Tribology
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Lecture No. # 26
Thermo – Hydrodynamic Lubrication

Welcome to twenty sixth lecture of video course on Tribology. Today's topic is thermo hydrodynamic lubrication. As the name indicates we are going to combine thermal effects with hydrodynamic effects and find out the pressure distribution based on that. In previous we have studied elasto hydrodynamic lubrication. Here, we combine elastic deformation with hydrodynamic lubrication.

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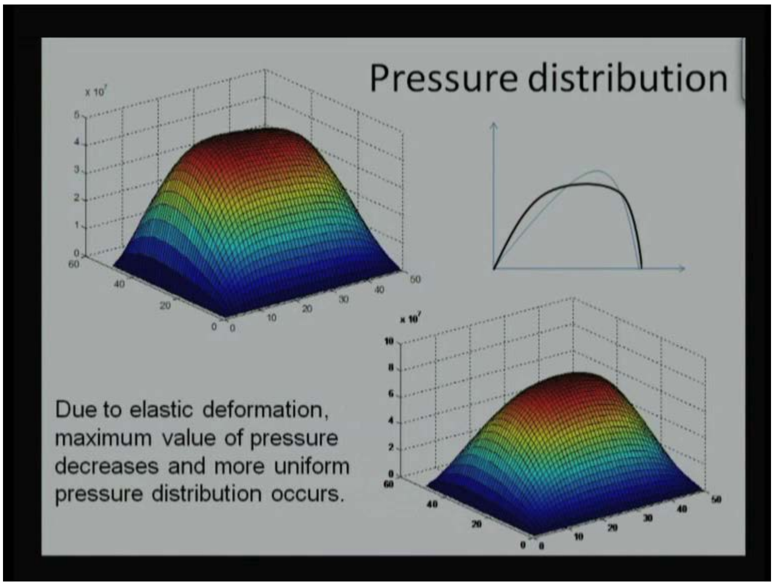

And our conclusions were that if we account plastic deformation then, pressure distribution will be or maximum value of pressure will be lesser. Compared to if we do not account plastic deformation, reason was given as the film thickness will increase with elastic deformation the increase in the film thickness pressure distribution or the maximum value of pressure will come down or contacting elements will be slightly more separated that will reduce the pressure generation.

It was now it was also mentioned that it was not a problem it is not going to reduce the load carrying capacity of the elements. In fact, indirectly it is going to increase the load carrying capacity of tribo elements and reason was given something like this; that if we do not consider elastic deformation pressure profile will turn out to be like this, which is to maximum value and comes down to 0 value. It is morel like parabolic distribution, offset parabolic distribution.

Well, if I consider plastic deformation maximum value of the pressure is reduced but that reduced maximum pressure value will be at more much more angular extent or towards the sliding direction. You can see that the maximum value remains to a greater extent of a space variable compared to this. So, elasto hydraulic lubrication is important. We are able to incorporate in any of the design. Elastic deformation combined with hydraulic lubrication mechanism that will be optimized tribo pair.

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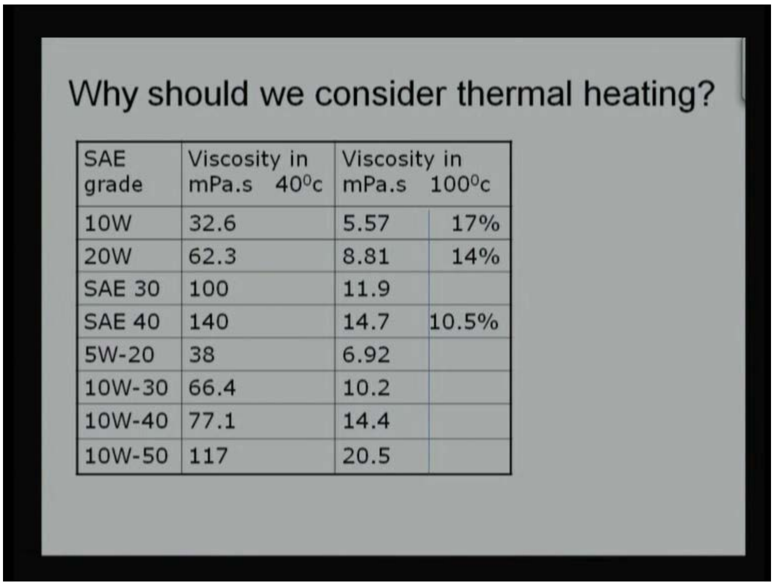

SAE grade	Viscosity in mPa.s 40^{0}c	Viscosity in mPa.s 100^{0}c	
10W	32.6	5.57	17%
20W	62.3	8.81	14%
SAE 30	100	11.9	
SAE 40	140	14.7	10.5%
5W-20	38	6.92	
10W-30	66.4	10.2	
10W-40	77.1	14.4	
10W-50	117	20.5	

Today we are going to discuss about the thermal effect. The question comes what should we consider thermal heating? We have been mentioned where ever there is a full film there will be some sharing of the liquid. But, does it really cost too much problem? Full film lubrication is used to minimize the friction. But, here we are thinking about even if slight friction is going to affect the performance. To compete that or to add that we again refer back to the our table which was mentioned which was discussed in previous model or lubrication. This table showed different oil grades it is a 10 W grade, 20 W grade say

30, say 40 W is strength for the integrate and this is a multigrade 10 W 30 10 W 40 10 W 50. These are the viscosity grades as the name given by SAE that is society of automotive engineers.

What these two columns are indicating the viscosity, absolute viscosity in mile Pascal per second. This first column says the 40 degree centigrade, second column gives the viscosity of 100 degree centigrade. Of course, the third column which is only the partially filled gives some sort of percentage variation or what will be the final viscosity in that case?

So, just to refer first one you say the 32.6 mile pecan, second is the viscosity of 10 W as a grade oil at 40 degree centigrade. This temperature is going to be hundred degree centigrade, viscosity decreases to 5.57 or in other words viscosity remains only the seventeen percent after viscosity which was at the forty degree centigrade. That is a huge change, almost 1 6th reduction.

Similarly, with the oil of the 20 WSA grade, viscosity is high 62.3 while viscosity is hundred degree centigrade 8.81, 14 degree reductions obviously, that 86 percent reduction and the final viscosity is only 14 percent of this viscosity at 40 degree centigrade. Of course, we cannot choose very low viscosity oil because of number of reasons. We say that load carrying capacity if we want more load carrying capacity we need to choose thicker oil. Another word this oil having lesser viscosity can be called the thin oil and if the oil which have a high viscosity can be called as thick oil. Thicker oils are required to increase the load carrying capacity of tribo pair and the load increases as the velocity comes down we should use thicker oil. If velocity, sliding velocity increases and load carrying capacity is lower or requirement is the lower we can choose low viscosity oil. It was conclusion from our previous model lubrication when we were discussing about lubricants. So, different kind of lubricants.

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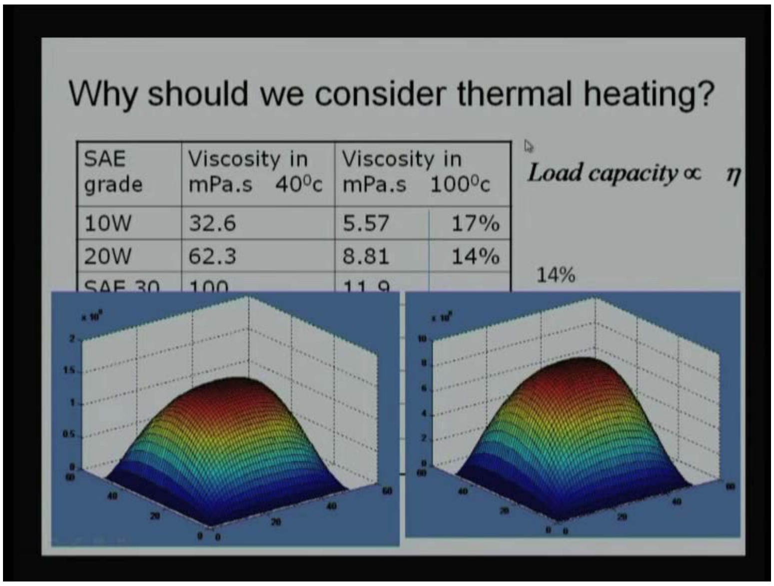

Now, if I do a model and I say that lets take this as viscosity, input viscosity because in Reynolds equation which we derive we assume the viscosity is constant is not depending on a space variable. The constant is oscillating then the temperature is not going to be accounted and whatever the input viscosity will be accounting in that avail or using this viscosity.

Using this viscosity of 32.6 Pascal second what we get the pressure profile accounting the elastic deformation in this case. This comes something of 5.8 or 58 mega Pascal. Now, if I apply account this viscosity this is even the viscosity grade had changed may be in one case tribo pair was operating in fort centigrade in other case its operating at hundred degree centigrade then this viscosity should be accounted that a 5.57 mile Pascal second. We can see as the huge variation or the decrease in load carrying capacity or maximum pressure or pressure variation.

Here it is roughly 5.8 over here it is roughly 0.758. So, 5.8 is coming only of the 0.8 there is huge decrease in this maximum pressure value. That means, if temperature is going to change viscosity as a result will change and if there is a change in viscosity, load carrying capacity is going to be much lower. Based on this comparison we can say that there is this load carrying capacity, this pressure distribution variation it is just only fourteen percent of this pressure variation. That means, eight six percent is the decrease. Now, viscosity decrease we say that viscosity hundred degree hundred degree centigrade is seventeen percent and if we are using this viscosity load carrying capacity may turn out to be only fourteen percent. Still viscosity has side compared to load carrying capacity.

But we can roughly say that the load carrying capacity is proportional to viscosity. If there reduction in viscosity load capacity is going to decrease or there is a more possibility of contact or metal to metal contact. That is a problem. When this kind of the problem comes then we need to account thermal effects. There is a possibility that temperature rise happens at only 2 degree 5 degree 10 degree or if the system design is not good or external temperatures are very high and that there will be going to increase the temperature of the system then viscosity thinning will happen, load carrying capacity will increase. And that may cause the failure of tribo pairs. So, this is the reason why should why we should consider thermal heating. We should account thermal effects either coming from outside external sources, generated within the sources whatever we should account it.

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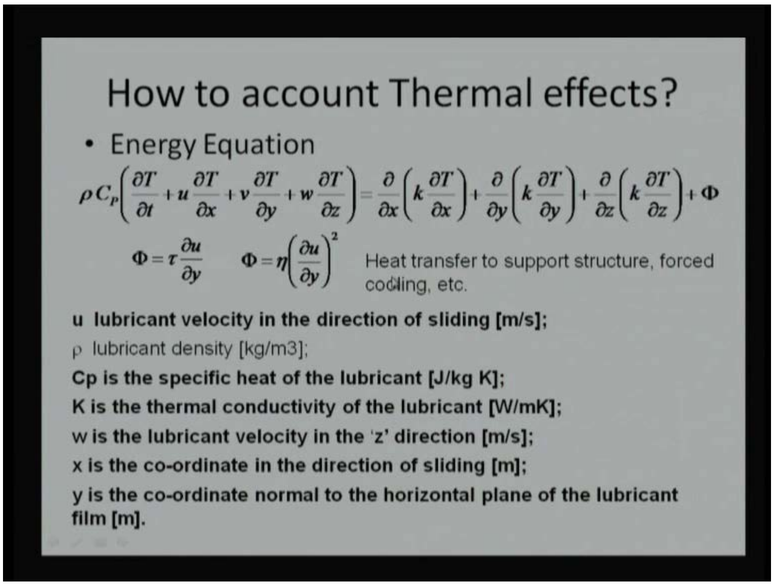

The question comes how to account these thermal effects? Is there any mean? Yeah there is a mean. We will call it the energy equation. That's simple heat transfer equation which we have studied number of times of fluid mechanics. We will say whatever the energy is generated are passed to the system it should be connected away, conducted away or radiated away. So, there should be some thermal equilibrium equation or heat transfer equation and that can be combination of a number of equations also.

That's simplified if I assume that there is no radiation effect what we can say? There is a connective term, conduction term and heat source term. Heat is being added within this term, heat has been conducted away by using this term, heat is convicted away by these terms. Now, this is the temperature variation with the time. Generally we account this when the heat source itself is a fluctuating. Sometime it gives heat of 50 watts to the system, sometimes 75 watts, sometime 25 watts.

So, when the five is going to be changed with the time naturally there will not be a consistent temperature. There will be a continuous variation in temperature and we should account that. However, if system is steady; heat source remain consistent whatever the energy is added it will be added continuously. It is not been removed; or we say it is not been discontinued. Then we can neglect this term.

Then these three terms is velocity in tangential direction or sliding direction or velocity in normal direction between the tribo pairs. right Velocity in perpendicular direction where if I compare what we say related to the sliding direction. This is the heart conducted in x direction, heat conducted in y direction and z direction. We are assuming the bearings or the tribo pairs are on z plane and the gap between those tribal pairs is in y direction. So, let us give some sort of expression for the heat source. So heat source, of course I can say that this whole equation is energy per unit volume and heat source this one is also energy per unit volume. This can be given by the shear stress shown or resistance shown by the liquid or the fluid which we are using into velocity gradient.

Now, this velocity gradient can be in number. Like it can be a in y direction can be in z direction, but, we already have done analysis using hydrodynamic lubrications and we found relevant or more appropriate term is only the velocity in sliding direction, perpendicular sliding direction or there was no velocity as such in W direction. We assume in W will be equal to 0. Now, if I assume the second thing that can be given by (( )) something like this; say eta into velocity gradient into velocity gradient which is already present here. So, this source term will turn out to be eta into velocity gradient square. That clearly indicates if there is a very high velocity missing gap, the term is going to increase in a square form. There will be more effect.

It is not a linear variation. It is parabolic variation or there is a square form coming over here. Now, if we give a nomenclature to these symbols we say that this is the density that means it is going to play row. Lubricant density which is in a kilogram per meter cube, this is specific heat of the lubricant. I am emphasizing it is of the lubricant express and the joule part in kilogram and Kelvin. Now if I multiply density with C p; what I am going to get is the joule kilogram, kilogram will be cancelled, joule per unit meter cube per unit Kelvin and here there is everywhere Kelvin Kelvin Kelvin coming. So, that can be cancelled out and in this situation is per unit time this is per unit time because of the meter per unit second, per meter per second and meter meter will be cancelled off. It will turn out to be per unit second this is also per unit second; obviously, the Kelvin per unit second in all the four terms will be in Kelvin per unit second and this is the joule per unit second turn out to be watt and when we say that overall this expression will turn out to be watt per unit meter cube. That is why I say this whole per expression in terms of watt per unit volume.

We should confuse with the units. Now, interesting thing is that this k. This k is the thermal connectivity of another tribe of pairs. Often it has been mistaken when we are using conduction term, k comes as a thermal conductivity of the material one, thermal conductivity of material two. In this situation we are not accounting the material connectivity we are still in fluid film zoning and that zone only. So, this thermal conductivity of the lubricant will not as high as solids, but, we need to account wherever there is a small gap; this whole conduction will be dominating compared to the longer or in another word we can say if this is the gap direction, y is the gap direction where the film thickness happens in microns while in sliding direction, x happens to me in mm and z also in happens to be in mm. So, the dimension in mm. So, the dimension in mm and dimension in microns. Initially this conduction term will be much more dominating compared to this conduction term compared to this conduction term.

We need to keep this in the mind. Similarly, in this case there is a tangential velocity. So, this term will dominate. There is a possibility of the turbulence. There is a possibility of some velocity along the y direction. It will lower gap, but, that may not be as high as this term. So, many times we can neglect for the simplification and if there is no tension and no velocity in z direction of tribo appear in vertically in that direction is only the unidirectional which we know that sliding happens in U direction only then this term also will be neglected. Right. So, we can simplify this thermal effect; obviously, that this energy equation to account the thermal effect with hydrodynamic lubrication mechanism. Now what is been done over here? We say that we have only one energy equation. We know there will be heat generation and that heat will be conducted to the surfaces or tribo pairs and there the thickness of the tribo pairs and thermal connectivity of the tribo pairs will be accounted.

But in time being we have neglected those aspects because that is going to give a second equation. We already have one equation related to pressure that Reynolds equation. Then we have elastic equation last (( )) lubrication mechanisms. Then we have third equation of energy like this and we had fourth equation that will be a too much or we say that particularly the elementary level courses it will take a much longer time to understand and it can be done only at the research levels and when we have very good computer high computational facilities.

Even those computers overall equations will take two days it will not give results in seconds, not in minutes, not in hours. It is going to take much longer time. That is why we are for time being neglecting it, but, whenever the situation comes you say there is at thermal heating outside the tribo piar and we need to account it. We take an example of ice engine where ice engine putside temperature is very high may be tribo tribo pair desnt come to that temperature then we need to account this temperature or if there is a forced cooling. Here is the water cycle system going around tribo pairs then we should account those things. But, to generalize to include here it will be difficult. Whenever the situation comes where the application comes we will account this.

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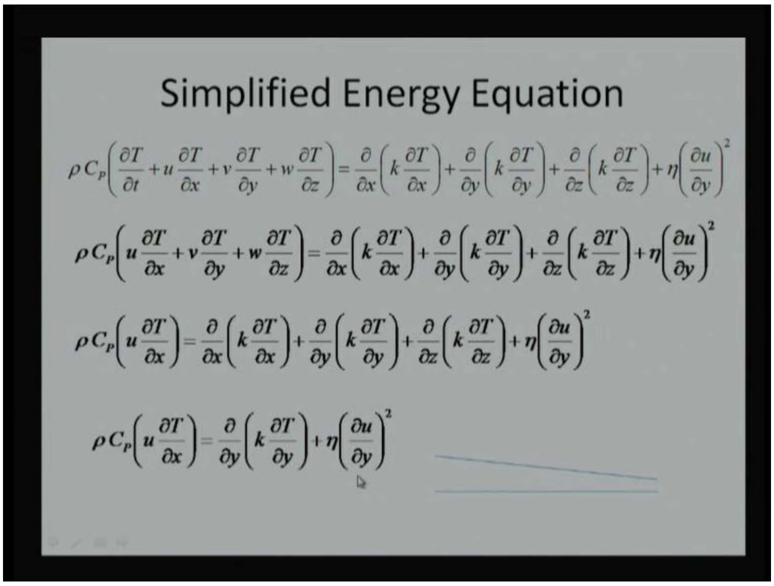

Now the time to simplify energy equation as I say that we are not considering the second energy or second heat transformation we are still on the simplified energy equation and we want to further simplify it. So, we want to neglect this term.

It is the transient solution we do not want to go ahead with that we want to go for a simple solution. Now this term is neglected yeah. So, simplification will be account only the tangential velocity along the x axis then I am going to account any other thing. So, these two terms also will be canceled or will be neglected. Simple? So, from so many terms we have reduced one term here. Now we have further reduced two terms. So, there are three differential terms have been neglected and we know there is a gap or film thickness happens in this gap or we assuming that this surface is sliding, this surface is stationary and this film thickness is in microns while this sliding distance is in annum. So, conduction is predominant only in y direction. So, I am saying thatthe arrow this is the y direction. This is x direction and perpendicular to the monitor, perpendicular to the our configuration that is the z direction.

So, variation or conduction in z direction is negligible. Conduction in sliding direction is negligible compared to conduction in film thickness direction. So, we will consider only this effect. So, this has been simplified earlier or we say that we are suing Newtonian liquid. You are not using the non Newtonian liquid and you know very well the most of the most of the liquids are non Newtonian liquids. If we introduce non Newtonian liquids here that is going to increase the complexity. It is always preferable to think simplification first, get a feel, get a impression how this equations is going to work, get a confidence and then subsequent we can improve or we can incorporate with slightly more complexity in that. So, this simplified energy equation say conduction in sliding direction, conduction along the film thickness direction, heat generation along the film thickness direction.

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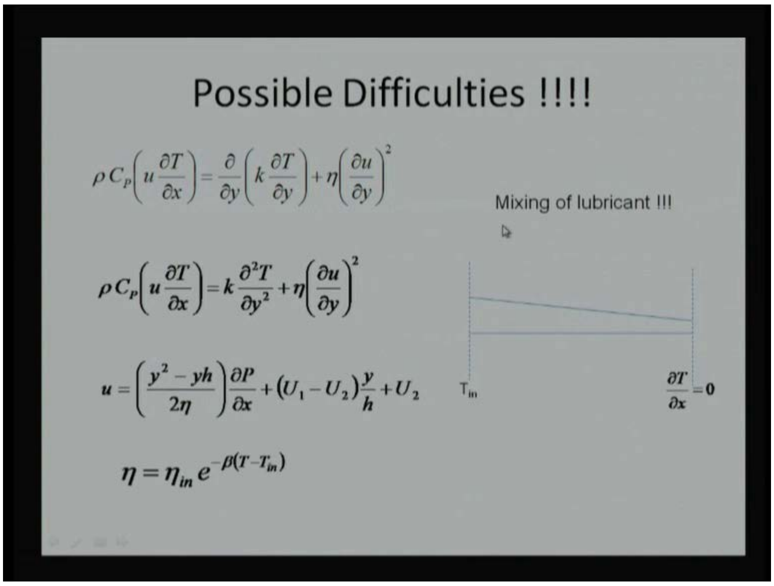

Now, what are the difficulties? I have already simplified so much, but, I am seeing there will be some difficulties I want to elaborate that, I want to discuss that with you. Is it really so simple as planning with lubrication? Is it so simple as elastic lubrication? I have seen a number of articles people feel that elastic deformation or counting elasto hydro dynamic lubrication is more complex compared to incorporating thermal effects.

But imagine if accounting thermal effects is more laborious, it is much more labour plastic deformation, compared to pressure distribution using Reynolds equation and that is why I want to just mention it here. In this case first thing is that how to find out thermal conductivity of the liquid. We know liquid is subjected to the pressure, molecules will be closer, heat conduction should increase and if there is some sort of similar formation there are some air bubbles in the liquid.

And that happened to be in a divergent domain. What will happen? Thermal conductivity of the liquid will come down will reduce. So, thermal conductivity of the liquid will not consistent. And how to account that? It is pretty difficult. Number of models are there, people have to just research have suggested for time being we have to assume that this k is constant. It is not going to vary because in this we are not including the pressure immediately. Of course, it will be done iteratively in a sequential manner, but, is not directly if we say that k is going to depend on the pressure and we are incorporating, it will be very lengthy work or the laborious work. So, for time being we are assuming k is constant. That is the first difficulty, unjustified unjustifiable assumption. Now, what is the second thing? See that how from where you you bring the U, this is variation or the velocity of liquid. It is not velocity of sliding surface. We know velocity of the sliding surface will be constant over here and may be in this case U is 0.

In this case is U is capital U fix is it we can account that way, but, to account the fluid viscosity we need expression. First one thing comes in the mind let we have already done this in dynamic lubrication. So, where is the problem? I can use this equation as it is we have done this, we have derived this equation or when we deriving the Reynolds equation we considered one surface as the velocity U 1 and other surface as velocity U 2 and overall velocity profile can be given like this. That is right, but, the problem is that we assume for the derivation the viscosity is faster. It is not dependent on any space variable. But, here we are not doing that you see the viscosity is going to vary with space variable. It will vary along the x axis, it is going to vary along the y axis also. And of course, if you consider third dimension it is going to vary along the z axis also. So, how we can justify this velocity?

It’s not possible. That means, if we incorporate this solution as it is, we are doing something wrong. We are not moving to the right direction, our assumptions are absolutely wrong. We cannot use this kind of Reynolds, we cannot use this kind of velocity profile in this and there is another difficulty. So, how do we incorporate this viscosity? yYou know viscosity is a strong function for temperature. You can you are able to see eta over here and this is the source term.

We substitute this eta over here. We say that initial viscosity is known to us initial temperature is known to us, this beta which is material property or in this case particularly oil property is known to us, but, still T is here T is going to here. T is in a differential form. So, it is going to give us overall complicated equation. It is the e exponential term here. Integration will not be that simple and that is going to introduce difficulty. So, what are getting difficulties are the energy level equation. We have not considered variation along the z direction. That is the first difficulty because if we incorporate that is going to increase our efforts.

Second we do not know how to incorporate velocity and whole energy equation is going to be dependent on that. The velocity expression is wrong; whole equation will turn out to be wrong. Next one is the our viscosity. How to incorporate our viscosity by using this and integrating I max exponential term when we have our second order equation?

These are the some difficulties. In addition there are difficulties of the boundary condition. See this is the partial differential equation where the variation along the x axis variation on the y axis and we require at least two boundary conditions to solve it.And what is the first boundary condition? We say first boundary condition is easy. We know very well what will be the temperature at the entrance because we are there, when we supply this lubricant. So, we can say that the entrance temperature is T in, this is in Kelvin that is why capital t. But, the problem comes. We have studied hydrodynamic lubrication we say there will be a pressure retardation there will be back flow at the entrance and whatever the liquid over here it has a high temperature.

It is going to get mixed with the entrance liquid. Even though I know what will be that supply temperature, but, not necessary at the entrance of this (( )) this will remain T in it will be higher than T in and which we do not know. We need to know how much lubricant is re circulated, S the back flow of the lubricant and how it is going to get mixed with fresh oil because fresh oil is not hundred percent. It is only the partly fresh oil will come, partly fresh oil sorry back flow will be there. So, they will get mixed and this temperature condition is again is a problematic. We cannot use as it is. We require rigorous analysis to find out what will be the temperature at the entrance and I am sure it will not going to be T in. It is not going to be supply temperature. What will be the temperature of the exit? It is not known at all. It cannot be predicted. (( )) the whole heat generation is going to happen over here. Although some heat generation is happening outside also we need to account that. So, defining exit boundary conditions is also difficult one. What is the easiest one?

Easiest one we will say that there will not be any variation at exit whatever the temperature is going to be there here may be more or lesser temperature will be outside assuming that there is no cooling outside. And may be temperature reaches to high level and after that its coming to the open air, it will not cool down immediately. So, there will not be much variation in temperature whatever the one mode before that same temperature should be there.

Which is again wrong we know that there will be some cooling, but, how to account that cooling? It is not possible. We are simplifying this we are using very simple assumptions right? So, these are the two most common used boundary conditions, but, there is no definite logic behind this. There is number of assumptions and assumptions are often short. There are other difficulties also and of course, this is what is already we mentioned, the mixing of the lubricant will be lubricant supply and there will be back flow and there will be some mixing and after mixing the temperature at the entrance will be different. But, we need to account that flow backflow conditions. This is another difficulty.

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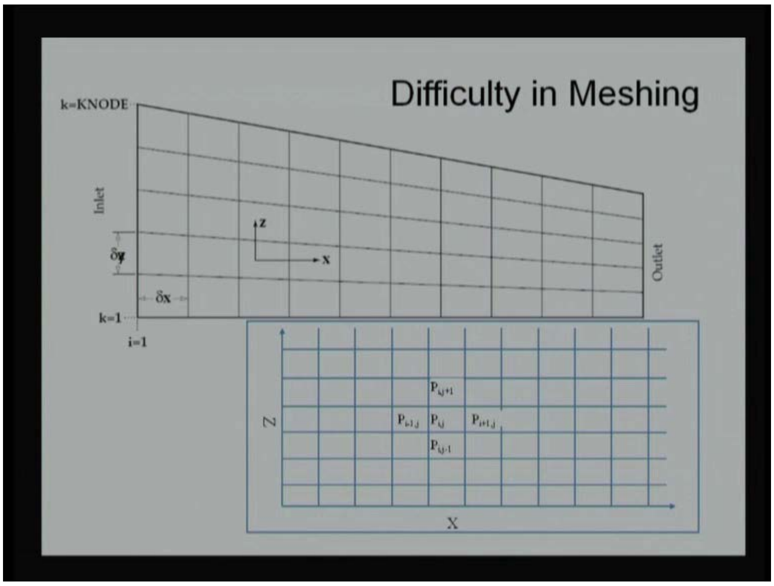

Meshing, what we done in hydrodynamic lubrication? We did mashing in x direction and z direction and we kept dell x is equal to dell z, always dell x bar is equal to del x del z bar. We tried to keep the same spacing, but, in (( )) there is a convergent. There is a continuous reduction in the spacing available to us and we cannot say let us have a six nodes here and then four nodes here and two nodes here. That will make a lot of difficulty for the final difference matter. O,ther one is that lets say all N number of nodes are same at this junction, this junction, this junction almost average number of this junctions are same. If I do that what is going to happen? This spacing is continuously decreasing and that is happening with the slope. It is nothing like I take some absolute value and then some definite decrement. This is going decrement is going to happen with the slope. Everywhere spacing is going to change.

And in computer program we need to account that as the average node spacing is going to change and its difficult to find out element as such in area unless we do some approximation in our (( )). This was the much more simpler situation what we did with hydronimoc lubrication x direction, z direction equally divide rectangular mesh. No problem we can account it we have for simplification we say that del z bar is equal to del x z bar there was not much problem. While here the problem starts. We have to account this, we need to account this in computer program and inaccuracy will also change. Of course in our one justification can come that towards the decreasing direction temperature is going to be more see spacing should be lesser from that point of view. It is justified from accuracy point of view right. It is the kind of degree of difficulty which is getting in is to have on higher side and we need to do a proper programming for this purpose.

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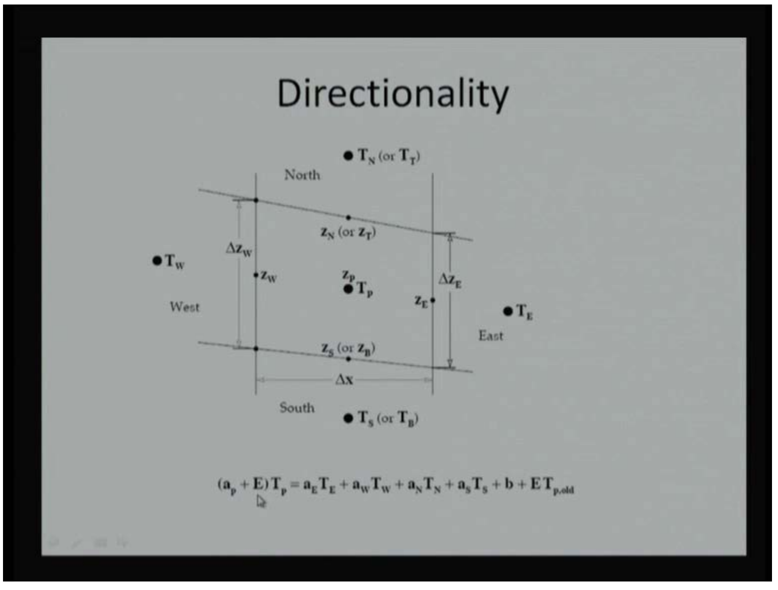

There are other difficulties what we call as directionality. Computer program does not know which side program is going to be in higher side and is the numerical iterations.

Anytime one iteration goes wrong; whole solution turns out to be wrong. That means, we need to give some directionality we need to tell the computer program. As we are moving towards x direction there will be increase in the temperature. Or easiest way is that you divide in the zone; say this is north zone, this is south zone, this is west zone, this is east zone. And each side there is slope and there is a convergent. That means, whenever the temperature is being calculated it should be compared between east and west. You see east needs to be higher than the west.

So, that kind of directionality is important. But, many times we say that if the external environment conditions are going to change, there is external cooling this kind of temperature profile may not be same because we are forcing the solution. We are not allowing it to naturally grow and this forcing is essential for our purpose otherwise we will not be able to get definite solution if they to do ten different equations they may get ten different results unless we do forcing. But, when you do forcing we give direction we are going to face problem.

Or we say that we will not be able to accommodate outside boundary conditions or everything need to be known to us and if everything is known to us then we do not have to do any simulation. Why do we do simulation when everything is known to us? Sometime people, sometimes researchers suggest d experiments as well as theoretical calculation. Few observation from experiments substitute this observation and then do modification. That is the best approach of course. We need to do simulation as well as experimental we know the experimentation on all the parameters it is very difficult. Which ever tern it is can be measured easily we should use experiments on that and remaining parameter can be calculated based on theoretical calculations.

So, absolute sense theoretical calculations are not going to give direct results because physics is not completely known. There are if you still the physics is known computational difficulties will give different results may not give actual results the way we want. So, there are number of difficulties and this is what we I mentioned that the way done the pressure p i plus 1, p i minus 1, p i j plus 1, j minus 1 same way the temperature has been calculated in the similar term and this e term sometime turn out to be negative sometime positive depends on external environment.

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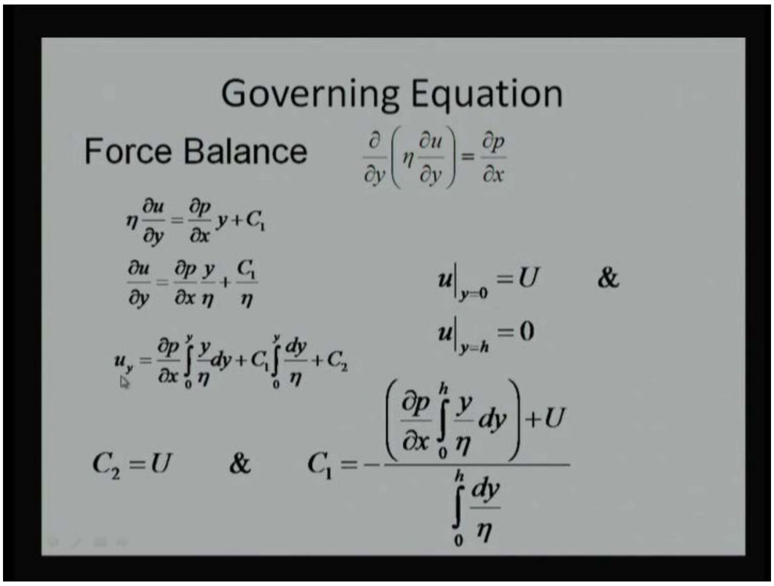

And this needs to be checked. It will not be in reverse direction. Now we are coming to the how to account velocity as I say earlier velocity of the liquid. We need to write if we are not able to calculate the value or estimate velocity of the fluid properly then whole energy equation will turn out to be useless. To develop the expression for velocity let us start again with hydrodynamic lubrications. You see they can take an element; we use a force balance equation. I hope you remember this equation. It is a very simple expression we say we are using Newtenon liquid and there is gradient of shear apace and that is equivalent to the gradient operation in x direction. Similar kind of expression will come for the direction also, but, in this case we are focused only one dimensional case. We are happy with the one dimensional then we shift to two dimensional.

When I am sure once you see all the equation for one dimensional you will not think about the second dimension at all. Now this is simple, integration is simple. You integrate you know the d p by d x which is not going to depend on the y. That is an assumption. If you remember in previous slide, we have assumed the pressure variation only in x direction and z direction. But, temperature variation in y direction. This is y this is x and that is a problem we are assuming the whole y direction pressure is constant, but, in y direction temperature is going to vary. There is a lot of difficulty in compatibility. One variable is not changing at all, other variable is changing only in that direction. Now, this is integration simply say that once you integrate, we will get integration constant as we know the eta is depending on a space variable we divide here and we try to integrate here.

Now, in earlier case what we did we simply took this eta out and then we integrated y d y as y square by 2, substitute the value of y as h and we got this simple expression. But, here eta is depending on the y and we do not know what is that. It has to come through the temperature it has to come directly. It comes simultaneous solution of equations. So, for time being we do not know how eta is going to vary with y. So, this is integration which is not known to us. Similarly, in this case also eta is depending on the y. We cannot integrate easily unless easily. We have this definite relation and this is the second integration constant and then at this integration constant C one and C two. What we need to do? We need to use two velocity conditions for simplification. We assume that lower surface has the velocity U and upper surface velocity zero. Just keep p in the mind in earlier case we kept both the velocity as U 1 and U 2. But, we want to simplify this expression that is why are assuming one surface as a stationary.

It is not going to move its not moving that is why velocity is 0 and when velocity is 0 calculating the expression for the integration or for the integration constant, we must be simpler. To simplify that we are using this boundary condition. Now, we have these twp boundary conditions substitute get the result C 2 and C 1 will turn out to be expression or an expression in terms of pressure variation in terms of viscosity variation where the viscosity variation is not known to us. We have do not have any relation for time b we have a relation we can substitute we can integrate it. But, otherwise we have a problem. Now, we have two integration constants we can substitute we can find out the expression for U.

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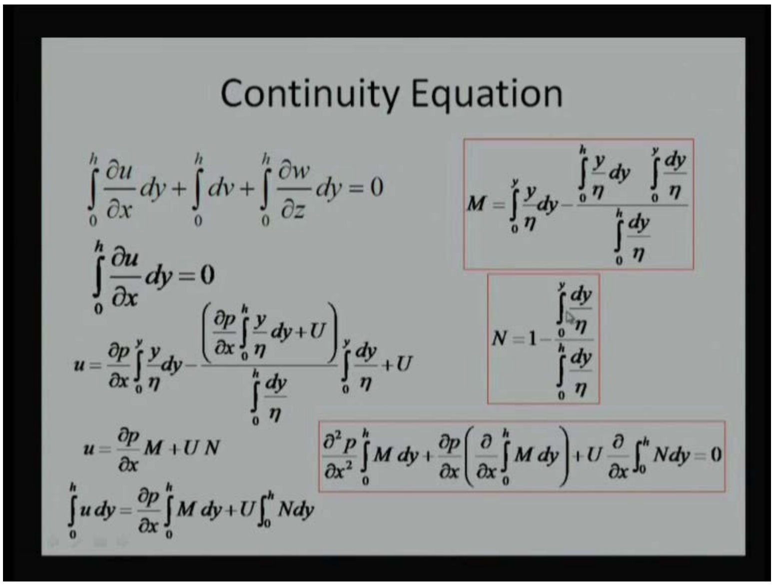

And we can use the continuity equation as we have done in hydrodynamic lubrication we must force equation with continuity equation. So, same thing we are doing here. We have earlier in previous slide we solved for the force equation now we are solving it for the continuous equation.

To be on a simpler side we assume these two terms as negligible. Otherwise expression will be much lengthier, much more complex to understand. So, simpler one; say this term is negligible, this term is negligible for time being for more or less q is not here or we say that there is no velocity in perpendicular to the sliding direction. Then this term will be negligible and there is no squeezing action, there is no turbulence in that disc velocity. That also will be zero. There will not be any change.

This is the only one term. Now, to utilize this we will be using this few as this has been expression which we have tried and C one and C two constants we have tried in previous slides. So, we are using those things. Now to be on a simpler side, to simplify this expression what e are doing we are separating U term and d b by d x term say d b by d x into some multiplication factor, velocity U and some multiplication factor n. So, M and N can be figured using this rearrangement of equation. If I do the rearrangement what am I going to get? M is equal to that is first integration, second integration, third integration, fourth integration and this is the term which we need to solve to get velocity profile. Similarly, and also can be figured out by rearrangement as one integration, the second integration. We are using integration limits zero to h and this is the other case this is the floating 1 0 to y. And y can vary from 0 to h depending on which node we are using. So, these are the not as simple terms if we have definite relation for eta and y these terms will be simple. But, we do not have any such relation available to us. That is why these terms will be slightly difficult to account these kind of terms.

Now, we have done like this. Now we need to integrate and differentiate. Integration will not be difficult. The way we have done this simply arranged in this manner. So, now, simple what we do is integrate this term whole. This will be having two terms d b by d x this integration should come over here, 0 to h d b by d x and d y second term will be U and d y. Now, if I do this integration by parts; I will get two terms. In first case there will be the sorry in this case differentiation is also there. So, we are what we are going to get this term with differentiation of the second derivation of the pressure and integration of m or the y and second term d b by d x will be as it is while this will be the differentiation term and integration term.

As we do not have integration, we cannot differentiate this term. Similarly, this term where U is differentiation of N d y is integration of this. As we again do not have this integration values available to us, we cannot differentiate easily. So, the whole thing is focused on what is the relation of viscosity with the y and we do not have any relations it has to be represented in time. It has to be represented in temperature. Once we represent in the temperature; that means, this equation and temperature equation or energy equation should be simultaneously soft which is a complex term. It will cause many iterations to come to the right solution number of number of examples.

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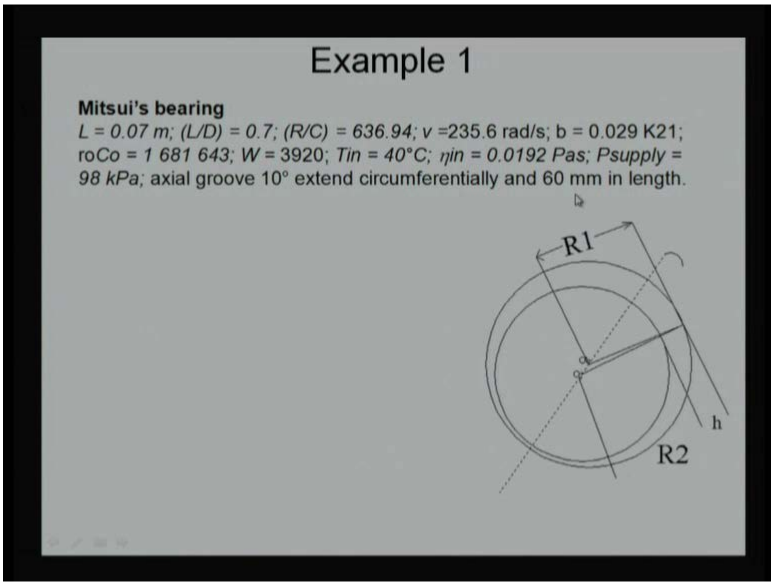

We can do also we can integrate and we can substitute what we call as a thermo hydraulic lubrication and have a couple of cases available which shows that how to crack this problem. So, I am going to show the three examples. Real data available and results obtained for this three examples. Say we are taking example of general bearing. What do the general bearing heart works we are going to discuss that in the next module in the application side. But, just the completeness I have shown this diagram. Let us assume as a shaft and this is the bearing and shaft has a radius R 2 and bearing has a radius R 1.

And they have some clearance. Obviously, the R 1 is the grater than R 2 and this clearance is need to be what we represent as the C and here the value is given R by C where radius either of the shaft or bearing we do not have much difference. It is in micron difference this ratio is 637. That is the ratio given to us and length is defined length of the bearing which we have shown I have shown on this case is the cross section and assume that perpendicular to that that will be length. The length is given as 70 mm and this length by diameter ratio is given we see that this is a 0.7 as the length is already known I will be defining able to find out the diameter right. So, diameter is known to me. Once the diameter is known I will know the radius.

Once I know the radius I can find out the C here this is omega. That is angular velocity of the shaft and we are assuming the shaft is sliding or is rotating about the own axis and there is a sliding velocity. And this constant b which is generally given for the velocity viscosity relation. These are the row when C row is 0 and C is 0 equal to some value is being given for liquid. This is the applied load.

We are assuming the applied load on the bearing is 3920 is the input temperature supplied temperature or lubricant to a supply. This is a viscosity or eta N. That is given as an 19.2 mile Pascal second. However, there are additional what we call as supply ration in this liquid is been supplied at some pressure the force (( )). There is a (( )) to pass the liquid from one place to other place. We will discuss all this when we discuss about the detail or the application of general bearing and how to design the general bearing using our topological knowledge and we have ten degree as circumferential length and 60 mm is the length.

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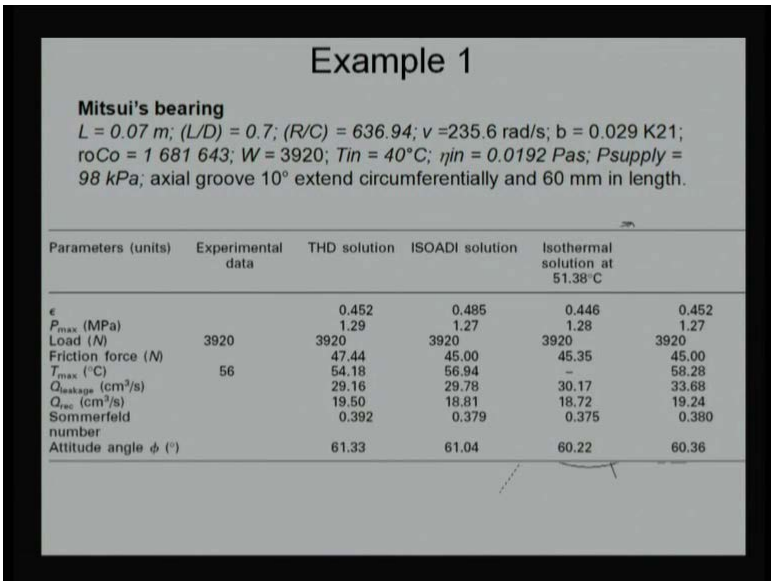

So, what is the purpose of doing this example? If I say that we need to be done with some experiment and more theoretical calculations. So, some experiments are available, we apply load 3920 Newton and maximum temperature on the surface which was measured, it was something like a 56 degree centigrade. Input temperature is 40 degree, and maximum temperature is 56 degree and there is sixteen degree temperature rise and sometimes 16 percent temperature rise can reduce a viscosity by 30 or 40 percent and that can reduce load carrying capacity or the parameters. However, this is one of the very likely loaded bearing that is coming, that can be judged from this value. Eccentricity value lesser than 0.4. It is known to be likely loaded bearings and we are able to see when we are doing a Th analysis. T is thermo, h is hydrodynamic and d is a dynamic yeah h is hydro and this is dynamic the thermo dynamic lubrication.

Here we are using a boundary condition. We are using the isothermal and adiabatic condition with the rotating element is not going to vary the temperature. So, it is a high convicted terminal there. So, that will remain the same as the constant temperature. There is isothermal temperature. Well stationary surface is not going to (( )) energy to surface that is adiabatic surface. So, here the boundary condition has changed by using solution. This expression is that we can use the same expression as hydrodynamic lubrication mechanism expressions or the pressure expressions.

What we need to do; we need to find out effective temperature and that effective temperature is estimated as 51.38 degree centigrade. This can be done my measuring the liquid which is coming out of the bearing. When general bearing is used there will be leakage of the liquid or liquid leakage of the lubricant. We can collect that lubricant, we can measure that temperature and that measured temperature is 51.38 degree centigrade. That is what we call as the effective temperature which is coming out of the bearing.

And performance is based on that and this is solution based on hydrodynamic lubrication and iteration and effective temperature is been measured by after this iteration. So, they are measuring the full results. One is the T S d result thermo hydrodynamic lubrication. It should show the very good result because it is going to take more than hundred thousand times the time compared to what this solution are taking. And starting is the hundred thousand times solution time. It takes hours to give the results. Now, this p applied load in all the cases same because whatever the experiment are measured we are giving these as the input. What we are going to get maximum temperature is experimental is as 56 then we are using a T S d analysis what we are getting is 54.18 degrees centigrade.

That’s lower than what experimental measured temperature. We know very well thermo couple which we use it acts as a generally an inaccuracy of plus minus 1 degree. So, this much difference is not a very critical assumption. However, coming to the, this boundary condition this isothermal and adiabatic boundary condition; this is able to give a 57 degree centigrade. It is not too far from T S d and this takes much much lesser time compared to T S d solution.

Now, this cannot give the maximum temperature because it is based on simple temperature collected at the exit of the bearing while in this case what we say the effective temperature in doing the iterations is a more like a (( )) but, it gives slightly better results because of the iterations because it takes slightly more time compared to this solution and it gives around 58 degree centigrade. So, near two extremes 54 measured is 56 and this gives a 58.

And there are other parameters. Now, what is to be important to be noted over here is the q recirculation. In all the lubrication algorithms, it has been calculated recirculation of the flow has been calculated. It is nothing like the whole liquid is moving out, it is going to be leaking out. Again entering back and that is why the inlet temperature of the temperature at the entrance will be a mixture of the temperature which we are supplying or liquid we are supplying some temperature plus this re circulated temperature. That means the algorithm needs to incorporate those. They can see here the q recyclisation in all the cases. In this case particularly is 19.5 units did is 18.81 18.72 a b d this case 19.42. In all algorithms these circulation has been accounted and this you can consider factor the leakage.

Leakage is 29.16 slightly more than what is circulated. Same thing over here same thing understand we need to account flow condition and when we derive the hydrodynamic condition or (( )) lubrication or thermo hydrodynamic lubrication. We are not accounted these recirculation liquids; obviously, the liquid which is getting recycled we should account that. However, developing expression, general expression for all those conditions is a not possible. That is why we take specified application or we solve all this by application point of view.

We can roll bearing and application can follow the application gears or the application general bearing as application and then we applied these algorithms. We know most of the situations will be simplified. Not necessary the geared pair will show similar behavior, the roller behaving will show the same behavior as general bearing. So, this will be having very specific equations which are that some assumptions which are specific to the applications.

And solution will not be that difficult as it appears in absolute sets. We will be discussing that kind of solutions you can say our next model will be on application whatever we have learnt in our fundamentals; fundamentals of friction fundamental of gear, fundamentals of lubricants, fundamentals of lubrication mechanisms are Reynolds equation or elastic equations. We will be utilizing those on application side and maybe we will be able to give better results where we are able to understanding when we apply it.

Thank you we will continue this module next time. Not this module we will be considering the application of course as I mentioned we have three examples I already showed you one example. I do not have much time, but, I can quickly refer the second and third example and these are more or less same to understand.

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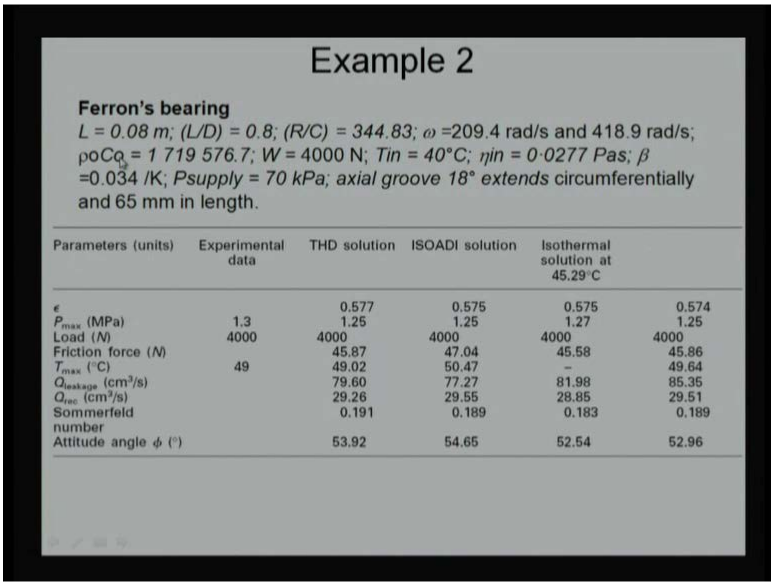

What is the basic example says that only the geometry parameters have been changed, load has been changed, actual groove arrangement has been changed. Now we can see here the initially the eccentricity was less than 0.5, while in this case eccentricity is slightly increasing. It is coming to the slightly moderate loads. We are able to see the temperature. T max maximum temperature is forty nine while T S d solution also gives 49. While this approach which uses effective temperature calculation gives the temperature as 49.64, there is another parameter which has been measured and have been quoted as p maximum. It is a 1.3 mega Pascal pressure was the maximum pressure calculated or measured.

And T S d gives 1.25 which is slightly difficult to understand unless the pressure is right it should not give the good results. However, it is been shown the 1.25 1.25 1.25. So, either there is some sort of inaccuracy in measurement all the solutions are showing the lower pressure. Lower maximum pressure or maybe there is some in accuracy we can calculate the maximum pressure.

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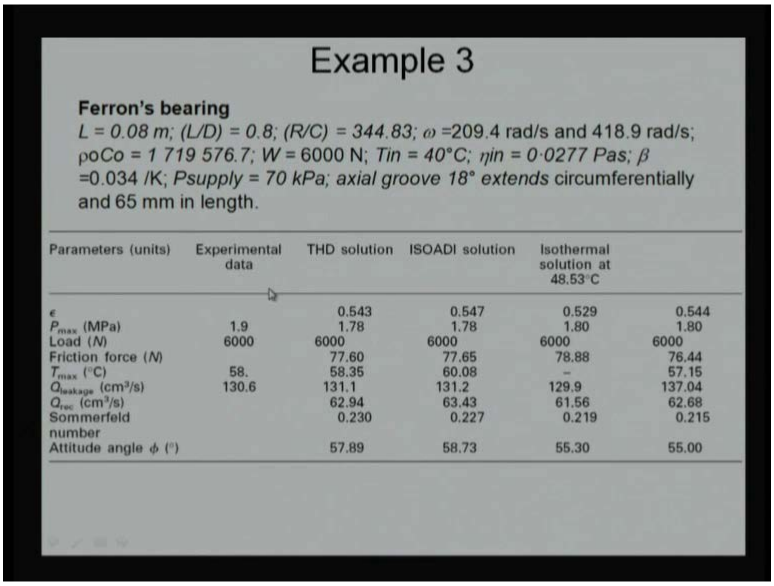

Similarly, there is a third solution, third example more or less again the same diagram where load has been increased to 6000 from 4000 and there are some changes in the input parameters. What we are going to get here the p maximum experimentally measured at 1.9, TSD gives the 1.78 and this approach gives the 1.8 So, again maximum pressure is slightly lower side compared to measured and temperature which is (( )) is coming to very close, is a maximum temperature measured 58 degree centigrade. TSD estimates one 50.35 degree while that this approach requires effective temperature and iteration gives a 57.15 degree. So, not a huge variation as such in this case. Results are more or less same. While in this case q leakage is also measured is a 130.6 and we are able to see that more or less same q is estimated by number of analysis.

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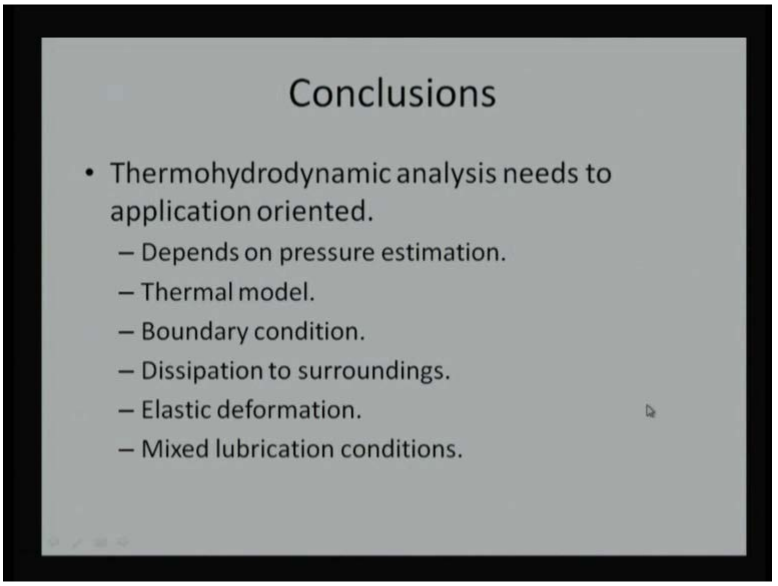

And in short, I can say that thermo hydrodynamic lubrication is generally a specific application or need to be a specific application whatever the equations we solve. It should not over burden our computer program, but, after understanding the application we can still simplify those equations and we can get a good solution.

And we need to know the boundary condition properly. Many times the boundary conditions can be used by exponentially measured data that will give good estimation. As far as possible we should account to elastic deformation because that is going to give the lesser pressure the way we have seen in the previous of the slides. Maximum pressure value is the lower and we have considered only the distribution within the liquid and we are not considered heat transfer within a solid surface. As far as possible if the situation permits us we should account that or in short it is a slightly on a higher side to think that we should get a complete TSD solution which involves a thermal elastic and hydrodynamic mechanism. But, if application is like that, like a gear application we need to account all this factors to estimate the right load carrying capacity.

If we do not account properly; there is a possibility of wrong estimation and which will not give us good results. So, it depends exclusively on applications. Which kind of equations is going to be properly fitted in that application? But, we cannot generalize cannot use all the equation and try to get a solution that will take much much longer time and may not give much more full results to us. So, thank you. Thanks for your attention. We will consider, we will discuss this application oriented model next lecture onward.