 2. So u which is the deformation at any point inside this element is given by N_1 q_1 plus N_2 q_2 where N_1 and N_2 are functions of zeta. What we are basically saying is that if I take any point inside this element, let's say I take a point here where zeta is equal to minus 0.5. If I put zeta equal to minus 0.5, I will get some value of N_1.

Similarly if I put zeta equal to minus 0.5 here I will get some value of N_2. For both these, for these values of N_1 N_2 I can put those values here and I will get some value for u, that will be the u at this point and since N_1 and N_2 are varying linearly this relationship gives me a linear variation of u along the length of the element. Let's say if I plot u along the length of the element let's say here it is q_1 and at this point it is q_2, along the length it will vary in a linear manner like this that is what I am assuming right now. And similarly if I plot N_1 and N_2, at this extreme I will find N_1 is equal to 1 but if I put zeta equal to minus 1, this will become 2 by 2 which will be 1 and at the other extreme N_1 will be 0 because if I put zeta equal to 1, I will get 0 so this is the variation of N_1. And if I try to plot N_2 I will find N_2 will vary like this, N_2 will be 1 at this extreme and it will be 0 at this extreme and N_1 is the other way round it is 1 at this extreme and is 0 at this extreme. At this point we have zeta equal to minus 1, here we have zeta equal to plus 1 and u is varying linearly from q_1 to q_2 along the length of the element. For one dimensional problem on the whole it's very straight forward but the same principle we will also be using for more complicated two dimensional and three dimensional problems. So we will be extending the same principle later on. These expressions N_1 and N_2 these are referred to as shape functions.

Basically these shape functions are defining, what is the variation of u that is the deformation along the element. I said it is a combination of the deformation at point one and the deformation at point two and I am assuming it to be a linear combination that is I am using linear shape functions. Let's say if in my element instead of two points, I consider a number of points let's say I consider three points then I can give my deformation to be a function of three points, we will see that later but right now we are taking a linear shape functions and we are defining only two points on my element that is why this element is also referred to as a 2-noded 1 dimensional element. So now I have u equal to N_1 q_1 plus N_2 q_2. If I know the expression for u then I can find out epsilon which is equal to du by dx, so let's find out an expression for epsilon.

(Refer Slide Time: 00:41:45 min)



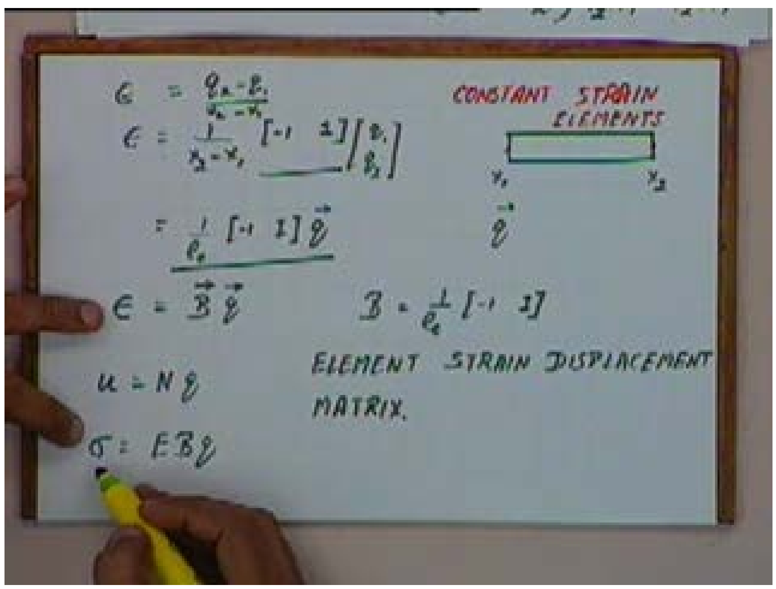
The equation that we have so far, our u is equal to N_1 q_1 plus N_2 q_2 or we will use a vector notation and write it as N times q where N is a vector and q is also a vector, where N is the vector N_1 N_2 and q is the vector q_1 q_2 transpose. I have also mentioned N_1 is equal to 1 minus zeta by 2, N_2 is equal to 1 plus zeta by 2 and I also mentioned that zeta is equal to 2 times x minus x_1 divided by x_2 minus x_1 minus 1. From this if I write try to write down d zeta by dx, this is equal to 2 divided by x_2 minus x_1. From this if I try to get dN_1 by d zeta is equal to minus half and dN_2 by d zeta it is equal to plus half and epsilon which is equal to du by dx, this will be equal to du by d zeta multiplied by d zeta by dx which will be equal to I have to differentiate this with respect to zeta. q_1 and q_2 are constants along the length of the element. If I have an element like this, q_1 is the deformation here and q_2 is the deformation here. Along the length of the element q_1

and q_2 are constant. So du by d zeta is going to give me q_1 multiplied by dN_1 by d zeta which is minus half so minus q_1 by 2 plus q_2 into q_2 by 2. This whole thing multiplied by d zeta by dx, d zeta by dx is this which is 2 divided by x_2 minus x_1 and this two's will cancel and I will get q_2 minus q_1 divided by x_2 minus $x_{1.}$ Again?

Student: du by d zeta.
Professor: du by d zeta.
Student: yeah, how do we get this?
Professor: How do we get du by zeta?
Student: Sir, how do we get this minus q_1 by 2…
Professor: minus q_1 by 2.

I am differentiating this term, q_1 is a constant so I will write it here, du by d zeta will be equal to q_1 multiplied by dN_1 by d zeta. dN_1 by d zeta is minus half plus q_2 multiplied by dN_2 by d zeta. Any other question? So now this is my expression that is there for the epsilon.

(Refer Slide Time: 00:45:43 min)

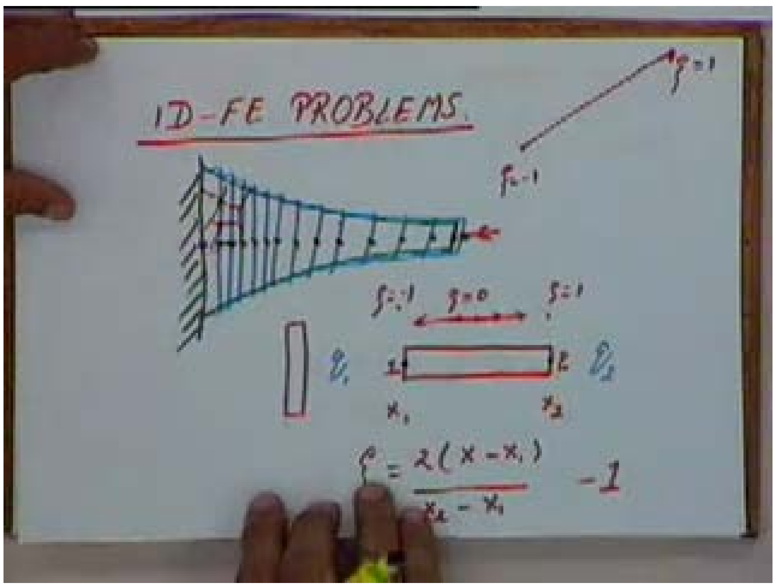


I will rewrite this expression, epsilon is equal to 1 over x_2 minus x_1 multiplied by q_2 minus q_1 or I will put it as in a vector form minus 1, 1 multiplied by q_1 $q_{2.}$ What I had before this was epsilon is equal to q_2 minus q_1 divided by x_2 minus x_1. I have just written that in the vector form like this and this now x_2 minus x_1, this is the element that I have. The x value here is x_1, the x value here is x_2, x_2 minus x_1 is nothing but the length of the element, so I will say one over the length of the element. This vector is nothing but the q vector that I had just introduced and this vector is what I get here minus 1 1 multiplied by q and this we will say is equal to B time's q, so we get epsilon to be equal B time's q where B is equal to 1 over l_e multiplied by minus 1 1 and this B is referred to as the element strain displacement matrix.
If you remember earlier we had u equal to Nq and epsilon is equal to B times q. So sigma is equal to EBq where this B is referred to as the element strain displacement matrix. And if you notice the strain within this element does not depend on zeta, either on zeta or on x that means

this strain is constant throughout the element. So what we have got in this particular case is the strain within the element does not change. We have assumed that this, we have assumed that the deformation is varying linearly.

If the deformation is varying linearly the strain within the element will be constant. That is why these types of elements are also referred to as constant strain elements. So for simple 2 noded 1 D element which are also the constant strain elements, we will get u equal to Nq, epsilon equal to Bq and sigma equal to E times B times q and if we remember what we have been doing earlier, if we can find out u, now we can see that we can immediately get the values for epsilon and we get the values for sigma.

(Refer Slide Time: 00:49:48 min)



So in order to solve a one dimensional problem, in order to solve this one dimension problem all that we need to do is find out the value of u in this element or to get the value of u, I need to get q. So if I am able to get the deformation at these two points, I can get the strain and the stresses within this element. Similarly using that you will see that if I can get deformations at each of these points and so on. So I can get deformations at each of these points, I can immediately get the stress and strain variation throughout this element throughout this body. So next we will see that using this formulation, how do we go about getting the deformations at each of these points in the body. Any questions on whatever I have covered today?

Student: sir, volume of each element same…
Professor: Volume of each element, need not be. And then the size of the element will be taken depending on… See if I take very very small elements, you will see that our computations will become complex, our matrices will become larger and so on. If we take very large elements, our accuracy will be less. So the size of the element will be taken on the basis that if in certain area, certain position I expect higher spaces, I give it a finer smaller element size. If I expect the stresses to be not very critical in that area then I will take a larger matrix.

Student: Sir, after we got the specified points, do we interpolate or we had…

See we are not getting the stresses at these points. Yeah between them. No, we are getting a displacement at these points. Once I get the, please please… once I get the displacement at these points, I can get the strains within the element by using this relationship that I had just given. Once I get the strains within the element I can get the stresses within the element. I will get the deformations at these points and then I will interpolate to get the stresses and strains and so on. So I am not getting the stresses or strains at these points, I will get stresses within the element. Any other question? In that case that's all for today. Next time we will do the detail formulation of…

Student: It is the only way we are solving for this displacement.
Professor: Again.
Student: The only way solving for is the displacement.
Professor: We will solve for the displacement and then using these relations we will get the stresses and strains.