Tribology

Prof. Dr. Harish Hirani

Department of Mechanical Engineering

Indian Institute Of Technology, Delhi

Lecture No. # 22

Hybrid Solution Approach (to solve reynold's equation)

Welcome to twenty second lecture of video course on Tribology. Today's topic is hybrid solution of previous approaches which we have used to solve a Reynolds's equation. In previous lecture, we took two very simple cases by neglecting one term, one case neglecting term in X direction, and second case neglecting term in Z direction. And neglecting these terms we could get solution much faster, and even in the closed form solution, those were the advantages of these two approaches, but sometime we need to sacrifice solution accuracy, which is not permitted.

That is why we are going to discuss one hybrid approach which combines those two approaches which we have tried in previous lecture, and gives a reliable result. So, in this lecture we will be given first what are the concepts behind the hybrid approach, and what is the validity of that hybrid approach.

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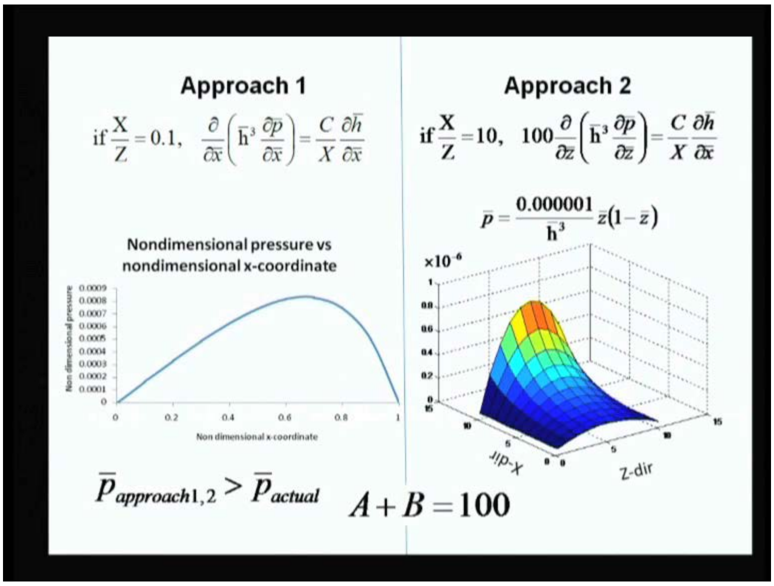

Just first slide this lecture says that lets summarize what was the approach when or when we were neglecting, determines that direction, you say that you are assuming X by Z dimension in X direction divide by dimension in Z direction, this ratio is equal to 0.1 that means, dimensions in X directions are ten times lesser compared to dimension in Z direction.

In that situation partial differential equation which contained 2 terms, 1 term will be cancelled out and or neglected and other will be retained and based on this (( )) equation we can find out pressure profile, non dimensional pressure profile in this term, of course this was for the some given data, given data for the clearance, given data for the velocity and given data for the dimensions.

For very general case we have to retrieve it and so, that we can utilize that derivation for any simple case which have a data and when we obtain a pressure profile distribution or say distribution of pressure profile something like this say non dimensional X coordinates we took we say when we are plotting we divided in a eleven divisions or ten domain 0 to 0.1, 0.1 to 0.2 and this is a non dimensional pressure.

You plotted these results. Similarly, we have followed approach 2 where dimension in X direction were very large compared to dimension in Z direction, as a dimension in X direction is ten times compared to dimension in Z direction. So, this initially for the approach 1 this was the case, but now for approach 2 this is our case.

You can see this is 100 times compared to this approach, as miss pressure distribution will change accordingly and we found the pressure distribution for the given data something like this by substituting h we can also represent in better manner, but 2 dimensional plot shows that this pressure distribution depends on X as well as Z coordinates, dimensions or we say non dimensional non dimensional Z and non dimensional X.

h is a function of x bar that is why we are getting X in this direction, Z this direction and you seeing that as X increases the pressure profile is increasing, that means, that convergent domain and more and more pressure will be generated. (( )) what are the drawbacks in this case we are not getting the pressure profile at the every points because we have plotted only the varying by Z (( )) and this is empty space, it is another thing which we can observe that you take either approach pressure non dimensional pressure obtained by approach 1 or non dimensional pressure obtained by approach 2.

In either of the case the pressure predicted by these equations will be greater than actual pressure that can be seen simple (( )) relation like this you see it is A and B; there are two terms and summation is equal to 100 just for this example, we are saying that there are two terms summation equal to100.

In this case there were two terms and we neglected one the neglect (( )) this second term in this equation. So, in other word I neglect B assuming the B value is low it may be 0.1, may be 0.5, may be 1, but is a negligible compared to A when we neglect this then whatever the A value comes that will be higher compared to what is given.

We know these are the non negative, they are positive quantities only even whatever the final value of the B, it will be having some finite value and that need to be subtracted to get the value of A, but if I am neglecting it I am directly getting value of a as a 100 naturally by neglecting any term whatever the term we get or whatever value we get for A will be higher than the actual.

This is the reason why we are written something like that, the non dimensional pressure obtained for approach 1 or approach 2 will be greater than the p actual non dimensional pressure actual.

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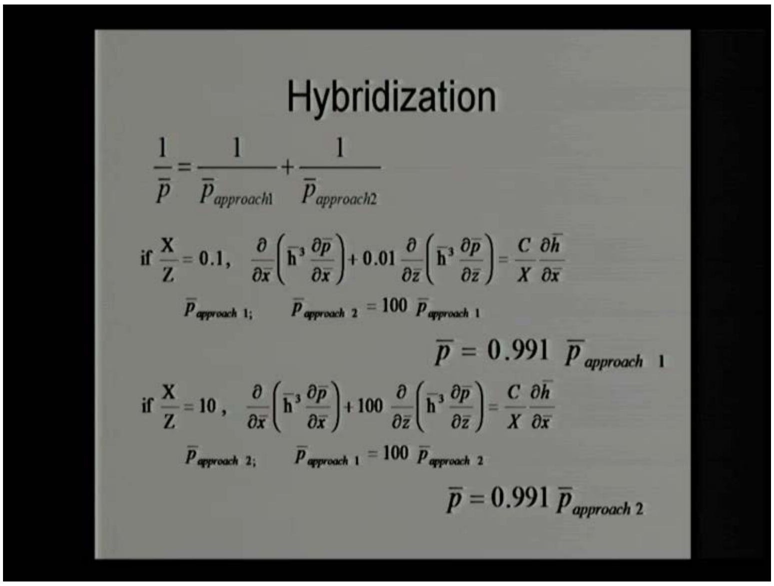

So, we need to rectify it that is why we think we can hybridize p. We can find out the pressure profile by approach 1, we can find pressure profile by approach 2 and go for the harmonic combination of these two pressures. Now, what is the advantage when we are using the inverse relation and we know if pressure (( )) calculated by approach 1 or approach 2 the pressure will be more or higher than the actual.

So, keeping that in mind we can find the pressure versus this calculated by hybrid approach will always be lesser than p approach 1, p approach 2, that is obvious. We can take an example also say that lets in the first case X by Z is 0.1, that means, dimension in X direction are ten times lesser compared to the dimension in Z direction, substitute it we neglected this and we evaluated the pressure profile by equating this term 1 first term equal to right inside (( )) we find some pressure profile.

Assume this is a pressure approach 1 now, if just for sake forgetting some data I neglect the second term and try to solve this term, what will happen? This 0.1, when it will come this side it will be 100. So, if I calculate by neglecting first term pressure profile I will find pressure profile approach by pressure approach 2 is equal to 100 times compared to non dimensional pressure obtained by approach 1.

Now, we can substitute in hybrid approach you see p approach 1 as it is and you can keep this as 100 p approach and to present pressure profile non dimensional pressure profile in terms of pressure obtained by approach 1, it will turn out to be 99.1 percent of pressure obtained by approach 1 which is the lesser than approach 1. So, that is giving some validity this result may give good results similar, case for the when the dimension in X direction are large. So, dimension in X direction is ten times large compared to the dimension in Z direction.

In that situation we neglect this and this is equal to 0 and will turn out to be this side whatever, the pressure profile comes we represent as a pressure non dimensional pressure obtained by approach 2, while coming to the pressure obtained by approach 1, we can get that will be 100 times this.

Thus here at least it was divided when we were neglecting this term, while we neglect this term this 100 figure is also going away and surely the pressure profile obtained by including this term with this term will be 100 times compared or we will provide the pressure 100 times. So, again the same thing now in the, we can substitute the value in this hybrid approach expression and we obtain pressure profile in terms of pressure obtained by approach 2.

Is again same thing comes, it is a non dimensional pressure obtained by non dimension hybridization will be 99.1 percent of pressure obtained by approach 2 and we know in this case approach 1 is reasonably correct and in this case the approach 2 is reasonably correct.

However, the situation may arise for the intermediate distance assuming the x is slightly higher value and is this ratio is 0.25 when this ratio is 0.25 quiet possible the pressure obtained by approach 1 will not be very good or will not be very reliable results. Similarly, in this case if I assume the access of four times compared to Z then again this approach alone will not give the good results, but hybrid approach generally gives a good approach because it is combining the characteristics of the two approaches and trying to give better results by minimizing the drawbacks of these approaches.

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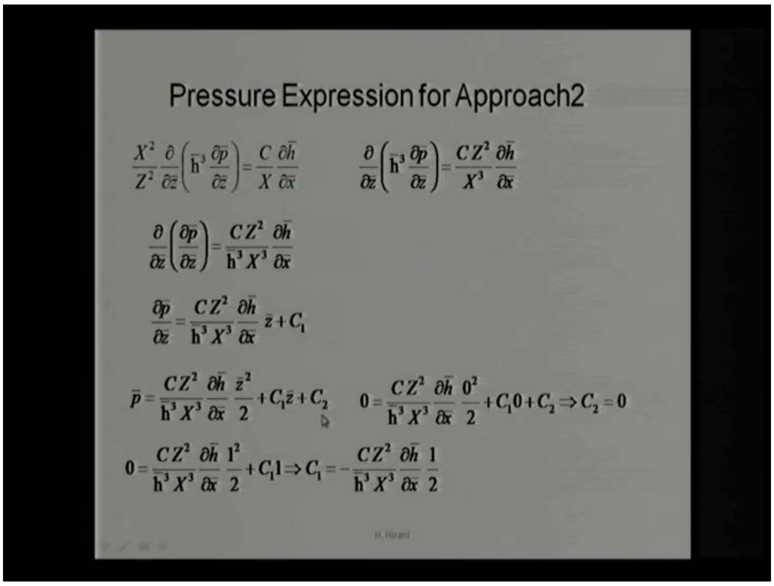

Now, as I in the previous slide I mentioned that those expressions for were for the typical one example, to apply hybrid approach we need to drive pressure distribution or pressure, non dimensional pressure without the dimension initially, assuming that we do not know dimensions I must say we have pressure profile we must substitute the dimension and get the overall result.

So, to do that let us start with approach 2 first say find the pressure distribution of pressure expression for approach 2 now using approach 2, this is expression your X earlier we were using in the 100 value while, here we are using X square by Z square, these are the non dimensional terms and we can rearrange, we can take Z square this side and divide it by X square re arranging the term say no other clearance dimension or maximum dimension in Z direction, maximum direction in X direction and additional point is that h bar film thickness, it does not depend on value of the Z or Z coordinates that is for this tedious stage 1 directional motion only.

Otherwise it may depend on the Z coordinates, for our case for present assumptions we can take it out without really differentiating with respect to z bar because h is not a function of Z. So, we rearrange this atom. So, here h power comes which is the only function of X. Now, we can integrate it as this whole term does not depend on the Z and this also does not depend on the Z, as it does not depend on the Z we can treat this as a for time being as a one constant and when integrated it will be z bar plus 1 integration cost because you are not using substitution or we are not using in any closed dimension on this.

First integration subsequent to that there will be second integration now, we can see that we are not substitute any value of any dimension and we are getting only an integration constant term. So, there is a 1 integration constant, there is another integration constant.

To get the pressure expression we require value of C 1 and value of C 2 which are integration constants and of course, overall pressure distribution we required geometry we required taper angle or all other necessary dimension which are illustrated or expressed over here which are expressed in this expression. So, finding the first constant we can we know very well that the end condition generally the pressure profile will be 0.

That will be ambient pressure, we can see 0 pressure at the start value of the Z that is Z equal to 0, Z 0, Z 0 that gives clear cut that expression for C 2 C 2 will be 0 in this case. Coming the second point let us say again when z bar is equal to 1 or we say that Z is equal to the maximum value of the dimension as the capital Z, then pressure will be 0 again right the z bar will be equal to 1, z bar equal to 1 here.

We rearrange and we find the constant C 1 integration constant, as the negative term and that is depending on the film thickness variation with X and clearance dimension in Z direction, dimension in X direction and film thicknesses itself. When we substitute this in Reynolds's equation or we say that we try non dimensional equation, non dimensional pressure expression p bar we can substitute Z 1, we substitute to Z , we can substitute C 1 and we can substitute C 2.

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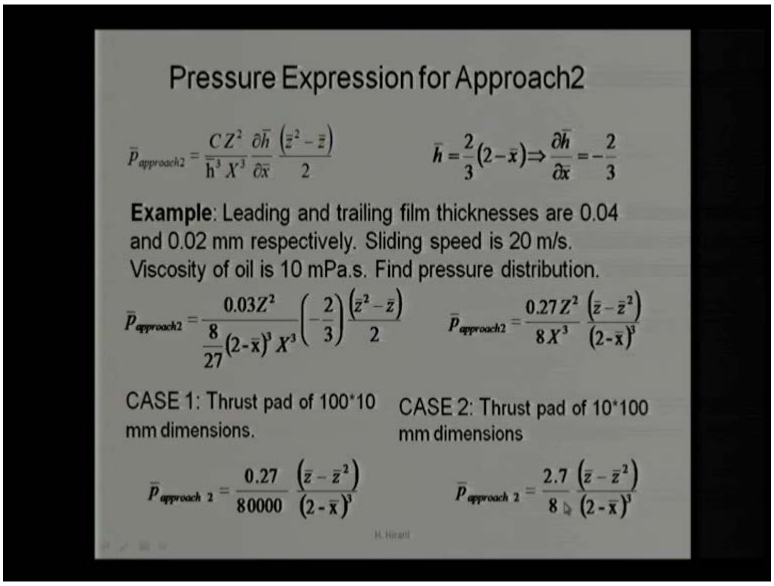

What we get something like this we can name it that this has been obtained by approach 2, there is a clearance single clearance as well as Z effect of the dimension is square form, effect of the dimension on X direction is in cubic form, film thickness is also cubic and here we are finding Z square minus z bar and this satisfied both, what we say initially pressure should be 0 when z bar is 0.

Substitute 0 here, substitute 0 here overall pressure will be 0, then we say that z bar is equal to 1 is a 1 here 1 square minus 1 that again will be 0 and with the satisfying this satisfy both the boundary condition which we substitute and we got this expression. So, this is expression. Now, we can substitute the value for some example we are continuing the same example which we have done for the approach 1 independently, approach 2 independently of course, this will allow us to find out whether we have made any mistake in derivation or not. So, here again same data which says leading and trailing film thicknesses are 40 micron and 20 micron respectively.

Sliding speed is 20 meters per second, viscosity 10 mile Pascal per second, find the pressure and as listed or given to us we can find out what will be non dimensional film thickness, that we did earlier also it is 2 by 3, in bracket 2 minus x bar, x bar will vary from 0 to 1, let me say in the final value whether x bar is equal to 1 film thickness is not 0 it is a finite value and that is a 20 micron.

When we differentiate because we require differentiation of h with respect to X for this expression we can differentiate it, we find this is the only negative minus 1 will be the differentiation of this. So, relative term has to be minus 2 by 3 now, we can substitute value of h bar, value of first gradient of h with respect to X and we can find this expression right. So, clearance value has also been used here we have assumed the clearance as a average value of 0.02 plus 0.04 divided by 2 that comes to 0.03.

So, it is square we are not substituting because in this example yet this does not give the dimension values about X direction or Z direction, Z square while, h is given that is comes up 2 by 3 to minus x bar. So, cube of that will be 2 cube of 2 will be 8, cube of 3 will be 27 and 2 minus x bar in brackets will give you like this, X cube we are keeping as it is.

Substituting the value of this gradient that turn out to be 2 minus 2 by 3 and this term will be as it is, after simplifications of this equation what we get is something like a 0.27 divide by 8 in terms of z bar sorry in terms of the Z and X because z bar and x bar are also there, but we are we know very well these will be varying to find out pressure distribution while this will remain constant we cannot vary this for pressure distribution.

Now, let us take case one in this case thrust pad dimension is 10 sorry 100 into 10, 100 mm in X direction and 10 mm in Z direction. So, 10 mm in Z direction, 100 mm in X direction, that is giving figure something like 0.27 divide by 80,000 and this is for the pressure distribution expression will retain it as it is, take case two here the dimension are just reverse dimension on X direction is obtained, while dimension in Z direction is 100. So, pressure expression will slightly change.

Here say instead of having 0.27 we are getting 2.7 and instead of 80000 we are getting only 8. So, pressure obtained by this approach will be much larger or in other words, we should choose dimension in such a manner which pressure gives a lot of value or maximum value in that.

This gives an indication we want to higher pressure we should go at with this approach, this kind of dimensions.

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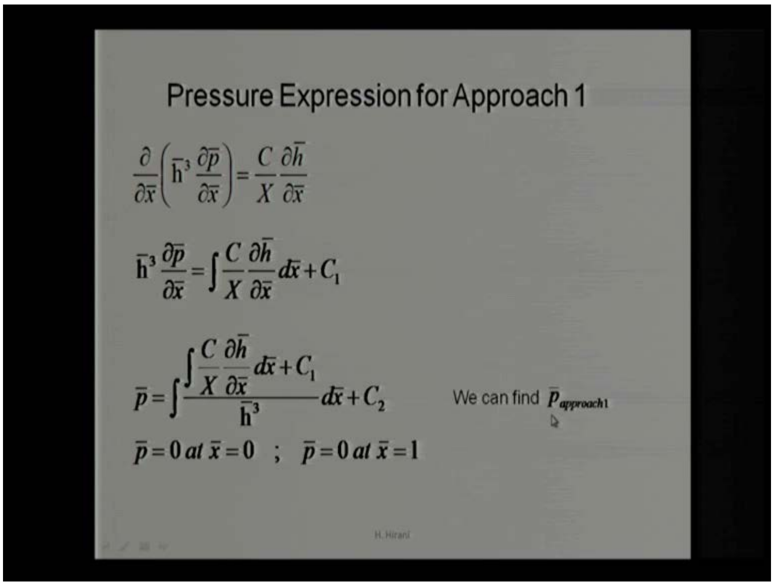

Now, that was for approach 2 similar kind of expression is required from my approach 1. So, pressure expression using approach 1, in this case this is the example relation, but here h bar depends on x bar, it is a function of x bar.

We can integrate it and when we integrate it, we initially we do not have expression for h. So, we are keeping as it is in the same form integration plus some constant integration constant on second time integration, this h bar q will be coming in the denominator, we need to integrate the whole expression numerator will come denominator plus another additional constant that is C 2 constant integration constant in this case is C 2. So, we have 2 integration constants or we need to figure out value of C 1 and C 2 to find the pressure expression.

To help that to find these values, we have pressure boundary condition, pressure boundary condition is 0 value and the p bar equal to 0 at x bar equal to 0. That means, at the initial, at the entrance pressure is 0. Similarly, pressure at exit will also be 0. So, we have pressure variation 0 value of the 0 pressure as 0 at x bar equal to 0 and pressure equal to 0 at x bar equal to 1, to find out this pressure distribution we required film thicknesses expression.

And that can come with some example only, it will this is the generalized one now, this on given problem we can find out value of h, we can find out gradient of h and substitute this integrate and get the results to do that again we can take same example I am using that example, we can say that we can find pressure distribution using in using approach 1.

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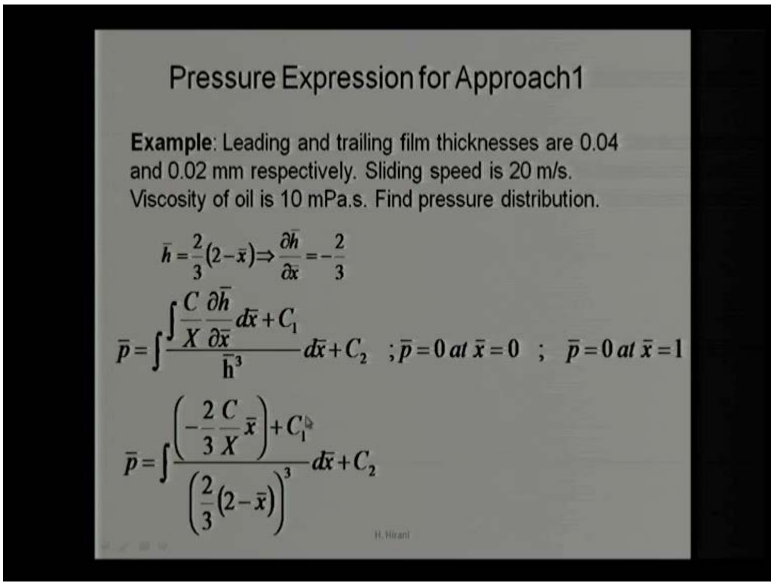

Same example here the leading and trailing film thickness 40 micron, 20 micron sliding is speed, viscosity of oil and based on this expression we know, film thickness expression will be, 1 dimensional film thickness 2 by 3, 2 minus x bar and gradient will be minus 2 by 3.

Now, we have these values we can substitute and we can substitute h also in addition to this values we have this pressure boundary conditions. So, let us start first thing we substitute value of gradient of h with respect to X that is minus 2 by 3 which is coming here minus 2 by 3 and this is C by X we are maintaining as it is, as this is constant we can integrate easily that as there is no pressure and there is no x bar term coming in this whole expression. So, it will be simple integration to x bar.

So, first term in this case, this whole term after integration will turn out to be minus 2 by 3 capital C is the clearance X capital X, x bar plus this integration constant and we substitute this value also 2 by 3 in bracket 2 minus x bar q bar now, lot of common terms in this expression we can rearrange and after re arranging we can take 2 by 3 common usually this will be multiplied by this constant 3 by 2 and we can change the symbol since instead of integration constant 1 we will say the integration constant C 3 because it will be multiplied with additional term which are constants. So, constant into constant means constant.

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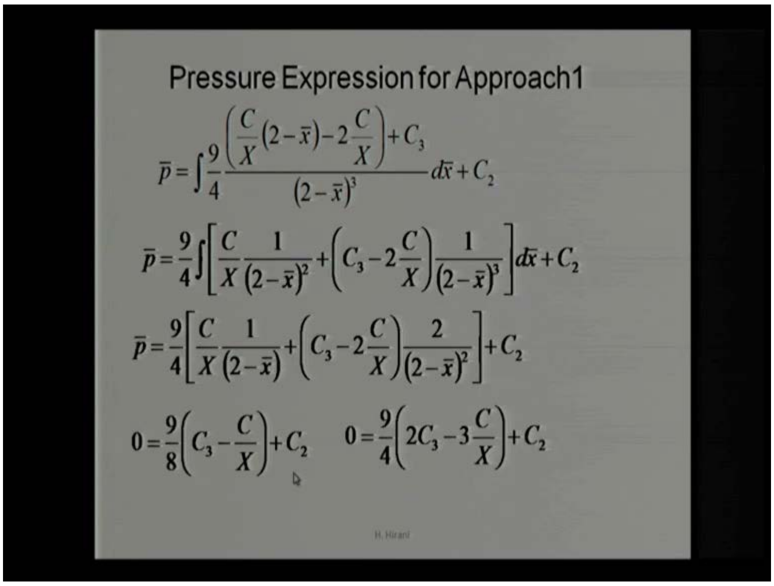

So, instead of seeing only the constant C 1 we say C 3 constant, but we will use only 1 term that is our after rearranging this taking common we instead of writing C1 we are writing C 3 which will be multiplied by 3 by 2 and we took a common term in addition, there is denominator 2 minus x bar which is a integreble and we need to convert numerator into simple form, what we did C by X, x bar is already one term,

We added a term 2 over here and subtracting the same term, we are adding one term we are subtracting the same term. So, that overall is a 0, but it will make much more meaningful integration to us. So, after arranging this integration constant is C 3 and plus another constant C 2 we need to determine C 2 value in C 3.

Right this simplification is required. So, that it helps us to get a easy integration and that simplification gives C by X, one term as a 1 divided by 2 minus x bar the whole square, second term is C 3 here and this is minus 2 capital C by X, that is also arranged and re arrangement gives this as a integration term.

This is a non integratable, it is just constant will be moved out similarly like this. Now, this one integration of this, another integration on this term and after integrating we know, this will be simply 1 by 2 minus x bar, minus term will be here and then this after integrating minus x it will be again minus 1. So, minus 1 minus 1 will turn out to be plus 1, same thing over here this term after integration will turn out to be 2 minus x bar whole square and minus 2 into minus 1. So, minus 2 into minus 1 will turn out to be 2.

This is a one, this is a term, we have this expression in terms of two constants C 3 and C 2 simply, we can find out what will be the if I see x bar equal to 0, what will be the pressure distribution, we know that the pressure distribution or non dimensional pressure will be 0 similarly, we can substitute another condition when x bar is equal to 1 then again the pressure will be 0.

Substituting this, what we are arranging the pressure is 0 here and here the 2 minus 0, 1 by 2 similarly, in this case also 2 divided by 2 minus 0. So, it will be 2 by 4, 1 half ,1 half, 1 half of this will be common, that is why it will turn out to be 9 by 8.

Right and here it will be C by X, here C 3 minus 2 into C by X, the 1 minus 2 C by X there will be 1 C by X over here, 1 will be cancelled out and our alternate will be C 3 minus C by X, this is not of any change it will remain constant C 2. So, first expression which contains C 2 and C 3 both are there, same thing for X equal to 1, p is equal to p bar equal to 0 this say when we say 2 minus 1 it will turn out to be 1, 1 by 1 is equal to 1. So, simply will be this way whole term will be equal to C by X.

Coming to the second this is constant term divided by sorry multiplied by 2 divided by 2 minus 1 that will be 1. So, here it will be 2. So, 2 C 3 minus 4 C by X and there is a one term here C by X. So, overall term will turn out to be 2 into C 3 minus 3 into C by X. Now, we have 2 terms C 2 and C 3 and we have 2 equations.

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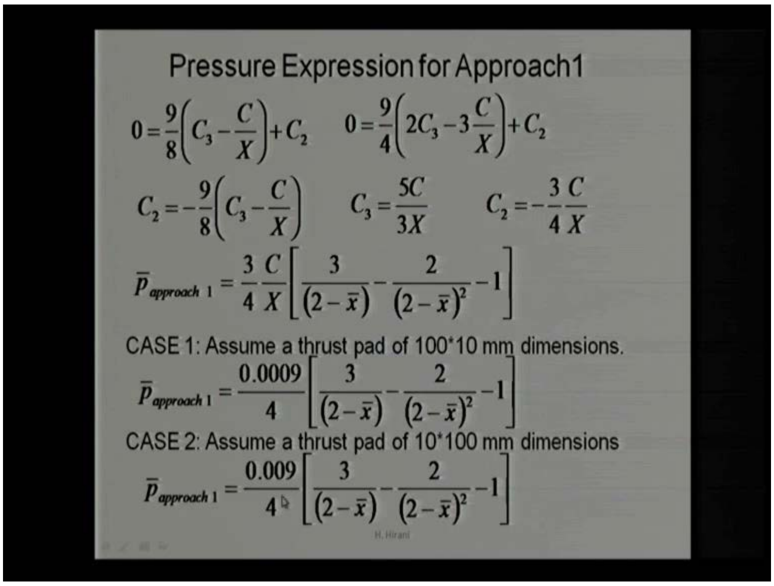

We can simultaneously solve these equations to find our integration constant C 2 and C 3, this is the shown out here when we can directly say C 2 can be represented in terms of C 3, we can say C 2 is equal to minus 9 by 8, C 3 minus C by X and when you substitute this value of C 2 over here, what we are getting C 3 is equal to 5 C divided by 3 X. Now, once I know this C 3 I can substitute value of C 3 in this expression to find out what will be the value of C 2 and that C 2 is turning out to be negative value, 3 by 4 capital C, clearance divided by capital X dimension in X direction.

Substitute and we get approach of something like this or it is a pressure expression something like this 3 by 4 capital C divided by capital X first term, second term and third term. We can verify that whatever, we have obtained earlier with the data, this is giving the same value, this expression is going to give the same value.

Now, we can assume we can start with a case 1 and case 2, first case where the dimension in X direction is much larger, compared to the case 2 or compared to the dimension in Z direction, substitute these values, what we get is the expression something like this and in second case we are assuming the dimension in X direction or one tenth compared to dimension in Z direction.

Naturally, so in this case expression will change, here the X is ten times more compared to this and that is coming in the denominator. So, this is 0.0009 while here it is 0.009 this clearly says that in this case dimension in X direction is not as high as dimension in X this direction or in case whenever, the sliding velocity is in X direction we should choose a lesser dimension in X direction to get a much higher pressure distribution.

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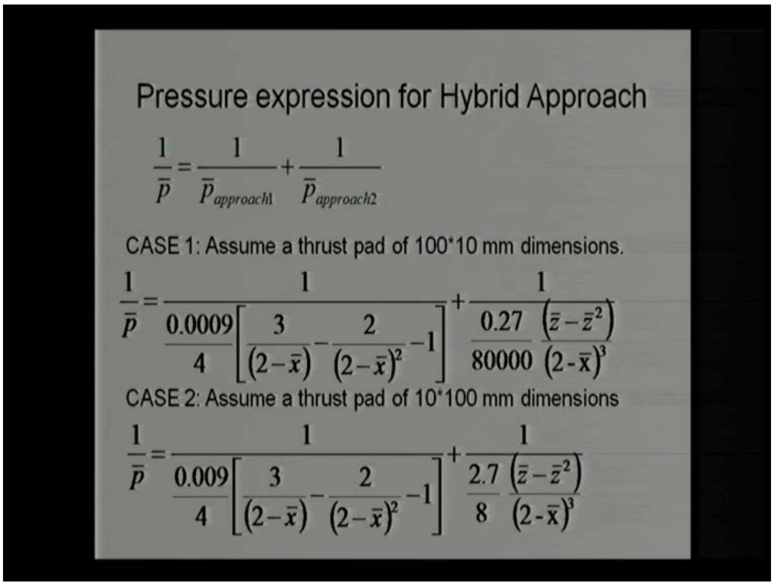

Now, we can apply these expressions for our hybrid approach and we already evaluated pressure by using approach 1 and pressure using approach 2 you can combine this onward, we get a expression slightly complex compared to approach 1 and approach 2, but this is going to provide the better results compared to approach 1 and compared to approach 2.

So, you substitute here and now we can see the pressure is coming we are getting in terms of x bar as well as the z bar both, both the dimensions are playing the role in this pressure expression similarly, for the case two here also we are getting the same thing you can see the pressure distribution in this case, this is higher compared to term this one similarly, this is higher compared to this one. So, pressure distribution will be on the higher side using this dimension this kind of dimensions or where ever the sliding velocity is there dimensions are lesser, we will get better result compared to tangential direction and the pressure or there is no tangential velocity in the direction. So, it should give the better results by choosing this, it can give the some formation how to optimize this kind of Tribo pairs.

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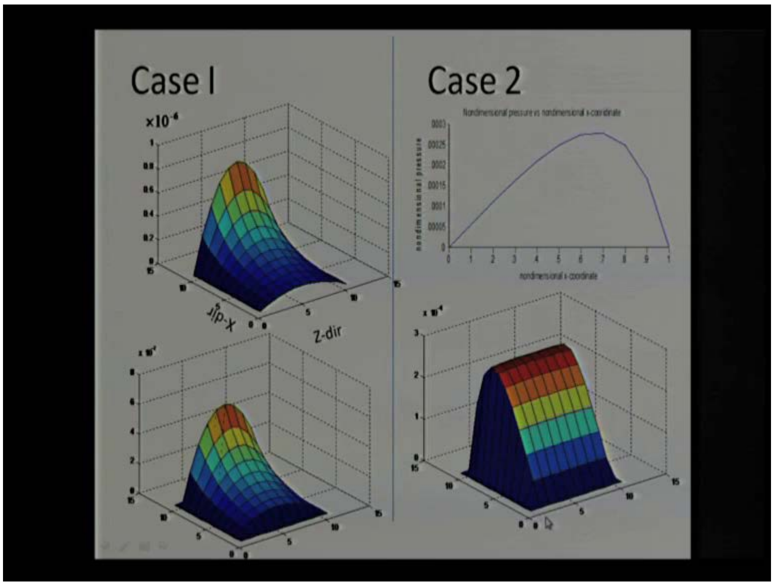

Now, if we try to evaluate the results by hybrid approach and compare with approach 1 and approach 2, the results turn out to be this is what we say expressions for the results obtained by approach 2, these are the results obtained by approach 1 and this are the results obtained by the hybrid approach.

You can see the slight variation only in this can clearly see over here, this is in a decimal form 0.6 while here it is not in decimal form, but this is 10 rises to minus 7, this is 10 rises to minus 6. So, only slight variation otherwise it will be ten time variation, coming to the this side again a slight variation compared to this. (( )) seeing that pressure distribution obtained by hybrid approach is pressure profile is lesser than the obtained individually by approach 2.

For a case where the approach 2 has been recommended and similarly, pressure distribution obtained by hybrid approach is lesser than the pressure distribution obtained by the case, where the approach 1 is recommended. So, this is approach 1 which profile obtained by approach 1 and which says pertaining a very high value X by 10 is a low value it should have recommended, but even after that we are finding it there is lot of variation in this approach. So, hybrid approach is good gives or reasonably good results compared to approach 1 and compared to approach 2.

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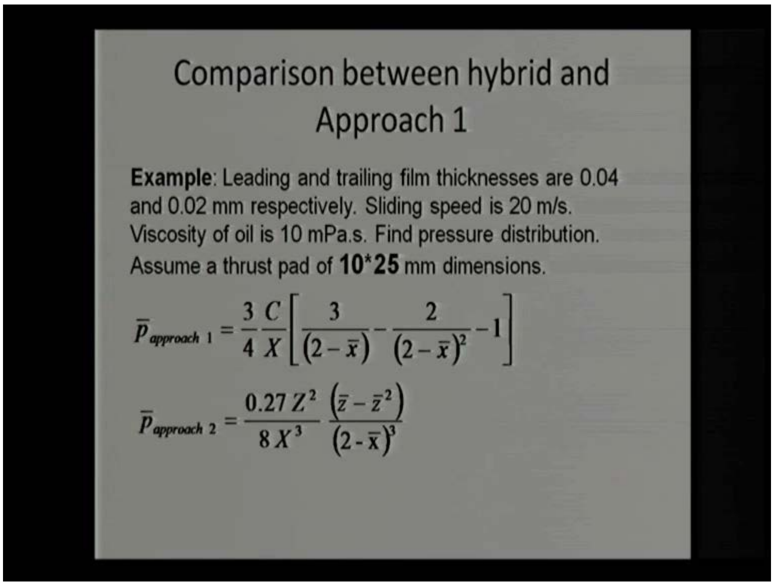

However, we cannot just believe on that, but we can get the more and more results and we need to verify this against some established approach, which does not take any assumption, say lesser assumption it is like final difference method which we say it is going to take more time, but can provide the reliable results. So, we need to understand what will be the comparison in that case situation or some.

Now, this is what we have mentioned earlier, the approaches, comparison of approaches this is a what previous slide I showed it was shown on this databases, here the dimensions are changed instead of 100 mm in Z direction, we take a dimension 25 mm in Z direction. So, only that is going to change. So, there is change in dimension only in Z direction there is no change in any other direction.

When changing in Z direction it will be final pressure approach, pressure obtained by the approach does not contain any Z dimension. So, it must not change the overall value and while pressure obtain by approach 2 must change the value, that is because it contains Z as well as X while approach 1 is containing only the capital X. So, if there is any change in the X direction only the pressure will change, how I change is the dimension in Z direction is not going to change anything whether the dimension is 10 mm, 15 mm, 100mm, 1000 mm the expression remains same it is not going to change which is the problem with approach 1.

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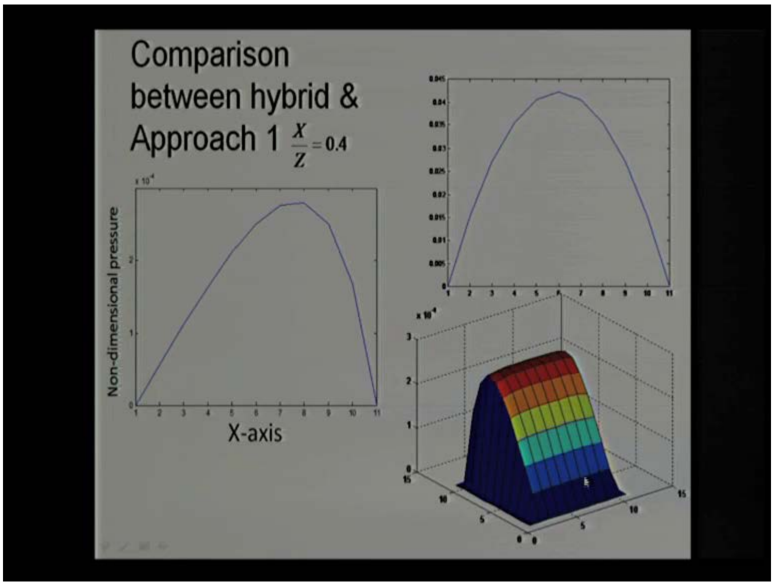

Now, when we compare this we can see here non dimension pressure obtained by approach 1 is higher value, it is coming roughly around 28 into 10 is to sorry 2.8 into 10 is to minus 4, while hybrid gives almost this value is around 1.8 and 10 is to minus 4. So, 2.8 versus 1.8 and if you go through a number of books they suggest that this is going to give a very reliable results but does not happen really, hybrid approach gives way much better results compared to this approach.

I agree that this approach, approach 1 is simpler compared to approach or hybrid approach, but hybrid approach provides a reliable results. Of course again and again, I am emphasizing that hybrid approach is providing you as a reliable results, but we need to prove it and that is we say that we will be taking help of final difference method, there are assumptions are lesser only assumptions is that, we need to choose a more number of nodes and assumption is that as space decreases accuracy will increase or nodal space will be higher then accuracy will be lower.

Well hybrid approach we get a quick solution and we do not descriptive, we do not divide the domains, obviously, number of nodes. So, hybrid approach has a advantage from that point of view and final difference from accuracy point of view we can establish hybrid approach, we can compare hybrid approach pressure expression with final difference pattern that is going to give us some confidence.

Of course in this slide we are showing that even when X is equal to 40 percent of the Z dimension, hybrid approach will be of much lower value the non dimensional pressure obtained by approach 1 which is drawback of the approach 1, even though this ratio is not a very low value sorry in this case it is not a very high value still this approach is not giving the good results.

However, just to in other extreme cases you can see Z, if I say that instead of pressure obtained by approach 1, I want to see what will happen when I obtain the pressure distribution by approach 2, you can see here the pressure approach of maximum value is coming roughly around 0.4, 0.04 to as almost 100 times or compared to we can say 200 times.

Here the pressure profile is at 2.8, 10 is to minus 4 while here it is 0.04 to 10 is to minus no, there is no 10 is to minus 2. So, we say that 4.2 into 10 is to minus 2. So, pressure distribution in this case is much, much larger compared to more than 100 times compared to what we obtained from this.

So, this should be neglected when ever, this kind of dimensions are there of course if I put an order this is the lowest worst case, this comes in the middle order, we do not have much time and we do not have computer facilities or we do not have, do not want to go to hybridized approach because it is not very stabilized approach or stable approach, but then we can go ahead with this, but to understanding the merits of the hybrid approach we can recommend that this approach is good approach and it gives a reliable results, only thing is that it should be able to give us a result comparable to the numerical result compared to the final difference method or final uneven method or whichever method we use it should be able to provide the good results to us.

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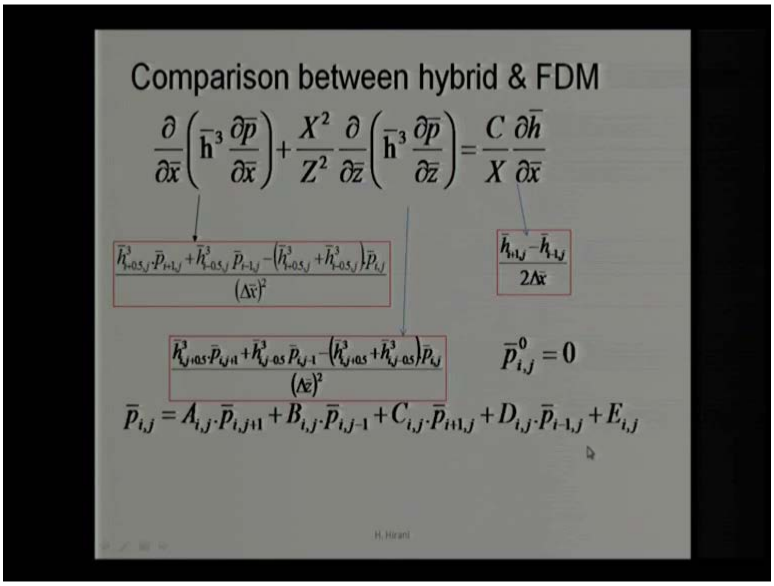

So, when we have that kind of thinking, this hybrid approach should provide us the reliable results and we can see let us start with the final difference method, it is a 2 dimensional in our earlier lecture we considered only the 1 dimensional case and tried to get the solution based on the 1 dimension, the dimensions in one directions are only sufficient to be considered, not in two direction.

However, in this case to get a more accurate result, more reliable result so, that we can compare with the hybrid approach we require dimension in about this both directions so, this is the first term, second term and third term, all are in non dimensional form, we already understood how to discretise, how to present the pressure profile at the nodes and here the i and j are the nodes. So, h here h bar was calculated as the some i node plus half of the node which is the hypothetical, but we can find out the geometry at the hypothetical nodes.

This is the P i plus 1, 1 node ahead of the desired node and here h bar i minus 0.5, that is a 1 node it is a half node behind our the node desired node, similarly, over here P i minus 1 node behind what we say, that which has already passed compared to the existing node or desired node.

This is the desired node and this is the spacing in X direction, del x bar is spacing in X direction, coming to the second term here now, one easier one thing is that, if this h bar does not depend on the Z then I do not have to do this kind of discretisation in terms of h

I can simply move out h q you see h i j q can be simply moved out of this differentiation and then we need to discretize only this term, but we are not taking the assumption here.

For derivation we are counting both the cases and where the h also depends on the Z. So, what we can write here is also the similar form instead of i, here we are writing j. So, i will be as it is and j will get additional node, j plus 0.5 that half node j direction similarly, here instead of i plus 1 j we are writing p comma j plus 1, same thing over here and again same thing over here.

Next similarly, we can take a central difference method for the dispersion of this film thickness gradients h i plus 1, h i minus 1. Now, many times what happened many times h as a given as a film thickness and this expression can be directly figured out. In that situation, we do not have to really differentiate we do not have to use the final difference method.

Assume, that we know already h as a 2 by 3 and in brackets 2 minus x bar, we can simply differentiate, we do not have any complexity in the value of differentiation. So, I can directly use those values, I do not have to really convert or say don’t have to discretize represent the gradient terms of the nodes, we can directly write in terms of i j, but not in this way we can simply substitute that. Now, what is what was the thought earlier that when we substitute all these pressure expressions, we should get the pressure P i j which is the desirable pressure.

At any instant P i j is pressure to be determined with the help of pressure at a four neighbor nodes and this are the constants, these are geometry dependent constants while this is the source term. So, we need to figure out what is the A i j from this expression we need to figure out what is the B i j from these expressions, C i j, D i j from these all expressions which we have written and finally, becomes E i j which can come from this expression.

And this if you see there here P i j here, P i j here they are 2 terms P i j, that means, we have to take common all these P i j and there will be some term with the multiplication factor.

And we are not writing any multiplication factor here, that means, whatever the factor comes over here that will be in denominator on this side or we need to re arrange these expressions in such a manner, who will geometry constants turn out to be in terms of A which involves the terms which are also on the left hand side.

To explain this we can use some expression, however, to solve this equation or with a there will be number of unknown to us, but there will be few known's also to us. So, we know the boundary conditions. So, ratio will be known to us without boundary conditions we will not be able to solve it, but in addition this will be in iterative scale and initially we all assume that pressure at the all end nodes is equal to 0.

Unless we know what is the pressure profile we are able to estimate by some other method, if we are able to estimate by other method, we can give that as an initial value. However, we are not able to estimate then you see simplest one is that we assume all the pressures are equal to 0 at the start and it through iterations only it will develop, it will come to the some solutions, but we need to provide some boundary conditions, what will the pressure exit pressure at the entrance which are generally known to us or what many times we are known what will be the supply of pressure at which pressure the liquid is applied to the interface.

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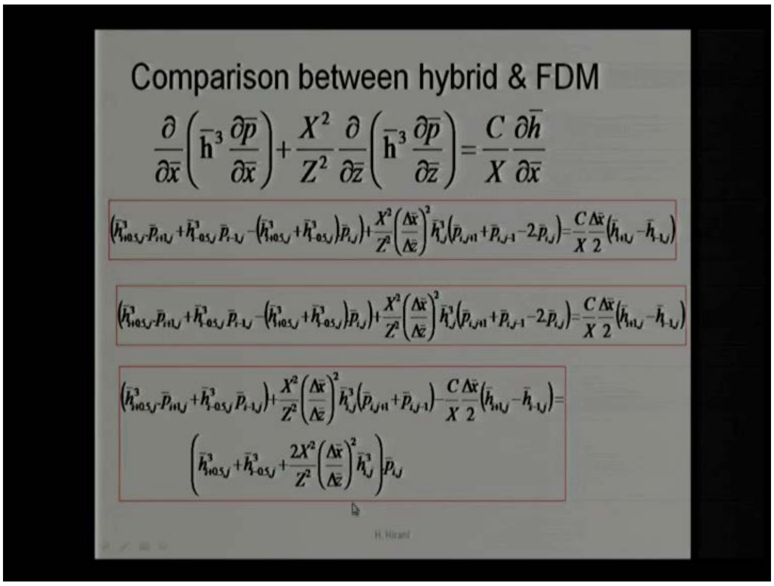

Those things are known to us. So, when we try to incorporate those values using (( )) find out the A, B, C, D in terms of geometry, we need to substitute all the expression what we got in the previous slide, expression for this first term, expression for this second term, expression for third term, now substitute all this assuming that h is not depending on the Z that is simplifying over the case, here in h bar q I, h i j and bracket is without any h.

So, it is a P i j plus 1, P i j minus 1, minus 2 P i j the simple one and in first case only all the h terms are appearing now, we can rearrange these terms by we can rearrange in terms of P i j, we substitute the values in P i j on one side and get the results accordingly and this is what you say that you take a common of P i j at the both the side and this has been done here P i j we had 1 term, 1 and 2 term over here that is come this side.

Yeah, this one and this one and similarly, this is alpha term which is multiplied with P i j that is the term which is coming over here and this is negative side. So, we have taken to the right hand side and this term will be on the left hand side. So, we are just re arranging now.

We know very well that this is the P i j and right hand side in the left hand side not a single term contains P i j, while on the right hand side only the term contains P i j, pressure profile and all other or geometry constants which will not vary with a iterations, it will not vary with iterations, however, all the pressure terms will vary with iterations, but all other remaining all other terms will remain constant.

So, once we know this expression we can equate, we can say we divide whole this term or to the this expression or we should get a pressure expression in terms of P i j, we will continue it this kind of approach in our next lecture and try to complete the difference method and compare with hybrid approach and try to establish whether the hybrid approach is going to give reliable results to us or not. Thank you, thanks for your attention.