Tribology

Prof. Dr. Harish Hirani

Department of Mechanical Engineering

Indian Institute of Technology, Delhi

Module No. # 05

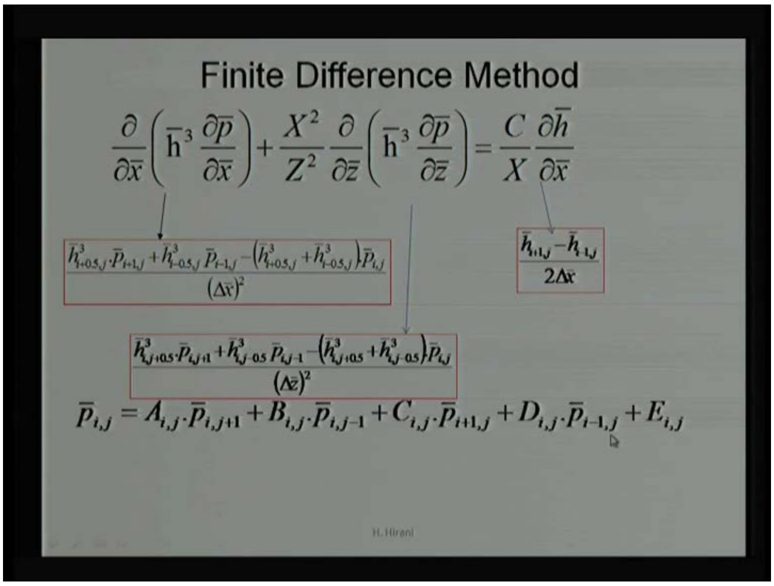
Lecture No. # 23

Finite Difference Method to Solve Reynold's Equation

Welcome to 23rd lecture on Tribology today is topic is finite difference method to be applied on Reynolds equation. Simplified Reynolds equation in previous lecture we learned the benefits of the hybrid approach (( )) better combat approach to existing two approaches.

Now, today is the time to prove it using some established technique like finite difference method only drawback with finite difference method is that. We need to have knowledge of computer programming and along with that it takes some computational time of course, with increasing power of computers. It is a immaterial now, the few seconds or increase in the few seconds while the computation time does not matter much.

(Refer Slide Time: 01:28)



To understand the first finite difference method to be applied for the Reynolds equation, we will start first right we say the central power difference method.

Which we learn in previous or previous lecture, how to apply central power difference method this question the first partial term say (( )) variation in X direction can be represented in central difference method using this formula.

Say there is (( )) at the hypothetical node is a virtual node i and j. We are assuming they are the nodes and there values are integer like 1 1 1 2 1 3 1 4, but this is a half node. Which we are taking as a hypothetical node as a geometry is known we can divide geometry any number of nodes without much problem.

However we are doing integer nodes for (( )) calculation and geometry is already known we can calculate at any distance. So, that is what we are utilizing that half node calculations. So, this the representing first term here we are able to see the P i plus 1 P i minus 1 and P i.

Along the X axis we are taking 3 nodes this node to be calculated with help of remaining 2 nodes similarly, we have transfer second term also we say that central difference method. For the second term second partial term here, we are getting again instead of i we are replacing instead of i we are writing j, i will be silent here. The (( )) in this case we have (( )) j without any alteration j remains same. j j in all three terms while in this case j is changing j plus 0.5 j minus 0.5 and here summation of that j plus 0.5 j minus 0.5.

In this case assumption is that the X is also the function of Z, but if X does not depend on Z. Then we do not have to write all these terms we can simply write one term as a h i j q. We do not have to write we do not have to calculate the half node plus and a half node minus the simple h i j node. We do not have to really calculate a 3 values if X does not depend on the Z.

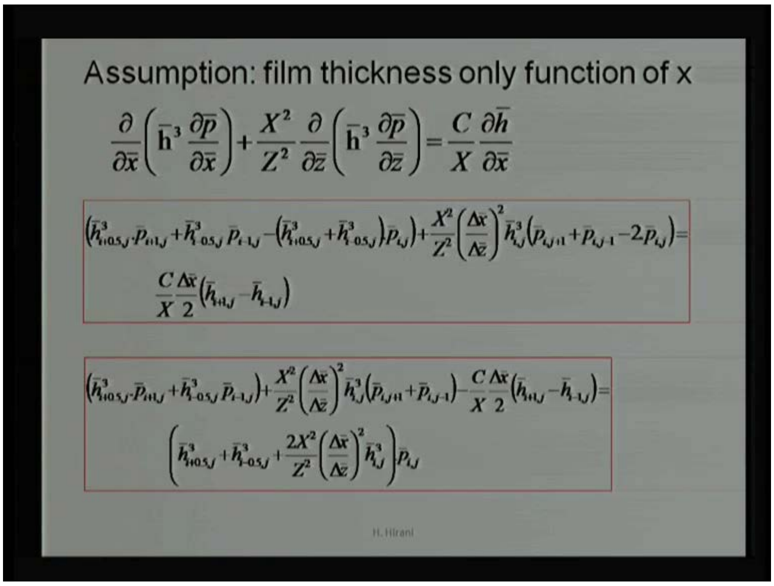
And last time in the central difference method they said partial derivative of h with respect to x can be simply represented by two nodes. (( )) Node we are not using in this case here h plus 1 and h minus 1 and i represent the X direction in this case.

So, we required 2 nodes help to evaluate what will be the discrete (( )) and when we started finite difference method in couple of lectures back. We say that we should represent pressure profile or nodal pressure in terms of 4 nodes as p i j is here the p i j plus 1, j minus 1, i plus 1 j, i minus 1 j and this is the rectangular (( )).

We are taking help of four nodes this more like a taking average of that. And what are these constants this constant can be figured out by arranging this equations proper order whatever the multiplication factor comes for the pressure unit. We think that will be the this constants or geometry constant may be say A i j B i j C i j D i j and last will be the source term like this it will be E i j.

Now, if we try to solve this we need some boundary condition and as well as initial conditions, if nothing is known to us. We take initial condition as a 0 pressure at the 0th iteration is 0 at all the nodes. (( )) If we know the supply of ratio, we can use that value and we know the boundary condition and boundary is some finite value we will put in that value otherwise we will be 0 value.

(Refer Slide Time: 05:44)



Now, if we substitute those terms which we shown on previous slide this overall expression can be represented something like this. You can see h i plus h i plus 0.5, j p i plus 1 j here h i minus 0.5, j e i minus 1, j and this is a summation of a this 2 (( )) and p i j.

Similarly, second term is represented here assumption is h is not a function of z in many a problems it does not really remain function of z. So, to simplify it you just checking it out common term and then as showing that h is only calculated i j node, we are not calculated that any other node in this case.

And these are the pressure terms and this is source term. What we are going to do now, the in this case particularly the denominator works like (( )) square we have simplified it and we taking this side and multiplied with here. So, this is simplified to some extend it can be further simplified by taking all the summing of all terms related to P i j.

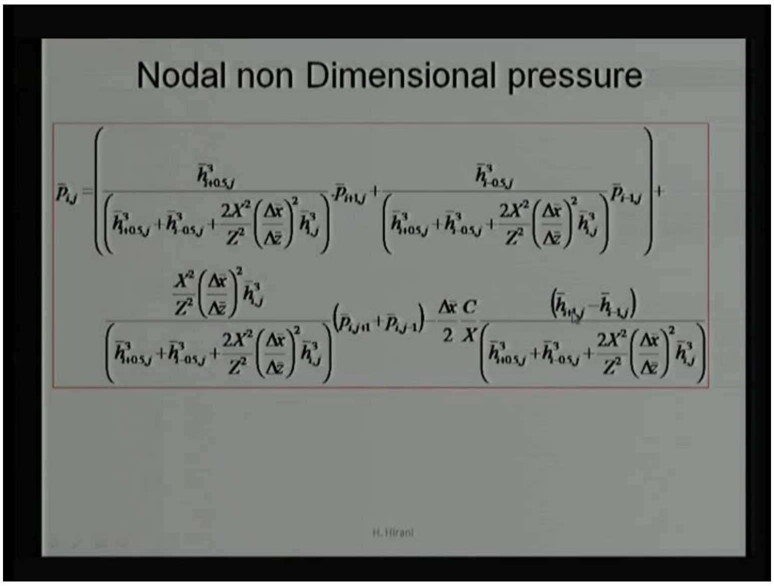
Pressure at i j node we know there are term in this P i j terms in this first bracket also as well as a second bracket also. And we required value of a P i j, that means, we need to take in this common sum of this two terms and both are the negative.

So, we can take it on right hand side, and right hand side term after summing it comes something like this h i plus 1 i plus o.5, j h i minus 0.5, j and here h i j here. The all three from (( )) where we had to calculate pressure and two other nodes neighboring nodes not neighboring nodes, but it is a middle of that is a hypothetically virtual half node.

We are summing up and we are concluding like that you know we need to find out P i j. So, we can take this on the left hand side and at this multiplication factor which is depending on the geometry does not require any iteration this are the definite.

We know what is the value of this terms. So, there we can divide this term this side we can bring out in denominator and bring over all expression for (( )) of course, in this case we are seeing this assumption is that film thickness is a only function of x our (( )).

(Refer Slide Time: 08:31)



Now, after bringing to the left hand side and dividing this whole term what we get as a multiplication factor for the p i plus 1 j this complete factor say h i plus fine 0.5, j h i minus 0.5, j.

Here h i j and here the difference h i plus 1 and h i minus 1. Now, this is a negative term this is positive, this positive and this positive. But, you can see the multiplication factors. This we know film thickness will decrease as a increase with increase in X that means, this will be negative h i plus 1 j minus h i minus 1 j. And this film thickness will be greater than this film thickness then only the conversion will occur otherwise it will not occur. That means, this term is a negative, negative, negative will be positive.

So, over all this term is positive, this term is positive, this term is positive and this term is positive. Now, we can arrange the way we thought earlier P i j should be represented in terms of nodal pressures and with their multiplication factor. So, here A i j is a multiplication factor for P i j plus one. So, this is factor here we are we are able to see P i j plus 1 and here is a P i j plus one. So, this factor whole factor will be equal to A i j.

Similarly, for B factor B i j minus 1 here this is the same term P i, j minus 1 and this multiplication factor will be B i j or another word in present situation A i j and B i j are equal they are same value they have a same expression.

So, I can write A i j like this term or we can say B i j also same term or we can say the institute of b i j. I will repeat A i j right A i j and again here A i j. Similarly, we have C i j that is A P i plus 1 here the P i plus 1 factor comes and the pressure term comes over here and this is the C i j.

This is C i j and here the last term in this case pressure term is A P i minus 1 j this is also same P i minus 1 j and this factor is t. And this is the last term that is e and we are writing in the plus sign. So, when we substitute we again need to write in the accordingly either you change this expressions something like the h i minus 1, j minus i h i plus 1 j. And then substitute or otherwise we can write a computer programming accordingly and get the results.

So, his is what we are trying to point out this E i j is this term similarly, A i j and B i j they are this term same term same A i j and B i j are equal this is C i j term where we have number of terms and D i j is over.

Now, from computational point of view, it is as well always advisable to non dimensional first. And when non dimensionalization is happening and both X and Z values are the same like a maximum value at the X axis and the Y axis and Z axis are the same. And we should take the same spacing number of nodes in X direction and Z direction should be same. Because, we have non dimensionalized in a such a manner they vary from 0 to 1 they have same values they have similar kind of magnitude values.

So, we should take an equally spacing if we take an equally spacing, that means del x square is equal to del X Z. What will happen is this term will be simplified this will be only 1 here, 1 here, 1 here, 1 here, 1 here. So, this number of terms which we need to be counted will be lesser and we required spacing only in this source term.

Otherwise all this is a ratio that will be simplified and (( )) calculation will reduce accordingly and it is a preferable if, you had non dimensionalizing we should reduce. We should take equally spacing of course, if we are not using non dimensional terms or we non dimensionalizing the Reynolds equation in that (( )) situation. We should take some other spacing depends on the requirement depends on the stability possibilities.

And another thing which we can observe from this terms are there whenever we are writing s term and we know very well that it is not varying in the j, it not varying in the z. So, we can simply drop the z terms.

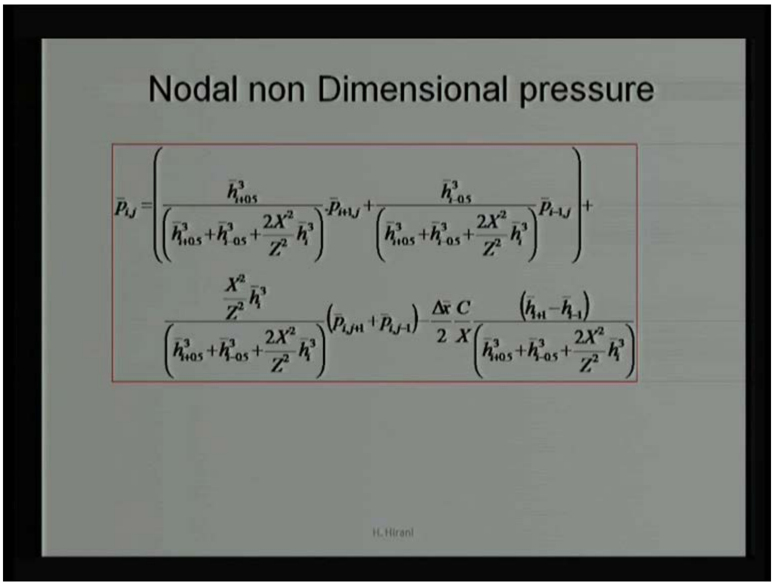
Everywhere only the j j is appearing does not carry any meaning here like A h i plus 0.5 h i minus 0.5. So, this variation is happening in X direction but, j does not remain s remaining constant it is not changing. You are not writing j plus 1 j minus 1 or j plus 0.5 or j minus 0.5 when we are talking about the film thickness.

So, we can drop this j index from this terms that will further simplify the equation not simplify the equation, but representation will be simplified. We can increase the font size and increase there is no they do not be requiring this s j the term.

So, another thing is that when we are dropping this j from h term then this constant also will not be requiring j term because, while the term is representing A. And here if I drop j after that there is no j at all in this term. So, we can drop this j from A i j also it will be the no only A i similarly, B i C i D i and E i.

Here you also you can see here, the i is only changing, but j is not changing. j can be simply removed it is not the representing anything else. It is because, there is no variation in Z direction which is accounted in the present case.

(Refer Slide Time: 15:29)



So, we can drop these terms and with that kind of thing we can see there is simplified equation here. h i plus 0.5 j has been removed h i minus 0.5 j has been removed similarly, in this case h i only j is not there again h i plus 0.5 j is not there h i minus 0.5 h is not there h i is not there.

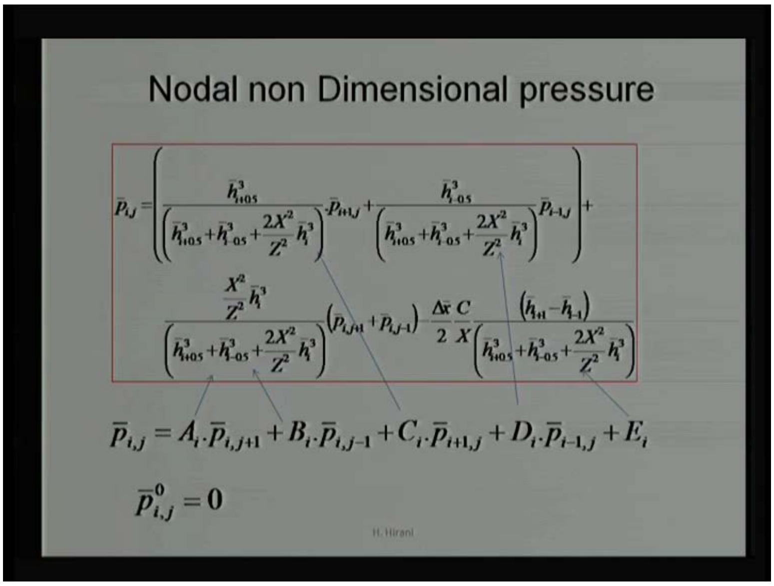
So, this is slightly simpler equation when we write computer program we need to see what kind of arrays are selected if I write a two dimensional array. It will be much costly compared to one dimensional array and this simplification helps to find out whether we should (( )) start with one dimensional array or two dimensional array.

Unless (( )) we write this symbols h i j which is not required in particularly in this case. We will shown that j is not required in that case, if I start with a two dimensional array computational time will be more and anywhere any mistakes happens that is going to impose be more and more penalty.

So, first thing here, is what is important and then we should start computational programming on this right. So, after again as I mention that h i j, j is not required h i is maintained B is maintained C i d i and E i j has been removed completely. However, pressure term we require this j terms because way we can see the variation j minus 1 here j plus 1 here and j also here.

Similarly, i is maintained i plus 1 i minus 1 and i also. So, we require two dimensional array for the pressure term, but we do not required a two dimensional array for other term. The one dimensional array is sufficient to represent the these constants these geometric constants or which vary as per the geometry, but will not vary as per the dimension iterations and will not require any iteration. We are definite we do not require any iteration for this kind of constant calculation.

(Refer Slide Time: 17:56)

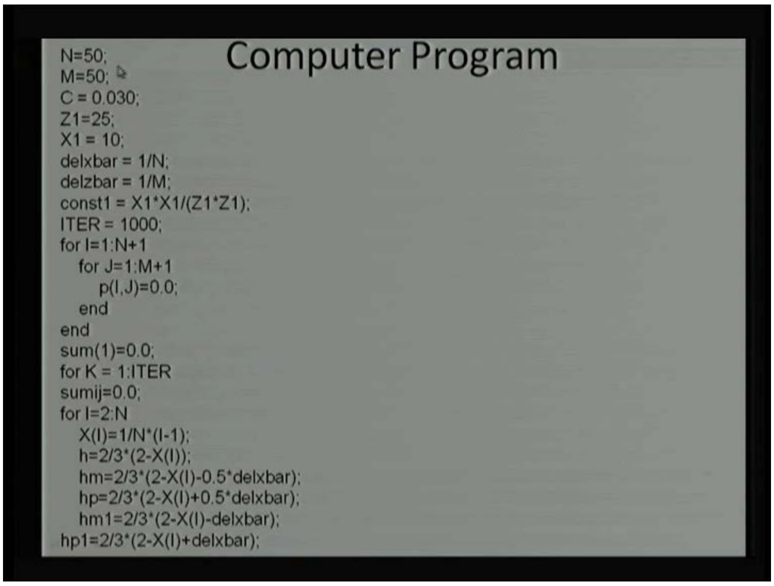


Is again the same representations, so that there should not be any doubt E has been represented something like this. This term A and E represented like this C representation and this D representation and as I mentioned that this iteration number 0. That the initial iteration p i j should be equal to 0 if nothing is been given to us.

However sometimes we evaluate pressure profile using some common approach like hybrid approach. We find out the pressure profile using hybrid approach and evaluate all the nodal pressures assuming that will give accurate results. And then we start doing iteration that kind of approach always helpful because, instead of a starting from 0 we are writing with some definite value and coming very closer to the actual values.

A number of iteration will reduce drastically and solution will be more stable. Because, starting from 0 there is a possibility (( )) stability somewhere solution moves away from the right point and we may not be able to get any conversions.

(Refer Slide Time: 19:13)



From that point of view this is important if you have a some good value available or we say we are nearly pressure value available to us and hybrid approach is when we able to provide those values. Write now we say that we can make computer program to get the pressure results. So, we should be ready now for computer program and this computer program has been written in the (( )).

See this computer program here, first two lines there is an N is a 50 and M is a 50. What we are talking is about the suppose the there is a X direction and X dimension is given or we say the X power is we are using (( )) dimensional term X bar. So, it is a 0 to 1 and we also assuming that there will be 50 nodes and spacing can be calculated between based on this and that spacing is a del X power in x direction.

We know there are 15 nodes and distance between two nodes will be 1 by N we know that is minimum value of X power is 0 and max value is 1. So, 1 minus 0 divide by N is going to give us what will be the spacing in X direction similarly, in Z direction again we are taking number of nodes 50 as I mentioned we are believing in equally spacing in Z direction and equally spacing in X direction. So, that term has been simplified in equation.

This is the C is a clearance we assuming that for the formula which for the example, which you been quoting for the different kind of approaches this is term or radial clearance for 30 micron. Which we picked up and here, we are also keeping the same thing we are keeping 30 micron as a clearance. But, for the different examples where the C is different we can use this that different volume.

However, you want to compare with hybrid approach that is why we are taking same example hybrid approach results are already available with us and when we take the same examples we can compare. (( ))

Now, this is dimensions now in the earlier we were using the word Z the maximum dimension maximum value of dimension in Z direction here. We are simplizing it with a Z 1 because generally X and Z are the axis we do not use those symbols and it is not case sensitive small x and capital X that is why I use the word Z 1 and X 1.

X 1 dimension is 10 mm and Z 1 dimension is 25 mm. Which were shown in earlier cases that when we are choosing the approach one it is giving worse results or is not giving as good results as the hybrid approach? But, we want to ensure there is a hybrid approach results are accurate or at least computable or better than approach number one.

Now, this is another we are writing the constant 1. That is ratio we saw that dimension X square divided by Z square. Which is appearing again and again number of times and it is not going to vary from iteration to iteration or node to node. That is why it has been treated as a constant. So, that remains out of the loop.

First we calculate otherwise we keep this in a loop it will be again and again calculated. And it will take more time maximum number of iteration. Which we are giving in past (( )) present case restricted to 1000 and my experience is that 1000 iterations are more than sufficient many times we get it solution in 30 iteration, 60 iteration, 90 iterations even a lesser that 100 iterations.

But, to be on a safer side and we are just keeping it 1000. If, the solution is not conversion to the1000 iterations then better we recheck the program something is wrong in the program otherwise it does not take that much time.

Now, this is what we say the initialization of the pressure if, the pressure is not knowing to us. So, this for loop we can see for i is equal to 1 2 and plus 1 now there are end divisions there naturally node total nodes will be 51 in this case. So, that is why there are 1 to 51.

Similarly, in j direction z direction again is a 1 to 55 and pressure as I mentioned nothing is known we can put equal to 0 to 0.0 that is initialization of the pressure. And it will repeat till it will complete whole matrix and this is a two dimensional array initialization of pressure using this loop.

Now, we need to find out the (( )) is conversion is happening iteration over iteration solution is improving or not to indicate that is why we start (( )) first situation or (( )) 0 th iteration in this case particularly summation is equal to 0.

And of course, sum we can write the sum in bracket 0 equal to 0.0, but we are using the integer. So, we are started with a one and second iteration will be two. So, 0 th iteration I have time being I have removed is only the 1 iteration the 2 iteration, 3 iteration and 1iteration we do not have any pressure value all the summation operation will be (( )) that is shown over here.

This is an indicator whether over (( )) every node whatever the pressure will be calculate will be summed up in this and over summation will be figured out that is why that there is (( )) with a nodal pressures we are able to sum up this will show the it will be repeated in a subsequent other lines.

And you can see that this is loop for the iteration you say the for (( )) for k is equal to k is variable representing the number of iteration it is varying from 1 to i t, i t e r and i t e r is a maximum value 1000.

Now, if any time conversions is satisfied this whole loop will be in broken or it will be (( )) there will be some indicator indicates that conversions is happen you move out of the loop and print the results. So, now, the real loop is start over here you say for i is equal 2 to n keep in a mind. We are using boundary condition we know nodal pressure or the first pressure first node as well as a N plus 1 node initial and final pressure is already known that is equal to 0. That is equal to ambient pressure.

So, using this kind of a loop we are in incorporating the boundary condition without any problem. We do not have to calculate or we do not have to reassign it is a 0. We say that it is a inbuilt 0, now you take all those three 0 value and account remaining pressure calculate remaining pressure.

That is why the first node and N plus 1 node is been removed here we know the pressure N plus 1 node and we know the pressure at the 1 node we do not have to recalculate those that is why the this loop is from i is equal to 2N.

This is representation of X power calculation of x power we know the value of X power is 0. (( )) Initial and maximum value will be1.0. So, this index when X i is equal to 0. It should give me the 0 value then x i 2 it should give me the del X, X i three it should give me the 2 del X, X i 3 as i this number increases that multiplication factor with a del X will increase this is a what has been mentioned over here has been shown over here.

And once we know this X i that x power pressure or X power location. We can find out the film thickness which again we are taking same example, which we have coated earlier for approach 1 approach 2 and hybrid approach. However if for the common purpose this h will change for if we have some other program other example that is expression for h will be given over here.

So, in this case it is h is 2 by 3, 2 minus X power that is X power is represented with this. Now, we are requiring h plus 0.5 node and h i minus 0.5 node. So, this is represent h m h p h p h m is for minus o.5 node and p is for plus 0.5 node. Excuse me, now this is what we are representing 2 minus x power minus point 5 del x bar we are subtracting point 5 node we say the point 5 spacing while an h p which is a positive side your saying the plus is there point 5 del X bar.

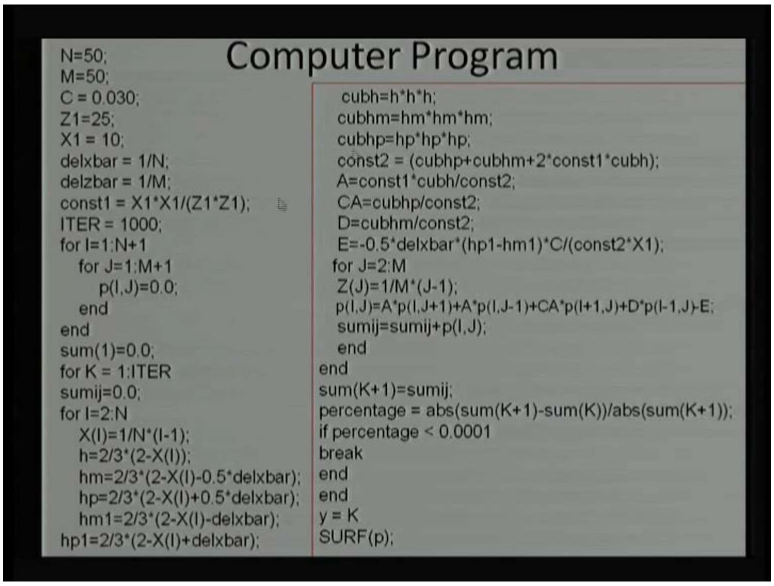
So, it has been added in this case; however, in addition to that in a source term we required a plus 1 and minus 1 h i plus 1 h i minus 1. So, that is been represented over here say h m i is equal to 2 by 3 2 minus x i minus del x power here the point 5 is not been multiplied it is a simple one.

So, it is a h i minus 1 and this will be h i plus 1 they are using this word. So, that no this is a calculated in the first situation, now if you want to save really space we can we can also remove this whole bracket out of this (( )) loop and we can keep in repeating this within iteration loop without any alteration. However, for time being I will not done it assuming that it will be the less second subsequent there will be elastic formation or elastic deformation will be considered then this loop has to come in another iterations.

That is why I am maintaining it otherwise if there is no change there is no elastic deformation calculation then this loop can simply move out and then after that subsequent to that only this for loop iteration loop can start.

How this was in this case in the every iteration will be calculate again which is important. When we are calculating the if we are calculating the elastic deformation variation in a film thickness because of the pressure.

(Refer Slide Time: 29:56)



However the pressure variation is not going to cause an any change in the file thickness that is why we should not use here. But, just for (( )) we are maintaining over here. Now, continuation this we have a now, when we were driving the expression most of the time we have cubic terms we say the h q h i plus minus 0.5 cube and h i plus 0.5 node cubic of that.

So, this is a cubic form and again when we were writing that expression this term was very constant (( )) was appearing in almost most all the five terms. And we can do directly in five times, but from computer point of view saving point of view once we calculate and we should repeat it that will be better.

This is (( )) constant E constant B and A of the same so, b is not been calculate again constant C. As we already use that symbol for the clearance that is why we are writing this new symbol as C A constant D and constant A. Based on this we can find out what will be the constants now this is X (( )) square divided by Z square because, repeated again and again that is why say that it should be directly used over here.

Similarly, cube h and this constant 2. Constant 2 is that you can see the number of times it has been repeated constant 2 here, constant 2, constant 2 and constant 2 that is why we kept only on calculation and just opted that whatever the answer comes we should be repeated.

It should not again and again calculate 0 this reason why we mention like this. So, this is for the i loop. Now, comes the j loop which is within i loop it is a bracketed in a i loop. So, j again we are saying that pressure is A 2 to A 2 to M we do not require (( )) calculation at the first node and the last node we know already what will be the pressure of those nodes.

And this is spacing between the two nodes in Z direction and this is a pressure nodal pressure you know a into P i j plus 1 here. We should write b, but b is equal to a that is why we are writing a into P i j minus 1 here, instead of writing C here we are writing C A as we mentioned clearly the C is been reserved for the clearance then D p i minus 1 j and last term is a source term as A E.

Now, in earlier slide we showed A i B i C i D i and G i that can happen this whole loop move out of this iterated loop. And then we need to maintain those i terms also if you are moving out of this then it has to be written in first one order array otherwise there will be problem.

However in this case we are keeping in iteration loop we do not need to represent this term as a array term. If we move this whole loop out of this iteration then we require otherwise we do not require in this case.

So, pressure is been calculated. Now, we required summation of pressure (( )) will be pressure will be calculated the (( )) and the summation whatever initially we say sum i j is equal to 0. It will be add up in this and this will continue from say 2 to N and 2 to M and all the pressure terms will be calculated figure out summation will (( )) and after this sum final summation of the completion of the both the cycles or both the loops. We can assign the sum as A k plus 1 iteration 1 is already here which as I mentioned we are a 0 value.

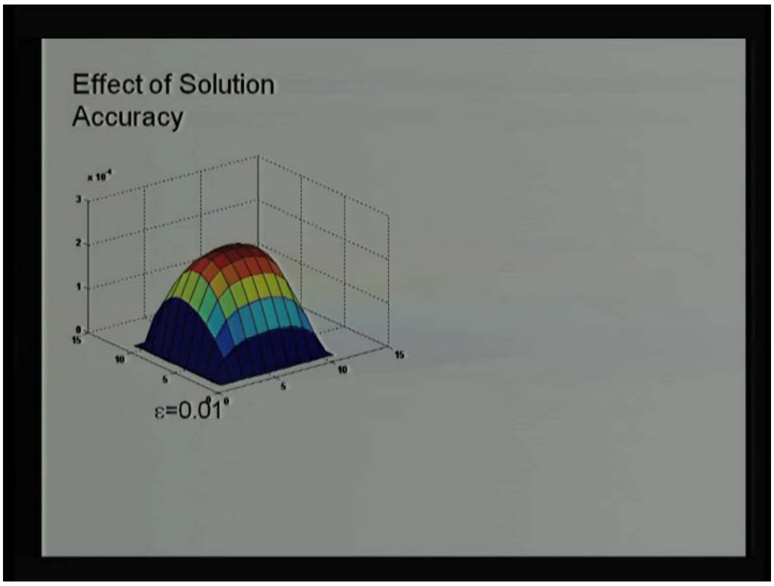
So, iteration will be this summation will be 1 plus 1 the 2 after first situation we are getting some 2 and is equal to this sum i j. Now, we need to find out the what will be the percentage error in this calculation you can simply say (( )) value A f c A f s y and sum k plus 1 minus sum k which is already known to us divide by sum k plus 1. So, this is going to give us what will be the convergence.

So, why did we fix for some criteria suppose in this case we say this percentage should be lesser than 0.0 1 percent. So, that is going to give a convergence criteria whenever this summation or this calculation comes out. And that is a lesser than this that shows that the solution has been derived or like we are getting the solution. And we should break the cycle. That is the what we say break the cycle if this is there then break the cycle and end it and this is the end of the if loop and this is the end of iteration loop.

So, this ends a program this very simple program and finally, we whatever the pressure we calculated here. We can plot that pressure as (( )) which is a two dimensional or we say that there use a three dimensional profile two dimension X and Z and p is height of pressure.

And we can find out how many iterations really happened in this whole loop that can be represented i th number of iteration may be say 13 may be 23 may be 53. It will come out and (( )) whether solution is may be of course, when it is reaching y is equal to 1000 maximum number of iteration may something is wrong. And we should recheck the program we should find out what is a wrong in this program.

(Refer Slide time: 35:42)



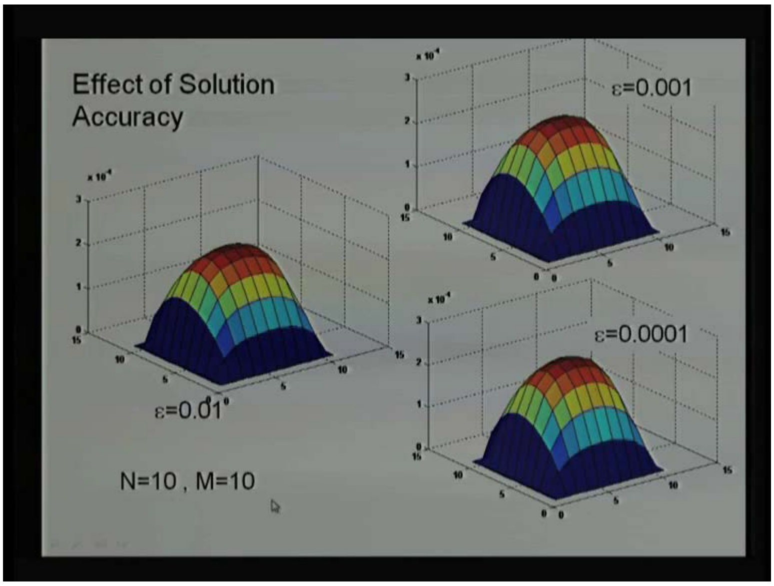
So, once we have this we can use our initial data which we are doing earlier in previous lecture, we coated as a previous lecture. However, in this case solution accuracy is reduced we can say in a program which we show point 0.001 here the solution accuracy we are maintaining that is 0.01.

Now, what I am trying is something like this here, we written the percentage should be lesser than point 0.001. And we say in this case let us we are relaxing it and we are maintaining that percentage error as a 0.01.

Once we do that we can see the pressure profile which is obtained (( )) non dimensional pressure profile obtained something like this can see the maximum value is (( )) 1.2, 1.3 into 10 is to minus 4.

Now, if i increase solution accuracy see the instead of 0.01 I want to maintain a solution accuracy as 0.001instead of 1 percent error in summation we say 0.1percent in a error in summation. We find slight improvement in maximum value at the pressure, but not that significant improvement slight improvement is happened and the pressure value is increasing to some value, so slightly high value.

(Refer Slide Time: 36:32)



Now, if we further improve we say that further reduce error instead of 0.1 percent we are saying now emphasis on 0.0 percent guess it is a some change in this here something was around 1.1 2 or 1.2 maximum here it is reaching roughly1.4. So, accuracy is improving with a improving accuracy magnitude pressure magnitude is improving in this case.

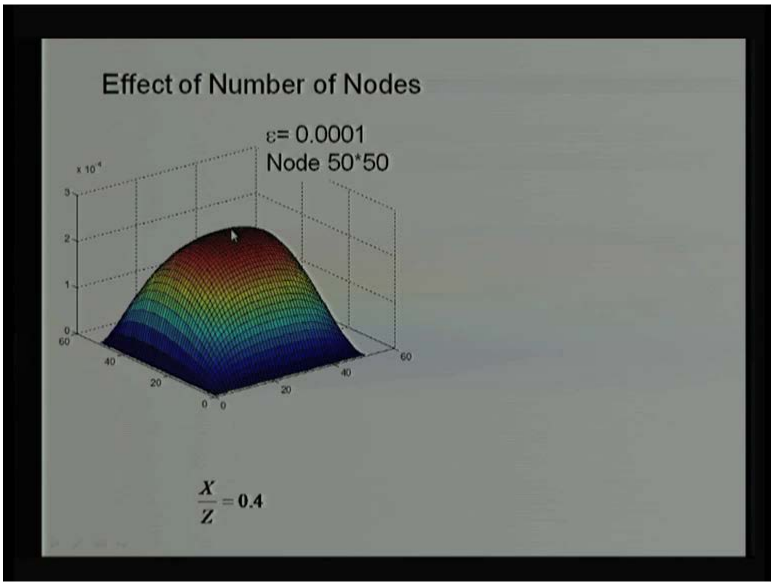
So, we can say till saturation comes whatever is a further accuracy is increase and we are not changing. We are not getting any variation in the pressure then we should (( )) we say the it has been we have tested accuracy and conversions is happening with this (( )).

And in this case particularly what I initially mention about we would have shown the program previous slide we started with A N 50 and 15. But, just for the comparison purpose how it vary with number of nodes i (( )) initiated or showed this all three solutions with number of nodes equal to 10 in X direction 10 in Z direction.

But to compare with 50 nodes these second slides show something like that here. The number of nodes is 50 in x direction number of nodes in x direction as well as z direction 50. I can see that slight change is happening and,, but, hours there is not much variation this is a straight linear problem and number increasing number of nodes is not going to vary much results.

It is not a non-linear problem film thickness is also linear variation and that is the we do not get much variation much significant improvement with increasing number of nodes, but we should try always if we are getting better results we should emphasis on that.

(Refer Slide Time: 39:03)



So, this a what we say the solution accuracy is a 0.001and number of nodes is 50 into 50 and it is reaching almost the same level as a roughly 1.5 over here maximum value 1.5 10 is to minus 4.

So, we can maintain the same thing we say that this is good solution. And it can be compared with now hybrid approach. Now, if you remember when we did hybrid approach we also did approach number 1 and for this example we say that when the X is equal to 10 mm and Z is equal to 25 mm we compare for that.

In this for that example we find the pressure profile that will be the constant all over (( )) maximum value was around 2.8. You can compare think of a 2.8 versus 1.5 or 1.6 is a huge variation this is of approach one.

We calculate using the approach 1 by neglecting the second term and we evaluate the non dimensional pressure profile (( )). I am talking about the maximum value of pressure profile maximum value of non dimensional pressure. That is here (( )) roughly 1.5 1.6 while here it is coming around 2.8(( )) significant variation.

However you compare with our hybrid approach can see hybrid approach of course, this 2 figures were already shown that is why I am just repeating just for the comparison purpose over here.

You can see that maximum value of the pressure coming over the 1.8 that is a far better than this approach and you can say it is coming near to this that is why many times we recommend that using this kind of approach will be providing the better results to us.

And second thing is that we started at the P i j equal to 0. 0 if, we are having computer program available. And we have we do not have much problem in the executing that computer program. So, in that case what we can do we can estimate the pressure profile using this approach instead of assigning value 0, 0 we can assign value this value. So, it will obtain from the hybrid approach and subsequent iteration should be done on the finite difference method.

It is going to give us better results. So, overall better results compared to or much faster results compared to doing only the pressure i j equal to 0. Initially now, why I am emphasizing here many times we required a complex solution the combination of solutions. So, we have been treating only the hydro dynamic case.

Fluid film is varying with towards the convergence gap, but many times we need to consider whether is there any thickening effect increase in a pressure is going to change viscosity if that is going to change a viscosity then we should account that also (( )) some extra efforts will be there. So, extra calculation will be required.

In addition we know viscosity is a strong function of temperature increase in a temperature will happen. if there is a increase in a pressure and there is a more friction at the surface. So, that (( )) going to change that going to change the temperature and the temperature is going to change a viscosity and viscosity is going to change a pressure profile.

So, they are dependent on each other that why when we have this kind of situation then we required simultaneous solution of the equations. And then in that situation in those situations particularly anything which is known to us. (( )).

Like in this case what I am saying that hybrid approach is giving a good pressure profile to us better. We use that profile is a initial pressure profile for the finite difference method. So, that we can achieve overall good results.

Now, just to elaborate what I mentioned about the pressure viscosity term is I am showing this slide over here is a this is a generally viscosity. Which (( )) initially mentioned it is a constant, but this viscosity can also be represented in terms of a pressure.

And this pressure viscosity constant that is a roughly value is a 1 into 1o into minus 8. 2 into 10 is to minus 8 depends on the which liquid we are using and this is a (( )) of the pressure profile is pressure is around 100, 1000 than multiplied this multiplier this multiplier particularly E power l for p is one.

One (( )) increasing 10 times (( )) this is viscosity is increasing by 1 percent or multiply will be 1.0. 1 as further increase in a pressure this is going to multiply is going to increase in a volume and maximum value will (( )) occur particularly in this table. And we are talking about 10 is to 9 pressure 10 is to 9 Newton per meter square pressure that is a increasing (( )) times.

So, we should use whenever there we know that pressure profile is (( )) side we should include this kind of effect. Now once we substitute this what iteration we are going to get we say the h cube dived by eta 0 that is (( )) ambient pressure into this factor.

Similarly, in this side and of course, this side there is no viscosity we can rearrange this equation it is something like this. Now that this is of on (( )) highly non-linear term pressures already the varying in A X direction and Z direction in addition there are non-linear terms coming over here, there is a multiplication factor exponential terms of coming over here that is a e power minus alpha and this term.

To account this to simplify this often use concept of the reduced pressure what is the reduced pressure we say reduced pressure is a q and q can be represented as a 1 minus exponential minus alpha p divide by alpha.

Now, what is the advantage of these assumptions? When we differentiate it with respect to x differentiate q with respect to X what we get this 1 is a minus here, it will be zero minus and minus will be cancelled out. So, e power minus alpha and the pressure gradient in x direction same thing if i differentiate this with respect to z. I am going to get a similar term e minus e power minus alpha and this will be D p by D z and that can be substituted here.

So, this is going to give us a simple 1 initially even for Reynolds equation without accounting this pressure viscosity effect we were getting the term p while here we are going to get term q thus is why the word (( )) reduced pressure we have changed this symbol we have changed the pressure in some other quantity and, but Reynolds equation remains same after this treatment.

Now, this is a same equation almost the same equation as this equation now only thing is that instead of p we are writing q here. So, if I solve this equation find out the q and (( )) that I can convert what will be the (( )) we can find out what will be the p from this.

That means, we are accounting pressure viscosity term without much modification we just (( )) we alter the equation in such a manner this pressure viscosity can be accounted in (( )) once it is done. We can find out what will be the results and will continue this topic on our next lecture. Thanks for your attention.