Computer Aided Design

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Lecture No. # 10

Perspective Projections

Today we will be talking of the perspective projection and then if time permits, we will go to hidden surface removal.

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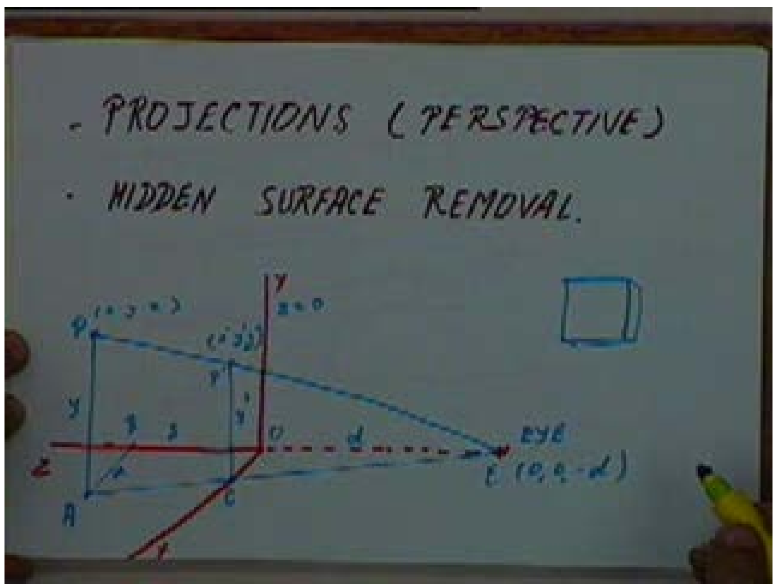

In a perspective projection, the basic idea is that when we are looking at any object we want to draw the object the way our eye sees it. That means if we have a cube and you want to draw the orthographic view from that is the front view let's say, we will just get a rectangle. But when draw the perspective view you won't get a rectangle because our eye is located at a particular point. And since our eye is located at a point, a view is going to be different. We will be able to see the side edges also. The side face is that is, so our view might look something like this. This is basically because the projecting lines from the object are not parallel. The projecting lines are going to be all focused at the eye. For example if you have any point here xyz, these are coordinate x is x and yz and this is the location on the eye, this is going to be the projecting line.

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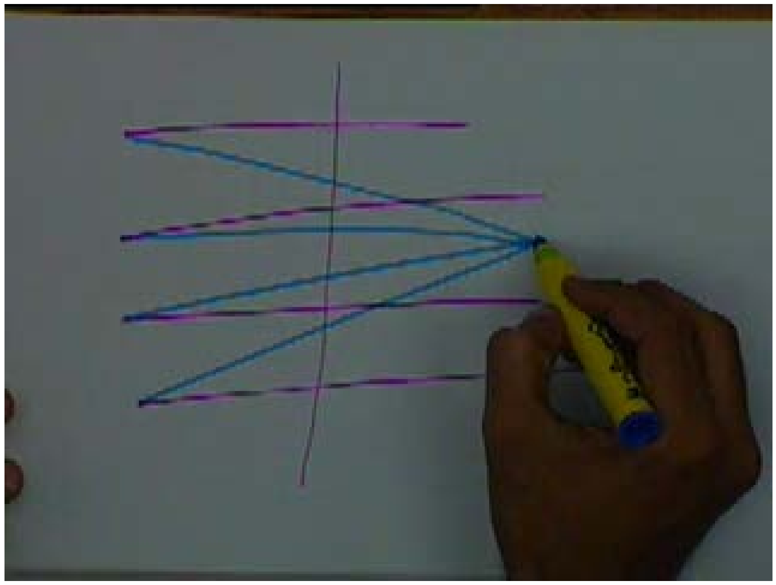

So if this is the eye, all the lines are going to be projected in this manner. Earlier when we are talking of the orthographic projection, the projecting lines were all going parallel and we have a projecting plane like this. So if this is the projecting plane and the projecting line instead of being parallel or focused on the eye in that case a point here will get transformed to this point, this point will get transformed here and this one will get transformed here and so on. So in the case of a perspective projection, we always define the location of the eye and we define a projecting plane.

In this particular figure I have taken the projecting plane to be the xy plane. The projecting plane is this xy plane that is the plane where z that is equal to 0 and the eye instead of being located at infinity, it is located at the coordinates 0 0 minus d where d is this distance from the origin to this point. I am assuming that the eye is right now located in the minus z direction. The projecting plane is a xy plane and we want to project any arbitrary point xyz onto this projecting plane. So this arbitrary point xyz will get transformed to a point x prime y prime z prime which will be on the xy plane. So obviously z prime will be equal to 0. So this way if you have a set of points, they will all get transformed such that if I join that point xyz to this eye to the location on the eye, the intersection of this line with the xy plane is going to give us the transformed point.

In earlier case the point xyz when projected onto the xy plane using orthographic transformation were just getting transformed to xy. The x and the y coordinate won't change because a projecting lines were earlier parallel and perpendicular to the projecting plane. In this case all the projecting points are going to make a different angle to the projecting plane. How do we find the coordinates of this point x prime y prime z prime? From this point p, I have drawn a perpendicular onto the xz plane, this vertical distance will be equal to y.

Similarly from this point if I draw a line parallel to the x axis, this distance will be equal to x that is because of the coordinates of this point and from this point to the origin, this distance is going to be equal to z. Is that all right? Now this is the origin, let's say this is the eye E, let's say this is my point A, this is B, let's say this is C.

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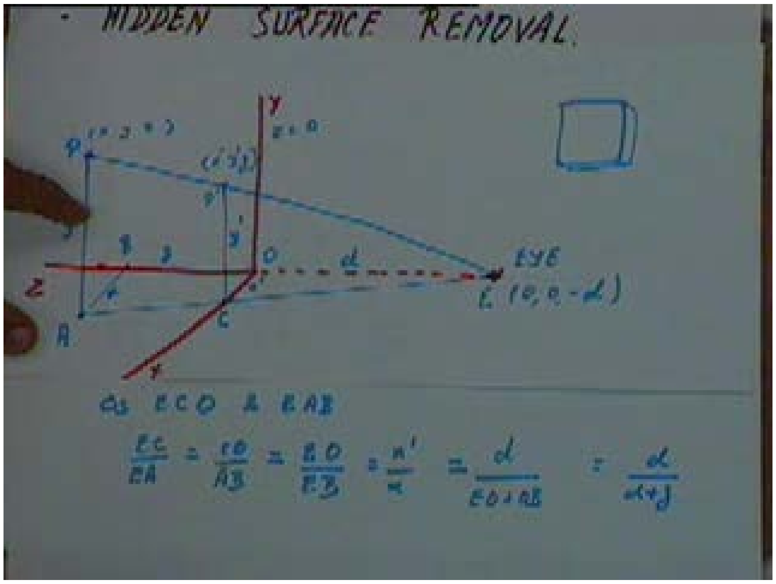

Now in this figure let us consider triangles ECP prime and triangles and EAP, ECP prime and EAP that is the eye C P prime and EAP. If you look at these two triangles, this distance from C to P prime this is a y coordinate of the point P prime, so this distance will be equal to y prime. So from these two triangles from these two triangles, we can say EC divided by EA would be equal to C P prime divided by AP which would be equal to C P prime is y prime, AP is y, put both of them together. Can you see it clearly both the figure in the text? So EC by EA that is this distance divided by this distance would be equal to y prime divided by y.

Now let's consider triangles ECO and EAB. Now we can say EC divided by EA will be equal to CO divided by AB and it is also be equal to EO divided by EB. So CO divided by AB that is CO is nothing but x prime, the CO is x prime divided by AB, AB is x. This is equal to EO, how much is EO? d, EO is the distance of the eye from the projecting plane. So d divided by EB. Now EB is going to be equal to EO plus OB which is equal to d divided by d plus z. Is that all right? And we know that z prime is equal to 0.

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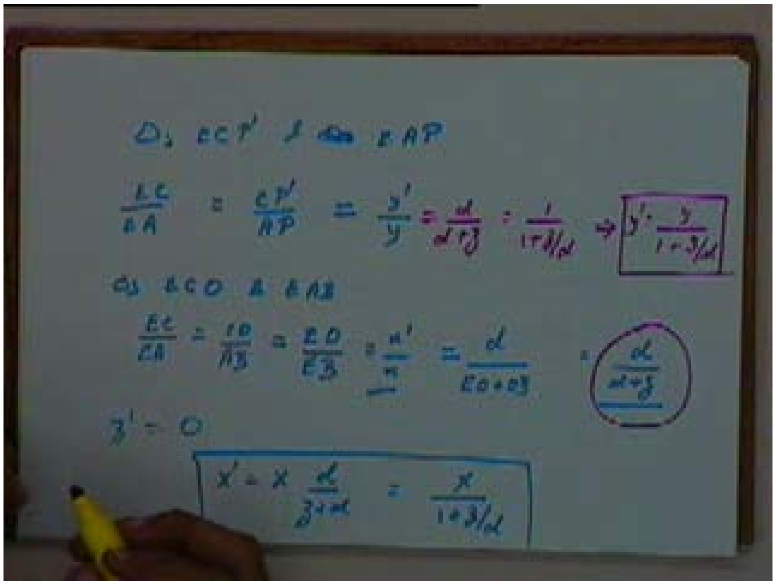

Now let’s look at these equations. from this and this we will get x prime is equal to x multiplied by d divided by z plus d which I can write as x divided by 1 plus z by d, just divided d in the both the I will just divide it by d both in the numerator and denominator, so this is one equation. Similarly if I take this equation EC by EA, I can take from this equation EC by EA is equal to this figure. So y prime by y is also equal to d divided by d plus z or is equal to 1 over 1 plus z by d and this implies y prime is equal to y divided by 1 plus z over d. So this is the second equation. Is that all right? So now we have got the x prime that is x coordinate, we have got y prime and we know that z prime is equal to 0.

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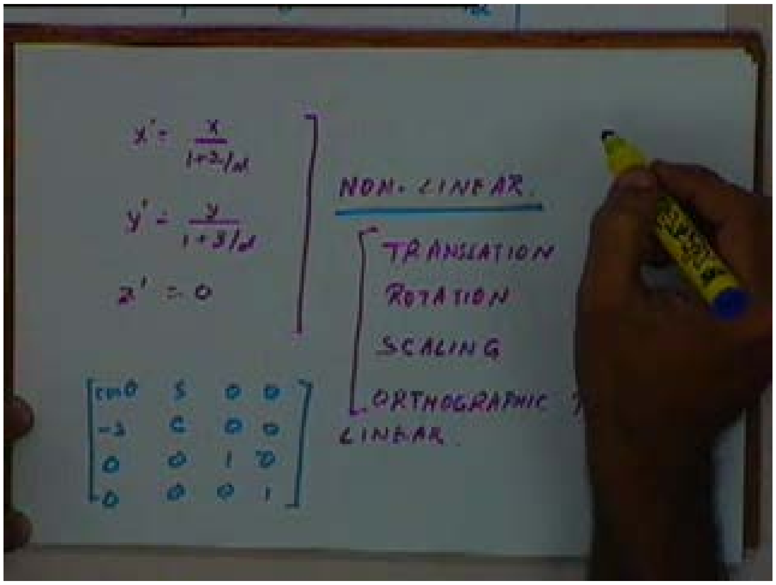

So if you take these three equations, we will get x prime is equal to x divided by 1 plus z over d, y prime is equal to y divided by 1 plus z over d and z prime is equal to 0. Now the first thing to be noticed is that these transformations, they are nonlinear transformations. All the transformation that we saw earlier that is we had seen translation, you have seen rotation and we have seen scaling and orthographic projections, orthographic and orthographic projections. All these were linear, for instance translation that was captured by this is simple addition of a translation vector. Rotation that was captured by a matrix which was something like cos theta sin theta 0 0 minus sin theta cos theta 0 0 0 0 1 0. In this the value of x prime is x cos theta minus y sin theta that is a linear combination of x and y.

Similarly a value of y prime was x sin theta plus y cos theta that is again a linear combination of x and y, z prime in this case was equal to z. So all x prime y prime and z prime were linear combination of x y and z but this is not a linear combination of x y and z, x prime is equal to x divided by a term involving z that means x prime is not a linear combination of x y z. Similarly y prime is also not a linear combination that is why these kind of transformation come under the category of nonlinear transformations and if we want to write them in the form of a normal transformation matrix, the way we have been doing so far.

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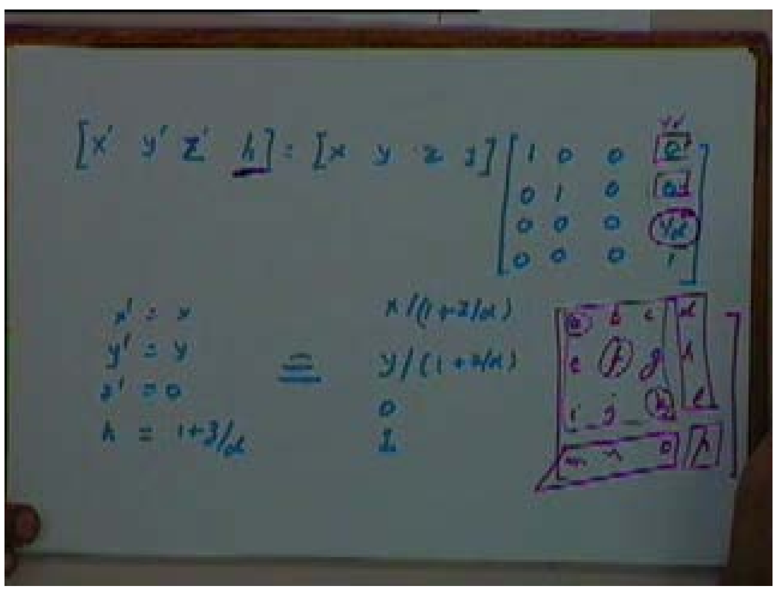

What we will have to do is x prime y prime z prime h, we will have to use the homogenous coordinate. Now both x prime and y prime are being divided by the term 1 plus z by d. So what we will do is we will get h equal to 1 plus z by d. For getting h to be 1 plus z by d in the last column we will say 0 0 1 by d 1. So that this row vector multiplied by this column will give us the term of 1 plus z by d, this way 1 plus z by d will be the value of h. If h is equal to 1 plus z by d, effectively x prime y prime as well as z prime will get divided by the same number and x and y will retain as the same values.

This way by having a 1 by d term here, we will get the homogenous coordinate to be equal to 1 plus z by d and effectively both the x prime and y prime will be x divided by 1 plus z by d. If you look at this we will get x prime equal to x, y prime equal to y, z prime equal to 0 and h equal to 1 plus z by d. So this point would be the same as the point x divided by 1 plus z by d, y divided by 1 plus z by d, 0 and 1. So this way we will be able to capture this nonlinear transformation. Is that all right? So capturing this perspective transformation, we put 1 by d term in the fourth column and then if we now look at a general 4 by 4 matrix which we have been using for transformation, we said this term will be used for uniform scaling. These 3 terms are being used for translation, these 9 terms are being used for different notations.

Then these terms, the diagonal terms is using for scaling and now we will see and we have seen that these terms would be used for the perspective transformation. So if these three terms are nonzero at any point, we can know for sure that the perspective transformation involved in it or some nonlinear transformation involved. By using these three terms we are able to get nonlinear transformations. If you remember when we are talking of homogenous coordinates, we had said that the homogenous coordinates will help us capture different types of transformations. By getting a different value of h, we managed to capture perspective transformation using ordinary transformation matrix. By using these positions, we can capture nonlinear transformations which involve the term of x y and z in the homogenous coordinate.

Right now we are getting only one term over here, if you want to get or rather in which or under what circumstances will these two terms be nonzero? Anyone? What is that again? When the eye is along the x or y axis. Right now we have taken eye to be around the minus z direction. If eye is the minus x direction or in the minus y direction, in that case we will get nonzero term in these two positions. If a eye is in the minus x direction then we will get a 1 by d term over here. Similarly for eye is in the minus y direction, we will get a 1 by d term here and so on. So this is how we capture the perspective projection in general. Any question on the perspective projection?

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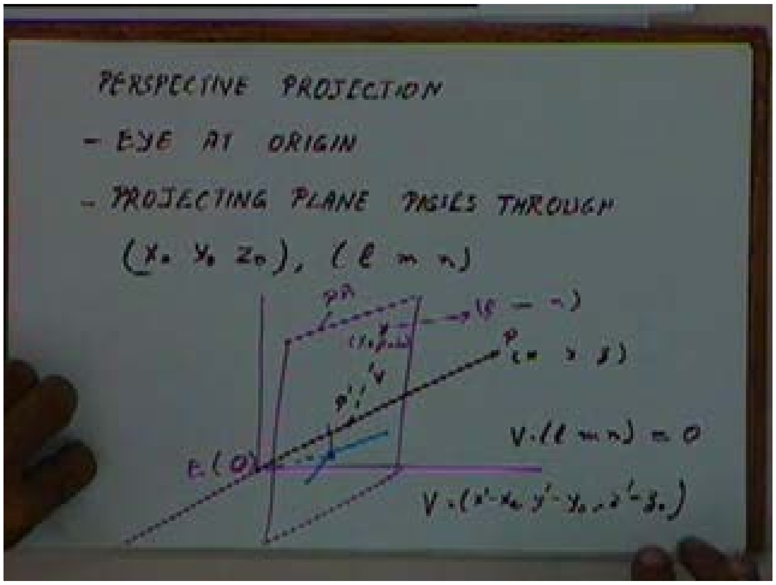

Then let’s talk of the perspective projection, when the eye is at the origin and the projecting plane passes through the point x_0 y_0 z_0 and has a unit normal vector given by lmn. If this is a projecting plane and eye is at the origin in that case how do we get the perspective transformation? This is a projecting plane and this is the location of the eye that is at the origin and now we want to find out the transformation matrix for this case. Any suggestions on how to find out transformation matrix? We rotate the origin, what do you mean by rotate the origin? So you have to take the coordinate system so that just by rotating the plane, so we rotate and then translate so that this plane this projecting plane becomes one of the principal planes. That can be one way of doing it, there is a much simpler way of doing that.

See what you are saying is that we will translate this point let’s say to this point or alternatively from this point we will draw unit normal sorry we will draw normal onto the plane and we will translate this origin to this point and rotate the axis so that one of the principal axis points in the direction of the lmn vector and then we can use a transformation matrix which we found in the previous step but that is going to be a very complex procedure. Instead of that we can again use a equation from geometry.

If we have any arbitrary point, here that is the point x y z. If this point is transformed onto this plane that will be the intersection of this point with this line. Let’s say the intersection is somewhere here that is my point P prime, this is my point P. now this point P prime will satisfy the equation of this plane and if this point P prime is satisfying a equation of this plane what is the equation that we will get for this point? What will be the equation for this plane? The dot product of this vector, with this unit normal vector has to be equal to 0. Is that okay? Let’s say this is my vector let’s call it V, so the vector V dot with lmn has to be equal to 0, this is because this vector V is a vector in the plane and this vector lmn is a normal vector to the plane.

So lmn has to be perpendicular to every vector in the plane and if two vectors are perpendicular the dot product has to be 0. So the dot product of V with lmn will be equal to 0. Let's say V dot lmn is 0 and what is this vector V? Vector V will be x prime minus x, y prime minus y sorry x prime minus x_0, y prime minus y_0, z prime minus z_0. So if we consider this vector V and is dot product with lmn that has to be equal to 0.

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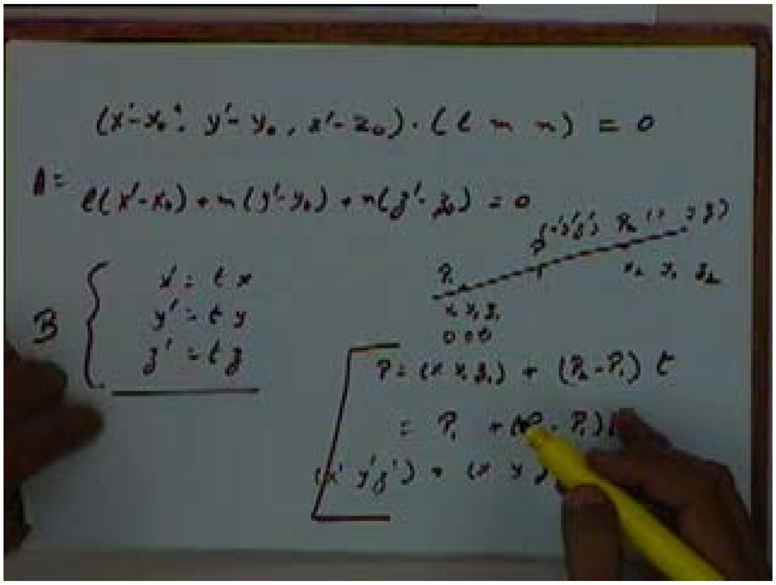

That means x prime minus x_0, y prime minus y_0, z prime minus z_0 dot with lmn is equal to 0. That will mean l times x prime minus x_0 plus m times y prime minus y_0 plus n times z prime minus z_0 will be equal to 0. The other thing that we have to notice is that if we have this point P and we are joining it to the origin, what is the equation of this straight line, what will be the equation of this straight line? So if you take any point on this line which is given by let's say x prime y prime z prime, x prime by x will be equal to y prime by y will be equal to z prime by z or we can say x prime will be equal to t times x, y prime will be equal to t times y and z prime will be equal to t times z, that's all right.

Essentially if you have any two points x_1 y_1 z_1 and x_2 y_2 z_2 and we want to find the location of any point xyz on this, we can write down the parametric equation of this line. Using vector notation let's say if this point is P, we can say that P will be equal to x_1 y_1 z_1 plus let's call this P_1, let's call this P_2 plus we will say P_2 minus P_1 times t or we can say it is P_1 plus P_2 minus P_1 times t. Any point P on this can be given by this solution. Same thing we are doing over here, a point P_1 is the point 0 0 0 and a point P_2 is the point xyz. So we are getting the point P which is x prime y prime z prime and the point P_2 which in our case is xyz, so x prime y prime z prime, P_1 is 0, so this will be equal to xyz times t. So from this we will get these three relations x prime will be equal to t times x, y prime will be equal to t times y and z prime is equal to t times z.

Now if we look at this solution we need to get the value of x prime y prime z prime which is the intersection of this line with this plane. So both these equation have to be satisfied let's say this is equation A and this set of equation is equation B. So value of x prime y prime and z prime, we will put into this equation.

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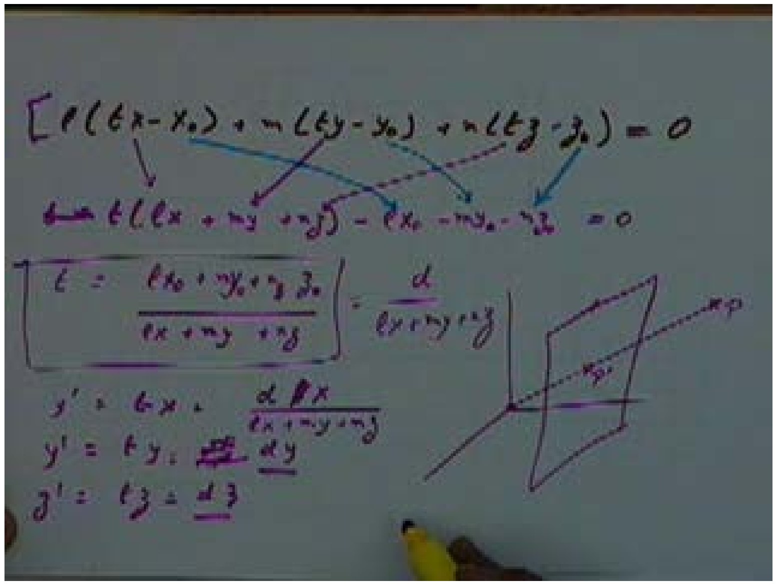

If we do that what we will get is l times tx minus x_0 plus m times ty minus y_0 plus n times tz minus z_0 will be equal to 0. I have taken this equation and these equations and I have got this. So I have put x prime over here, y prime over here and z prime over here, I will get this equation. So now if I find out the value of t from this equation, I will get t or let's do this step by step, t into lx plus my plus mz minus lx_0 minus my_0 minus mz_0 is equal to 0. This term is coming here, this term is coming here, this term is coming here and this term is coming here, this is coming here and this is coming here.

So from this I will get t is equal to lx_0 plus my_0 plus mz_0 divided by lx plus my plus mz. And projection point in this figure, this point P prime is given by x prime is equal to t times x, y prime is equal to t times y and z prime is equal to t times z where t is given by this relation. So this is how we will get the perspective projection in this case where the eye is at the origin and a projecting plane is any arbitrary plane. This is my point P, this is my point P prime, a point P prime will be given by these relationships. Now in this if we look at lx_0 plus my_0 plus mz_0 what are this correspond to? Again. Distance of this plane from the origin lx_0 plus my_0 plus mz_0 is a perpendicular distance of the origin from this plane. Is that all right?

This we will say is equal to d divided by lx plus my plus mz where d is the perpendicular distance of this plane from the origin and if we want to write this relationship using a matrix multiplication transformations, how do we do that. We have to say x prime y prime z prime h, x prime is t times x but t is given by this. Again lx plus my plus mz is coming in the denominator. So this is also a nonlinear transformation because lx plus my plus mz is in the denominator. I can put these terms here, this is equal to d times t divided by lx plus my plus mz.

This will be equal to d times t divided by the same term this is also equal to sorry. This is equal to d times x divided by this term, y prime is equal to d times y divided by this term and z prime is equal to d times z divided by this term. So again we will take this complete term into the homogenous coordinate. To get this complete term in the homogenous coordinate, we will say l m n 0, x y z 1 when multiplied by this vector l m n 0 will give us the term lx plus my plus nz and here we will just say d 0 0 0 d 0 0 0 d 0 0 0.

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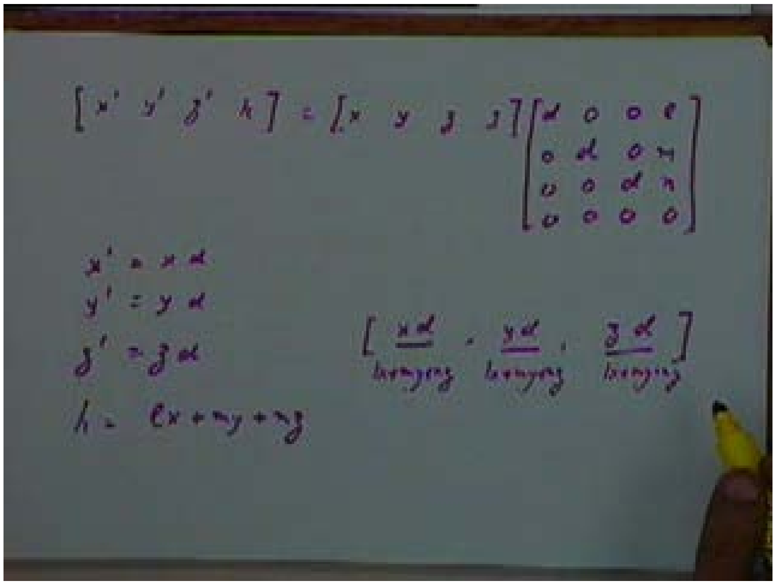

This set of equation will give us x prime will be equal to x times d, y prime will be equal to y times d, z prime will be equal to z times d and h will be equal to lx plus my plus mz. So this point will be the same as the point xd divided by this term, yd divided by this term and zd divided by this term, this term is lx plus my plus mz. Similarly you get that here also. So this would be the, this point which is the perspective projection of the point P that is what we have got over here. Any questions on how we have got this perspective projection? Any question on this? So if the eye is at the origin, I am going back to this figure.

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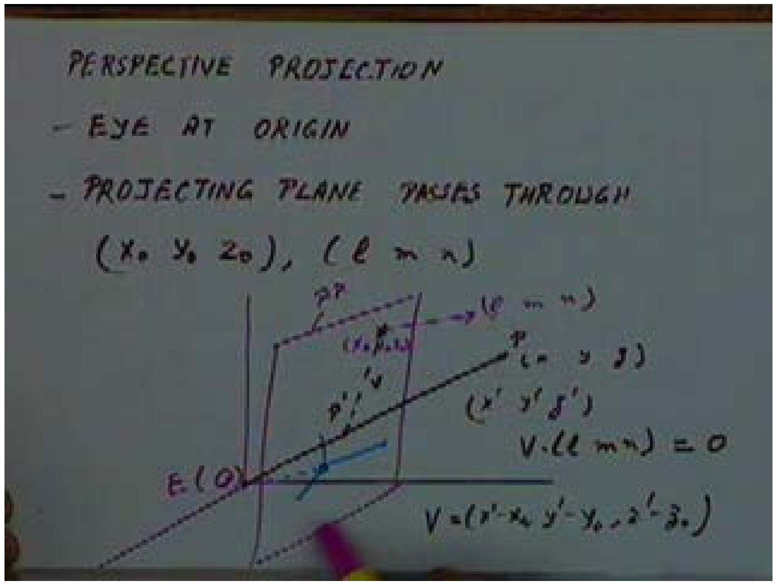

If the eye is at this origin, the projecting plane is any arbitrary plane in that case we can project a point P onto this projecting plane using this set of equations. So now in perspective projection let's see one more thing and that is the general perspective transformation.

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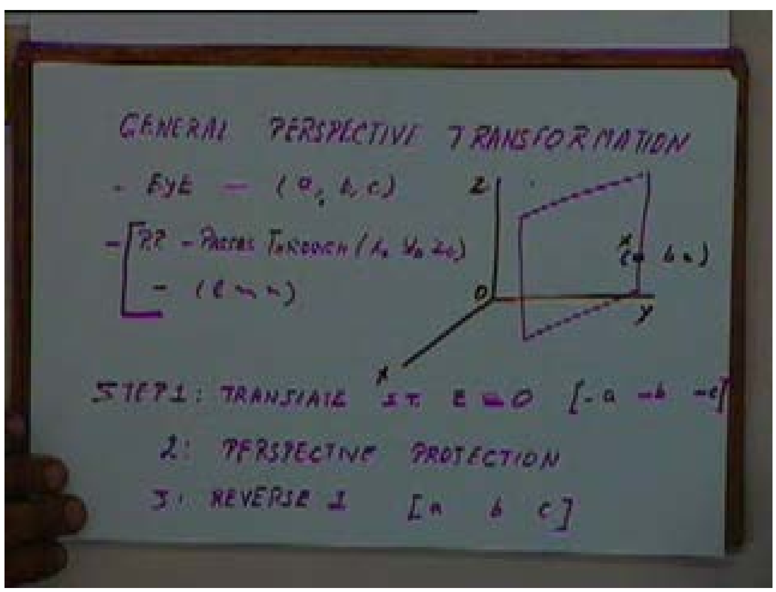

In this case we say that the eye is at any arbitrary point a b c. If I draw my, this is my origin so my eye is located at any arbitrary location which is given by the point abc and projecting plane a projecting plane, this passes through some arbitrary point x_0 y_0 z_0 and has unit normal vector of lmn that means a projecting plane is any arbitrary plane like this. In such a case if we want to take any point and transform it out and get the perspective projection of that, how do we do that? In the last case that we had, the last case that we had, the projecting plane will be same as this but the eye instead of being at abc was at the origin.

Now what I am saying is that the eye is also at any arbitrary location, the plane is also any arbitrary plane. How do we get the perspective projection? Now in this case we will use a standard projections. The first thing that we will do, the step one will translate such that eye becomes at origin and that is translation by minus a minus b and minus c. The step two would be then general perspective projection that we have seen and this projection is given by this vector. And then in step three we will just do the reverse of one which is translated by abc and then the combination of these three will give us the general perspective transformation. Any question up to this point? Just one more thing in perspective projections and that is the concept of what we call as vanishing points.

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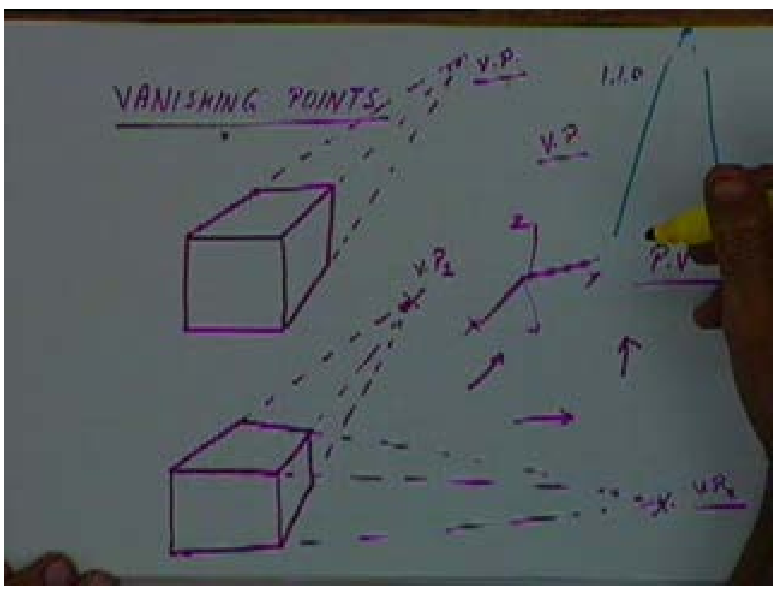

You know what is the vanishing point in a perspective projection. Anyone? One behind the other. No. Do you know what are vanishing points? I think that, what I will do, I will define the vanishing points and how to get the vanishing point that we will briefly see in the next class. See vanishing points, if you have a set of parallel lines that means let's say if I want to draw a cube, now this, in this case I am taking the projecting plane to be parallel to the front face of the cube. So if you remember in one of the earlier classes, I had given a example of the railway track. If you are standing in the middle of the railway track, you look at the distance you will get a view something like this. These two lines will seem to come from this point which is actually a point at infinity but when you project it onto the vertical plane, this will be a finite point.

Similarly in a cube the two edges which are at the back, they will seem to come from a finite point. So let's say if this is my axis, a projecting plane is right now the yz axis and all lines parallel to the x axis will seem to come from this point. So the view of the cube would look something like this, excuse me because the lines which are parallel to the x axis would seem to emerge from this point which is a vanishing point. So vanishing point is a point where all lines, all parallel or set of parallel lines will seem to come from. In this case since my projecting plane is parallel to the yz plane, lines in the y direction will continue to remain parallel, lines in the z direction will also continue to remain parallel.

Two parallel lines in the z direction and the y direction or in the z direction, it will always be parallel but two parallel lines in the x direction will seem to come from a point which should be a finite distance away. This is one vanishing point. Similarly if a plane is at a slight angle then possibly you might have a situation like this. In this case all my lines parallel to the y direction seem to come from this point, all the lines parallel to the x direction seem to come from this point and all the lines parallel to the z direction continue to be parallel. So here we have got two vanishing points, this is vanishing point one and this is my vanishing point two. This vanishing point corresponds to lines parallel in the x direction, this one corresponds to lines parallel to the y direction and for lines parallel to the z direction they will continue to be parallel. So this is the case where we have one vanishing point, here we have got two vanishing points.

Similarly we can also have three vanishing points that means lines parallel to the z direction; they will also seem to emerge from a finite point. Now these vanishing points corresponds to or correspond to lines parallel either to the x direction or to the y direction or to the z direction, so these are called principal vanishing points. If you have a set of lines which are let's say in the direction given by the vector 1 1 0 which is a vector like this, all lines parallel to this axis will also seem to come from some other point. That will also be a vanishing point but that will not be a principal vanishing point. A principal vanishing point is a vanishing point corresponding to the three principal axis. A vanishing point is defined as any point or a point from where a set of parallel lines would seem to emerge in a perspective transformation. The set of lines may or may not be parallel to the principal axis but in the principal vanishing points, the set of lines have to be parallel to one of the principal axis, so this is how we define vanishing points. For the general perspective transformation, how do we find out vanishing points? We will briefly see that method next time and then we will go onto the next topic that is hidden surface removal. That's all for today.