Computer Aided Design
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Lecture No. # 11
Projections and Hidden Surface Removal

In the last class we were talking about the perspective projection and we had seen the case where the eye was located in the minus z direction at a distance of d that is at the point 0 0 minus d. And we had seen how we can transform any arbitrary point p that is x y z to a point p prime, which is x prime y prime z prime. And we had said that the transformation matrix for this was like this where we were having this 1 by d term in the last column. And we had noted that these kinds of transformations are nonlinear in nature because the homogenous coordinate will get transformed to z by d plus 1. Therefore these transformations were nonlinear in nature.

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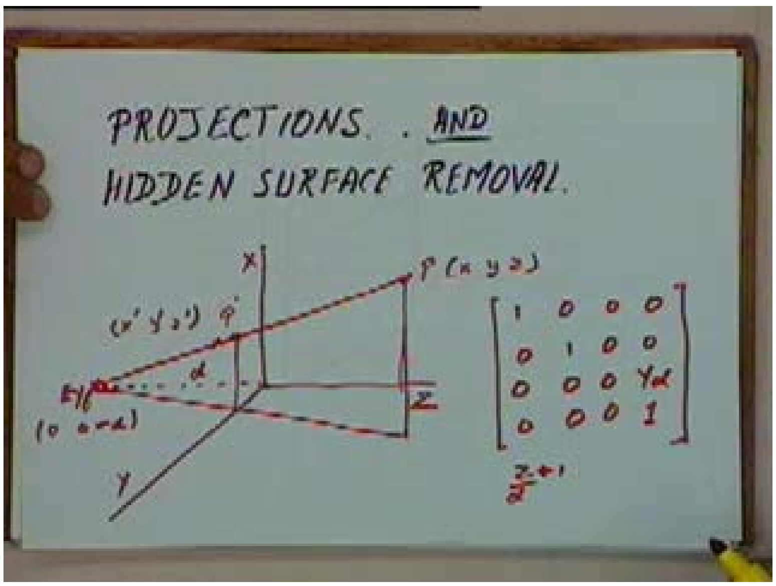

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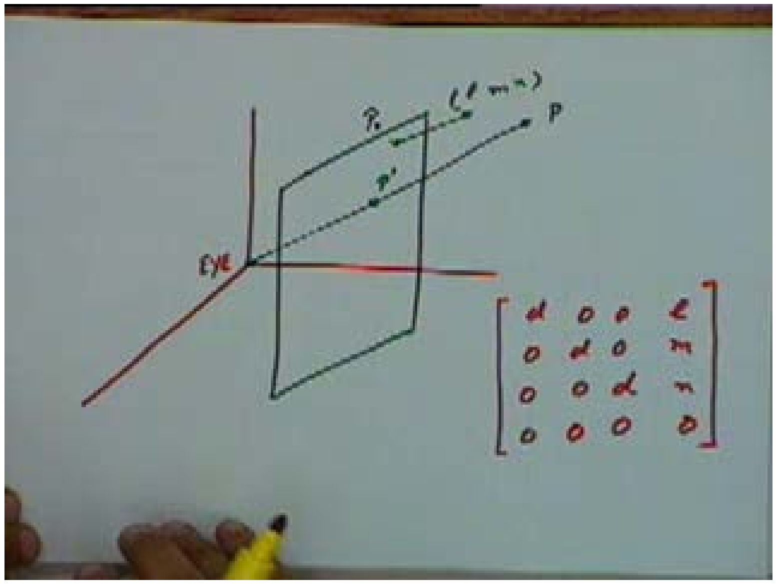

Then we had seen the perspective projection in a case like this where the eye is at the origin and the projecting plane is any arbitrary projecting plane. And for this case we had found out the transformation matrix as this where l m n are the direction cosines of the normal vector of the unit normal vector of the plane and d is the perpendicular distance between the eye and the plane.

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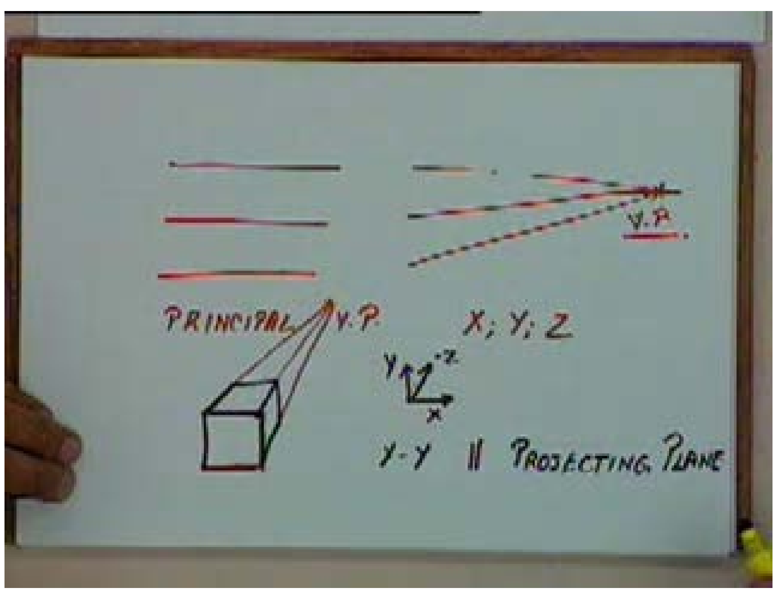

Then we were defining vanishing points and we had said that if we have a set of parallel lines, when I take a perspective projection they will appear to be appearing like this. They are seen to come from a point which is only a point at infinity but which will be a point within the visible region. And this point we said is the vanishing point. So vanishing point is a point in the perspective projection from where a set of parallel lines seem to emerge. Then we had seen a case of the principal vanishing points and we said principal vanishing points will be the vanishing points in the principal axis in the principal directions that is either x y or the z direction. And then we had seen the case like this where we have only one vanishing point that is in this direction. So if I call it lets say x y and this will become minus z direction. In this case we have a vanishing point only in the z direction. This is because at the x-y plane is parallel to the projecting plane. Only lines parallel to the z direction will appear to be coming from one point or lines parallel to the x or to the y direction will continue to remain parallel.

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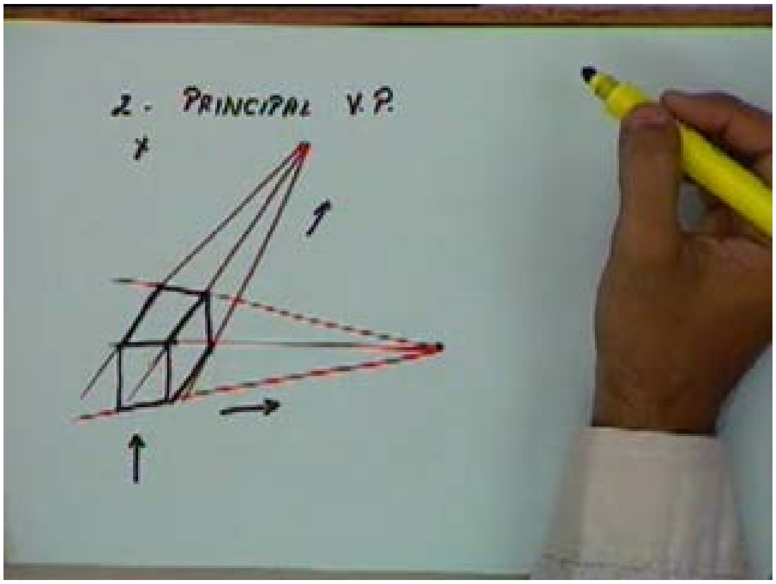

We can extend this to a case where we will have two principal vanishing points. And when we have two principal vanishing points let's take. The vertical lines in this case would continue to remain vertical. So that's if I get something like this, this point has not come up correct, it will be something like this. All lines parallel in this direction will seem to come from this point. All lines in this direction will seem to come from this point and all lines parallel in this direction will continue to remain parallel.

Similarly, we can have three principal vanishing points and in that case even lines parallel in this direction will seem to come from a particular point here. But the question that we come to now is that when we have a general perspective projection that is a projection like this. In this projection how do we find out which are the vanishing points? That means let's say from this point P, we have a set of parallel lines.

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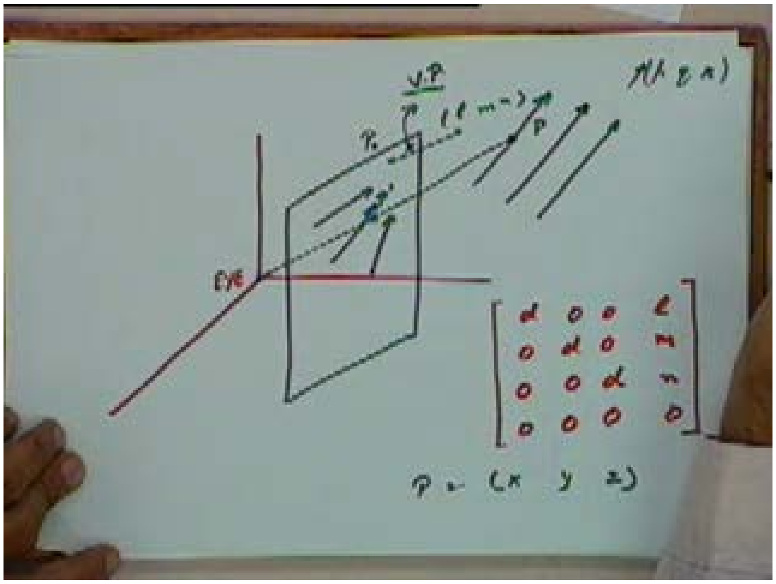

A line like this and other set of lines in this direction and in this direction and so on. Now these lines will get projected on to this plane, may be they will look something like this and for this direction we will get the vanishing points somewhere here. This vanishing point, how do you find out the coordinates of this vanishing point? Why do we want to find out? See the vanishing point in fact if you can join the perspective projections earlier, the principal vanishing points from the first step in drawing any perspective view, when you are drawing it manually. You decide the three principal vanishing points and then draw the view of the object. Just like something similar to what I was doing. If I know the location of this principal vanishing point, I know the location of this principal vanishing point, I can draw the figure of the cube by taking rays in that direction.

So traditionally, whenever a perspective view is drawn, the principal vanishing points are displayed along with that, that gives a designer a good feel of what is the orientation of the plane and so on. One would always like to show that this is the principal vanishing point or if in a particular object, a face is paralleling up in a particular direction then in that direction where the principal vanishing point lies, that we will normally like to show. Otherwise it can become very difficult to check whether lines appearing like this they are actually parallel or not unless I know that this is the vanishing point. So the question is how do we find out this vanishing point? For finding out the vanishing point what we will do. Let's say if I take any point P. This is my point P, P has coordinates of x y z and let's consider a family of lines which are passing or the family of lines which are parallel in a given direction p q r. So I have a family of straight lines, what will be the equations of those straight lines?

Just one second, what I will do is pqr is the direction. Let's take any arbitrary point here which is a b c. If I want to write down the equation of a line passing through the point a b c in the direction of p q r that equation would be, x would be equal to p times t plus a, y will be equal to q times t plus b and z will be equal to r times t plus c.

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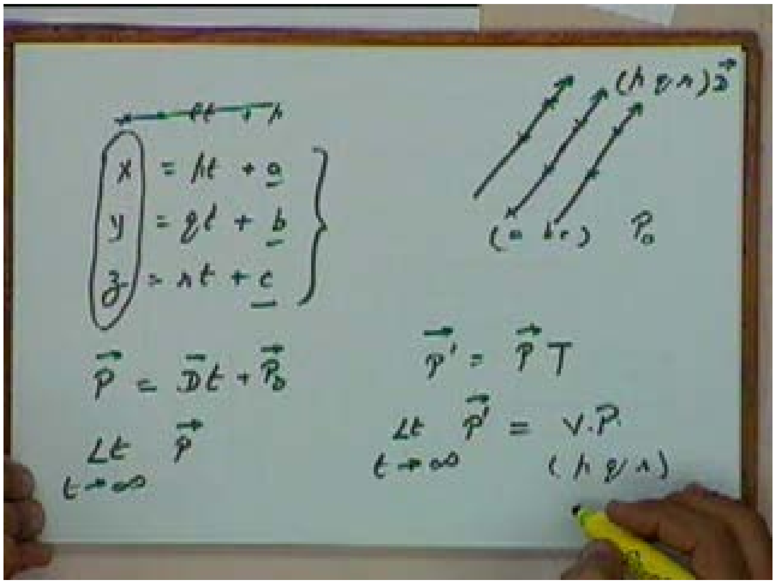

Now here a b c was any arbitrary point. So this was my point a b c and this is a direction I am taking. So this is the parametric equation of this line. Now if I want a family of straight lines in that direction all that I need to say is that this set of straight lines for different points a b and c will give me a family of straight lines parallel in this direction. Is that okay? And this point x y z is any arbitrary point x y z lying on any of these lines. Is that okay? If I want the point on these lines at infinity, now this I will write it as P, this direction let's call it as a direction vector D. And this point let's says any arbitrary point P_0, I can write this is as P is equal to the direction vector D times t plus P_0. These three equations I can write it like this. If I want the point on these lines at infinity, I will just say it will be limit t tending towards infinity of P, this will give me a set of points at infinity on these lines. Is that okay? So what we will do is we will transform this point P x y z using this perspective projection that means this set of parallel lines, we will find out what is their projection on to this projecting plane. We will get these straight lines, these ones. So let's say if this transformation is the transformation T then the transformed points that is the vectors P prime will be equal to P multiplied by T. So this vector P prime is going to be a family of vectors corresponding to lines parallel in the direction p q r. Now if I take limit t tending towards infinity P prime, I say that this will be equal to the vanishing point in the direction of p q r.

See my point P is any arbitrary point P on this family of lines. I am projecting these on to this projecting plane, I will get a family of lines like this. So this point P will correspond to some point P prime over here. So this point P prime corresponds to the projection of the point P, so in a general parametric form this point P prime or this P prime will be the family of lines which are the projection of parallel lines in the direction of p q r. And now if I take the limit of tends to infinity, the projection of that will be my vanishing point. As this point goes towards infinity, the point at infinity is going to be projected on to this point. A point at infinity in this direction will be or will be mapped on to this vanishing point like this. That is how a vanishing point is defined. All these lines will seem to come from that point. Because that point corresponds to this line extended to infinity.

That is what I am doing here. If I take a limit of t tending towards infinity of this P prime, I will get the vanishing point in the direction of p q r. is that? No, not at all. The observer is at the origin in this case. The observer is here and he is trying to locate a vanishing point on the projecting plane. He is trying to locate the vanishing point on this projecting plane. You have a set of lines parallel in the direction for p q r, these lines will meet at infinity but in the projection they will meet at the vanishing point. So I am finding out the equation of these lines, taking the projection and taking the limit of that as t tends to infinity that will give me the vanishing point. Is this process clear? All may points in the three dimensional space will get projected on to this plane because this is my projecting plane. See how I define my perspective view.

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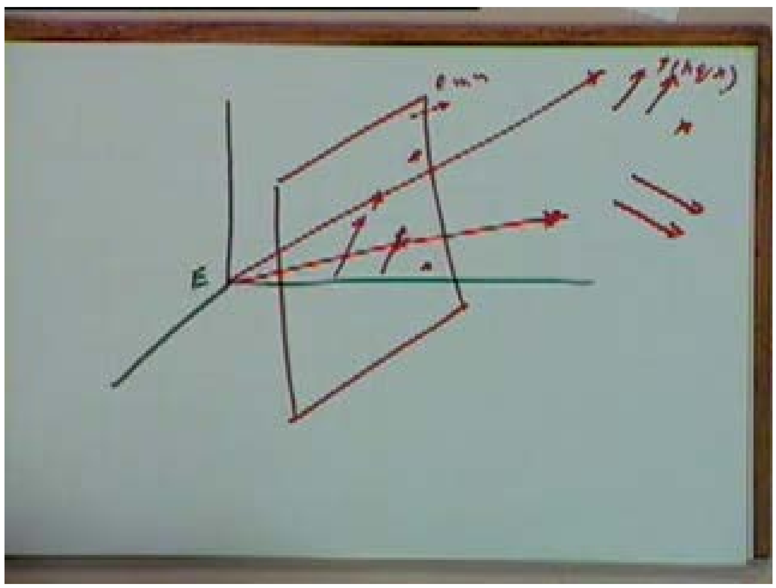

Let's say these are my axis and this is my eye and I have any arbitrary projecting plane like this. If I take any point, the way I am projecting it on to this plane, I am joining the eye to that and the intersection is the projection of this point and so on. So even if I have a set of parallel lines in this direction, a point at infinity will get projected to this point, so this is the vanishing point for lines in that direction. Similarly if I have a set of lines let's say in this direction, a point at infinity here will get projected on to some point over here, so this will be the vanishing point for lines in this direction and so on. No, there any arbitrary parallel lines in any arbitrary direction that is why I said the normal vector for this I had taken as lmn and this direction I had taken as p q r. If lmn is perpendicular to pqr then our vanishing point does not exist. If this direction is parallel to the plane, you can imagine you can visualize that, if you have a set of lines which are parallel to this plane, if you take the projection the vanishing point will also come at infinity. Only if these lines are at an angle to the plane we will get a vanishing point at a finite distance.

See these are a set of parallel lines. As said they have defined a whole space but we are only concerned about the vectors. We are only concerned that these set of parallel lines will seem to come from a point at infinity. We are only concerned with the direction. That is why I took any arbitrary point a b c and I took a line from that. Yeah, if the plane is perpendicular to the lines you will still get a vanishing point. But if the plane is parallel to the lines then you will not get a vanishing point that means if the normal vector is perpendicular to the these lines p q r. If the plane is perpendicular to the lines pqr then you will get a vanishing point. The plane is parallel to this direction p q r then you will get a projection of these lines all right but that projection will also appear to be parallel. Look at it this way let me take this block. If you are looking at it from front in orthographic projection, you will just get a rectangle but if you are looking at it from a particular point and you take a cutting plane which is parallel to the front face. The front face will still look like a rectangle. Look at it from your point of view from where you are sitting and you consider a plane which is cutting it like this and consider lines coming from each of the four corners, this edge and this edge with the way the position in which it intercepts this plane, they will still give you a rectangle. But if I change it at an angle like this and you maintain the same projecting plane then that parallel parallelism in the projection will not be maintained. Just see from your point and take a projecting plane which is parallel like this and which is like this and the block is at an angle. In that case you will not find the edges of this block to be parallel but if my blocks are like this and the plane is also parallel to it then these edges will appear to be parallel and so their intersection would seem to be at infinity.

So that is why I said in my projecting plane is parallel to these lines then there will be no vanishing point. But if a projecting plane is at an angle then I will get the vanishing points. If my projecting plane is like this and my block is like this vertical but at an angle, for the vertical lines they will still appear parallel but the horizontal lines they will appear at an angle in the projection. So in the vertical direction there will be no vanishing point but in the horizontal direction you will get a vanishing point. Is that clear?

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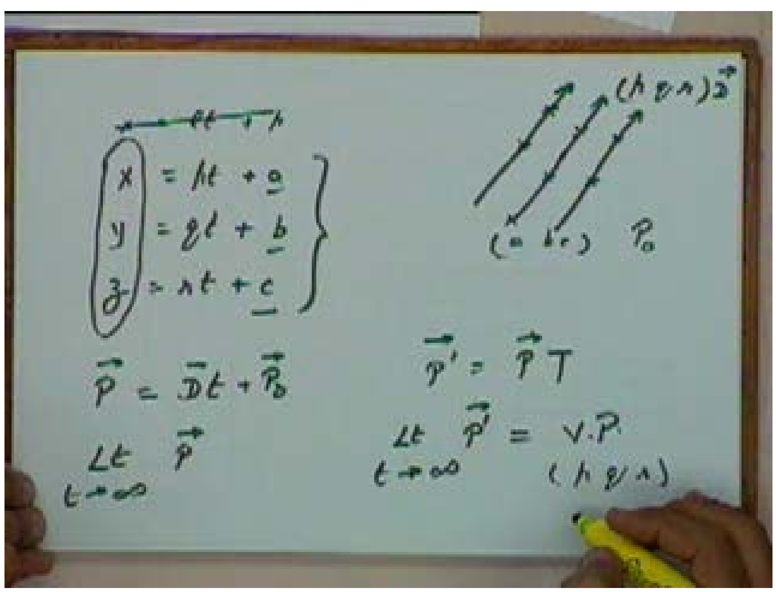

So we come back to this. For finding the vanishing point, we will take a limit of t tending towards infinity of this point P prime.

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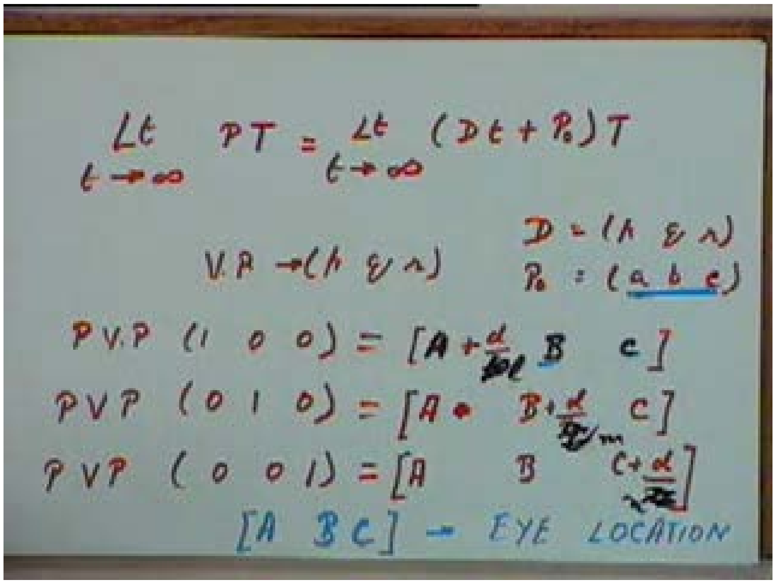

Now, so we have said limit t tending towards infinity of P prime which is the same as P times the transformation T which is equal to limit t tending towards infinity, t was the direction t D times t plus P_0 multiplied by the transformation T, where D is equal to p q r and P_0 was equal to a b c. This will give me a vanishing point in the direction of p q r. This is giving me a general vanishing point, if I want the vanishing point in the direction of 1 0 0, all that I need to is instead of p q r I will replace that by 1 0 0. I will put p q r equal to 1 0 0 respectively and I will get the vanishing point in the x direction or the principal vanishing point in the x direction.

Similarly, I can also get the principal vanishing point in the y direction and the principal vanishing point in the z direction and you can carry out the detail proof, the principal vanishing points will come out to be the points a. Let me use a different, let me use a capital A, A plus d by l B C, this should be A plus sorry A B plus d by m C and A B and C plus d by n. What I will suggest is that you carryout this step to completion and find out what are the actual limits as you take t towards infinity and then put these specific cases for the x direction y direction and z direction and try and get this result. In this the capital A, capital B and capital C they are different from this a b c, this capital A B C is the location of the eye. So you can try and derive these results by using this method, I have outlined the method the derivation will be straight forward after that. All that you have to do is expand this out and then take the limit of t tending towards infinity, it will be a simple case of taking a limit. So this is how we can find the vanishing points in a perspective projection and the principal vanishing points also. Any question on the perspective projection? Eye location was origin. No, if you take the eye location to be origin, you can make the changes. Oh sorry sorry sorry sorry. lmn is the unit normal vector of the plane.

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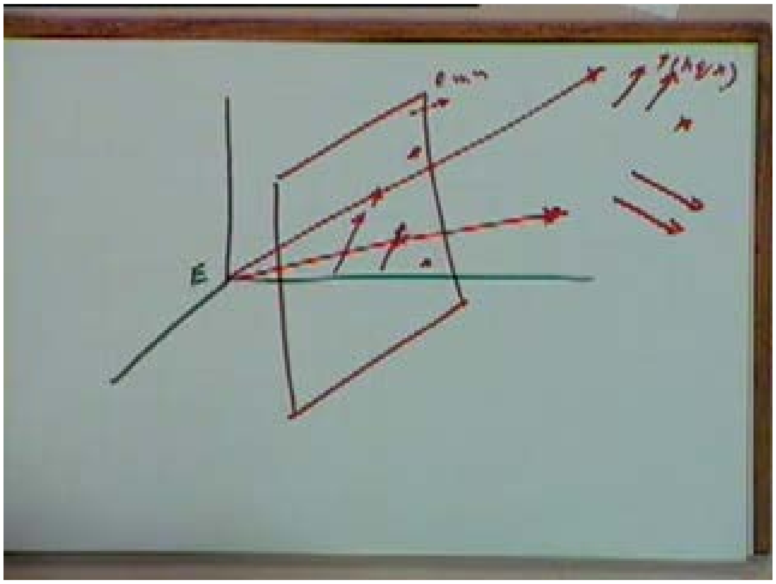

P q r, what was p q r? p q r were any general direction. Now the result that I have given is that this general direction is 1 0 0, so this is p, this is q and everything will be with respect to the eye. You can check these results, the p q r will not appear because p q r was a general case and I have taken specific value for getting this result. Now before we go on to the next topic of hidden surface removal, there is just one thing that we have to see and that is.

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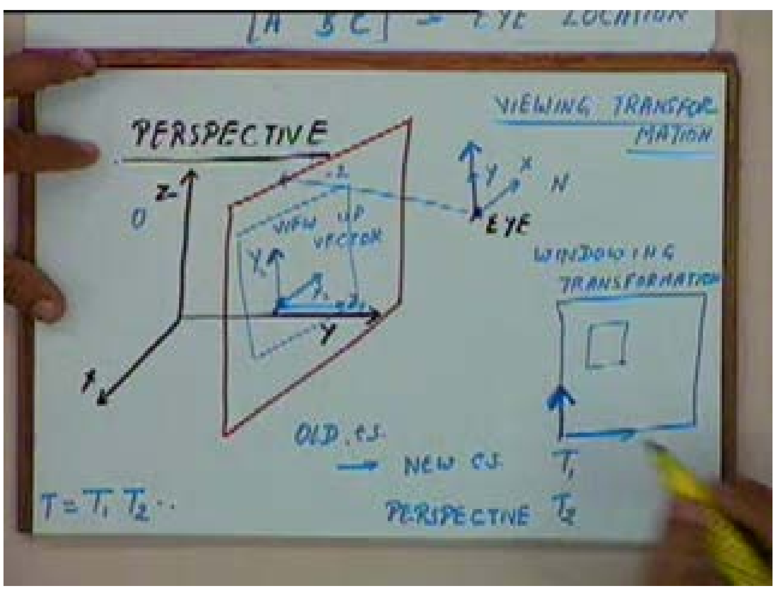

We have seen that if we have a perspective projection, in the case of perspective view if you have modeled objects in a particular reference frame for getting the perspective projection, what do we need to specify? We need to specify the location of the eye and we need to specify a projecting plane. We specify the location of eye and we specify this projecting plane, we can get all the transformation on to this projection plane. But eventually our aim is to transform this view on to the screen coordinates. So let's us say our screen is like this. So if we have to transform this onto the screen coordinates, what we need to do is on this plane we will have to define where the origin of the screen is going to lie. Let's say we define this as the origin, we have to decide that this is the vertical direction and this is the horizontal direction or some such we will have to decide a particular orientation. So just transforming or just taking a perspective projection is normally not going to be sufficient. What we will have to do is we will have to define the screen coordinate system on this plane and then finally we will able to get the coordinates on the screen.

So typically what would be done is that we will define that this is the eye, we will assume that this plane corresponds to the screen itself. And we will define a origin on this and we will define a upward direction, we will say that this is the upward direction or some, the term that is sometimes used is the view up vector. If we define that this is the upward direction, we can get which is the horizontal direction. So we will have to define the origin here and the view up vector. So if we specify the origin and the view up vector, the corresponding z direction can again be found out where this is my x let's say x in the screen coordinate system, this is my y in the screen coordinate system and the z will come like this.

Sometimes what is also done is that you place your coordinates at the eye, from the eye you take a direction perpendicular to the plane. Let's say my direction of plane is like this or in the perpendicular direction of the plane is like this and you take this to be the negative z direction and you define your, again define a view up vector let's say you define this as a view up vector. So let's say this is my y and we get an x like this, this would be defined normally as a minus z. The direction looking into the plane, into the plane like this is normally defined as a minus z direction. So if I define this as the x direction, this as the y direction and I define a minus z direction as the direction perpendicular to the plane. In that case the first thing that would be done is that this coordinate system the x y z that we have will first transform that into this coordinate system with the eye at the origin and x y z in this direction.

So let's say this is my new coordinate system and this is the old coordinate system. So we will transform the old coordinate system to the new coordinate system that will involve some transformation T_1 and then in this coordinate system we will apply the perspective transformation. So T_1, T_2 is the perspective transformation and the complete transformation involved is T_1 multiplied by T_2. Now this complete transformation is referred to as the viewing transformation. So viewing transformation is that transformation which will transform the world coordinate system into the coordinate system which is finally required at the screen. And out of this, the transformations which will, actually the viewing transformation will finally transform it on to the screen coordinates. We shall also require some transformation after this, viewing transformation is the complete thing. And out of this just the transformation which will convert a part of this plane on to this screen is referred to as the windowing transformation. The windowing transformation you transform this plane or a part of this plane on to a particular window on the screen or a particular view port on the screen.

Therefore taking a finite area here which is the window, transforming them onto a finite portion of the screen which is the view port is the windowing transformation. And the viewing transformation will transform the world coordinates on to the viewing coordinates system. That is the viewing transformation, this was with respect to the perspective transformation. x and y of the eye along with y x of the previous screen, see normally this view up vector that you are specifying is the view up vector that you will see on the screen that is what is meant.

So if I am specifying this as my view up vector, it means that on the screen I want to see this as the upward direction. Yeah, practically yes xs and ys, they will be at least parallel to those directions. Student: And both are chosen, both are I mean. Yeah, if you specify one the other can be found out. Yeah, both are to be specified. One has to be specified both are to be found out both are to be found out yeah, you have already specified the z direction, if you specify one the other can be found out, anyway a simple cross product.

So the view up vector is normally specified, we can specify either with respect to a point on the plane or with respect to the eye. But the view up vector will normally always be specified. This is how if you have an arbitrary coordinate system, you can transform that onto the coordinate system of the screen using the perspective projection. Almost a similar thing can be done, if you are using parallel projections.

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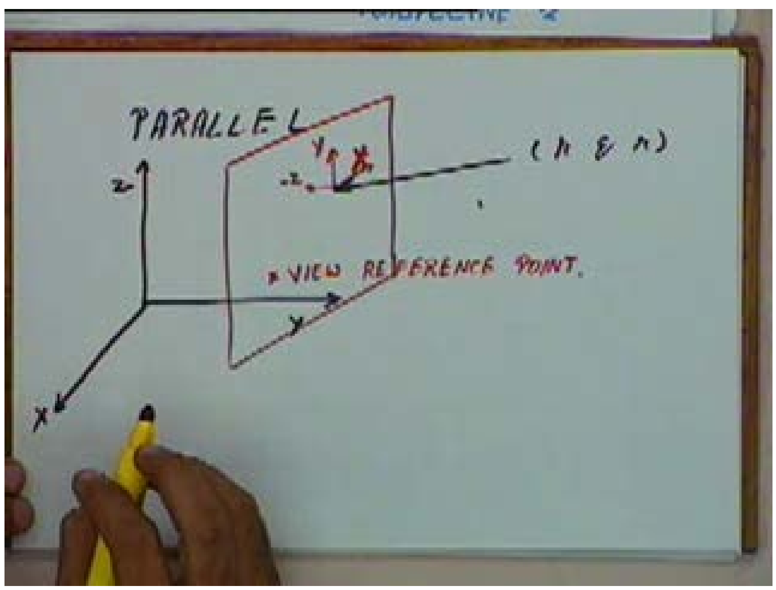

What we will do is in the case of parallel projection if this is our coordinate system; we have been taking x y and z. Now we don’t have a particular eye, a particular location of the eye in this case. But we do have a viewing direction; let’s say we define this as the viewing direction given by some vector p q r. If you are following the orthographic projections then our plane, projecting plane will be perpendicular to it. If you are following oblique, if you are following an oblique projection then it can be any arbitrary plane.

Let’s take orthographic projections. Again in the orthographic projection also in this plane, we will have to specify a view up vector and get a corresponding y direction. Again we will take this to be the negative z axis normally. The direction which we are looking into will be taken as a negative z direction, this will become y and we will get as minus z, this is x. Again on this we will have to specify a particular origin which will be our view reference point, this will become our view reference point. And at this point we will take the direction of viewing as the minus z direction, the view up direction as the y direction and the other direction as the x direction. So we will get a z this way, a y this way and x this way. Normally the direction into the screen is generally taken as the minus z direction, so there is no hard and fast rule about that, you can also take it to be plus z. any question up to this point?

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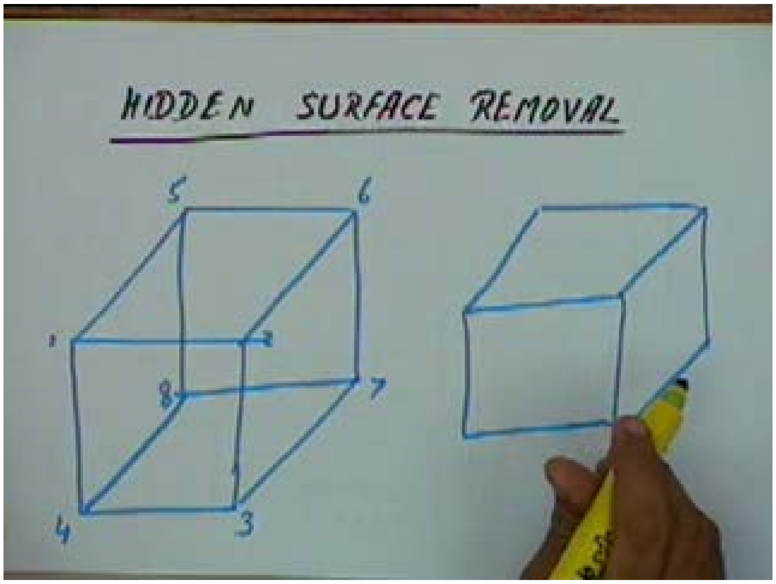

Then what I will do is I will introduce the topic of hidden surface removal. When we are talking of hidden surface removal the, if I draw a simple cube a simple block, I have drawn a simple block which consists of 6 faces and equivalent number of edges, 12 edges and so on. When I draw this on the screen, right now I have done a very simple thing. For each edge I have drawn a line. From a view like this, it will be very difficult to make out which edge or which face is in front and which face is at the back.

Let’s say if I number them whether the face 1 2 3 4 is in front or whether the 5 6 7 8 is in front it is very difficult to make out. Instead of this if I just draw a view like this, things will be much clearer. Since I have not drawn the faces or the edges which are hidden, clarity is much more. So what we have to do in this is whichever are the hidden edges or the hidden faces, they should not be drawn. Now this is a very expensive task, it’s not easy to find out which are the faces which are hidden and which are not. In this particular case it is very simple to say that the three faces which are at the back, we will just remove them and draw the others. But normally such a simple thing need not be the need not be or need not always be available because our faces can be curved. If we are talking of let’s say cylindrical surface, we cannot say that the complete cylindrical surface will be invisible.

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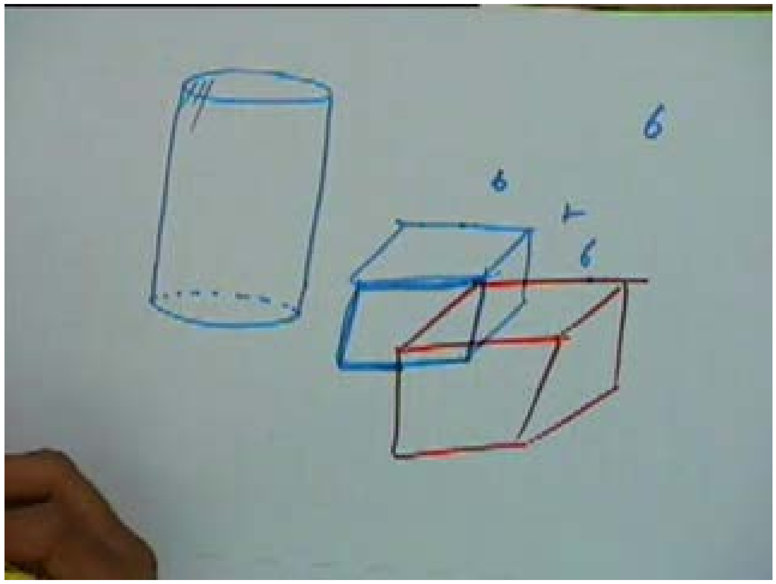

If you have a surface like this, we have only three surfaces in this case. Right now I have just drawn the edges. Let’s say if I also do a complete shading of this, complete filling in this figure. I will probably end up just getting a uniform a shaded area right through which is not what I want. So what we like to do is whichever faces are at the back or whichever surfaces are not visible, I should not be displaying them.

Typically what we will do is if we have any curved surface, we will split the curved surface into a set of small polygons, a small planer surfaces and then carryout the hidden surface removal. For instance in this figure I can consider each of the faces separately and decide which faces are to be displayed and which are not to be displayed. If I am just deciding which of these surfaces are to be displayed, in a single object whichever surfaces are at the backside I can simply decide not to display them. I can use an algorithm what is referred to as back face removal, whichever surfaces or whichever faces are on the back side I will just not consider them while displaying. And how do I decide which surfaces are on the backside? We have a particular viewing direction, with respect to the viewing direction we will see the direction of the face and see whether it is pointing towards the viewing direction or away from it. Details we will go into later on but we can easily decide which surfaces are on the backside.

But if we carry out a back face removal like this, that back face removal might not work in all the cases because for simple objects like this, it’s all right. So let’s take a case. We have one block like this and another block lying in front of it like this. now we have got two blocks. As a result of that what is happening is this face, for this block is a front face but it is partially hidden by a second face. So just a simple back face removal might not be sufficient, back face removal will definitely cut down some planes or some surfaces, surfaces which are at the back of this plane or at the back of this sorry surfaces which are at the back of this block or at the block of this block will definitely not be drawn.

But surfaces which are partially visible those kind of surfaces cannot be taken care of. This surface is partially obscured by other surfaces, so we cannot account for such surfaces by the back face removal. Back face removal will definitely cut down the total number of surfaces which we have to display, total number of planes which we have to display. But it will not solve the complete problem. Even then an exercise like back face removal is very desirable, basically because even if I am able to cut down the number of surfaces I am handling by a factor of half that is considerable because you will soon see that hidden surface removal is going to be a very expensive process.

So in this case initially I had 6 plus 6 surfaces 12, out of that if I can cut it down to 6 surfaces 3 from here and 3 from here I am reducing the number of visible surfaces by half that is also a considerable improvement because hidden surface removal is going to be a very expensive process. In fact if you have worked on Unigraphics or any other CAD package, you will see that whenever you give any shading command or whenever you want to see a complete view without any hidden surfaces it takes a lot of time. Basically because it is very difficult to decide computationally which surfaces or which edges are not visible. What we will do is in the next class, we will see some specific algorithms for removing hidden surfaces. So that's all for today.