

Tribology
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Module No. # 06

Lecture No. # 42

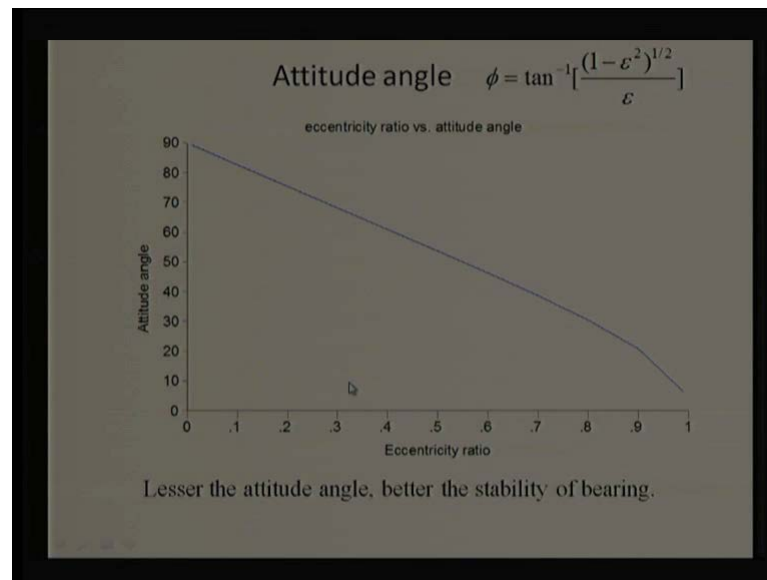
Design of Hydrodynamic Journal Bearings

Welcome to the last lecture of video course on Tribology. So, final lecture, we are going to complete bearing design or hydrodynamic bearing design based on short bearing approximation assuming bearing length is a much shorter than length diameter of bearing much short term is may be say 0.2 5 times of the diameter.

And most often in the present situation, we prefer short bearings, even then previous lecture it was pointed out increasing length is going to increase load carrying capacity and sensitivity of the length is much larger or so stronger; but increasing length has a some problem like a misalignment more heat decipitation or more heat generation increase on a temperature wise increase in a coefficient of friction.

So, we need to account this, when we are talking about the design is not only the load carrying capacity, we need to think from temperature point of view, coefficient of friction point of view. And again we are always aiming for the **miniaturization** shorter is the better that means, smaller size is the better, is from that angle length need to be reduced.

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So we are continuing with first slide from the previous lecture you say attitude angle which was shown or which was mentioned in previous lecture can be given as a 10 inverse of 1 minus epsilon square **square** root of that divided by epsilon. By using this relation we can find out attitude angle. And once we know the extensity and attitude angle we say that, shaft center is fixed is been located and it shows that, when you try to plot this e verses attitude angle or we say that, epsilon verses attitude angle what we are going to get, we can change extensity from 0.0 1 to say 0.9 9 and try to find out what will be attitude angle this is turning out to be one extensity ratio is very low attitude angle is very large, it is almost 90 degree.

When extensity is very high, attitude angle is much smaller in real, in practical situation we do not want much smaller attitude angle, that is why from designer point of view (O) keep extensity ratio up to 0.8 8 or say 0.80 more than that will not be recommended because, we know there will be some sought of fluctuation in a load, that may increase extensity ratio. So, from design point of view I prefer 0.8 and minimum value as a 0.5 or 0.4 that means, this should be the operating zone for the bearing under operation. Reason being lesser the attitude angle more will be in a stability of operation, that is why whenever there is attitude angle as much larger extensity is much smaller, we change the bearing, we reduce the bearing length, we make a necessary grooves in the bearing, so that extensity ratio is greater than 0.5.

And bearing is going to be little more stable and this is often the case with bearing operating at the high speed applications, because we know that, as speed is increasing load carrying capacity of the bearing is going to increase; and if the load carrying capacity of bearing is going to increase, because of the increase in the speed extensity will decrease, extensity will decrease for the same equilibrium load and that decrease is in a extensity is going to increase attitude angle which is unfavorable, that is why many times for high speed operation, we do not go higher with a cylindrical bearings, we will go higher with a half set bearings, we go higher with elliptical bearings, four load bearings, we try to disturb the bearing clearance, we try to reduce the bearing clearance and try to increase stability, increase extensity **in those ratio** in those zones.

But that is a totally a separate subject for us, we are going to discuss about the bearing design based on the short bearing approximation and this is a gives an indication try to keep extensity ratio, we say 0.4 more than 0.4, but lesser than 0.8. If any time bearing extensity is increasing beyond the 0.8 try to modify the design change the parameter, if it is a turning out to be lesser than 0.4 try to change the parameter, increase this extensity, this should be more operating zone for the bearing.

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Friction force

- Petroff equation --- inaccurate

$$\tau = \eta \frac{U}{h} + \frac{dp}{dx} \frac{h}{2}$$

$$F = \int_{-L/2}^{L/2} \int_0^{2\pi} \left(\eta \frac{U}{h} + \frac{dp}{dx} \frac{h}{2} \right) R d\theta dz$$

$$F = \frac{2\eta ULR\pi}{c\sqrt{1-\epsilon^2}} + \frac{c\epsilon W_\phi}{2R} \quad W_\phi = W \sin \phi = \frac{U\eta L^3 \pi}{4c^2} \frac{\epsilon}{(1-\epsilon^2)^{3/2}}$$

If $\epsilon \rightarrow 0$, $F \rightarrow$ Petroff solution

$$F = \frac{2\pi\eta ULR}{c\sqrt{1-\epsilon^2}} \left(1 + \frac{\epsilon^2 L^2}{16R^2} \right) \quad F = \frac{2\pi\eta ULR}{c\sqrt{1-\epsilon^2}}$$

Now, we discussed about the load carrying capacity, we can use a short bearing approximation to the find out that and the addition mention the bearing length should not be extended, because its that is going to increase the friction force.

Question comes how? That is a what we are trying to find out how to drive the friction force we know, we have discussed Petroff equation, but for the bearing we are saying that is inaccurate equation I will demonstrate that, using the after deriving this friction force formula. Friction force will be depending on whether there is a metal to metal contact and we know for hydro dynamic bearing there will not be any metal contact that means there is a only sharing of liquid lubricant, that sharing can be sharing the stress can be given by using this relation is eta higher the viscosity higher will be sharing, higher the velocity higher will be the sharing, lesser the film thickness higher will be the sharing, then comes the pressure gradient and a h by 2.

To find out friction force, naturally we need to integrate this sharing is fix over its area and that area is $R d\theta$ because (θ) coordinate into design and extra length and we have only to demonstrated, that bearing effective bearing they say only half of the bearing, that is why the integration for the θ will be θ equal to 0 to θ is equal to π . We again mentioned about the z that at the mid plain we are assuming z is equal to 0 that means extreme will be minus L but, $2L$ plus L by 2, L is a bearing length.

Once we integrate this what we are going to get this expression it is interesting to note that, F friction force itself depends on the load, this is a W sign 5, whatever the load applied load that is going to affect the friction force, may be to lesser extent but, it is going to affect friction force.

And we try to see what is this friction force or relation that is a W sign 5, that can be given in terms of other parameters like eta, U , L , cube, c , square and we are able to see they are smaller terms are common in this two, so to simplify it what we can do we can take some common terms and compare this term, first term with a second term in the second term is negligible for our simple calculation we can neglect this term, but if it is not negligible, then we need to account slightly more complex problem. And we talk about the Petroff equation, the Petroff equation can be derived from this relation itself, if I use epsilon is equal to 0 we are going to get the Petroff equation same which we have derived in our lecture or earlier lectures.

Now, gives when am using the epsilon equal to 0, that is giving the Petroff equation but, is the situation there is no extensity or we say that the shaft in that bearing surface is a center are conceding there is no load carrying capacity of that bearing, how friction force

will be generated? Friction force cannot be generated that means Petroff equation which is predicting the friction force, because of that its inaccurate no force, no normal force, but still there is a friction force that is **a that is a that is** not a good option that is why, we say that Petroff equation cannot be used for the bearing design. But with the modification when we say using an integration of shear stress can give provide a good results, off course this is based on the short bearing approximation, so it cannot be 100 percent reliable but, it gives a reliable results.

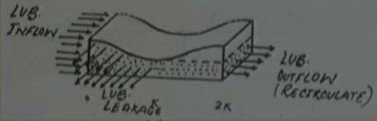
Now, as I mentioned there number of terms common in this two relations, this expression we can take common term this common term turn out to be $2\pi\eta U L R$ divided by c and in bracket also in this square root $1 - \epsilon^2$. So this will be $1 + \epsilon^2$ into $L e^2$ divided by $16 L R^2$. So, if I assume $4 R^2$ is equal to d^2 that means L by d^2 term is somewhere here and we are talking about the short bearing approximation so length maybe equal to 0.25 times of the diameter, so what will turn out to be this L by d^2 will turn out to be 0.25 square.

We know extensity, may be 0.8 maximum value we are choosing, so 0.8 square will turn out to be 0.64, so 0.64 into 0.25 into 0.25 divided by 4 that means 0.25 into 0.25 into 0.25 into 0.64 will turn out to be much lesser than 1, it can be neglected for our simple calculations for detail analysis this can be incorporated for simple class room calculation it can be neglected. That is why in our calculation we are going to treat this F or we are going to write F or estimate F as $2\pi\eta U L e R^2$ divided by c in square root of $1 - \epsilon^2$, so this is a simple friction expression we can be utilized **yeah** we can utilize this.

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Oil flow

- Flow due to velocity & feed pressure
- Circumferential flow $Q_\theta = \left[\frac{Uh}{2} - \frac{h^3}{12\eta r} \frac{\partial P}{\partial \theta} \right]$



$$Q_{\text{leakage}} = Q_H = \int_{-L/2}^{L/2} (Q_{\theta=0} - Q_{\theta=\pi}) dz$$

$$Q_H = \frac{U}{2} c(1 + \epsilon - 1 + \epsilon)L$$

$$Q_H = U c \epsilon L$$

Mass flow rate, $m = \rho U c \epsilon L$

Now, even though we were discussing of the friction force from temperature point of view, we want to estimate temperature so that, we can modify viscosity we can properly estimate the load carrying capacity of bearing as viscosity is going to affect load and temperature is going to affect viscosity and friction is going to affect temperature.

And this is that common change, so naturally we required a good approximation do a complete iteration procedure to find out what will be the final result, perfect combination of temperature viscosity, friction force and the load that is why we require a flow rate also. If there is a possibility the temperature or heat generation is there and it is getting convected and conducted simpler one we say assume the conduction is 0, that is going to give slightly conservative design to us but, that is fine for us.

And to proceed in that direction we require what is a flow rate, we discussed earlier if there is a hole arrangement or slot arrangement we can estimate what will be the flow rate, that is a Q P in addition to that flow generally occurs because of the velocity, that velocity the circumferential flow can be given by this relation that is a Q theta flow rate passing from any location, theta location can be given in this term again when this situation also this term is a almost negligible compared to this dominating feature or dominating term.

So, for our calculation we are going to neglect this second term we are going to account only for first term and this overall leakage we say that, when we are finding the flow is

coming in may be θ is equal to 0 and $\theta = \pi$ film thickness is minimum the most of the liquid is leaked out not most of **most of** the whatever the liquid is a some portion of that liquid is leaked out and remaining portion is getting circulated.

So, we require this leakage rate, that is going to give us cooling effect because this liquid will be again cool and come back or whether we can be return back using the pump arrangement and that Q leakage as it is happening, because of the hydro dynamic action can be given in an other form also we say that instead of writing Q leakage I can write Q_H due to hydro dynamic action and what we say that here the feed pressure flow due to the feed pressure will be Q_P .

Overall may be a combination of Q_H plus Q_P there will be some sought of a disturbance when the pressure comes there will be slightly decrease in a Q_H or we say overall flow rate for time being we are neglecting that we are saying the Q_H can be calculated by integrating both the length for two situation what is the exit condition that Q θ is equal to π and what is the entrance condition that is a θ is equal to 0, so θ is equal to 0 θ is equal to π that difference is going to give us what will be the leakage from the bearing Q_H coming out of the bearing which is going to give cooling effect.

And in fact, it has been observed 80 to 90 percent heat is been carried away by liquid lubricant which are going to get leaked from the surface, so this is going to give us a reliable results reliable in the sense 80 to 90 percent results when we integrate it we know the U is not depending on z h is not depending on z , so it will be turn out moved out and the dz will turn out to be complete length that means $L/2$ minus **minus** $L/2$ that minus **minus** will turn out to positive, that is a $L/2$ plus $L/2$ is equal to L that comes out here this will be maximum film thickness or we say that, Q as another that will be h as a at θ is equal to 0.

So this will be c into $1 + \epsilon$ this point h will be minimum so that will be c $1 - \epsilon$ and when we arrange it is turn out to be Q_H is equal to velocity, clearance extensity ratio into length it depends on all and this is a volume flow rate we can find out there relation here this is the meter per second meter **meter** so that will turn out to be meter cube per second.

But if you are interested in mass flow rate, what we are going to do we are going to multiply with the density, that is generally k g divided by meter cube and meter meter cube will be cancelled out they will turn out to be k g per unit second, the mass flow rate this is a important for calculating the temperature raise.

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Temperature Rise

- Friction, due to shear of lubricant film, generates heat ($F \times U$) in lubricant oil and increases the temperature of lubricant.
- Assuming that total generated heat is carried by the oil flowing through bearing

$$F = \frac{2\eta ULR \pi}{c\sqrt{1-\epsilon^2}}$$

Rate of Heat generated = Rate of Heat convected by oil flow

$$\frac{2\eta (2\pi RN)LR}{c\sqrt{1-\epsilon^2}} (2\pi RN) = m C_p \Delta t$$

$$m = \rho U c \epsilon L$$

or $\Delta t = \frac{\eta \pi^2 (2R)^3 N^2 L}{c\sqrt{1-\epsilon^2} m C_p}$ $\Rightarrow \Delta t = \frac{4\eta \pi N}{\epsilon\sqrt{1-\epsilon^2} \rho C_p} \left(\frac{R}{c}\right)^2$

So, finding the temperature raise we know we need to find out what will be the heat generation, that the friction force into velocity is going to give us what will be the generated heat and that is going to be carried away by liquid lubricant. Assumption we are saying that there is a no much conduction and most of the heat is been affected from using the liquid lubricants. So, we required a thermal equilibrium we say rate of heat generated is equal to rate of heat converted by oil, heat rate of heat generation can be given it as a F into U, F is given like this 2 eta U L R into pie divided by c square root of minus epsilon square that is been given over here and there is a velocity that is can be given as a 2 pie R into N or say that pie d N is equal to as a m C p as a mass flow rate specific heat into temperature raise for heat connection and that is going to give us the total temperature raise is equal to this relation.

Now, this temperature raise is going to depend on viscosity and viscosity is going to depend on the temperature, that is why we are keeping close loop of temperature and viscosity, then it depends on radius very sensitive temperature raise is very sensitive towards the temperature radius larger the radius more and more temperature raise and it

depends on the clearance, larger the clearance lesser will be the temperature raise, this is a overall relation for temperature raise we can calculate using this relations.

And off course, we derived m as a mass flow rate, in previous slide that can be incorporated over here mass flow rate which was derived in previous slide it was given as row U c that clearance epsilon into length, substitute rearrange after substitution we are rearranging this what we are going to get this is a ratio R by c square, that means and this is a related to the liquid specific heat and density this is a speed and viscosity.

So, larger viscosity larger temperature raise, larger in this ratio R by c larger will be the temperature raise, larger density which is generally not happen by large this row and C p will remain constant it is not going to much affect this whatever the temperature is that if I change the liquid lubricant it is not going to change, because of this product, but because of viscosity it will change.

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Design of Hydrodynamic Journal Bearing

<ol style="list-style-type: none"> 1. Guess eccentricity ratio 2. Calculate load capacity, friction force, temperature rise. 3. Modify lubricant viscosity. 4. Repeat steps 1-3 so that average viscosity and load converge. 	$W = \frac{U\eta l^3 \pi}{4c^2} \frac{\epsilon}{(1-\epsilon^2)^2} \left\{ \left(\frac{16}{\pi^2} - 1 \right) \epsilon^2 + 1 \right\}^{1/2}$ $F = \frac{2\eta U l R \pi}{c \sqrt{1-\epsilon^2}}$ $\Delta t = \frac{4\eta \pi N}{\epsilon \sqrt{1-\epsilon^2} \rho C_p} \left(\frac{R}{C_r} \right)^2$ $\eta = \eta_{in} e^{-\beta \Delta t}$
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Now, we can think what we have done we have derived the relation for load relation for flow, relation for friction, relation for temperature and flow incorporate the relation of sorry temperature involves the relation of the flow that is why, we say how to do design first guess extensity ratio extensity ratio been guess may be 0.5 will be an initial approximation.

We know minimum value 0, maximum value 1 I will take 50 percent of that, 50 percent is a epsilon ratio, epsilon is equal to 0.5 or extensity ratio as a 0.5 that is a initial guess we need to go ahead with that. Once we know extensity ratio we can find out the load we can find out the friction force flow has been already been incorporated in temperature raise. So we do not have to calculate separate flow rate if there is no flow due to supply pressure or we say supply pressure is 0 in those situation, otherwise when we are supplying with a some feed pressure naturally need flow rate need to be counted and separately accounted for the temperature raise.

So, for present case we are calculating the load for given or estimate extensity ratio, estimate the friction force and temperature raise. Once we know the temperature raise we need to use lubricant viscosity relation or we say temperature viscosity relation, to modify to modify the viscosity. Once we modify again we need to do calculation, we can think about extensity calculation, load calculation, friction calculation, temperature raise, so it will be continuously iterated that is why we say repeat steps 1 to 3, so that average viscosity and load are going to convert after that, even you repeat results are not going to change that is a convergence it may be 3 step may be 5 step may be 7 step may be 10 step depend on 15 steps depends on your initial approximation of this.

But most of the complicated situation we use this procedure to provide initial good approximation, however if I use a finite difference method and start with a some orbiter extensity ratio is going to take long time to convert, but if I use this kind of short bearing approximation, estimate the results and use those results as input to the finite difference method, number of calculation will reduce significantly and overall there will be better we say the overall there will be lot of (()) on that.

So. what we say is we assume some extensity ratio substitute this find out the load carrying capacity, that is one find out the friction force, then find out the temperature raise. Once we know the temperature raise delta t modify the viscosity that is a what we have mentioned there are 3 steps extensity is a guessing, so that is does not require any calculation as such it is a guess than load calculation friction force calculation temperature calculation then finally, viscosity calculation. And this viscosity is going to be input to the load, naturally it requires iterations it requires overall iteration to convert to one final solution and demonstrate to demonstrate this procedure which is been discussed over here, let us take one example.

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Ex: Determine the minimum film thickness, maximum pressure, coefficient of friction for a hydrodynamic journal bearing, which supports a 600 N load at rotational speed of 2000 rpm. The shaft dia is 40 mm. Assume bearing length = 10 mm, oil viscosity at room temp (30°C) = 15 mPa.s, $\beta=0.029$, and radial clearance 20 μm .

Given: $U = 4.19 \text{ m/s}$, $\text{Factor1} = \frac{U \cdot L^3 \cdot \pi \cdot 0.25}{c^2} = 8227 \text{ m}^2/\text{s}$,
 $\text{Factor2} = 2 \cdot U \cdot L \cdot R \cdot \pi / c$,

$$W = \frac{U \eta^2 \pi}{4c^2} \frac{\epsilon}{(1-\epsilon^2)^2} \left\{ \left(\frac{16}{\pi^2} - 1 \right) \epsilon^2 + 1 \right\}^{1/2}$$

STEP 1: Assume $\epsilon = 0.5 \rightarrow W = 118 \text{ N}$
 Assume $\epsilon = 0.75 \rightarrow W = 562 \text{ N}$

STEP 2: Assume $\epsilon = 0.8 \rightarrow W = 900 \text{ N}$, $F = 6,58 \text{ N}$, $\Delta t = 8.65^\circ\text{C}$

$$W = (\text{Factor1}) \eta \frac{\epsilon}{(1-\epsilon^2)^2} \left\{ \left(\frac{16}{\pi^2} - 1 \right) \epsilon^2 + 1 \right\}^{1/2}$$

$$F = \frac{\eta (\text{Factor2})}{\sqrt{1-\epsilon^2}}$$

$$\Delta t = \frac{4\pi \eta N}{\epsilon \sqrt{1-\epsilon^2} \rho C_p} \left(\frac{R}{c} \right)^2$$

$\rho = 860 \text{ kg/m}^3$
 $C_p = 1760 \text{ J/(kg}^\circ\text{C)}$

What this example means see in this case we are trying to find out number of bearing parameters, see its determined estimate or calculate minimum film thickness maximum pressure coefficient of friction we are not done coefficient of friction, but we know if we are able to estimate friction force and we know the normal load take that ratio F by W that is going to give me the coefficient of friction.

And this is been, what we say that it is been required to estimate all this, when bearing is supporting 600 Newton load or the rotational speed of 2000 rpm, rpm rotational speed is defined, applied load is been defined, shaft diameter is a 40 mm there is a similarly, bearing dimension also will turn out to be almost 40 mm. Assuming the bearing length is a 10 mm that the L by D ratio is 0.25 all viscosity is a room temperature is been defined as a 15 mille Pascal second, beta that is a constant for calculating the viscosity at any temperature raise is been given as a 0.029. And radial clearance c is been defined as a 20 micron, it shows clearly the radius of the shaft is 20 mm and clearance will be 0.1 percent of that **that** is a 20 micron so we are following that same scheme radial clearance is a 0.1 percent of radius.

Now, some important calculation we say that U will be utilized again and again, so instead of directly calculating in a formula you separately calculate what is the Q and that is a calculate a pie d n, we try to find out that is a 4 0.19 meter per second or say for 0.2 meter per second, **(O)** factor U i Q pie 0.25 divided by clearance square can be

should be calculated separately, because we are going to do iterations we do not know exactly what will be the extensivity ratio, we are going to assume and calculate W based on that once we calculate W we are going to find out the friction force temperature raise and viscosity based on that again that means there will be iterations.

That is why we use this calculation separately $U L Q \pi 0.25$ divided by c square this is not going to be calculated again and again, we can directly use this factor that 8.227 meter per second directly.

Same way for the friction force we use a separate factor that is $2 \pi R L$ into R into π divided by c square again this will not be calculated again and again, but friction force will be calculated again and again, because we require iterative scheme we require perfect iterations to evaluate what will be the friction force. To demonstrate it as I say the first is approximation for me will be extensivity as a 0.5, maximum value is 1, minimum value is 0 will take $1 + 0$ divided by 2 that it will turn out to be 0.5, I will be using that as first approximation.

So, when epsilon is 0.5 what we are going to get W as a 118 lower, I do not have any other idea we can use some sought of a interpolation, but for that purpose what we required again the mean value for interpolation we require two values at least first load and then subsequent second load, so what do we do we know maximum value of extensivity 1 this epsilon mid value 0.5 and we are not getting the desirable load **load** carrying capacity which is 600 Newton and what we are getting is a 118 Newton what I will do I will again do $0.5 + 1$ divided by 2, take a mean value that is turning out to be 0.75.

So, extensivity 0.75 what we are going to get W as a 562 meter, now I can use some sought of interpolation to evaluate or I can find out again averaging value $0.75 + 1$ divided by 2 I can go ahead with that, but that rough approximation says that, generally we do not recover more than 0.8, so am just taking the value as a 0.8, epsilon value as a 0.8, am want to find out whether the load capacity is a really exceeding 600 Newton and not.

So extensivity 0.8 what we are going to get W as a 900 Newton which is a 50 percent higher than this naturally we will be we can decrease this take intermediate value of these two but, we know viscosity which is been used in this calculation is a 15 mille

Pascal second and that is happening at the room temperature, we are not calculated that operating temperature and viscosity is going to be lesser than this at a operating temperature that is going to reduce a load carrying capacity.

So, for times being we are saying this was assuming extensity is a 0.8 calculate W calculates friction force, find out the temperature raise and that is turning out to be 8. 65 degree centigrade.

Now, what is the next step to find out what will be the viscosity for this. Off course, we have use this relation we say factor one how it is been utilized factor two how it is been utilized we are not doing this calculation again and again we are simply substituting the factor, we know epsilon is going to change viscosity is going to change, that is why except these two we have assuming one constant factor. Similarly, for friction force viscosity is going to change epsilon is going to change so keep apart from these two factor thing all other will be constant so, that is what the factor 2, this is required for simple calculation to reduce efforts. And temperature raise again can be given in this case again we can find out viscosity **sorry** speed is a constant 4, 5 is a constant R by c is constant row C p is constant we can take this as the factor 3.

And here nothing is been mentioned, but we are using the word density of the liquid as a 860 kilo gram meter Q and c P as a 1 c 1 6 0 joule per unit kilo gram per centigrade, this is been utilized and most often for liquids these are the results for the liquid lubricant these are the relations or we say that these are the parameters for liquid **lubricant**. So, we can substitute we can find out another factor 3 in this case which will be 4 pie into n into R by c square divided by row c P, that will cannot be remain constant and after that we can **(())** we can keep changing eta and epsilon to find out final temperature raise.

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STEP 3: Modify viscosity using

$$\eta = \eta_m e^{-\beta \Delta t}$$

at $\Delta t = 8.65^\circ \text{C}$ $\eta = 0.0117$

STEP 4: For $\epsilon = 0.8 \rightarrow W = 703 \text{ N}$, $F = 5.13 \text{ N}$, $\Delta t = 6.75^\circ \text{C}$, $\eta = 0.0123$.

Now it is preferable to decrease ϵ . Let us assume $\epsilon = 0.78$ and $\Delta t = 7^\circ \text{C} \rightarrow W = 599.1 \text{ N}$, $F = 5.13 \text{ N}$, $\Delta t = 6.92^\circ \text{C}$, $\eta = 0.0122$.

Answer $\epsilon = 0.78$ $\phi = 32.2^\circ$

$$\tan \phi = \frac{\pi \sqrt{1 - \epsilon^2}}{4 \epsilon}$$

Now, once we know the temperature raise we can use this to find out what will be the viscosity, what will be the operating viscosity, that is viscosity is given as this is defined η in is already known, β is defined, Δt we calculated that is going to give viscosity as 11.7 mille Pascal second, in our calculation earlier we assume this is a 15 mille Pascal second, so almost 20 to 30 percent change in a viscosity.

Now, we need to substitute this value this viscosity in the load say for ϵ is equal to 0.8 with a modified viscosity what we are going to get W as a 703 Newton, earlier we got 900 applied load is a 630 by 50 percent variation, but after incorporating this viscosity change what we are getting W as a 703 Newton, which is still higher than applied load, naturally we need to change extensity we need to decrease extensity ratio for here for this kind of extensity and viscosity what we are getting F as a 5.13 Newton calculate temperature raise. So it has reduced from 8.65 percent to 6.75 percent an **sorry** there are not percent is a 68.65 degree centigrade to 6.75 degree centigrade it is reduced.

Now, at this temperature we will again try to find out what will be the operating viscosity and that is slightly more, that is turning out to be 12.3 mille Pascal second, we did calculation for 11.71 Pascal second naturally this viscosity W will be on a higher side. That is why now it is a time has come to take an average 0.75 and 0.8 take an average of that turn out to be 0.75 for time being we are taking extensity ratio as a 0.78.

So 0.78 when you calculate this temperature raise will turn out to be 7 degree centigrade, calculate load at this is turning out to be fortunately it is turning out to be 599.1 Newton very close, we know we have done approximation if you want we can keep slightly more load we can increase this 0.78 from 0.78 to 0.79 and keep a higher load or depends what we want. In this case, friction force is almost a same temperature raise is also almost coming to the 0.7 degree centigrade, viscosity operating viscosity is turning out to be 12.2 mille Pascal second this is a fine design for me.

Or we say if you want to go for the finite difference method this design is going to give reliable results or we assume that load carrying capacity estimated by short bearing will be slightly higher, then we can think about 0.78 also 0.79 also which will give roughly may be say around 640 Newton load capacity applied load is on a 600 we say that is fine for us we can go ahead, so this is a word we say the how we do calculation.

Next comes how to find out attitude angle we are following the short bearing approximation we have calculated epsilon as a 0.78 substitute this value find out attitude angle and attitude angle is turning out to be 302.2 degree that is fine say extensity 0.78 attitude angle 32.2 degree. So this is all about, but problem what we have asked in question find out minimum film thickness? Find out maximum pressure? Find the coefficient of friction? And we are not discussed those things, till now this is a first step locate the shaft, find out extensity and attitude angle and after that do remaining calculations. So, this is we have completed we have converged to the final results extensity 0.78 attitude angle 32.2 degree now time has come to find out the minimum film thickness.

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$$\begin{aligned} \text{Minimum film thickness, } h_{\min} &= c(1-\varepsilon) \\ h_{\min} = 20(1-0.78) &\Rightarrow h_{\min} = 4.4 \text{ micron} \end{aligned} \quad \Lambda = \frac{h_{\min}}{\sqrt{R_{q1}^2 + R_{q2}^2}}$$

Maximum pressure will occur at $z = 0$

$$\begin{aligned} p &= \frac{3U\eta}{c^2(1+\varepsilon\cos\theta)^3} \left(-\frac{\varepsilon\sin\theta}{R} \right) \left(z^2 - \frac{L^2}{4} \right) \\ \Rightarrow p &= \frac{3(\omega R)(\eta_{\text{eff}} e^{-\beta\Delta x})}{c^2(1+\varepsilon\cos\theta)^3} \left(-\frac{\varepsilon\sin\theta}{R} \right) \left(0 - \frac{L^2}{4} \right) \\ \Rightarrow p_{\max} &= \frac{0.75 \omega (\eta_{\text{eff}} e^{-\beta\Delta x}) L^2}{c^2(1+\varepsilon\cos\theta_{\max})^3} (\varepsilon\sin\theta_{\max}) \end{aligned}$$

$$\theta_{\max} = \cos^{-1} \left(\frac{1 - \sqrt{1 + 24\varepsilon^2}}{4\varepsilon} \right) \rightarrow \theta_{\max} = 105.43^\circ$$

$$p_{\max} = 7.24 \text{ bar} \quad \mu = \frac{F}{W} = \frac{5.13}{600} \Rightarrow \mu = 0.0086$$

We know clearance film thickness can be determined based on epsilon as a c **minus** in bracket 1 minus epsilon, substitute this value 20 micron is a radial clearance 0.78 is a epsilon value is turning out to be 4.4 microns. Question comes whether we have done everything right? We are talking about hydro dynamic lubrication and we know hydro dynamic lubrication we need to will be valid for the specific film thickness by enlarge more than 5. If surface softness of shaft and bearing surfaces are given to us or provided to us, we can find out this is specific film thickness using this relation, that will be ratio of minimum film thickness to the composite self surface softness.

Now, if I assume roughness of the shaft is a 0.4 micron may be on higher side and bearing may be even 0.6 microns, take a composite one and find out what will be the composite surface softness if this ratio is turning out to be more than five is say hydro dynamic lubrication is fine bearing design is as per the hydro dynamic lubrication there is no problem.

If that is not the case then we can think about adding hydro static lubrication, we can supply pressure that is going to reduce temperature and that is going to reduce film thickness or **sorry** reduce extensity when epsilon is decreased naturally h main is going to increase. So, that way feed pressure is going to increase film thickness is going to bring **is going to bring** you say mixed lubrication domain to hydro dynamic domain so we can do calculation when we do calculation we find out whether everything is fine and

not if it is not then we should use some sought of (θ) pressure to supply liquid to cooler liquid to cool the liquid lubricant can reduce the temperature make into a factor viscosity more than what is been estimated and (θ) .

So this is a film thickness then next comes a maximum pressure, how to find our maximum pressure this is a short bearing approximation, we know maximum pressure will occur when z is equal to 0 that is condition which we are used to derive this relation the mid plain dP by dz will be equal to 0.

Now, we have used z equal to 0, but we need to find out at which angular position maximum pressure is going to be there or maximum pressure will be generated and pressure profile what will be location of maximum pressure, that mean θ_0 max θ_0 , because of short bearing or we say θ_0 as a we are using a short bearing approximation. So start with the pressure relation first you say that this is a pressure relation we say that maximum pressure will occur at z equal to 0, substitute this, rearrange this and after that in h differentiate with the respective θ . After differentiating with respective θ equate to 0 that is going to give us that is going to give us θ for maximum pressure once we know substitute those value over here and find out maximum pressure.

In our case, this been done and we find this location or maximum pressure θ_0 max depends only on extensity ratio that is a what is shown over here, it is a $1 - \sqrt{1 + 24\epsilon^2}$ divided by 4π . Now we know this will be always greater than 1, that means θ is going to be greater than 90 degree this is going to be negative. Now ϵ can be 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 whatever you take this will be negative, if this is a negative our value will turn out to be negative sorry this will be more than 1 and if it is more than 1 minus more than 1 naturally will be negative that is why the θ will be always greater than 90 degree.

In present case, this is turning out to be 105.43 degree and this is a different than θ is equal to π , this is difference that θ is equal to π that means location of minimum film thickness is different than location of maximum pressure. Otherwise in many books there is been confuse have been treated the maximum pressure location and minimum film thickness location are the same it is not, there locations are different.

Now, based on then we substitute this value theta o max in this expression what we are going to get maximum pressure as 7.24 bar not very high value is well within permissible limit mega Pascal it is only 0.724 mega Pascal its not very big, this is a big quantity or larger any material can be used any (()) material can be used in this situation.

Now finally, comes the coefficient of friction we needed to find out coefficient of friction, we know what is the friction force, we know what is the applied normal load, take a ratio I have divided the W that is what is going to get, we are going to get that is a 0.0086, this is very low coefficient of friction 0.0086 almost negligible pressure that is why the hydro dynamic bearings are most popular, whenever we require good (()) good damping local self friction 0 wear hydro dynamic bearings are on the top place.

Now, what we can do to find out if length is doubled what is going to happen with this parameters, just now of a present case we took length is a 0.25 times of the diameter, but if we are going to increase this length, we say doubled the length instead of 10 mm we can think about 20 mm what is going to happen.

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Ex: Same data which were defined in previous example, but bearing length = 20 mm.

STEP 1: Assume $\epsilon = 0.5 \rightarrow W = 943 \text{ N}$
 Assume $\epsilon = 0.45 \rightarrow W = 741 \text{ N}$

STEP 2: For $\epsilon = 0.45 \rightarrow F = 8.84 \text{ N}, \Delta t = 10.33^\circ\text{C}$

STEP 3: Modify viscosity using

$$\eta = \eta_{in} e^{-\beta \Delta t}$$

at $\Delta t = 10.33^\circ\text{C} \quad \eta = 0.0111$

STEP 4: For $\epsilon = 0.45 \rightarrow W = 548 \text{ N}, F = 6.6 \text{ N}, \Delta t = 8.0^\circ\text{C}, \eta = 0.0119$.
 $W = 588 \text{ N}, F = 7 \text{ N}, \Delta t = 8.2^\circ\text{C}$.

That is shown over here, the same data which we defined the same viscosity, same load, same speed, same everything right. Now only the variation is bearing length instead of 10 mm we are considering as a 20 mm, if we do that, again we need to assume as a first step what is the extensity ratio we need to guess as I mentioned earlier for me simple

case is summation of 1 and 0 divided by 2 that is an average value at the first step and that is epsilon giving us as a 0.5 this is turning out to be 943 Newton.

Now, I can take a again average 0.5 plus 0 divided by 2 that is a epsilon as a 0.25, but we feel epsilon lesser than 0.4 should not be recommended for in this case, we have recommended a 0.45 minimum value, it should not be lesser than that. However, we need to change the design. So epsilon if I assume the 0.45, we can find out the load as a 700 40 Newton, what is a more than 600 we can reduce it further, but we what we want to do that calculation we know this 741 is at that 15 mille Pascal second viscosity and if we account the temperature raise this is will be **this will be** lesser than that any b if viscosity will be 12 mille Pascal second or 30 mille Pascal second so this will be reduced.

So, for time being we can assume the extensity of initial approximation as a 0.45 calculate the friction force and friction force calculation gives the result as a 8.84 Newton, based on that we can find out the temperature raise, that is turning out to be 10.33 degree centigrade. Based on this we will modify the viscosity this is a 15 mille Pascal second temperature raise is 10.33 degree centigrade and beta is given to us in example or we say in a question so based on that, what we can find out the viscosity that is turning out to be 11.1 mille Pascal second, we substitute this value to find out what will be the load carrying capacity. Now, with this viscosity extensity 0.45 load turn out to be 548 Newton which is a lower than applied load, I can change viscosity or I can change extensity immediately here.

Other one is that calculate again the friction force recalculate temperature raise recalculate viscosity and again see whether that is going to increase this load carrying capacity reaching to the final value which we require, if it a case you say that we do the calculation, now find the friction force evaluate temperature raise based on that find out the viscosity that is a instead of 11.1. Now is turning out to be 11.9 mille Pascal second substitute W is turning out to be 588 Newton, again we can do calculation, but we know this extensity is not going to increase to 600.

I can think about this extensity may be say 0.46 and find 4 6 also can give me some result which is slightly more than that, now it is my choice whether I go for slightly higher value or convergent (()) you say the 588 is a value within 5 percent of the load I

can converge it or we can think about slightly more than that so when we are talking about the epsilon as a 0.46 and the load carrying capacity is a 606 Newton, F is a 6.94 Newton.

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STEP 4: For $\epsilon = 0.46 \rightarrow W = 606 \text{ N}, F = 6.94$

Answer $\epsilon = 0.45 \quad \phi = 57.3^\circ$

$$\tan \phi = \frac{\pi \sqrt{1 - \epsilon^2}}{4 \epsilon}$$

$$\theta_{O_{\max}} = \cos^{-1} \left(\frac{1 - \sqrt{1 + 24\epsilon^2}}{4\epsilon} \right) \rightarrow \theta_{O_{\max}} = 110^\circ$$

$P_{\max} = 13 \text{ bar}$

$$\mu = \frac{F}{W} = \frac{7}{600} \Rightarrow \mu = 0.0117$$

$P_{\max} = 7.24 \text{ bar}$

$$\mu = \frac{F}{W} = \frac{5.13}{600} \Rightarrow \mu = 0.0086$$

So, I can find out, I can close this example with extensity as a 0.46 or extensity of 0.45 both the options are open to me this is giving me high load carrying capacity, that is giving slightly lower load carrying capacity. So from conservative point of view I can choose this or for the continuity we say no we do not require too many iterations then iteration is itself is giving a reasonable good results (O) choose those results.

So, depends on the whether we have freezing 0.46 or 0.45 we can find out attitude angle and attitude angle for 0.45 is turning out to be 57.3 degree, which is a higher value we can increase this we can change the bearing length, we can reduce the bearing length, we can change the parameters so that extensity is coming out to be 0.7, 0.6, 0.7, 0.8 that will be the more desirable level, that is a giving indication bearing length increasing from 10 mm to 20 mm is not full filling the purpose extensity ratio is going down its more like we have a capabilities and but, we are not doing our best we are not giving our best performance.

So, bearing have a lot of capabilities but, because of the larger length bearing is not able to give the best, which is not desirable first thing is a increasing a going for a larger bearing length naturally cost of the manufacturing is going to be increased, cost of the

material is going to increase, in addition there will be some sought of **the some sought of** misalignment in addition there will be friction force I use the word there will be additional friction force, but we need to check it we say that for a this if I assume 0.45 as a freezing point calculate what will be the maximum pressure.

For that purpose we require θ_0 max or θ_0 max and that is turning out to be 110 degree 110 degree in this case, that is going to give me value of something like a P max or the 13 bar or 1 point here mega Pascal, if you remember the length was 10 mm this pressure maximum pressure was only 0.7 mega Pascal.

So, what we are doing increasing the bearing length maximum pressure is been increased to 1.3 mega Pascal, one way it is a negative side another way is that bearing material mostly they are able to sustain more than 5 mega Pascal, so why to worry about 1.3 mega Pascal let it be like that, if bearing length is giving all other advantages we should go ahead with a larger length, but temperature from coefficient of friction point of view, say that coefficient of friction in this situation, because friction force is a 7 Newton in this case divided by 600 is going to give us a result as 0.0117 coefficient of friction is a 0.0117 while in earlier example this coefficient of friction was a lesser than this 0.0086.

So, increasing length is not full filling lot of purposes, first extensity is going down lesser than, we say that 0.5 may be recommended fine, but that is increasing maximum pressure larger length is giving lesser pressure compared to **sorry** smaller length is giving larger pressure, smaller pressure compared to larger length which is a quite reverse. While coefficient of friction for the smaller length is lesser than larger length, naturally I will prefer lesser length smaller length for the bearing it rest it is says a space, say the cost gives optimum performance.

And that is what we gain from a studying lubrication mechanism understanding tribology. Otherwise, if somebody says the load carrying capacity is go ahead with that maximum length that is a wrong, here lesser length, smaller length is giving more benefits lesser, maximum pressure, lesser coefficient of friction, lesser space, lesser cost, so that is essential for us or we say that that is going to give us overall economics.

So, with this am trying to close the course I hope you understood the course and in future you will be able to say lot of cost incorporating tribological principles, tribological guidelines **thank you**, thank you for your attention.