Tribology

Prof. Dr. Harish Hirani
Department of Mechanical Engineering
Indian Institute of Technology, Delhi

Lecture No. # 25

Estimating Elastic Deformation

Welcome to twenty fifth lecture of video course on Tribology. So, we are going to complete today quarter century of the lectures on Tribology. Topic name is estimating elastic deformation. It was started long time back it was started more like a (( )). When they come in a contact there will be elastic deformation plastic deformation. We are restricting our discussion to elastic deformation, we do not want plastic deformation, we do not want permanent deformation on the surface it should recover back once the load is removed or after unloading.

In previous lecture we started with high dynamic lubrication and explore the how to model hydrodynamic lubrication between the (()) and in present lecture we are going to combine elastic deformation with hydro dynamic action. That is what the topic name may be also can be termed or can be called as a elastro hydro dynamic lubrication. Here we are going to mix elastic deformation of the materials, hydro dynamic reaction due to velocity and lubrication because of presence lubrication of lubricants.

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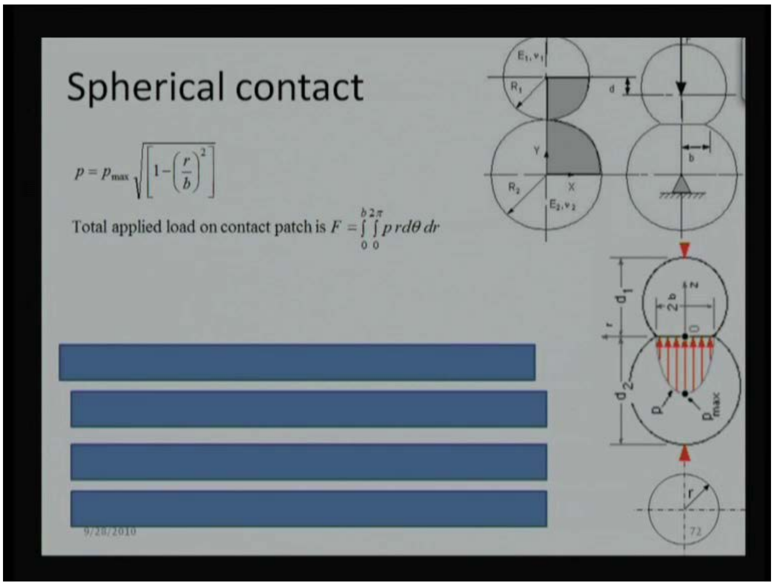

Couple of the few slides which we discussed in previous lecture and I am just going to repeat for the continuity. Say that this is figure which is showing the two spheres they are in contact and of course the circles are as shown as a front view of the those two spears in our contact. And appears initially that there is a point contact, but, if you apply the node f, point contact cannot be sustained. There would be infinite stresses and we know there will not be infinite stresses there will be some sought of deformation, material deform to sustain the load and that deformation to bigger the scale is shown out here. Of course this deformation will be very very low.

That is why we can say that the half width if I say 2 b total of deformation which is shown out here or we say this is the by b and this is symmetry both the side and this b must be very very low compared to d 1. Or we say b need to be lesser lesser than d 1. It may even be lesser then one percent. It does not reach even the one percent. Just just resketch it. So, that we can demonstrate there is a distribution we have kept in a bigger scale or we say that these figures are not on this scale. It is just the demonstration is a schematic diagram.

And when this kind deformation occurs, you can see there will be contact area and that contact area in this case particularly is sphere versus sphere will be a circular. Or to is there is a some sought of miss alignment it may be electrical, but, in this mostly there is a circular and we are assuming that this radius is start from 0 to max value of b where the maximum deformation occurs. And external reports indicate that pressure will be parabolic. Maximum value will be at the centre or whether the load is coming the maximum pressure will occur there and it will gradually decrease or maybe say parabolic decrease, not a gradually, is parabolic is decreased to 0 value and the extreme bond is b or when r is equal to b in this pressure will be 0.

Now, what is our interest? To find out how much elastic deformation really has happenned or what is this contact area. Now, there are two variables; we say that it is a readily compressed as well as the flattened also. So, we have two about this is the d is a deformation and b is a width of the contact or we say that this is a circular area, r will be variable as well as this deflection will be variable. For convenience, we start with parabolic distribution we say the pressure at any radius will be equal to p maximum. That will which will occur at the center with r is equal to 0 and a square root of 1 minus r by b square r is going to vary from 0 to b. When r is 0 pressure is going to be maximum when r is equal to b; that means, b by b it will be 1 1 minus 1 will be equal to 0; that means, pressure is equal to 0. And this is the what we are showing through this diagram. Pressure will be 0 at the boundaries and maximum at the center. That means, this kind of the present and this kind of relation is representing the pressure distribution which is generally observed in experimental results.

Now, as a as a common thing to given to us is the force. How much force is applied on a materials. Of course with the elastic properties you can this cylinder we have young's modules and knew that poison ratio as well as geometry radius. So, these two were lasting properties; this is the geometric property the material property and geometry property, we need to account this. And force is generally given to us naturally all the parameters like a related to b or deformation can be figured out once we find out the expression for the F which are that the total load on can be represented as a integration of p or that the p into d a and integrate all the compete area. What is the complete area? You say this is a circular. So, at any point if the starting is 0 theta is equal to 0 it will compete at as a theta is equal to 2 pi.

That's why it is says that first integration limit for theta is 0 to 2 pi and second is a radius 1 and this radius is going to vary from 0 to maximum value or the b. This assumption, the maximum batch the width is b or radius maximum value of the b radius maximum value of the r is equal to b. So, that is why the this integration is 0 to b. Now, p we are suppose assuming that it is equal to the p max square route of 1 minus r by b square. We can substitute over here and we can integrate.

However, we can see that there is no theta in this p. That means, p r d r can be kept separately and this first integration can come out with the d theta and integration will be theta and we can substitute value of the theta that is the 2 pi minus 0. It will turn out to be two pi and that is shown over here.

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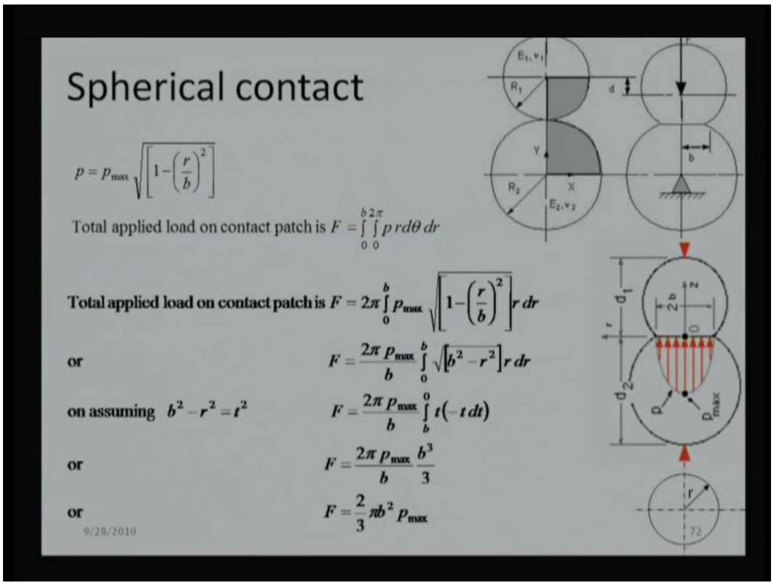

The force F is equal to 2 pi. We have already integrated out theta substitute in the values of the boundary 0 and 2 pi and the 2 pi minus 0 equal to 2 pi. That is a 2 pi over here p max. Now, we can rearrange the p max is also does not depend on r now. We can take it out and we can rearrange in addition we have rearrange this bracket also. We are saying that now it is b square minus r square or we say that we are taking 1 by b as a common from this which is also constant for given force, given geometry, given elastic limits, b will remain constant. So, this bracket remains which is need to be integrated that is b square minus r square, limits 0 to b. We know this is simple integration. We can substitute b square minus r square is equal to some variable differentiate it and substitute this value.

For time being your assuming the b square minus r square is t square. If you differentiate it will turn out to be minus 2 r d which is here r d r we have to substitute and this side will it turn out to be 2 t d t. That means, we can substitute easily the this is t and r d r is given as a minus t d t. Now, this integration if it is simple. So, t square integrating it will turn out to be t cube divided by 3 and we can substitute the values of this d t integration limit is been changed here an integration limit was a 0 to b while this is a negative side. So, that is why the integration limit is turning out to be different b b and 0.

However we say that when b is equal r is equal to b square t will turn out to be 0 that is been shown over here and when b is equal to 0 t you can turn out to be plus minus p. We are neglecting the term minus b because there is no negative geometry in this. It has to be only b cannot negative b. That is why we are writing something like that 0 to b integration. Now, as I mentioned that t square integration is simple now at the t cube by 3. That is shown over here t q by 3 another substituting the values b instead of t. It turn out to be b. So, it is showing results at b cube by 3. After sending it to sign will change because of this limit to turn out to be 0 minus b cube by 3 and negative negative will cancel all or will turn out to be positive. That is where overall expression turns out to be this.

However, there is a common one term that is a b here and b cube here we can simplify and we can express this relation something like this. So, after you applied force is equal to 2 by 3 66 percent into pi that is constant b square p max. Now, many times we know the, what is the p max. What will be maximum compressive force which you can be applied on this a never lasting body or at which is pressure which pressure it will be started to forming. We can substitute these values. However, if we do not know this value then we need to use some other equation also.

Now, I will say that we know force, but, they are two variable b and p max. If I use elastic properties and material; I know that b p max what will be value of p max again to simply find out what will be the b. If I do not know with assurity what will be p max then I have to use some other relation. I have to try with using some other relation. However in this case we have discussed about this spherical contact.

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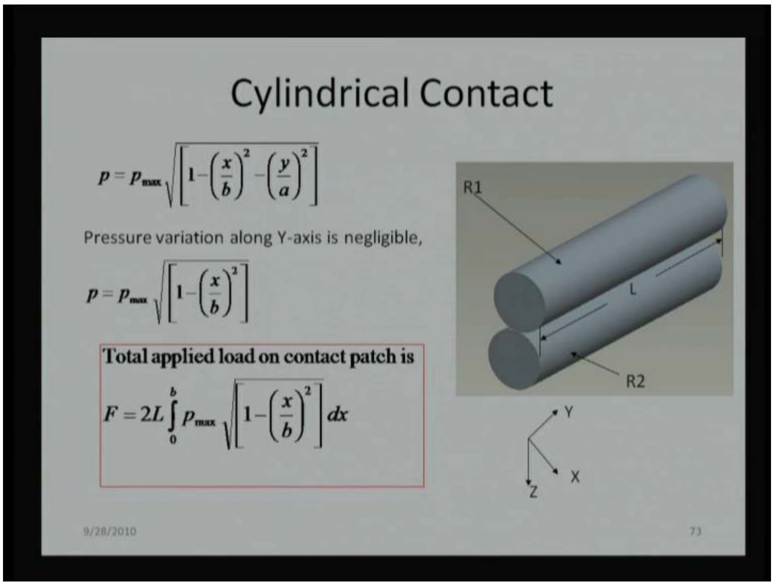

The second general contact is a cylindrical contact. That is shown in this slide. You say that in the cylinder what you we are seeing the cylinder because in this case the length is much larger more than four times of the diameter and radius is given in R 1 and R 2. Both the cylinders have the same length. It is a typical example of the roller bearing. In roller bearing many times we keep alpha d is a one, but, in this case we have assuming this much larger. That may be L more than four times compared to the diameter and pressure distribution because there are two (( )); component x component and y component we can give pressure distribution something like this. Again the p will be maximum at the center and x as direction shows here. As x varies it will vary and when it reaches to maximum value of the contact patch that is equal to b.

Pressure it will not be 1 minus 1 or 0. Whatever the value remaining here, that will we can be shown out can be represented. However, this was our assumption. This length is much larger the pressure distribution or pressure variation along the y axis may not be very dominant. So, we may neglect this for the time being. You can say that y by a square term is negligible compared to when compared to the x minus x divided by b squared. If we neglect this and pressure term will turn out to be most simpler in the way in which we have solved in a previous slide. Same thing. We will in previous slide, we use a word letter r instead of x. In this case your showing only that (( )) system in previous slide do we consider polar coordinate system.

Now, the p is known to us. We can substitute in a force equation and integrate that force equation to find out what will be the relation between the F, b and p max. So, when we do that, its F. Here d y the way we have done earlier. The d theta this same thing in the d y will be simply integrated it turn out to be L and here it is 0 to b p max 1 by 1 minus x by b square b x. p max is also constant it can be taken out easily. And then we can substitute something x is equal to b sin theta, b cos theta in some other term and get integration done.

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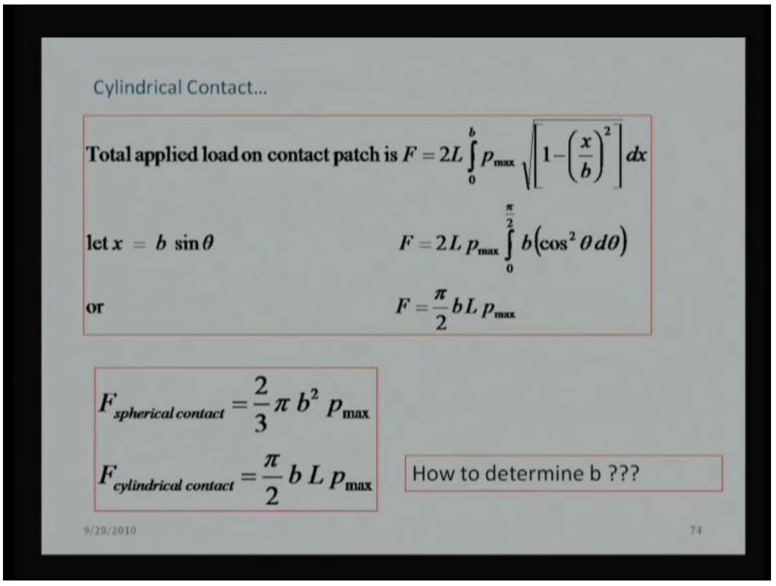

For time being in this case, we are substituting x is equal to b sin theta So, when you substitute x is here b, x x is begin substitute b divided by sin theta sorry b sin theta divided by b. b will cancel out it will now turn out to be 1 minus sin square theta. When you find out it will turn out to be cos theta only. When integrated d x is equal to it will turn out to be b cos theta. So, overall this term turning out to be the cos square theta. We know because integration of cos theta is square cos square theta is simpler one. It can be termed as a cos 2 theta plus 1 divided by 2 and we can integrate because we have integration limit given over here.

So, when we integrate cos 2 theta and integration limits are 0 in pi by 2; the overall integration will turn out to be 0 because sin 2 theta is substituted by 2, substitute 0 both the values will be 0. I have a plus 1 whatever which will come out to be the cos 2 theta plus 1 and plus 1 can be integrated easily. That will turn out to be theta.And when you substitute this boundary condition or this integration limit it, will turn out to be pi by 2. We can substitute that those values and this overall expression turn out to be in terms of pi b L p max.

Now, again the way we can we have treated a spherical contact, we can similarly, code the same thing about cylindrical contact. We say that if we know the p max I can find out what will be the b value. L is already known geometry. This is a constant p max if I not have a material I can simplify now it what to be the contact patch with when we apply load F for the some materials. Now, we can compare this two we say the spherical contact with this cylindrical contact. You can see this two that there is a 2 by 3 and here it is one half. So, the 66 percent this will be 50 percent pi, pi is a common.

While b, b 1 b is common, but, second b there is b while it is a L. Length is generally very large compared to the b (( )) or more than the 10 times and p max is the material is the same the p max will remain same. So, what what is the conclusion coming out from this comparison? We say we can say that as a b is much lesser compared to L and this difference is not very high. This is 66 percent, this is a 50 percent this difference is not very high, this is a 16 percent difference. But, b will be ten times. So, we say that more than ten times the 6.6 while this sorry in this case the b will b 1 time, but, l will be ten times. So, we can say this this is a 0.66 and this will turn out to be 5. So, 5 verses 0.66. In other words that cylindrical contact can bear much more to larger load compared to the spherical contact.

So, then when there is a larger load limit we need to carry out with their larger load; we need to carry larger load it should sit from a spherical contact with the cylindrical contact. We know very well the friction force will increase with a spherical contact cylindrical contact. In a spherical contact friction force or friction resistance will be lesser compared to cylindrical contact. But, load carrying capacity is higher. So, which is a advantage if you we have space restriction we know that load dimensions are permitted that along the length direction, but, lesser dimension is permitted over longer dimension or diametrical direction and we can choose a cylindrical contact. Of course these things when we discuss about the roller bearing or selection of the rolling element bearings.

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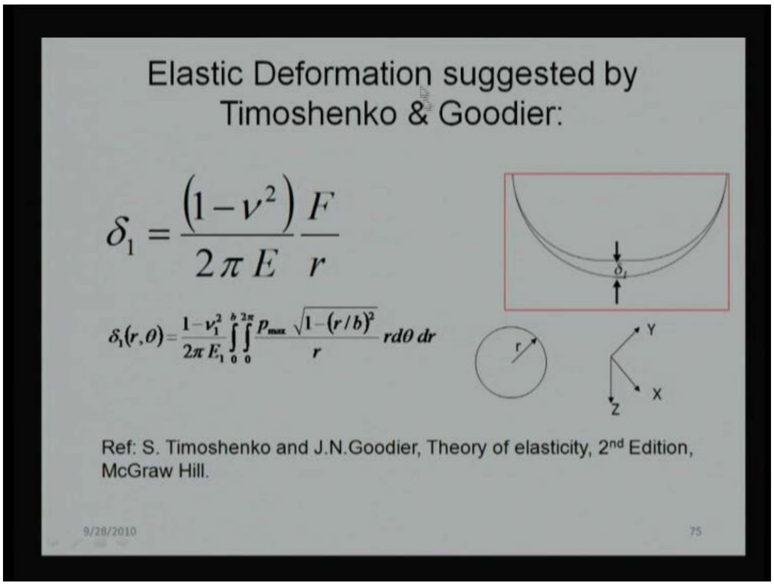

The present terms may be I do not know the p max and then I do not know b. How to find out this b when we do not know the p max. So, we can start with some (( )) material equation. We say that this assume this is spherical contact and under deformation this delta one is the deformation. When we are applying the force there is a deformation over here. This can be represented with a applied force geometry or material parameter and geometry. What is the radius? If I keep equal to b I should be able to find out what will be the relation.

However, if I do not know f as it is and if I know the pressure distribution the way we have done earlier and previous example. In terms of pressure we can find out in the that term. Now, why we are emphasizing on the pressure because when we discuss about the hydro dynamic lubrication; we had not we had not discussed in terms of force we have discussed in terms of pressure distribution. So, we will know what will pressure distribution after solving the hydro dynamic equation.

However, if we do not require hydro dynamic we require simple lasting deformation then we can treat directly f. We do not have to come to the pressure. What we required elastic deformation to the combined, to be joined with hydro dynamic lubrication. That is why we are talking about the pressure term. We are giving explanation term. So, the pressure. So, when we do that represent f in terms of p max pressure distribution and area d r theta d r. So, this overall can be represented in terms of b now because we required relation also in in reflection and extent of the zone or extent of the contact zone. And this is what the same distribution will be shown as a coordinate system and this is formula available in number of text books. One reference is given over here.

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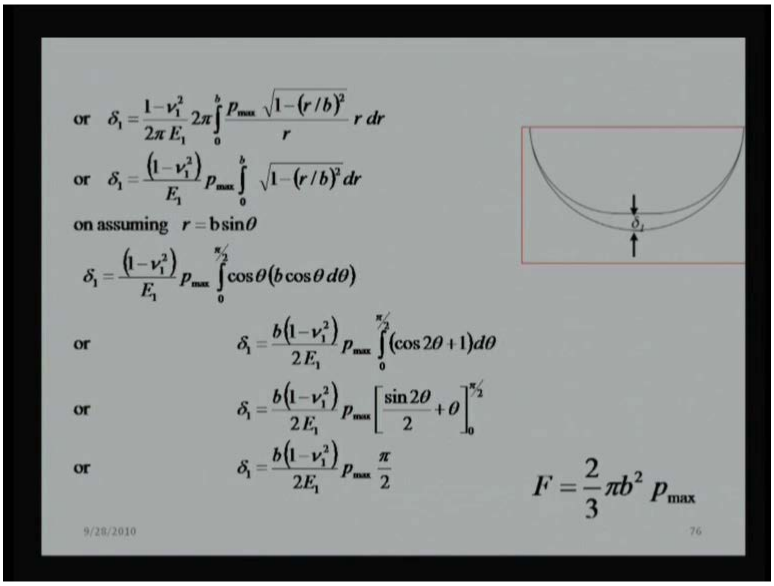

Now when we substitute this, rearrange equation what we get in integration bracket is one minus r by b square. Square root of that d r you can substitute or as a b sin theta simple to get the value of integration. When you substitute here and rearrange we are getting this term in this form of the cos 2 theta plus one. You have done earlier, but, here I have written in more explicit from. Now, when you integrate cos 2 theta it turn out to be sin 2 theta and when you put this limit 0 and pi by 2; both the terms will be turn out to be zero and this is only important to integrate. One when you integrate 1 into d theta is when you integrate it will turn out to be theta and you substitute this value it will turn out to be pi by 2 minus 0.

So, we can substitute this values and integrate that the way it is been shown here sin 2 theta divided by 2 plus theta. This is turning out to be 0 and this will turn out to be pi by 0 when you substitute this integration limits. So, now what we have? We have this deflection in terms of b, in terms of p max and we can use a previous relation, but, we have done for this spherical contact that is the 2 by 3 pi b square p max. Now, we have also p max p max and b here we can use this relation and we can find out the in terms of b relation.

However there is another possibility. We can go ahead with some sought of simplification. We can use a teller series to find out exclusively what will be the be in terms of f, neglecting not neglecting and replacing p max with other geometric parameters.

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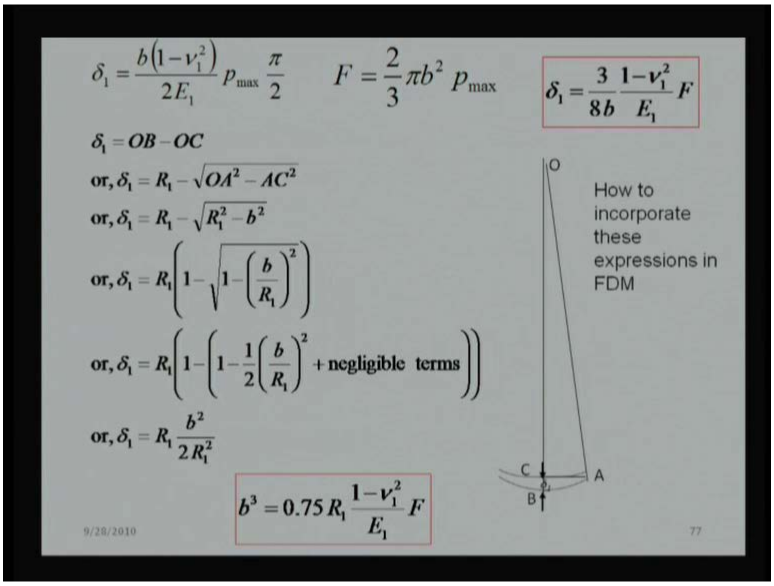

To do that, we can simplify it. We can say first am substituting p max value of the p max here I want to replace p max f and represent this deflection or radial deflection on this surface that is turning out to be radial deflection delta one as 3 by 8 divided by b. So, this is the related, 1 minus mu square by E 1 f.

Now, we are considering only the deflection of the one surface. You cannot consider deflection of both the surfaces. There is a possibility the other surface is rigid enough. It does not deflect at all or if there is a deflection then we should consider delta two also. And this will be changed one minus mu square divided by E 1 all other things will remain same because the contact patch will remain same. b will remain same for the both the material it is something like the one material will have one b, b 1 and another material with different b we are coming in a contact b r making a pair. So, they will have same value of b.

Now, what do I say the simplification; that simplification can come from a geometry here. We know very well that this deflection we need to be much lesser then the radius of this pair. That is why I say that OA is the radius, OB is the radius and when we deflecting it to some extent it is a after deflection this spear is getting shape of OC or there is a deflection this much inside. Now, that can be represent in terms of OB minus OC. This is a OB minus OC that is a deflection. We know OB is a radius R 1 while OC can be represented with the this triangle, right angle triangle. That is a OA square minus AC square OA already we know that is R 1 and this is in terms of b that half with or to the we considered with overall deformation or the extend of the contact zone of the 2 b.

So, this will turn out to be b. So, this is a R 1 minus b square and we know b need to be much lesser than r one. So, we can rearrange, we are taking R 1 common from this and overall common from the brackets it turn out to be one minus square root of one minus b by R 1 square. Now we can use a Taylor series for this. It will be 1 minus half b by R 1 square plus high order term as b is much lesser then R 1. So, this term high order term will be negligible that is why we we are writing here somewhere the negligible terms.

Now, the delta 1 is can be represented here in terms of R and in bracket is a 1 minus 1 and this minus 1 and minus 1 will turn out to be plus one sorry plus and this is what will be in the R 1 divided by 2 into b square divided by R 1 square. So, this is a same thing what we have mentioned R 1 b square divided by 2 R 1 square. Now, there is a one relation over here in terms of delta and b. There is another relation delta and b over here and we know the force is given to us. Elastic properties or material will be known to us, geometry of the contacting surfaces will be known, generating body will be known to us.

So, we can simply find out deflection in terms of the force or what will be the contact patch in terms of the force. That is a very clear in from this relation I can find out either delta or b it will not be much problem. There are 2 equation 2 variables both will be a straight forward relation to us. So, here the b cube is a given as a 0.75 R 1 1. This is a elastic properties into f or in another word what we say that if f increases b is not going to increase in the same order means, if you make force 8 times than only the b will increase by two times. The sensitivity of the deformation is much much lesser in this case or we say with a slight deformation load carrying capacity of this material will be very high. Here, b need to be lesser than R 1 and much lesser then R 1. If the any time that kind of relation is violated then we will not be able to get the results properly.

That is why we say in b need to be much lesser than R 1 that you mixture whenever there is a deformation and we need to compute, if any time b is turning out to be more than 1 percent or 2 percent of R 1; we should stop it, we should go ahead with non-linear analysis with this kind of simplification will not be valid.

Now, the question comes how do we incorporate in hydro dynamic lubrications? We are talking about only the solid mechanics in this case. We talk of the deformation in terms of the force, but, how to combine with our hydro dynamic lubrication which we have discussed in previous lectures and we have more interest in that. When we cannot combine hydro dynamic lubrication with elastic deformation then only the name can be given as a elastro hydro dynamic lubrication which is important as I say that is the one of the optimum tribal sphere mechanism.

If we are able to achieve elastro dynamic lubrication throughout the life of the component, we cannot achieve better then that results. Hydro dynamic lubrication is not as good as a elastro dynamic lubrication. Mix lubrication is not as good as a elastro dynamic lubrication. It is the only the elastro dynamic lubrication which gives the best results. And most of the high performance tribal spheres are designed based on the elastro order lubrication. Talk of the gears, talk of the rolling element bearings, talk about the (( )) follower mechanisms they have been designed based on elastro hydro dynamic lubrication mechanism with the load carrying capacity is enormous with a less, much lesser area they are able to sustain much larger load.

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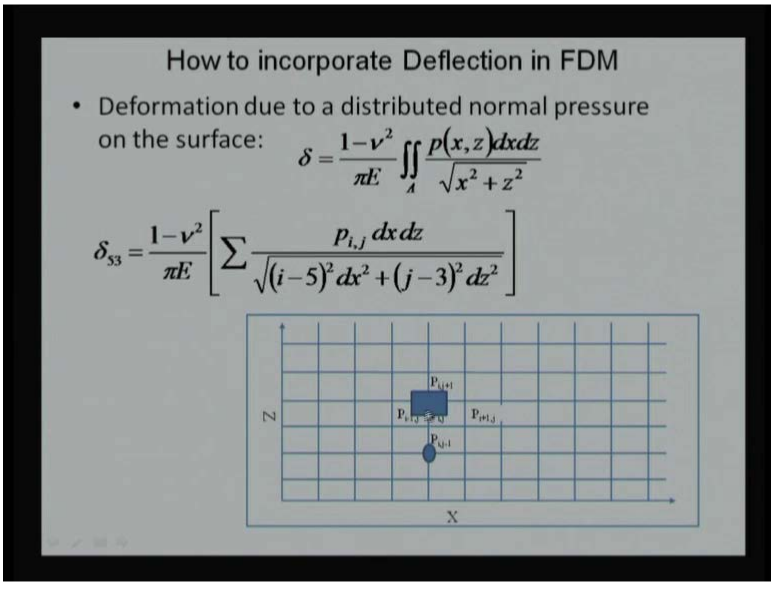

Now, it is a question need to be answered hoe to incorporate deflection in f d m or finite difference method which was used for hydro dynamic lubrication. We derived deformation in sorry we derived I mean a FDM pressure distribution or found nodal pressure. What we did in a finite difference method; we develop the surface, divide the surface in number of division we say there are number of nodes and those number of nodes we found pressure distribution.

Now, if I know the pressure distribution at the those nodes, I can integrate those pressure to find out the force. If I know the force of the difference nodes I can find out the what will be the deformation because of those. However, there is a surface connectivity whole solid surface need to be integrated. It is nothing like that they are individual molecules individual pieces and they will simply deform under the load. Here we need to incorporate or combine load.

And that is comes what we are trying to convey this is a deformation which which we have done in a earlier case and in a previous slide in polar coordinates. Here we are writing a rectangular coordinates because the pressure what we have expressed in our hydro dynamic lubrications it was terms in terms of x and z. And. So, there is a area d x into d z and this is the main main thing. We say that this is the distance. What is the distance between the pressure and the deflection point with the pressure is applied and what where are calculating the deflection. Now, we cannot calculate the deflection of the same point where the pressure is been applied because that will simply penetrate, will give the infinite stress.

That is why we consider the movement equation. We take some distance from the pressure point and value of the deflection. We do not calculate the deflection of the same point now more and more accurate can be the achieved when we go ahead with more and more number of divisions. This is what we am trying to convey. See now the pressure suppose p i is somewhere here. I want to calculate the what will be the fact of this pressure over this nodes in this node, this node, this node, this node and keep in a mind pressure at a any point is going to a going to influence the deflection the surface with the whole surface is a connect. It is a solid surface they are separated. Think about the diaphragm when we apply a pressure any point of the diaphragm. Whole diaphragm gets deflected. It is not only that point can get deflected, whole diaphragm get deflected and we are taking going to take those consideration. So, that we get over all good results

Now, just for the example I have I have given in this case if I want to find out the deflection at the fifth node in x direction and third node in z direction, what we are saying this is a 1 is starting is 1 1 1 2 sorry 2 1 3 1 4 1 5 1 5 1 5 3. That means, if I want to find out the word is deflection at this node due to p i j, p i j that is a somewhere here. I can be anything j can be anything depends on number of divisions how will I find? Or we say the pressure p i j into area because we are taking very small areas; that means, there is no need of extra integration it will simply will be p i j into d x into d z.

That is going to give me force and this is going to give me the distance. What is the distance here? In this case what I will do, I will subtract. i minus 5 square. If there is a fifth node then, it will turn out to be 5 minus 5 there will be 0. Similarly, j minus third node that is here the pressure is a j node and minus where we want to evaluate since a minus three will be square root here and now this can clearly see when we want to find out the deflection five three due because of the pressure five three this will turn out to be infinity. When we do computer programming mean to avoid that. If I want to find out the deflection under 5 3, I will not find due to pressure of five three whenever the pressure of 5 3 number comes, I have to simply bypass that. I do not have to add and is only in may be say 50 by 50 matrix I have to avoid one point.

See whenever that this deflection and this deflection, this node sequence and this node sequence are same, I have to avoid this summation. Remaining all other points I have to account it as summation 50 into 50 except one. Say that 2004 100 99 pressures we will find out what will deflection because of 2004 100 and 99 pressure at this; however, if you know the boundary condition the pressure is zero at the extreme boundaries. Then, we may avoid those points also because we know the zero pressure will not cause any deflection. We can avoid the calculation of boundary we can say start from the two go for the 49 if the we have our node limit from 1 to 50 (( )). One it will give a pressure 50 again it will give 0 pressure.

So, we will start from 2 to 49 in x direction as well as in z direction. So, that will reduce computational efforts. This is what we are saying that when we use this kind of a device. Now, if I already solved the pressure equation or we already solved a Reynolds equation we already know what will be the value of p i j at every node. Of course we can intermediate we can put in between also when we are going through iteration loop. However this is what I showed there is a there is a p i j here we calculate pressure with area near by the d x and d z and now we are trying to find out what will be the effect of this pressure force on this point and will be a number of points over here.

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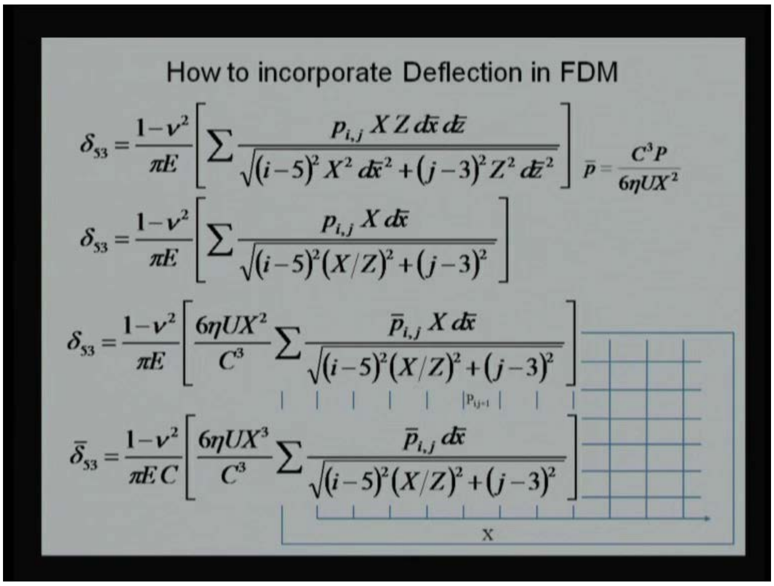

Now, we continue with same and when what is the difference when we were discussing about the hydro dynamic lubrication; the whole hydro dynamic lubrication equation have been represented in terms of non dimensional numbers and this is a dimensional. We cannot simply combine non dimensional with the dimensional numbers. So, what we need to do; this whole expression needs to be converted to non dimensional numbers. Now to do that, instead of x d x square we can use a non dimensional number that is a d x bar square and there will be multiplication of maximum dimension in x direction. Similarly, for the z side also instead of z we will write a d z into z and whole square of that.

Similarly, here instead of d x we are writing x and d x bar. Instead of d z we are writing capital z as a maximum value of z in z direction into d z bar. Now, we can use a relation the way we have done in hydro dynamic lubrication. We normalize x by z we are using that bracket x by z bracket. So, I can take out this complete expression out. So, the z square, d z square take common out of this and the results are numerator z d z and here it is a square root. So, z d z bar will be cancelled out and expression will be modified something like this. You say that capital x d x bar, this a non dimensional, this is a dimensional. Here it will turn out to be x divided by capital z. So, x divided by capital z is turning out to be non dimensional and we do not have any other term over here right.

Now, this is a why we have taken because we know d x bar and d z bar are equal. In hydro dynamic lubrication we took d x bar equal to d x z bar. If those are not taken in the equal, then we cannot take this as a common or we say that in this case we are taken both d z bar out and d z sorry d x bar and d z bar both out. Assuming d x bar is equal to d z bar otherwise we will not be able to take out. As we have already done in hydro dynamic lubrication mechanism or when we are solving this equation with a non dimensional numbers or in a non dimensional form of the Reynolds equation we already took those as assumptions ,we are just repeating here.

Now, this is a what? This is the whole non dimensional number I minus 5 square. I is a number x by d is a non dimensional is not going to have any dimension z minus three is a real number it is not dimensional. Now what is only dimensional here? Here the p i j is a dimensional x is a dimensional and e is dimensional. And if we try to non-dimensionalize it then, it will we will get deflection in terms of non dimensional number. We did normalization in hydro dynamic lubrication using this relation. We say the p bar that is the non dimensional pressure is given as p that is a dimensional pressure divided by six eta u x square by c cube. This way we non-dimensionalize the temperature we can substitute same relation here instead of p i j we can write p bar that is a non dimensional pressure into six eta u x square divided by c cube.

When we do that what we are getting now, this is a non dimensional number and this is a constant coming over here there is a six eta u x square divided by c cube and there is a capital x over here. That can be transferred outside. The whole this summation will turn out to be non dimensional number. That will be a once we know the pressure p i j, we can simply find out what will be the deflection.

This is a what am trying to convey over here when we are moving out and further deflection this is a what 6 is a number. One minus eta eta square is a number and when my mu square is a number, eta is a Pascal second u is a meter per unit second, the second and second will be cancelled out this will turn out to be Pascal into meter. Here it is a meter cube meter cube. So, meter cube and meter cube will be cancelled out still we have Pascal into meter and e is in a Pascal. So, Pascal Pascal will be cancelled out here it will turn out to be only meter and we are dividing by c to normalize it. So, to non-dimensionalize. So, meter divided by meter. So, whole this turn out to be non dimensional number.

Now, we have converted whole dimensional deflection in non dimensional deflection. So, this equation can be simply used in film thickness expression wherever the film thickness we have calculated. We can add this expression, we can find out film thickness distribution over a x direction and as well as z direction.

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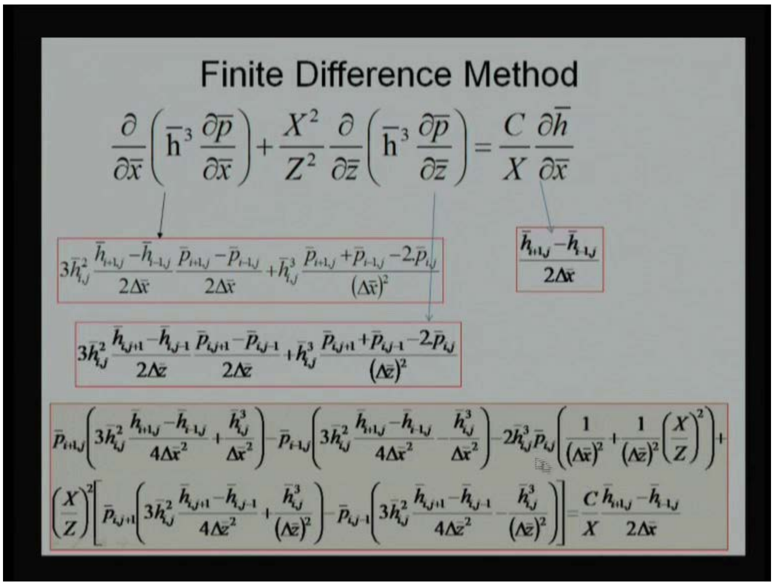

Now, we need a slide modification in a finite difference method which we have done in hydro dynamic lubrication. If you remember what we did? We converted finite difference method we use a finite difference method as center for central difference method and we evaluated h at the half node. Now, in a when we calculate h at the half node, deflection cannot be added in that case again we have to calculate deflection of the half node which is slightly more cumbersome. That is why we are trying to achieve this expression. So, that we know we do not require any half node calculation, we can go ahead with a full node calculation.

What is a meaning of that? Let us start with this first term there is a pressure distribution in x direction which depends on the film thickness and the pressure. What we are doing? We are going with the parts first we are taking differentiation of h. That will turn out to be three h square divided by d h by d x bar into d d d p bar by d x bar. So, first term will be that second term will be h bar cube out and this will be second differentiation of pressure.

So, we will be having two terms and we need to do central difference for the two terms. Now, you can see the first one we are first only integrating first h that is the three h square three h bar square and that is comes over here into d h by d x bar which is shown over here. H bar i plus 1 j minus i minus 1 j, this is what a central difference of film thickness gradient. This is a second term into not second term it is a multiplication factor d p by d x that is can be evaluated here. So, something like the p bar i plus 1 j p bar i minus 1 j divided by 2 x. So, this is going to be complete the first term, second h cube h bar cube is common that is shown over here.

And after that this is a second derivative of the pressure that can be simply given as p bar i plus 1 plus p bar i minus 1 minus 2 p bar i j and divided by delta x square. So, this is the for first expression in x direction, same thing for the z direction. This is the same thing we have just change only I and replace I with a j and j with I; you can see here three h i j square the same. Here is a h i plus 1 j instead of that here we are writing i j plus 1. Again somewhere here this is i minus 1 j, what we are writing here is i bar j minus 1. That means, we are taking variation along the z direction.

In previous case we assume that there is no variation in in film thickness along the z direction. Variation was only in along the x direction, but; however, we when we are calculating the pressure and pressure is deforming the surface and that will deform also in the z direction. It is nothing like the way need to avoid we need to account that also. So, this is given over here same letter is the term, in this will remain same it is not a much variation the way we have done a hydro dynamic lubrication same thing. Now, we substitute we get vey lengthy equation very big equation too many terms, but, we have to bear it. We we do not have any escape route for that. If you want to get a solution from the, for this kind of expression; Ah what is been done? Terms have been separated, you can see p i plus one j term p i minus one j term p i j term p i j plus term p i j minus one term. Here they have been separated out and in bracket we are writing in terms of only the complete node. It is not a half node the way we have done in hydro dynamic lubrication there was a half node while in this case we are writing complete terms complete node is a i plus 1 i minus 1 i j and similarly, j plus 1 j minus 1. There is no half term and interesting thing is that after rearranging, we are getting much lesser number of cubic terms. Cubic term is coming only in h i j we do not have any other cubic terms.

So, number of calculation will be reduced with this kind of formulation. The only problem with the this kind of formulation is that it is involving more number of terms compared to the terms which we have seen in hydro dynamic lubrication mechanism and we use a finite difference method in hydro dynamic lubrication. Now, we this can be rearranged because at any time we need to find out what will be p i j for given value of p i plus 1 p i minus 1 p p i j plus one p i j minus 1 and the source term. This is source term.

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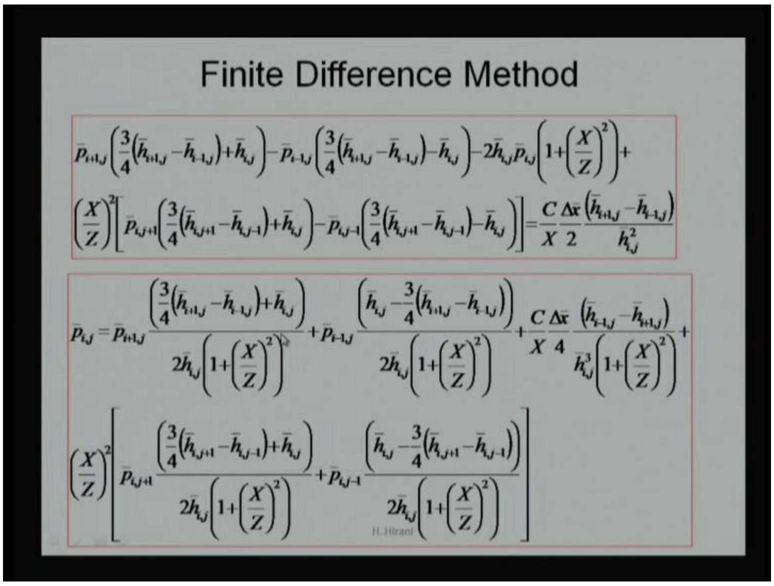

That's why we need to rearrange equation in a such a manner we can find out p i j. That is that that can be arranged something like this now. We had a smaller equation now is turning out to be much bigger equation right. So, because we have divided this one also 2 h i j and one plus x square by z square or everywhere we are divided here, we are divided here that is why the number of terms are increasing and this expression is turning out to be slightly lengthy. But, when we do computer programming it is not going to be much lengthier the compared to earlier expression what we have done we are going to get a good results without much problem.

Now, first point after doing this kind of derivation, first point will be that let us compare with this formulation and previous formulation without accounting elastic deformation.

In this case we have never we have not added any where elastic deformation for time being only thing what we are mentioned that film thickness will change.

And the deformation there will be more separation from the film thickness will increase that need to be accounted. But, yet we have not accounted here anywhere the delta 0. It should give me the same result as the hydro dynamic lubrication mechanism. If it is not giving the same result; that means, I have made some mistake in derivation I need to verify it again.

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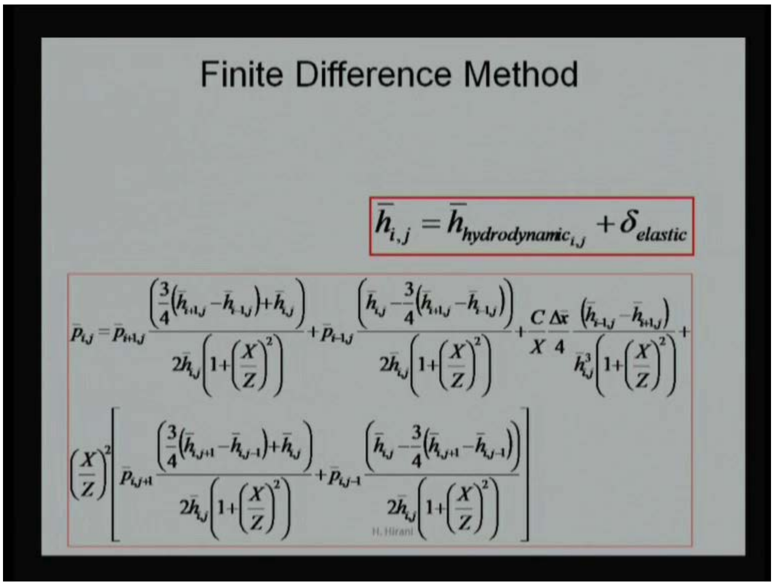

However what am trying to convey something like this. Here h i at any point will be given as a h hydro dynamic i j. That means, whatever the film thickness was calculated for hydro dynamic lubrication plus whatever the deformation is happening and that will be also in a deformation at the i j. Aat the every note will the deformation will be calculated and will be added in this case.

So, this will turn out to be more complex much more time consuming compared to the hydro dynamic lubrication. But, is still there is a point we can verify this full formulation keeping elastic deformation as a 0. Using this coding may be add with a elastic deformation become complete code.

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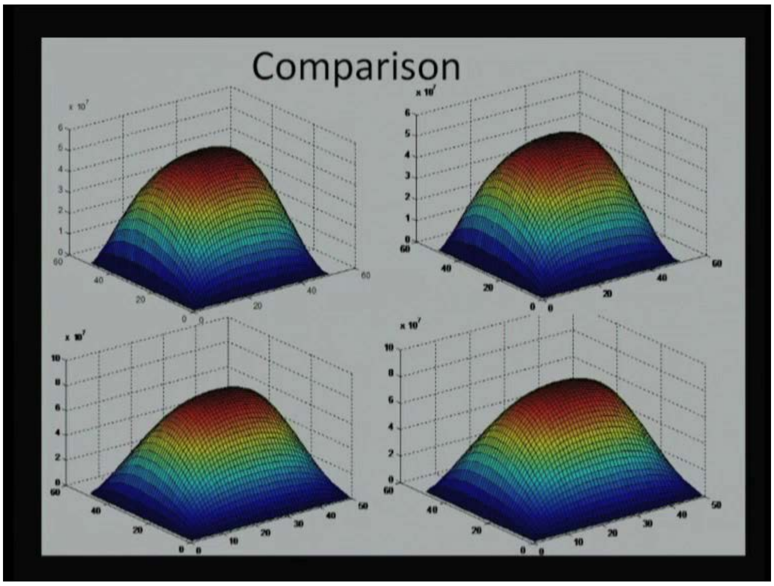

And after that use put everywhere elastic deformation as 0 and we can compare this new formulation which we have done with full node without accounting half node with earlier program which we have done with a half node. Now, that this slide is going to show the comparison is there is any change this is done with half node calculation. When we are counting pressure distribution and this cause effect of this viscosity on the pressure due to pressure; obviously, this viscosity thickening the first time, this viscosity thickening is not accounted while in second figure viscosity thickening is accounted.

Now, this is a with a previous formulation or the half node formulation. Now, this is with a complete formulation without accounting half node. We are using only the full node. I am not able to see any difference over here. This is out this viscosity thickening is also without this viscosity thickening. This is by using the using the expression for the half node this is using full node.

So, that means, w are not made any mistake in our formulation results are almost same not a single percent not a .1 percent change, 0.01 percent change in the results. Same thing with this also we are saying here the 4. Something, this is around forty bar pressure or we say that forty mega Pascal pressure here and the same thing over here.

So, we are not finding any difference in this curve and this curve. We here in this case we are not accounted this viscosity thickening effect. While on this case this viscosity thickening effect have been accounted and we are not able to find any difference in this results. That means, what is the formulation whatever we have done in previous slides that is all right is not going to change results, it is not affecting the results.

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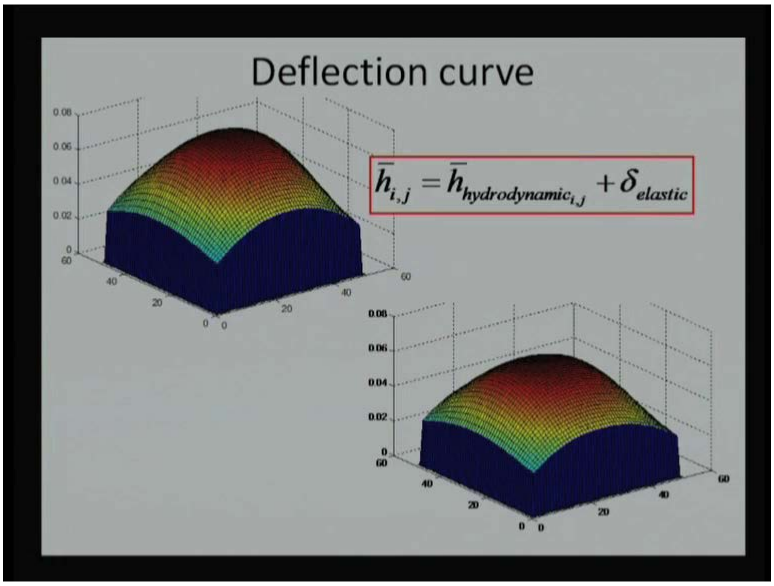

Now, the time has come to find out what will be the elastic deformation. So, this is elastic deformation. What is been done over here? We calculated. We calculate the pressure at the every node and simplify what is the deformation without going with a iteration loop, without accounting in the film thickness expression, this is been done only in once we know the pressure find out the deflection at every node.

Now, we know the 50 by 15 node. At the initial there will be 0 deflection because the pressure is 0. After that is going high and reaches to some value maximum. Now, if I compare with a non dimensional film thickness. Non dimensional film thickness we had a roughly 20 by 30 micron and 2 by 3 the 0.66 value at the exit and at the entrance it will it was a four by 3 it was 1.33.

Now, what we are saying here in that film thickness this will be added. So, whatever the two by three in that case the film thickness say this expression because this is going to add because of deformation, two surfaces are going to get most sufficient due to deformation and you account this kind of film thickness variation.

Now, what will happen if I increase the film thickness in any point? Pressure is going to go down, pressure will decrease and we require really iterative loop to come with a final results. What am trying to say, use hydro dynamic lubrication mechanism, use that algorithm find out the pressure at every node, from those pressure find out the deformation of the every node, redo it. Now, you can come back calculate the film thickness at every node use those film thickness in the expression to find out hydro dynamic pressure from hydro dynamic pressure again find out what will be deflection. And it do it need to go through a number of iteration to find out the final conversion solution.

Or what we are saying initially assuming elastic deformation zero; after that we know what will be the hydro dynamic lubrication film thickness and using this expression we find out pressure, finding after finding the pressure we find this elastic deformation we go back and change the film thickness expression.

And again redo it, again find out the hydro dynamic pressure, from hydro dynamic pressure again you find out the delta, from delta again come back to this equation, find out the film thickness a couple of iterations, not couple of iteration may be say hundred iteration plus it may give some solution and that is given over here. When we are not accounting iterations, we are not accounting in a change in geometry and after evaluating the pressure and here we are going through number of iteration and this is a final curve.

What we are able to see here the deformation after a number of iteration is again reduced. That is clearly the pressure will come down, reduction in pressure will reduce a deflection and that is why the overall deformation will turn out to be lesser. You can see here it is crossing the four and reaching may be say 5.2 or something like that .052 while here it is a much lesser then that even 0.038. So, there is a variation. What this means? That we need to account this variation in film thickness. So, that we can find out the right results otherwise it will be problematic.

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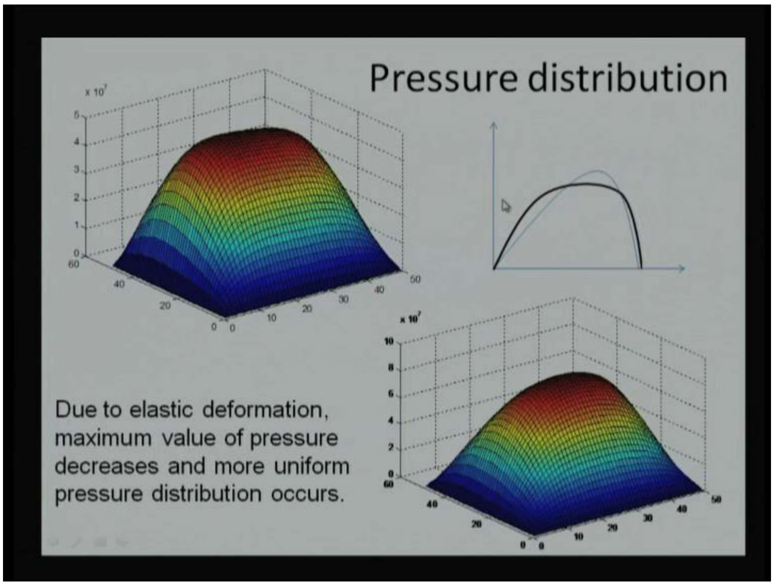

Now if we account the pressure distribution with elastic deformation; accounting elastic deformation what we are going to get is something this. Pressure distribution something like this shows a three point something as the pressure. And if we do not account pressure elastic deformation use only the pressure viscosity relation. Do not be account pressure deformation of the surfaces we are going to get much different results. You can see here when we are not accounting elastic deformation pressure shown over here is something more than four. When we are accounting pressure deflection due to pressure the pressure has come down.

Or another word; we need to account this kind of elastic deformation or we need to go through an elastro hydro dynamic lubrication to find out the right results, the right pressure distribution because the pressure distribution is a first point to find out the friction losses also.

If we do not account pressure, right pressure distribution the friction losses will be different, mass flow will be different and most of the parameter which are depend on the pressure will turn out to be different. It will not give accurate results. So, as far as possible if the pressure is high and this is going to the find the elastic deformation or its going to deform the elastically that surface we need to account.

What we say due to elastic deformation, What is the maximum value of pressure decreases and more uniform pressure distribution occurs is the meaning of that you can see there is a this stretch is much larger then this stretch or there is a more uniformed distribution. That means, load carrying capacity not going to decrease load carrying capacity of the surfaces will remain same only in the maximum pressure is going to decrease. And more surfaces are going to sustain same pressure. Or in other words if I say this is a without elastic deformation if I account elastic deformation pressure distribution will change something like this.

However, area under this cure is not going to change. That is what the load carrying capacity of this will remain same only. What is happening? The maximum peek pressure the peek pressure here is different than peek pressure over here or pressure is going to decrease in this case. So, this is a recommended case we say the elastic deformation of the surface is needed to be accounted properly. If we do not account we may we may not get good results.

So, we will continue with film full film lubrication mechanism. We will be considering energy equation because we have considered yet only elementary treatment of thermal heating. While we are going to consider now in a slightly deeper or more description will be given in next lecture.

So, most of the next lecture will be more focused on the energy equation, how we are going to account the temperature difference, how we are going to evaluate the temperature difference and how the temperature difference is going to change in the load carrying capacity. Thank you.