Tribology

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Lecture No. # 24

Viscosity Variation

Welcome to 24th lecture of video course on tribology. Today’s topic is viscosity variation. We want to incorporate viscosity variation in Reynold’s equation or at least get the feel of, need of that variation. How that variation is going to change the pressure distribution? To continue with this topic, I prefer to refer previous lecture slides, couple of slides, we say that we tried Reynold’s equation, and then we found hybrid solution, and to establish hybrid solution we discussed about the finite difference method.

Compare finite difference method results with hybrid approach and approach 1 and approach 2. We figure out that hybrid approach provides a very good result without any computational time compared to finite difference method. Of course, in addition to that we refer it is a computational time in these days is not very costly that is why we do not have to worry about the computational efforts, but at least the time but efforts are important how to develop computer program and how to proceed the iterations? Those things were discussed. And in previous lecture when slide was given on a viscosity variation with respect to pressure; so I am just going to repeat that slide.

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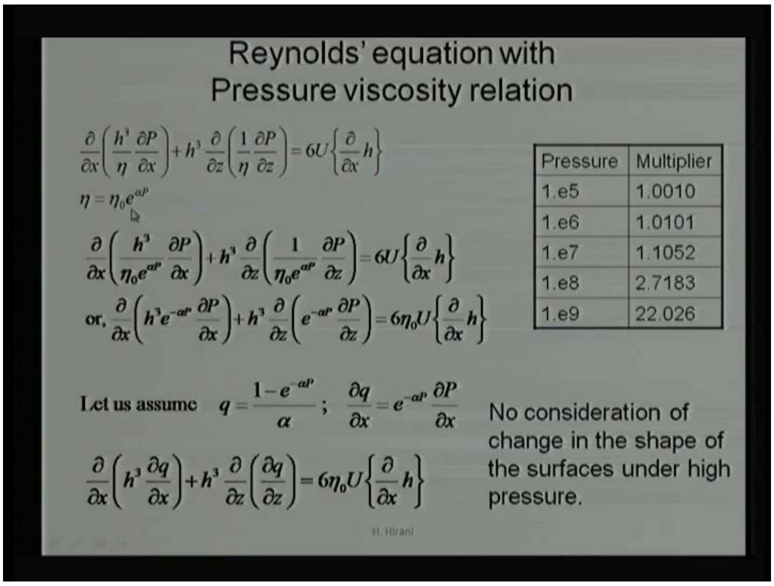

Reynolds' equation with Pressure viscosity relation

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\eta}\frac{\partial P}{\partial x}\right)+h^3\frac{\partial}{\partial z}\left(\frac{1}{\eta}\frac{\partial P}{\partial z}\right)=6U\left\{\frac{\partial}{\partial x}h\right\}$$

$$\eta=\eta_0 e^{\alpha P}$$

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\eta_0 e^{\alpha P}}\frac{\partial P}{\partial x}\right)+h^3\frac{\partial}{\partial z}\left(\frac{1}{\eta_0 e^{\alpha P}}\frac{\partial P}{\partial z}\right)=6U\left\{\frac{\partial}{\partial x}h\right\}$$

$$\text{or, }\frac{\partial}{\partial x}\left(h^3 e^{-\alpha P}\frac{\partial P}{\partial x}\right)+h^3\frac{\partial}{\partial z}\left(e^{-\alpha P}\frac{\partial P}{\partial z}\right)=6\eta_0 U\left\{\frac{\partial}{\partial x}h\right\}$$

Pressure	Multiplier
1.e5	1.0010
1.e6	1.0101
1.e7	1.1052
1.e8	2.7183
1.e9	22.026

Let us assume $q=\frac{1-e^{-\alpha P}}{\alpha}$; $\frac{\partial q}{\partial x}=e^{-\alpha P}\frac{\partial P}{\partial x}$

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial q}{\partial x}\right)+h^3\frac{\partial}{\partial z}\left(\frac{\partial q}{\partial z}\right)=6\eta_0 U\left\{\frac{\partial}{\partial x}h\right\}$$

No consideration of change in the shape of the surfaces under high pressure.

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So, in this case the Reynold's equation with pressure, where viscosity relation; in other words viscosity is a strong function of temperature as well as a pressure and this slide shows viscosity variation with a pressure.

Generally we use exponential law, we say that viscosity is viscosity (( )) pressure into exponential of alpha P depends on the pressure as a pressure increases viscosity is going to increase that is a simple case also we say that if any liquid is subjected to completion viscosity will increase. And using this relation this table was shown we say that when the pressure is a 10 raised to 5 newton per meters square or 10 rise to 5 pascal viscosity multiplied (( )) ambient pressure particularly this.

So, viscosity will be U 1 unit, I am not just multiplying with the (( )) stroke or (( )) or anything it is just that viscosity is an infinite unit. For time being we are using only multiplication factor then if the pressure increases by 10 times from rise to 5 pascal to 10 rise to 6 pascal, I will say 1 mega pascal. In that case, we were find able to find 1 percent increase in viscosity or multiplication factor is turning out to be 1.01.

So, 1 percent increase in viscosity. Further increases in the pressure by 10 times you say 10 mega pascal we are able to find 10 percent increase in viscosity. If I follow this pressure viscosity relation, many times we call as a (( )) relation and given by the authors and they have used extensially. As pressure further increases viscosity is going to increase we say that at 1 giga pascal pressure viscosity variation or viscosity increase will be at 22 times.

That is significantly increase in viscosity which is substantially positive variation and that need to be incorporated in Reynold's equation. This is a simple Reynold's equation it should be straight we are assuming only tenatial velocity in x direction and film thickness variation is also along the x direction. There is no viscosity film thickness variation in z direction. In addition, we are assuming the plates of the fixed they are not are not moving related to each other in y direction. That means, there is a no squeeze film term. If we substitute this relation in Reynold's equation what we get is something like this.

Instead of eta we are writing eta 0 e power alpha P (( )) we say that eta 0 will be constant, its atmospheric viscosity is constant. In that case it can be taken out and we can shift to the right hand side and this instead of 6 U we can write as 6 eta 0 U and this alpha or say e power alpha P can be rearranged and the numerator and in this case it will be e power minus alpha P, in x term as well as z term. And we need to rearrange the expression in such a manner that linearity comes back. While in this case we know very well that alpha P is here and P is here also.

So, this is a high non-linear term, it will be difficult to solve this term. So, that is why we require some sort of mathematical manipulation and rearrangements of situation. Say let us assume, one quantity q and name as reduced pressure and expression of reduced pressure is 1 minus e power minus alpha P divided by alpha. Coming to the advantage of this term when you differentiate q with respect to x, 1 will be differentiated to 0, negative will remain there and this when differentiated e power alpha P negative- negative will be cancelled out this will turn out to be positive.

And this will retain e raised to minus alpha P and this pressure term, pressure grating term. We require this term only here we are able to see e raised to minus alpha P into d P by d x and we are able to get equivalent term for that that is a d variation of q with respect to x, we can substitute over here and we can rearrange. Now this is the same equation as equation number 1 when we started with that equation.

Mathematically, there will not be any change to solve this pressure equation or this pressure equation, efforts will remain same except the boundary conditions because in this case the pressure is 0 at the boundary condition is again substituted to 0 here it will turn out to be 1 and q will also be 0.

So, pressure variation can be simply replaced with the reduced pressure variation and boundary condition also can be maintained the same boundary condition. And we can solve this equation with any of method which we have discussed in approach one approach two hybrid approach or finite difference method approach.

It is time to think whether when we are doing this kind of conversion from P to q or we are trying to incorporate viscosity variation will there be any change in results. As this table shows that as a pressure increases from 10 atmospheric to the 100 atmospheric viscosity variation initially increase in not substantial may be initially is in 1 percent to 10 percent then 171 percent.

So, we need to see these results. For that purpose, we have a couple of examples and of course, in this case it will before moving to the next slide. I want to emphasize there are no consideration of the change in the shape of the surface and high pressure is assumed or considered. We have not considered change in the film thicknes, we have not considered change in surface of the solids which are coming into conflict. However, in pressure we know very well the surface will deflect or surface will be deformed and film thickness variation will be there.

But we are not accounted for time being we want to go ahead one by one initially we are using only the viscosity variation.

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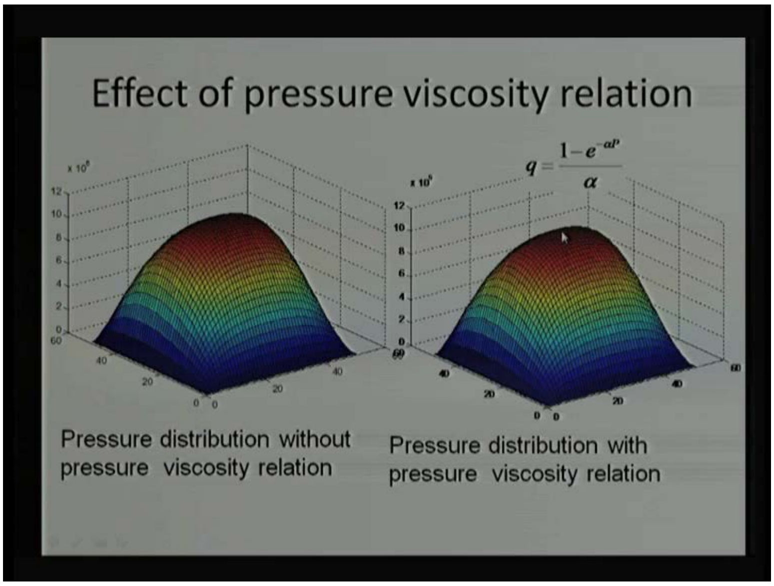

Effect of pressure viscosity relation

$$q = \frac{1-e^{-\alpha P}}{\alpha}$$

Pressure distribution without pressure viscosity relation

Pressure distribution with pressure viscosity relation

Now, this is pressure relation what do you see in this case earlier I showed a non dimensional pressure while here we are showing dimensional pressure. So, we know what are the what is the magnitude of the pressure what is the value of maximum value of the pressure in this case.

This is may be x direction, z direction we have same number of modes it does not matter much to us, but, this is the vertical x. So, this is z x in this case is. Obviously, in this same case particularly we can choose y x because we have selected x axis and z axis and along the axis and perpendicular to the axis this is the pressure axis.

Maximum value comes some roughly here is around 7 into 10 is to 5. Obviously, 7 bar or 7 atmospheric pressure. This clearly indicates the viscosity is not going to be very significant because earlier in the previous slide we showed that it increased by 10 times. If viscosity ratio increases by 10 times viscosity variation is only one percent and that is the what we are trying to understand from here.

Now, when we are accounting only the pressure variation, we are not accounting the reduced pressure, we are not substituting the relation eta is equal to eta 0 e to power alpha P. In that situation what we are gaining almost same value of pressure even if it is hardly any percent change in the pressure. Here we are using q or this chart is P but, we have used q in Reynold’s equation.

For the results of q and then we have finally, converted it in P. So, this is again a P relation or P axis, this is a P axis we can see the magnitude almost the same magnitude in both the cases. We say that in case we are accounting the pressure viscosity relation in this we have not counted pressure viscosity relation or magnitude change is not happening or we say what are the pressure in this case same pressure is happening in this case also, not significant variation.

So, when we do not get a change in the result and there is no point to account relation say that low load bearing cases, you should not account to this pressure or viscosity coefficients we can simply use this as in internal fluid having constant viscosity.

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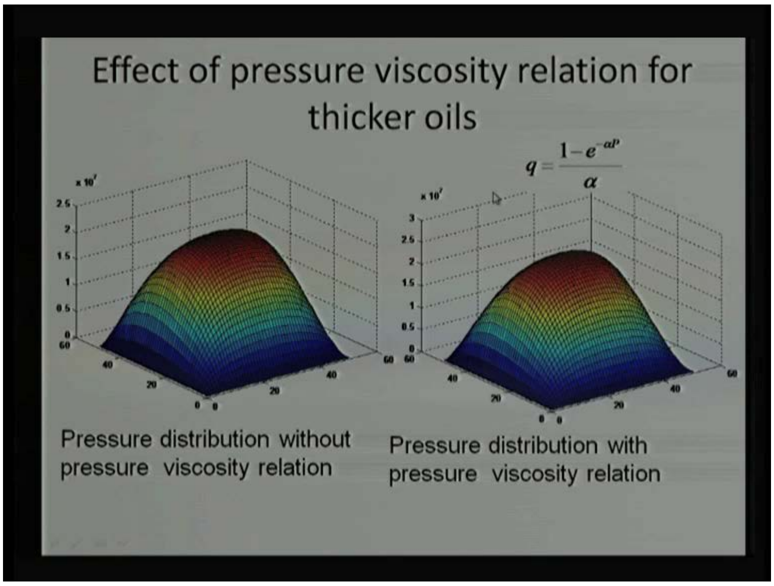

Effect of pressure viscosity relation for thicker oils

$$q = \frac{1 - e^{-\alpha P}}{\alpha}$$

Pressure distribution without pressure viscosity relation

Pressure distribution with pressure viscosity relation

However, there is a point you see if viscosity increases or if you are thickening the oil, we are going with a thicker oil. We we go for the thicker oil, we are able to see there is pressure is increasing in magnitude. In earlier case in the previous slide, it was 7 into 10 is to 5 now in this case we have increased viscosity by 20 times. It is earlier was a 10 mile pascal second here we are considering it to 200 mili pascal second.

And the viscosity of the lubricant has been increased by 20 times. This oil viscosity is itself increasing 20 times, then in this situation what we are able to get roughly 1.25 into 10 is to 7. So, there is a point or we say that in this particularly 12.5 mega pascal pressure comes. Viscosity will vary through may be say some extent 1 percent or slightly above than 1 percent.

And we should be able to get effect on pressure variation also. So, this chart is without accounting variation in viscosity and this chart is to show or account the variation in viscosity or in this case first q has been obtained from the Reynold’s equation and subsequent to that the pressure has been calculated using this relation and this vertical axis is representing the pressure variation.

We are able to see the difference, when we see in the case gap between the peak this red peak and this line is countable. While in this case this gap is reduced and its pressure is increasing or another word when we are accounting the pressure viscosity relation maximum value of the pressure is changed, it is increasing.

So, there is a point to account the pressure viscosity relation when maximum pressure is going to be more than 10 mega pascal. In this case particularly it is 1.25 into 10 is to 7 pascal or 1.25 into 10 that means 12.5 and 10 is to 6 that is the mega pascal. 12.5 mega pascal is the max pressure and we are able to see there is some variation some 12.5 going for 13.5 or 13.75 mega pascal there is a one, there is a change in pressure in this case.

Now, there is another thing to see, this is the only thing viscosity we can increase the viscosity by 20 times. We say the initially in previous example, viscosity of oil was 10 mili pascal second. While in this present example or in the present case we have considered this as 200 mili pascal second, 20 times increase in viscosity.

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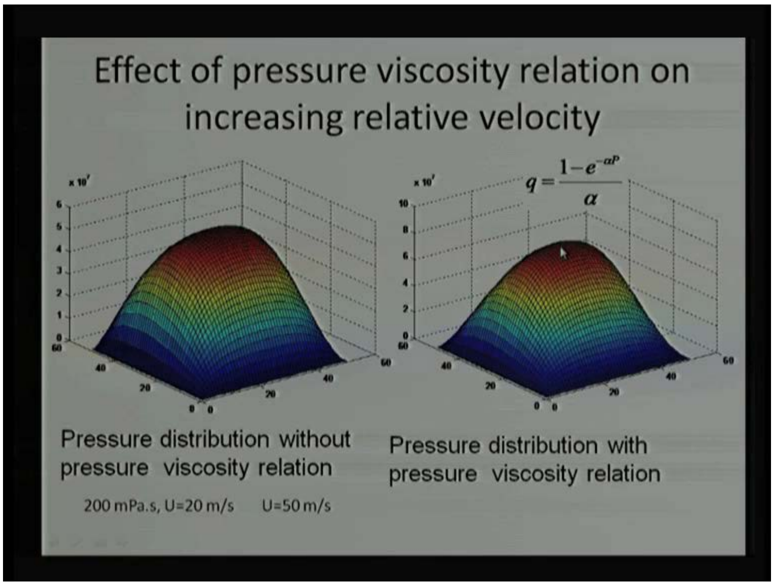

Effect of pressure viscosity relation on increasing relative velocity

$$q = \frac{1-e^{-\alpha P}}{\alpha}$$

Pressure distribution without pressure viscosity relation

Pressure distribution with pressure viscosity relation

200 mPa.s, U=20 m/s U=50 m/s

Now, we can see the variation with increase in velocity also. We know very well hydrodynamic reaction basically depends on the relative velocity. If velocity is continuously increasing then pressure will vary, a load carrying capacity of that tribo pair will increase. So, in this case we are trying to see the fact of relative velocity if I am increasing relative velocity what is going to happen.

With the increase in the relative velocity initially it was 1.25. Of course, we are maintaining the same viscosity which we have shown in previous slide. That means, 200 mili pascals second. Now, when velocity is increased initially this before the increase in velocity the maximum pressure was roughly 12.5 mega pascal without accounting pressure viscosity relation. While in this case it is turning out to be 35 to 37 mega pascal, that is 2 times. That means, the velocity has been increased by 2 times or 2.5 times.

Now, if we account we account the pressure viscosity relation we say that we in Reynold’s equation we use a q and once a q has been determined. We change to the pressure we convert to the pressure and that pressure is being plotted over here with x and z axis.

We are able to see the maximum pressure is crossing the four or is crossing the 40 mega pascal. While in this case it is not crossing them 40 mega pascal, there is substantial change. This is the what we tried to convey when we incorporate viscosity variation in Reynold’s equation there is going to be change in the pressure and if you do not account we are going to predict the low load carrying capacity compared to what it should be what it should be.

This is the reason why most of our human body joints are made with elasto hydrodynamic lubrication where the pressure viscosity coefficient is also comes into picture and we need to see the thickening effect or viscosity thickening effect in those situations.

We are able to see here the clear difference the more than 4 may be say 4.2 or 4.3. While here it is a lesser than that its 3.5 or 3.7 and substantial change in pressure in this case. Here we are saying that viscosity, we have maintained 200 mili pascal second initially viscosity velocity was 20 meter per second. Now velocity has been increased by 2.5 times its turning out to be 50 meters per second.

Of course, in this situations we have not accounted change in the geometry due to pressure. We know very well under high pressure, everybody is subjected to deformation. If we account that deformation film thickness will vary, film thickness will increase, film thickness increases, then load carrying capacity will come down or pressure will come down.

So, to some extent they are self adjusting first one cause is the pressure increase deformation will be subsequent to that of, deformation will occur because of that. And that because of the deformation film thickness will increase because of the film increase in the film thickness pressure will decrease. So, there will be self adjustment or what we are predicting because of this viscosity there is a possibility when we account to deformation pressure again will come down to some level.

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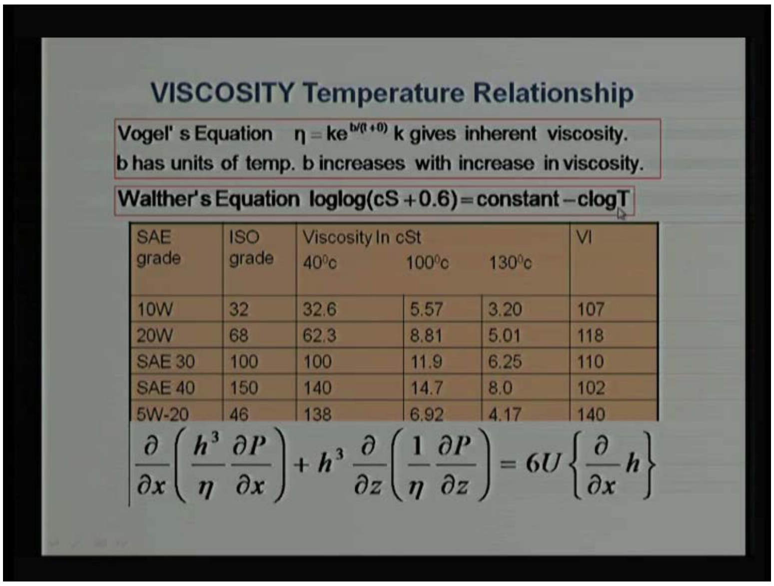

VISCOSITY Temperature Relationship

Vogel' s Equation $\eta = ke^{b/(t+\theta)}$ k gives inherent viscosity.
b has units of temp. b increases with increase in viscosity.

Walther's Equation $\log\log(cS + 0.6) = \text{constant} - c\log T$

SAE grade	ISO grade	Viscosity In cSt 40⁰c	 100⁰c	 130⁰c	VI
10W	32	32.6	5.57	3.20	107
20W	68	62.3	8.81	5.01	118
SAE 30	100	100	11.9	6.25	110
SAE 40	150	140	14.7	8.0	102
5W-20	46	138	6.92	4.17	140

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\eta}\frac{\partial P}{\partial x}\right) + h^3\frac{\partial}{\partial z}\left(\frac{1}{\eta}\frac{\partial P}{\partial z}\right) = 6U\left\{\frac{\partial}{\partial x}h\right\}$$

Now, this was related to pressure viscosity relation. We know very well, we have studied when we were discussing about the liquid lubricants. The liquid lubricants are a strong function of temperature with increase in the temperature. Viscosity of liquid lubricant decreases and we give a two relation we discussed, they are Walther’s relation and Vogel’s relation.

What about the Vogel’s relation? It was the viscosity of dynamic viscosity is equal to some viscosity of the ambient temperature. Again exponential, this is another a constant and theta another constant. And in this case t is the temperature in centigrades or we can say how k is interpreted that it is the inherent viscosity may be 20 degree centigrade or 20 degree centigrade what will be the reference temperature which we are accounting initially.

Now, if I account, if I want to substitute this viscosity relation in this Reynold’s equation then what will be the problem. If I substitute directly I will find when there is the temperature viscosity is not represented in terms of pressure here but, it is represented in terms of temperature and in Reynold’s equation we do not have any temperature account.

This is the reason why we required an energy equation to be accounting to be incorporated where the pressure and temperature relation may come one way or the another. There is some sort of coupling between pressure and temperature that can be accounted and then this one simultaneous solution of pressure and temperature. We can find out what will be the variation in viscosity with respect to temperature.

That is slightly difficult slide but, we can drive those equation. So, before coming to that we will try to take very simple case which is more popular and people initial of the first case they start with that kind of relation. This is what I showed us this slide during the discussion with the lubricant or discussion on the lubricant, we said they are different sacred oils they are multigrade oils, monograde oils, these two are the monograde oils this is multigrade oils.

And what we express that viscosity will vary at 40 degree centigrade suppose, this is 32.6 centistoke viscosity, a 100 centrigrade we are able to see that the significant decrease and at 130 degree centigrade further decrease in viscosity and that need to be accounted.

And most popular method, they mentioned from measurement point of view because mostly the viscosity is measured in centistoke and that kind of viscosity is not viscosity we are using in Reynold’s equation. And it has a log log clogt you say there is a using Walther’s equation. First good thing is that we have only two constant, while using Vogel’s relation we have three constants k b and theta there are three constants, while Vogel’s equation has two constants namely this constant and c as a constant.

What is the difference in Vogel’s relation we have to substitute temperature in terms of centigrades, while in Vogel’s equation we can substitute in terms of kelvin that is a 273 plus centigrade will be temperature in centigrade will be expressed over here.

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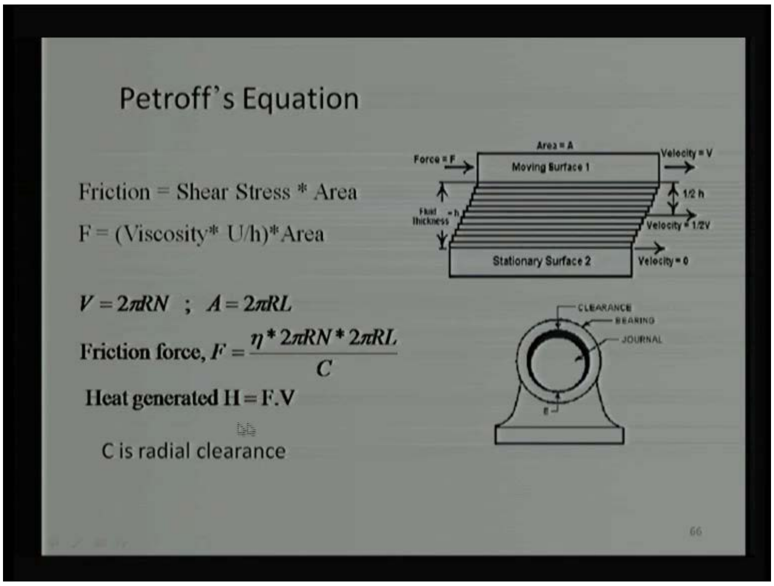

Petroff’s Equation

Friction = Shear Stress * Area

F = (Viscosity* U/h)*Area

$V = 2\pi RN$; $A = 2\pi RL$

Friction force, $F = \frac{\eta * 2\pi RN * 2\pi RL}{C}$

Heat generated H = F.V

C is radial clearance

So, when we incorporate these relations we can see the variation in temperature. Obviously, variation in viscosity with the temperature, but, we need to temperature in one way or the another either in Reynold’s equation or with some other accessory equation. We see one of the equation is energy equation where we can find out heat is generated? How much heat is converted? How much heat is connected away?

Well simple or most common use in the beginning people generally prefer that is Petroff equation. Petroff has used a very simple formula you see that how to calculate friction because temperature comes because of the heat generation and heat generation comes because of friction. And friction can be represented with a shear trace of liquid into the operating area where ever the this operation is happening.

So, area into sheer space and (( )) can be represented as viscosity into the velocity gradient here the velocity is U or V whichever is the because most of the Petroff relation show that the he represented velocity as V not U.

But in this case we are use U by h viscosity, of course, it can be interchanged easily. We can see here what are the diagrams indicates velocity is a V, there is gap film thickness and there is a velocity V here.

But here the velocity is 0. That means, relative velocity is v and this is what Petroff has developed in this equation. It was generally from one (( )) and we can develop a similar kind of relation for all other bearings or we can say that they open up this complete circumference the old circles, there we can open it up and develop a surface something like this. So, we can calculate what will be the friction force at the centre of S. So, F friction force can be given in terms of viscosity in terms of velocity per unit film thickness into area.

Now, for this kind of geometry we can say that pi d n, velocity is a pi d n with a rotational speed of the shaft which will be accounting. Surface speed or of the shaft will be pi d n or 2 pi into R, R is the radius of the shaft. In this case and area can be given as a pi d l which again the whole circumference into the length of the shaft which is a within a bearing. So, area which is really sharing the liquid is can be is given as 2 pi into R into l. Now we substitute this relation what we find here friction force is equal to viscosity or we say dynamic viscosity into 2 pi R N, this is the velocity.

In into the service area which is responsible for the sharing of the liquid as a 2 pi R into l. l is the length of the bearing or the where the shaft is shearing the liquid particularly that area and here the C is the clearance between the two surfaces geometric clearance or particularly radial clearance when we are referring this kind of tribo pair. So, when we know the frictional force we can simple find out heat generation we know that the friction force into velocity to initial velocity is going to give us heat generated what is the heat generated at that interface that can be calculated easily.

Now, once we know the heat generated we need to find out how this heat has been conducted away. So, we are able to see that from this situation we are using additional equation it is not Reynold’s equation. In this case we are not using a Reynold’s equation at all. We are using additional equations simple algebraic equation we know the frictional force is sheer stress into area and shear stress for the liquid can be represented in terms of viscosity into the velocity gradient and to area and we have represented those values. We finally, figure out heat generated is additional force into velocity.

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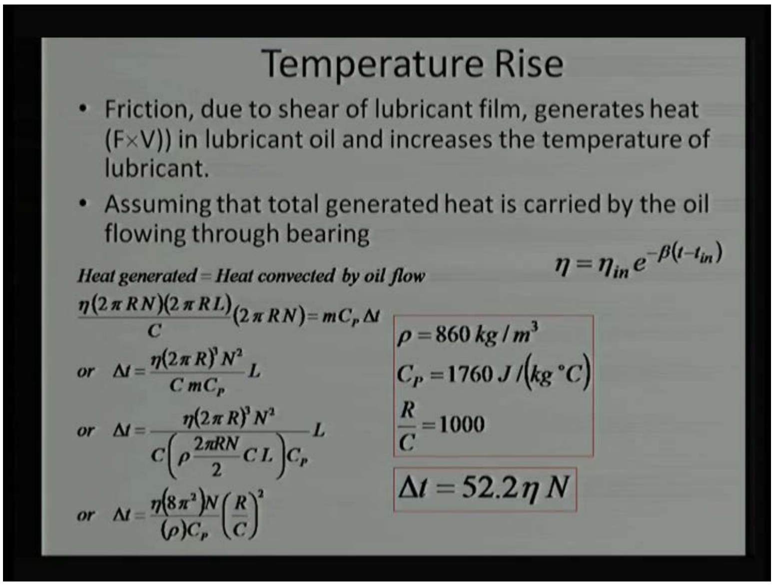

Temperature Rise

- Friction, due to shear of lubricant film, generates heat (F×V)) in lubricant oil and increases the temperature of lubricant.
- Assuming that total generated heat is carried by the oil flowing through bearing

$$\eta = \eta_{in} e^{-\beta(t-t_{in})}$$

Heat generated = Heat convected by oil flow

$$\frac{\eta(2\pi RN)(2\pi RL)}{C}(2\pi RN) = mC_p\Delta t$$

$$\text{or} \quad \Delta t = \frac{\eta(2\pi R)^3 N^2}{C\,mC_p}L$$

$$\text{or} \quad \Delta t = \frac{\eta(2\pi R)^3 N^2}{C\left(\rho\frac{2\pi RN}{2}CL\right)C_p}L$$

$$\text{or} \quad \Delta t = \frac{\eta(8\pi^2)N}{(\rho)C_p}\left(\frac{R}{C}\right)^2$$

$$\rho = 860\,kg/m^3$$
$$C_p = 1760\,J/(kg\,^\circ C)$$
$$\frac{R}{C} = 1000$$

$$\boxed{\Delta t = 52.2\eta N}$$

As I mentioned that temperature rise can be determined by finding how much heat is conducted away or convicted away. If full film bearing and mass circulation continues flow of the liquid most of the heat will be conducted convicted away by the lubricant. There will not be heat convicted away because there is a continuous mass flow rate and if that is the situation, we can take a very simple case we say that whole of the heat is convicted away by the liquid lubricant and we need to account the properties of liquid lubricant.

We can assume the density of the lubricant is 860 kilogram per meter cube. Slightly lower than water density this is the heat capacity the 1760 joule per kilogram centigrade. This is one of the very common figure uncounted of all the liquid lubricants if you do not have anything we can go up with this kind of heat capacity. And another thing we have to use it in Petroff equation we have just taken assumption that radius, geometry sharp radius of a bearing radius is much larger than the clearance between the surfaces or clearance at the interface.

That is also genuine because that this ratio is not very high Reynold’s equation cannot be used. So, to make sure the Reynold’s equation is applicable we can find out this we can use this assumption which is R by C is 1000, it can be lesser, it can be more or it can be 900, it can be 1100 depends on the different configurations, but, just for the first initial exposure or to get a feel of the temperature rise at appearing interface of the type of interface we can assume this data.

So, in this case what we are seeing that heat total heat generated was earlier in previous slide calculated and whole heat is conducted convicted away by the oil. So, heat generated in this case is related to viscosity, related to the velocity. Velocity is coming to two times, the velocity squared 2 pi R N here and 2 pi R N here and this is the area of surface area and clearance.

While heat convicted is mass fluorite whatever we assumed into heat capacity into temperature difference. So, with this we can find out what will be the temperature difference. If we know the mass florate if we know the C p, the mass florate can be figured out depends on the volume available into density. Density is here rho and what are the volume available which are the pi d n into clearance into length.

That is the volume and we assume that only half of the surface is convergent and half of the divergent. So, we are taking 50 percent of that and this is what we say the mass fluorite can be represented in terms of the geometric parameter. Is of course, basically derived by the velocity or rotational less speed if we are feeding then some pressure then we should add that term. Here in this case basic assumption is that there is no feeding pressure, there is no supply of pressure to the liquid.

Is a happening at the suction impact which is generated by the velocity because of rotation and rotation itself is pulling the liquid in tribo pair or in this interface. If I say that some liquid is open or the hole is open to the liquid and due to rotation there will be some sort of vacuum creation there will be some section creation and that liquid is coming in the bearing because of that, coming in this geometry because of that.

So, this is mass fluorite, heat dissipation and then temperature difference can be given by this relation. Now we know very well that most of the data are known to us in this case R by C. we are already assuming C p is for liquid then the constant value can be used directly here density terms and eight pi square is simple constant.

So, we can substitute these values and can say that temperature rise can be given in terms of viscosity and in terms of rotational speed. Of course, if there is a no radial bearing or we say that radial geometry is not there then that case we should use a tangential velocity for the term.

If we substitute this value what do we get temperature difference is given as 52.2 times viscosity into rotational speed. Now we can clearly understand from this relation if viscosity is increasing temperature rise is going to increase. That is very interesting thing, increase in viscosity is going to increase the temperature rise, increase in temperature rise is going to decrease viscosity.

So, it will be a iterative loop to find the sum of what is the result. Higher the viscosity higher will be the temperature rise, higher the temperature rise lower will be the viscosity. So, it is iterative loop, it is a closed loop system. In addition what we say here the velocity or rotational velocity, see that if the n is higher temperature rise will be on higher side. That is the genuine and we know very well and most of the tribo pairs if there is a high sliding velocity then temperature rise will also be higher side.

Of course, in the previous slide there is another release has been presented. So, it will say the simplified Vogel’s relation here. We have seen that instead of k we are writing n at the initial level in light temperature and t is inland temperature. Obviously, the ambient temperature, here b is only the constant if we know the 40 degree temperature or the environment temperature or we can directly use viscosity in addition to that there will be one additional constant as a beta. We can figure out the viscosity is known for some other temperature.

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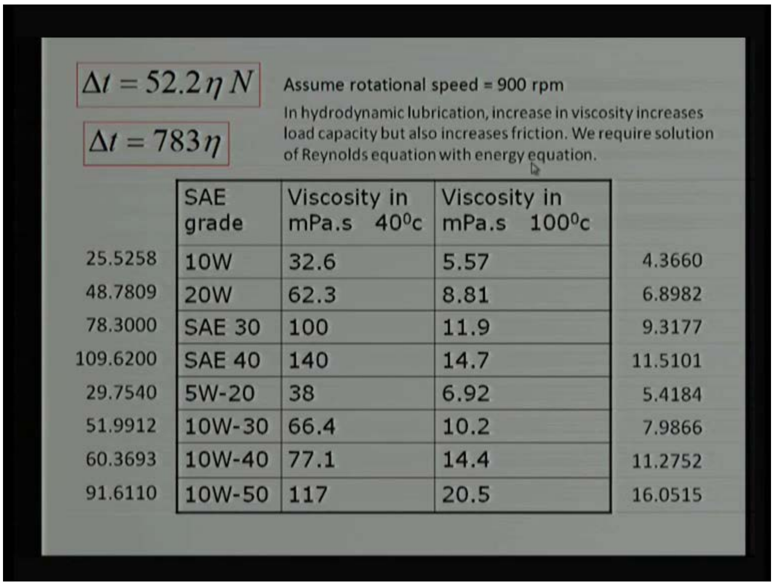

$$\boxed{\Delta t = 52.2\eta N}$$

$$\boxed{\Delta t = 783\eta}$$

Assume rotational speed = 900 rpm

In hydrodynamic lubrication, increase in viscosity increases load capacity but also increases friction. We require solution of Reynolds equation with energy equation.

	SAE grade	Viscosity in mPa.s 40⁰c	Viscosity in mPa.s 100⁰c	
25.5258	10W	32.6	5.57	4.3660
48.7809	20W	62.3	8.81	6.8982
78.3000	SAE 30	100	11.9	9.3177
109.6200	SAE 40	140	14.7	11.5101
29.7540	5W-20	38	6.92	5.4184
51.9912	10W-30	66.4	10.2	7.9866
60.3693	10W-40	77.1	14.4	11.2752
91.6110	10W-50	117	20.5	16.0515

Now, if you understand further we can simplify we say let us assume N is at 900 r p m and substitute that what we get from this. Of course, we have converted to r p m and then covert found this figure. So, the temperature rise is equal to 783 into viscosity and this viscosity will be in pascal second. We have represented viscosity here in mili pascal. So, this viscosity will be in pascal seconds and we can see the effect of this variation. When operating temperature is this or viscosity is this and we try to find out what is the temperature rise. We are able to judge the temperature rise something like this.

So, the viscosity is 32.6 mili pascal second you convert it you are able to see the temperature rise at 25 degree centigrade for this viscosity increase in the viscosity temperature rise is increasing. You can see here that 140 mili pascal per second is causing the temperature rise goes to more than 109 degree centigrade.

Initially, there will be drastic reduction during operation. However, if we account actual temperature or we say that if I account the temperature is 100 degree centigrade, then we are trying to figure out based on that what is the effect of this viscosity. What we get as the temperature rise is this much. That means t in we are assuming 100 degree centigrade and when we are accounting this viscosity, we are able to find only temperatures 4.3 or 4.4 centigrade. To complete this 4.4 degree centigrade, 25.5 degree centigrade when we account in this viscosity keeping everything else all the geometry parameters same, all rotation speed everything is same.

With this viscosity temperature rises this much, with this viscosity temperature rises this much. All other parameter remains same and that is also visible how it can be judged form this relation.

Similarly, in we go for the high viscosity say 20.5 percent centistokes. Then we are able to see that this is 16.5 temperature rise. While here 117 and temperature rise is 91.6. Only one point to be kept in the mind is that the economic viscosity and we need to account the density of the liquid when we use this temperature rise the fact.

That is important and another important which we going to emphasize is that there is a significant change in a temperature rise because there is change in the viscosity we should account this viscosity variation in the Reynold’s equation. We need to really drive the energy equation to consider these results. We do not use the energy equation and assume there is isothermal analysis there is no temperature change or there is no variation in temperature may not get good results.

That is what we say in hydrodynamic lubrication increase in viscosity increase in load capacity, but, also increases of friction. We require solution of the Reynold’s equation with energy equation by solving Reynold’s equation alone we will not get the good results. We need to use some supporting equation, in this case we showed this effect or effect of the temperature rise using Petroff equation, but, in reality we need to account all the facts we need to account to connect energy connected away, convicted away and may be if there is a possibility of radiation in some environment we need to account those. And then find out oral with temperature rise and based on the temperature rise we need to figure out what happened to the viscosity which is going to affect overall pressure distribution.

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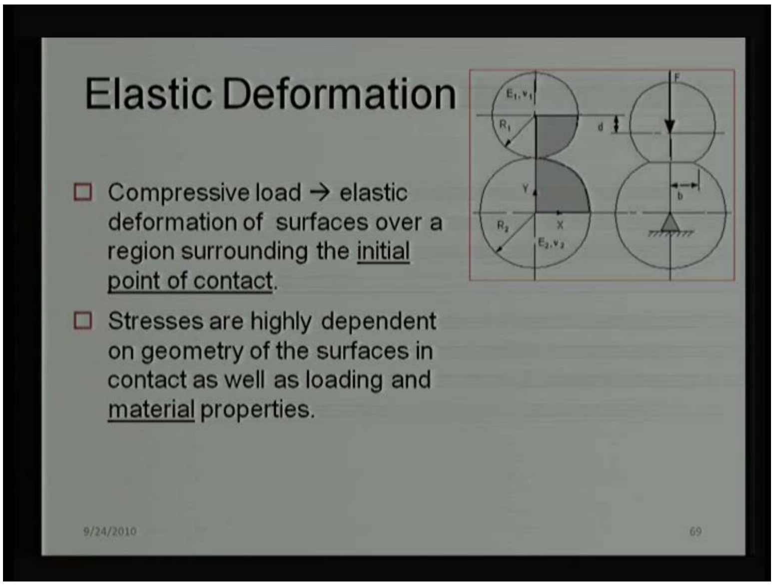

Elastic Deformation

- ☐ Compressive load → elastic deformation of surfaces over a region surrounding the initial point of contact.
- ☐ Stresses are highly dependent on geometry of the surfaces in contact as well as loading and material properties.

This was related to viscosity variation and viscosity variation with the temperature, viscosity variation with the pressure, but, when we were discussing viscosity variation with the pressure we say that we are not accounting the change in surface geometry change in the film thickness change in, obviously, the deflection of the surfaces.

Without that it is only the half treatment giving to the elastrodynamic lubrication to account both the cases viscosity variation as well as deformation. We need to find out what will be the induced deformation and conditions. That is why the slide comes, you see that we need to find out what will be the elastic deformation of the surfaces.

Initially we are taking very simple geometry say sphere on a sphere or cylinder on cylinder and more or like we have a convex geometry. We can after driving the equation we can extent this to the concave geometry also or more like flat geometry also. What is the meaning of this elastic deformation we say that in under compression loads surfaces will be subjected to elastic deformation.

Over a region, very narrow domain particularly near the initial point of counting, what I am trying to convey here is one is faced here, other is faced here and E is young’s modulus of one, nu 1 is poisson ratio of one and this is r 1 is radius of this fare one; same parameters for the sphere 2 e 2 and u 2 and r 2. Now we are subjected to the load what will happen there will be a deformation of surfaces. They will undergo the elastic deformation and you say that it is not only one surfaces subjected to elastic deformation both the surfaces are subjected to elastic deformation and this is a force. Now question comes is this really when they come then only they get deformed? No. Even the liquid film from which is generating the pressure if you apply the pressure on a surface we can deform the surface.

So, it is not necessary solid to solid deformation occurs, but, the liquid to solid deformation also occurs. However, hertz was the first person to develop this kind of relation and he mentioned deformation due to solids. That is why we are just repeating those things over here. Otherwise liquid also can deform the surface we need to account that deformation or the elastic deformation which is going to finally, generate elasto hydrodynamic lubrication mechanism which is the most demanding, most optimum lubrication mechanism.

Now, second line of this slide says that, if this is highly dependent on the geometry of the surface. We can see here very clearly here the geometry is sphere to sphere, but, we change the sphere to the cylinder on cylinder. We will be able to find different stresses. Geometry is playing a very important role when we come to the elastic deformation. Otherwise in ordinary mechanical engineering we assume that geometry is not going to play that much important role as material property are going to play an important role.

Obviously the elastic deformation generally the young’s modulus, poisson ratio will play an important role compared to the shape of geometry. While in this case we are going the geometry is going to play the important role even if we try the relation for sphere on a sphere and cylinder on a cylinder there will be variation in expression itself.

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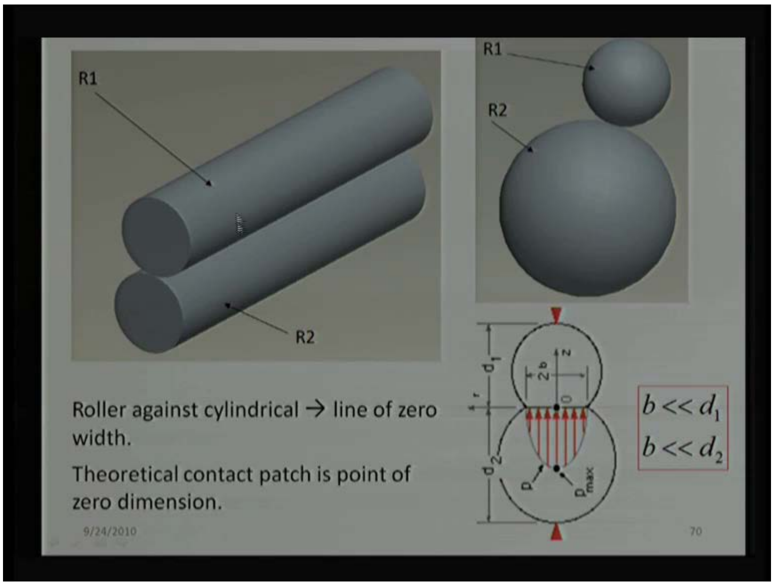

Roller against cylindrical → line of zero width.

Theoretical contact patch is point of zero dimension.

$b << d_1$

$b << d_2$

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Now, these are the two examples, as I say this sphere on a sphere and just for the nice sphere I can say this is a nice picture may be some sort of cartoon. Of course, for our case we assuming this is sphere 1 and this is sphere 2; while in this case the cylinder 1, cylinder 2, we are subjecting to the load. So, that there is a deformation of the surfaces. Let us take a first example, says his sphere on a sphere when I subject to the load there is elastic deformation. This is again not on the scale we are showing this deformation. So, that we can demonstrate what will be the pressure variation. However, this deformation is very very low.

That is what we say that this b otherwise in this deformed zone which is having dimension 2 b in general case in real practice is much much much lesser than dimension of this sphere. It is not even matchable, it is not even comparable not, even one percent of this. It will be much lesser than this, I will say that this kind of deformation agree localize. Particularly, when there is a convex geometry there is a non convex or concave geometry then deformation will be more, but, pressure generation will not be that high.

While in this case, the deformation is a low always to the extent of the deformated zone is low, but, maximum pressure will be very high and this max pressure can go even a giga pascals. And these are the words we say initially when we read any book what we get the roller against the cylinder when we say cylinder against the cylinder when they are subjected to load they will not be any with counting there will be only line counting.

But that is those are the ideal cases in reality does not happen, but, we will subject it to elastic deformation and there will be some sort of elliptical counting instead of line contact. When come to this sphere again in many books, we see that sphere around a sphere this gives only the line contact but, in reality it is a point contact generally a circular contact and we need to account this kind of contact as this is given over him in this case the perpendicular to this also.

Dimension is b or 2 b, then it will be naturally sphere on a sphere, circular contact while perpendicular direction if the contact area is contact patch is at 2 a and a is different than b then we can say that is elliptical contact.

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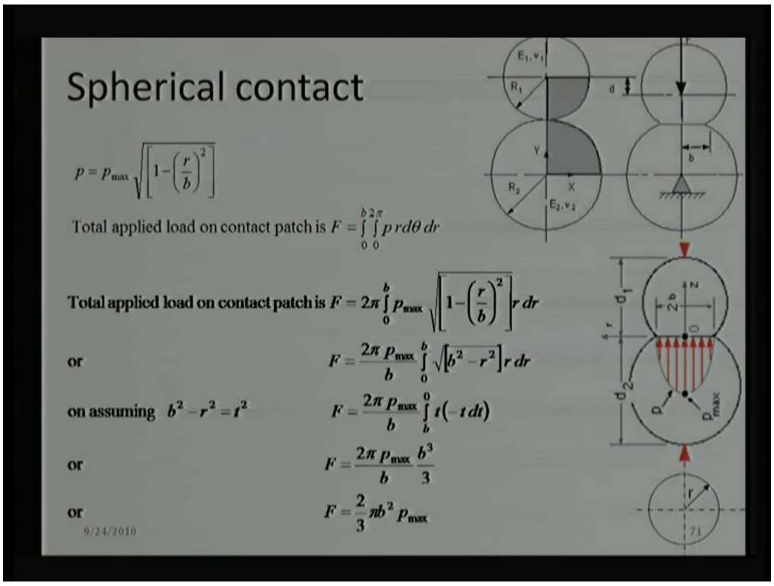

Spherical contact

$$p = p_{\max}\sqrt{\left[1-\left(\frac{r}{b}\right)^2\right]}$$

Total applied load on contact patch is $F = \int_0^b \int_0^{2\pi} p\, r\, d\theta\, dr$

Total applied load on contact patch is $F = 2\pi \int_0^b p_{\max} \sqrt{\left[1-\left(\frac{r}{b}\right)^2\right]}\, r\, dr$

or $F = \frac{2\pi\, p_{\max}}{b} \int_0^b \sqrt{\left[b^2 - r^2\right]}\, r\, dr$

on assuming $b^2 - r^2 = t^2$ $F = \frac{2\pi\, p_{\max}}{b} \int_b^0 t(-t\, dt)$

or $F = \frac{2\pi\, p_{\max}}{b} \frac{b^3}{3}$

or $F = \frac{2}{3}\pi b^2\, p_{\max}$

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Now, we are able to see there is a parabolic variation and we can take this as an assumption. So, that p is equal to pressure distribution in contact can be given a p max square root of one minus r by b square, b is a maximum value of r, r is the starting from 0. that is shown over here, we see when there is a contact patch if I take a top view of that and it will be a circular and this radius like a solid circle like radius can vary from 0 to maximum value of the b.

Whole pressure will be generated in this domain only. It will not be more than that it will not be lesser than that if only thing the boundary cases pressure will be 0 and midpoint pressure will be maximum as a parabolic pressure and that why we are taking this assumption p is equal to p max is square root of 1 minus r by b square.

To find out, what will be the this value of the b and what the maximum pressure we can take a help of the load which is have been applied which is have been we know the load how much load is being applied and we know the geometries. So, we can integrate it comes from 0 to b if I am assuming on one side and there is a whole circumference.

So, it will vary from 0 to 2 pi. It is 0 to b it is the radius and 0 to 2 pi will be to the 10 to and this the elemental area r d theta into d r. If we substitute this p max, p max will be common which the constant is and this is represented in terms of r by b. So, we can get the integration easily.

This is what is being given over here; we know whole expression does not depend on theta. So, integration with respect to theta is simple it will be theta outside and substituting value of theta as a 2 pi minus 0. It will be the 2 pi it will be the 2 pi will be common and v max this is in terms of r under the d r.

So, we can take p as a common again and p max as the common form this whole expression is b square minus r s square into r into d r. Now, we can substitute some value for the b square and minus r square that is what we assumed over here b square minus r square is equal to t square. Now, we can differentiate and substitute the value instead of d r we can substitute value of t and that is being given over here. So, that will be minus t squared d t.

And then this boundary conditions will change wherever the value of the b it will turn on to b, t is equal to 0 and wherever the value 0 well be t will turn out to be b and we are assuming these is no negative dimension, there is no negative patch. Minimum value of this contact is 0, it is not negative. So, we can find out from that angle and substitute we can drive the expression based on the pressure.

If I know the load applied load I can find out in terms of b and maximum pressure if I know what will be the hardness or at which pressure this surface is going to deform I can find out directly b value. Form this expression, where p max is known to me I can directly load is known, p max is known, b can be simply figured out from this relation. How many times we do not know the value of p max at which pressure this surface is going to get deformed in this situation we need to use additional relation what we sue the deflection relation maybe we can take a simple formula from the material books.

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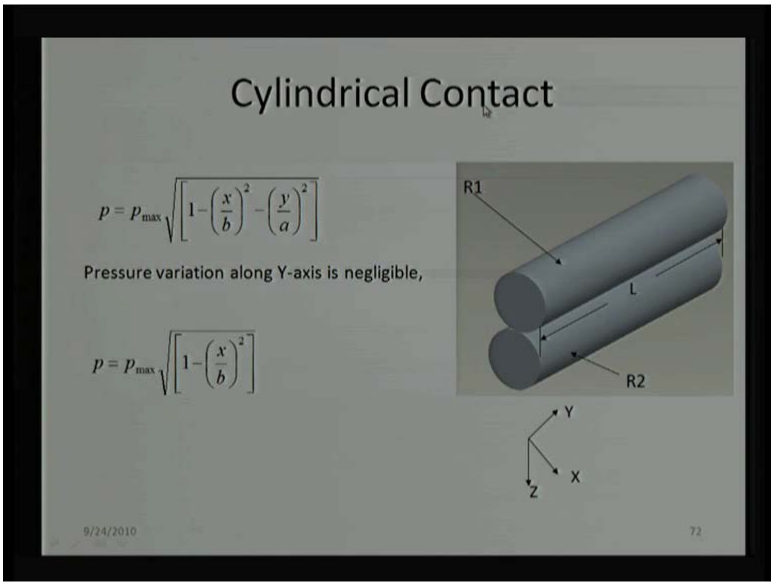

Cylindrical Contact

$$p = p_{\max}\sqrt{\left[1-\left(\frac{x}{b}\right)^2-\left(\frac{y}{a}\right)^2\right]}$$

Pressure variation along Y-axis is negligible,

$$p = p_{\max}\sqrt{\left[1-\left(\frac{x}{b}\right)^2\right]}$$

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And we can write based on that, however, we have discussed only when a spherical contact. In this case what we are showing is cylindrical contact also. So, in cylindrical contact there will be two axis circumferential as well as the longitudinal axis in this case that is why x by b and y by a square comes over here. However, pressure variation is not going to be significant, but, this geometry or this circumference is much smaller than this length. So, this pressure variation along the length is negligible if I am assuming, if we are assuming in this situation then what will be relation p is a same the way we have represented for this spherical contact.

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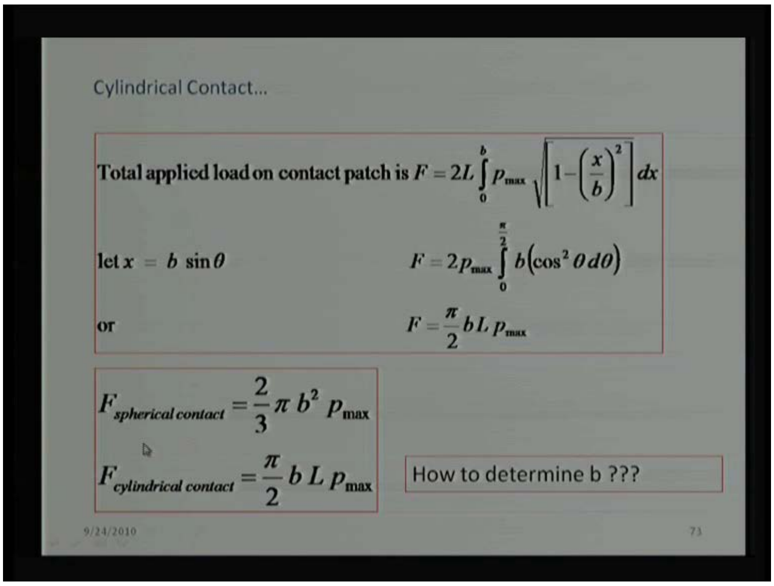

Cylindrical Contact...

Total applied load on contact patch is $F = 2L\int_0^b p_{max}\sqrt{\left[1-\left(\frac{x}{b}\right)^2\right]}\,dx$

let $x = b\sin\theta$ $\quad F = 2p_{max}\int_0^{\frac{\pi}{2}} b\left(\cos^2\theta\, d\theta\right)$

or $\quad F = \frac{\pi}{2} b L p_{max}$

$$F_{spherical\ contact} = \frac{2}{3}\pi b^2 p_{max}$$

$$F_{cylindrical\ contact} = \frac{\pi}{2} b L p_{max}$$

How to determine b ???

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And we can integrate in this situation the same the way, we have already done the p max this is one minus x by p square into d x. We can for a easy integration we can substitute something like x is equal to b sine theta. So, that we can get easy integration and by doing this we are able to get this expression.

See F is equal to pi by 2 b into L into p max. That means, again length of the cylinder will be known to us b is unknown and p max is unknown. If we know the definite geometry if we know what pressure the surface is going to deform then we can directly find out b value, if those things are not known we need additional relation as I mentioned earlier that we need to drive based on elastic deformation relation some elastodynamic relation or you can use a book on (( )) book.

So, this is what we have tried today F is equal to for a spherical contact this is a relation for cylindrical contact. This is the relation and we need to determine b that will be done in the next lecture and we will continue the same topic in next lecture, but what is the interesting thing in this case is that in a spherical contact length is much smaller or we say the b value is smaller, while in cylindrical contact length is much larger.

So, what will happen in this situation when L is much larger for the same contact same area while in this case its happening 2 by 3 into pi its n 1 by half. So, 2 by 3 is 0.66 while this is a 0.5 pi, pi remains same value of the b assuming for time remains, same p max in the more material will have the same geometry same material strength remains same. So, the version comes in the b versus L. That means, in cylindrical contact load carrying capacity will be significantly higher compared to the spherical contact we can predict from this relation that this when we are trying to understand this relation. So, because L is much larger b is not very large and we already know that, when we drive the relation is that b need to be much lesser than radius.

If it is not much small value then we cannot use this relation at all. That is why we say that cylindrical contact load carrying capacity will be much larger than a spherical contact. However, you keep the value of b we will start on the next lecture on elastic deformation of materials; we try to derive the relation. Thank you.