Tribology

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Lecture No. # 20

Reynolds Equation

Welcome to twentieth lecture of video course on tribiology. Today's topic is the Reynolds equation. In previous lecture, we derived this equation; assuming few concepts or assumptions based on those assumptions, equation was derived, it is not generalized one, but restricted to systems, which fulfill the assumptions. Today, we are going to discuss this equation or some variation of variations of this equation.

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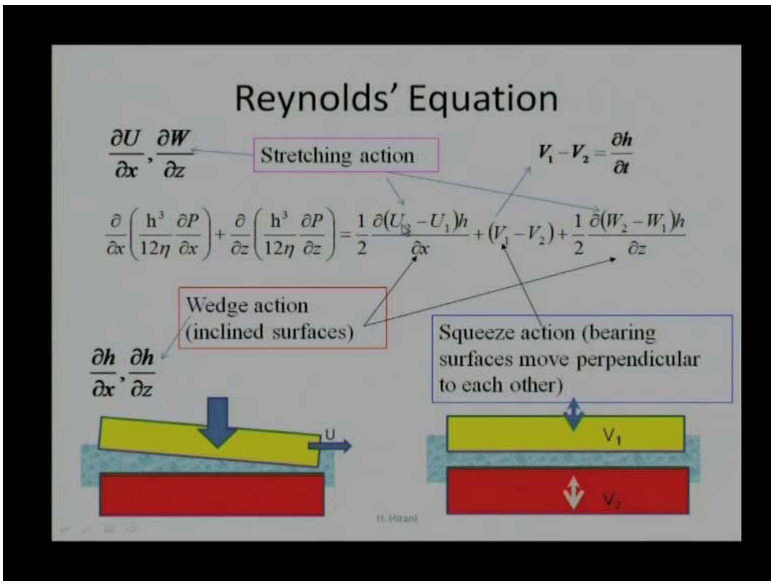

So, first this slide shows Reynolds equation, which was derived in previous lecture, left hand side shows pressure term. We need to drive pressure distribution over the (( )) surfaces. Well, right hand side shows the source terms, there are three source terms, first and last source term, where the derivation of film thickness h is coming, that is known as the wedge action. Basic assumption is that two surfaces are inclined at say on some line, it may be one degree, may be two degree, may be find, find degree also.

Some inclination is required to develop the pressure that is the wedge term. We can simplise also the wedge term as derivation of h with respect to x. To find or you say to define the wedge action x direction similarly, we can write wedge action in z direction. In this figure, we are showing wedge action in x direction. If I assume perpendicular to this inside this (( )) that will be the z direction and variation of film thickness in z direction. Because of this inclination there is a possibility of pressure generation.

As far you say that for hydronamic action wedge action is essential. We drive the second term also which is known as a squeeze term. Basically depends on a squeezing action of plates of relative movement along the film thickness.

Sorry say that, if I am assuming the tribal base, they move perpendicular to each other. And this figure illustrates that yellow plate has a variable viscosity in the both the directions. Similarly, red block, red plate has a relative velocity in between there is a viscous fluid.

If yellow plate is moving in this y direction is going to generate some pressure and it can be say there is a pressure term. And third source is and third source is again stretching term. But before that we simplizing here, the V 1 minus V 2 is a relative velocity can be expressed also the variation of h with respect to time.

As velocity changes movement of this plate will increase or decrease the film thickness. And as a velocity is a constant, they say time term comes into the picture. That is why we say, it can be expressed as partial variation of h with respect to time.

As I mentioned third term is a stretching term, we are assuming plates are not solid plates. They stretch and vary the velocity along x direction and z direction that is why we are representing the stretching term as variation of U. Then you show velocity along the x direction with respect to x direction or variation in W with respect to z that is represented over here.

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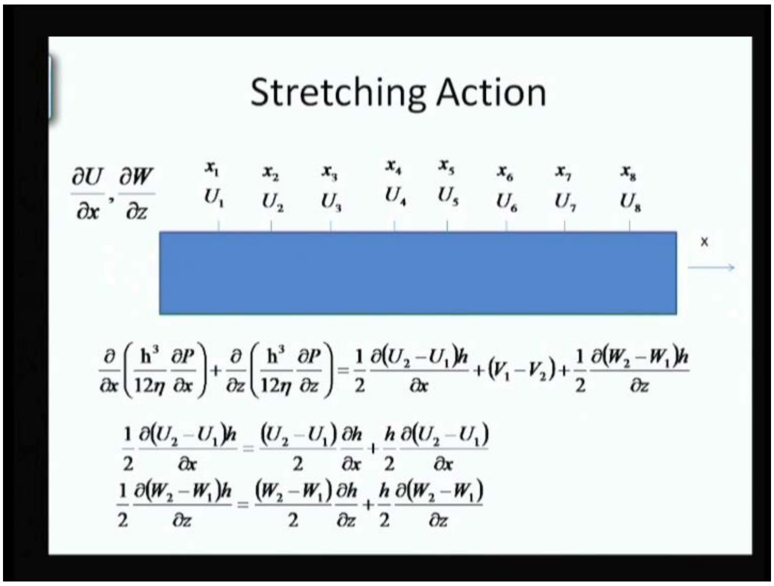

To elaborate this stretching term, we have one more slide (( )) in the cases like stretching action. I am trying to show with example of one plate, say there is a velocity along the x direction, but, this velocity is not constant. If I take different sections, say plate as a section x 1, it has a velocity U 1, plate as a x 2 velocity U 2, plate at the x 3 position has a velocity U3. Similarly, plate at x 8 has a velocity U 8. What is the meaning of this? This plate is not rigid whatever the velocity over this surface is not the same velocity of the slightly ahead of that, it may be increasing or may be decreasing.

As a stretching term, typical example is a rubber material, which case is stretched. And we find slight variation in velocity along the surface and that is also a source of pressure generation. So, this wedge direction can be represented every believe, that is the most of the, in most of the cases is negligible.

If you want to represent in Reynolds equation, which was explained in previous slide. We can write this term or partly in differentiate this term, first differentiation with respect to h, differentiation of h with respect to x and then in second case differentiation of velocity with respect to x. So, this term can be represented of this first term can be represented I as a relative velocity divide by two. And partial variation of h with respect to x plus here h is not integ differentiated. So, h by 2 and differentiation not give 2 minus U 1 with respect to x.

So, one term is converted into two terms and what is a stretching term is the second term, first term will be wedge action. So, this term has a two components, one is a wedge term and second one is a stretching term. Similarly, in z direction this term can be expressed in two terms. In first case differentiation is only with respect to differentiation of h with respect to x and in second term differentiation of velocity with respect to x. I think there is some mistaken, this will be say differentiation, with respect to z and differentiation with respect to z. So, this will be your refrected to z terms, variation of velocity with respect to z and variation of h with respect to z.

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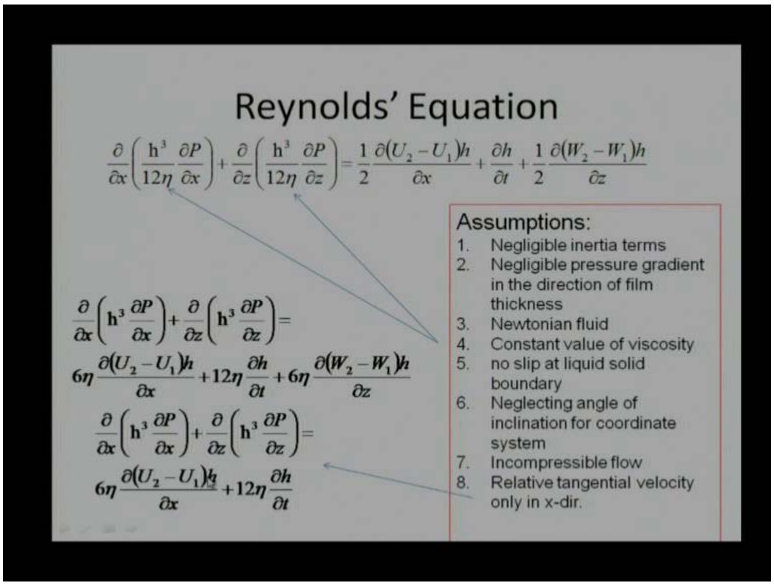

Now, coming back to the Reynolds equation, we want to find out the various variations, we derive this equation with a number of assumptions. We say that initial term is a negligible, negligible of pressure gradient along the y direction that means, there is no variation along the y direction. Whatever we choose pressure will remain same, at any y location, then Newtonian fluid we took as the example of Newtonian fluid, we use (( )) relation than constant viscosity value, no slip neglecting angle of inclination for coordinate system and incompressible flow.

Ever, when we are saying that there is a constant value of the viscosity, while in this equation is still this viscosity term is in a differential form. As these terms are the constant, it can be taken out easy without differentiation, and this equation can be rearranged. So, this is what we are saying that if we take out this 12 eta rearranged into the right hand side, this equation will be slightly modified. We say now instead of half this term as the 6 eta, 12 eta divide by two, it will turn out to be 6 eta. Similarly, in this case there is noth no multiplication, but, here after taking this in the right hand side is turning out to be 12 eta into d h by d t.

Here is again only half with multiply with twelve eta, it will turn out to be 6 eta and term along the z direction, which represent the wedge action as well as the stretching action. Now, to simplify this equation, we assume (( )) velocities along the x direction in other word, there is no relative velocity in z direction. So, this term can be neglected in this situation.

Because, there is no velocity allowed there or there is no relative velocity along the z direction. In that situation, this equation is simplified one term has been eliminated that gives the simplified expression. The pressure term on the left hand side and this term express the wedge action plus stretching action and this is squeeze action. Now, we have a three source terms over here, one is a squeeze action term, one is wedge action in x direction and stretching action in x direction.

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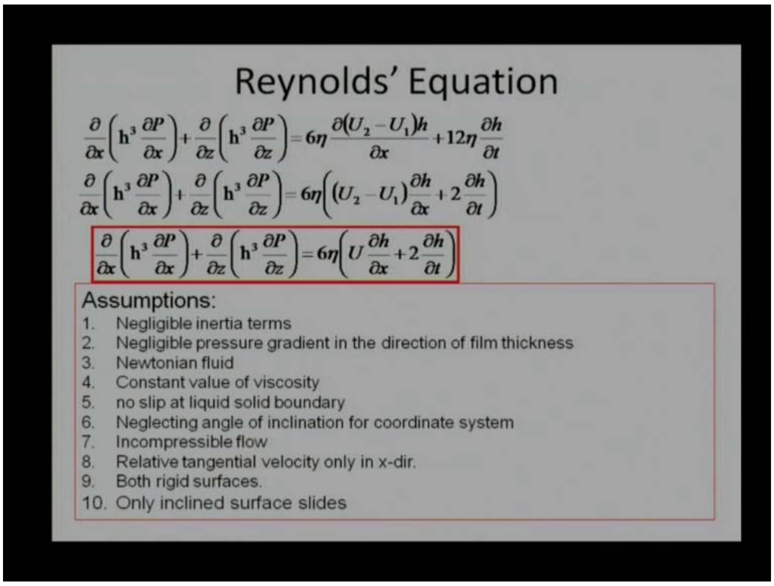

We can further simplify this equation, by assuming both the surfaces are rigid surfaces. There is no stretching action, we are not talking about the rubber material, we are talking about more like a metals or ceramics which cannot be stretched. So, easily I know you are (( )) stress stretching action is negligible for our calculation.

So, assumption with assumption the both rigid surfaces, this term can be say variation of relative velocity with respect to x can be neglected. And as we expressed in previous slide, previous to previous slide that this term can be expressed in two terms, one is stretch term and another wedge term.

Stretch term is negligible, because of the rigid surface. So, then only we will be getting the wedge term, that is given by definition and this is one of the common Reynold equation number fourth is simplification can be made. So, we are talking about the relative velocity.

Again assume one of the surface is a (( )) or particularly, we say only inclined surface is incline, inclined surface is sliding at some velocity (( )) surface. Which is a straight surface does not move and U 1 can be neglected, U 1 is 0. With a that kind of thing, we say that only inclined surface slides, equation will be modified U 1 will be 0 and in that situation it will be only U 2.

As there is only one velocity there is no point to say it is a U 1, U 2, U 3, we can say directly it is a U (( )) velocity of sliding surface. With that equation will be modified in this form, there is a pressure term in x direction, pressure term in z direction constant viscosity and this is a relative velocity in of inclined surface in x direction. This is the wedge term in x direction and this is the squeeze term and this is one of the most commonly used Reynold equation.

In literature, in number of books Reynolds equation is starts with this kind of form. Where the number of assumptions are there in understand this da, this to this equation, we have made a number of assumptions. We assume there is a negligible in a short term, this may not be necessary in particularly high velocity terms. In a short term will not be negligible, then we assume the film thickness direction of pressure gradient is negligible. That happens only when there is a very small film thickness infinitely small thickness.

And we assure Newtonian fluid and interesting thing most of the liquids are non Newtonian liquids, that they are not lia Newtonian liquids. Then we assume the constant value of the viscosity which we know very well velocity. Viscosity will not remain constant. It is a function I say it is a strong function of temperature then there is no slip on the liquid and solid surface.

What do you with the solid surface? Is the relative velocity in the same sliding, the speed is imparted to liquid without any loss. Which most of the cases, most of the liquid cases is true, but, is not true for the gases. Where the molecular distance is much and there is a earnest possibility of the slip. Then we assume the neglecting angle of inclination for the coordinate system in the angle of inclination is the very small than that is true, but, if the angle of inclination more then it may not be true.

We assume the incompressible flow, even in liquid cases many liquids can be compressed to 1 percent or 2 percent. So, assuming incompressible flow, provides may be 95 to 90 percent 98 percent good results, but not hundred percent results. If you require really 100 percent accurate results and we should account the compressibility of the liquid itself.

I ever for the gases, this assumption cannot be made gases are compressible and the pressure they reduce the volume. Then we assume the relative velocity in x direction that to simplify it. Because, we can super impose to separate equations, but, will effort will increase in this situa in those cases.

Finally, we assume two assumptions the rigid surface which is most of the case times is a true even for rubber material stretching does not happen in that much. So, that is a reasonably good assumption. And finally, we say that only the inclined surface squeeze to simplify the situation. But, whether the solution we can get for x direction and z direction. And you can super impose. So, that is not a very bad assumption, it can be made and we can get a good results.

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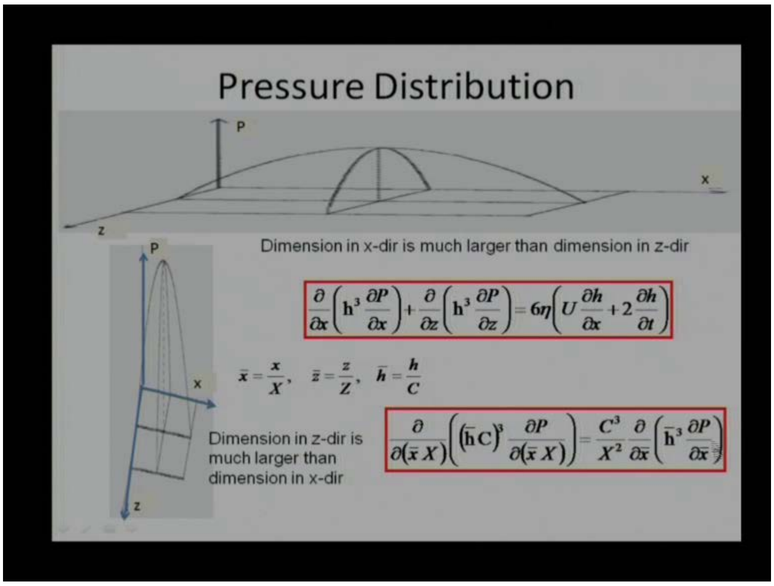

Now, there is a some interest to think over. What this equation is really imparting, what this equation really conveying to us to understand that, what we will do will non dimensionize this equation, what is the reason for the non dimensionalizing? On this slide is going to present now, you say let us take a two assumptions in one case, x direction dimensions are much larger compared to the z direction, while in second case x direction term or dimension in x direction is much smaller than z direction, they are two extreme cases.

In one case we are assuming the x direction, dimension is much larger compared to dimension in z direction. In second figure, we are assuming x direction dimension is much smaller compared to dimension in z direction. So, we can write the dimension in x direction is much larger than dimension in z direction and the distribution comes something like this.

Now, what we are gaining from this. You are saying that pressure maximum, pressure remains same. As x value is more p by x will be lesser, while in this case z is a low p remains same p by z will be low higher than p by x let I asked to neglect one of this term I can if I say that value of x is much larger. So, p by x will be smaller in that case this term can be neglected.

While in other case, while we have short length in x direction and p by x will be there. So, in that cases second term can be neglected. We can assume that based on dimensions, we can get good results.

To understand this better (( )) less non-dimensionalize, what is the reason for non dimensionalization say what is the maximum value of x? Maximum dimension in x direction assume that is a capital x. That means, this x will vary from 0 to maximum x or x bar value will be varying from 0 to 1. Second term same thing maximum value of z or you say dimension in z direction, maximum value is same, this is a variable, it will vary from 0 to capital z in that situation z bar will be 0 2 1.

So, what is the advantage of non dimensionalization, irrespect to half expression, we are trying to use the first term, where the x will x bar will vary from 0 to 1. Similarly, z bar will vary from 0 to one. So, we can do order analysis what will be multiplication terms comes that will decide, which term is going to be dominating. To get the meaningful results, we will also non dimensionalize film thickness say film thickness is a h divide. We say non dimension of film thickness of h bar is film thickness at any location divide by clearance between two surfaces is a mean clearance.

If maximum and minimum value are given, we can average out we can get the results to some meaningful (( )). Now if we go ahead with the, this kind of non dimensionalization, while in this case like a instead of, wirte x I write x bar into capital x. Similarly, instead of h, I write h bar into c. Similarly, z instead of z we can write z bar into z and substitute over here. So, this is the same thing instead of x, we are writing x bar into capital x instead of writing h. We are writing h bar into c, here x instead of x, you are writing x bar into capital x that is for the first term. And when we rearrange, what we get term x bar h bar x bar over here and multiplication factor comes as a c q by capital x square.

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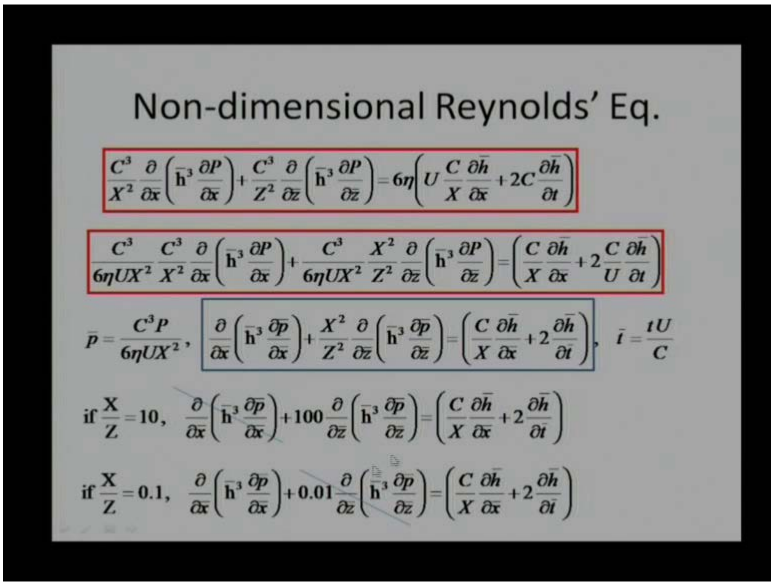

This is what I was talking about this multiplication factor is going to give us order analysis. If we do similar kind of for the z direction, what we get, c cube by z capital z square. So, in this case we are getting similar kind of term only, the non dimensional form except the p in p is still in dimensional form, multiplication factor comes as the c cube by capital x square. While in second term is the c cube c cube is common in both the term while instead of x capital x square in second term is coming capital z square.

Coming to this side, which we have normalize x as well as h we are getting multiplication factor as a capital c divide by capital x. Now here 6 eta is term and there is a interest to non dimensionalise all these terms. We already non dimensionalized x, non dimensionalized h, non dimensionalized this x, but, pressure is still in a dimensional form. So, there is a need to non dimensionalize this pressure.

So, one way to non dimensionalize, it is taking U as a common of this is a 6 and eta into U, while here it will be without U and here it will be dividing 2 c divide by U, that is coming over here 2 c divide by U. That means, U has been taken common from this bracket, the whole 6 eta into U has been shifted to the left hand side. That is right here the c is cube remains same capital x square and 6 eta U is added this is pressure unit.

This is m m q this is m m square of in the meter cube meter square. So, meter remains over here, while velocity is expressed in a meter per second. So, meter meter will be cancelled and eta is generally pressure unit into second. Second is a over here and divide by second, that will be cancel out. So, overall this expression will be per one divide by pascal, one divide by pressure unit. And which means if we multiply this full c cube into p divide by 6 eta U capital x square, it will turn out to be non dimensional pressure term. So, that is why we how rearranged. In a such a manner, this equation can be rewritten in this form say x bar is a non dimensional term, h bar is non dimensional term, p bar is non dimensional term, x bar is non dimensional term.

Now, in this situation whatever the dimension we choose, equation will remain same, solution procedure will remain same, only the magnitude will change. That will bring a lot of stability in numerical analysis, which we can express later. However, what is the better thing coming over here is in this case, we are non dimensionalizing with a capital x square term. And here we have a capital z square term in that to bring that capital x square term, we have divided as well as multiplied.

So, this is going to make a non dimensional pressure term, while this term remains, here is a capital x square, divide by capital z square. This is pressure unit non dimensional pressure unit, where x will vary from 0 to 1 here pressure term again, where z will vary from 0 to 1. And maximum value of the pressure will remain same, in z direction and x direction the same pressure maximum value.

So, they can be treated as same unit may be say overall pressure term, comes from as a one unit, one same this will come also unit one. But, this is square term x by z is square is going to make difference. Again say this is going to give me the quatitative analysis, whether second time is going to dominate or first term is going to dominate.

If one term is dominating much larger extent compared to other, I can neglect one term. That will simplify solution to greater extent ever in this term, we are not non dimensionalize the time. Which was important also because to get all non dimensional term is also important to non-dimensionalize time. That is why we did U into t divide by c at the meter per second into second divide by meter that is turning out to be in non dimensional, that is non dimensional time term. So, we have a non dimensionalize this complete equation.

As I mentioned this equation is going to give us order analysis a quantitive results also. So, we are going to see that now assumption is a, if dimension in x direction is 10 times compared to dimension in z direction or we say x by z is 10. I substitute over here, what we are getting here is 100, if this is a 1 unit, this second term is only 1 unit, it has been multiplied with the 100. That means, this term is very rich compared to first term or even if you neglect this term I am going to get a accuracy problem upto 1 percent. I am going to get 99 percent accurate result that is advisable.

We need to see, how much time really require to solve this equation and I am getting even neglecting this term. I am getting 99 percent positive results and there is no point or there is no meaning to account this relation, unless I am requiring accuracy much much larger extent.

While the second and this source terms are remaining, the same there is a non dimensional terms. Similarly, if I assume the reverse up this, we in this case, we say that this term will be negligible that. That is very clear after non-dimensionalize before a non dimensionalization.

It was not very clear after non dimensionalization, is very clear that this time is negligible if capital x by capital z is 10 or dimension in x direction is 10 times compared to dimensions in z direction, reverse is also possible. You say that x is the much smaller compared to z. In that situation if we substitute over here I am getting 0.1. That means, when this dimension is much smaller, x dimension is much smaller, we are getting low value of z.

That was expressed in previous slide with a figurative form. When x dimension is much larger, then only this term is meaningful, first term is not carrying much weight. Here the 100th weight compared to one weight or we are going to get 99 percent accurate result, even if we neglect the first term.

While in second term, case this is negligible, this is getting one way while this second term is getting only 1 percent of that. Even I think in this case if I neglect this term, I am going to get accurate results upto 99 percent. So, we can say this term is negligible.

This is going this kind of analysis is going to help us, to solve the, this equation. The solving partial differential equation requires some effort, require some numerical method to be implied. And numerical method whenever we imply, we need to think how to direct that solution.

Numerical solutions are not easy to solve without understanding it. If we keep giving instruction to the computer to solve the equation, without understanding that without bring a good understanding about that, we will not be able to get more meaningful results. That is why I am emphasizing to understand system first, understand the dimension first and then you think which numerical analysis should be applied. If it does not matter much your time, does not matter much you can go ahead. But, if time matters to us then we should the think solution based on dimensions.

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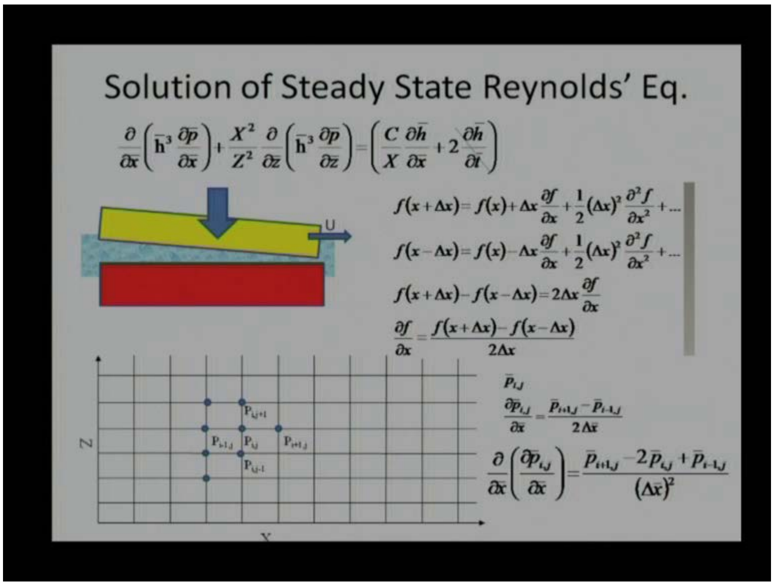

Now, there is a one modified version also you say that we are assuming that, there is a steady state Reynolds equation. Plates are not going to change the position with respect to time. That remains inclination, remains same film thickness, remains same with respect to time. So, in that case this second term say you can source term will be negligible and that will be call a steady state Reynolds equation. So, this term is negligible.

Now, this is what we are talking load is applied, velocity of the yellow plate along the x direction that is u velocity. And this plate is a stationary and relate to film thickness between this two plates at any x position is remaining fixed, is not going to be change with time. Anyway this is a proved assumption we know very well because of the (( )) condition some change will occur, but that may not be much problemity. We can do some sort of a statistical analysis. We can assume the pressure will vary to within a some bad.

So, for starting purpose is always advisable to start with the steady state analysis, unless turn the pressure generation. And see how the results are, we are able to get what kind of modification in design should implement, so that we can get much more desirable results.

Now, this is a shown the two plates and if we want to think about solving this. We can use some numerical method, one of the numerical method is the final difference method. What we do in a final difference? We divide full surface in number of nodes. You say big zone cannot be taken easily, if we divide the zone in number of sub zones, then tackling each sub zone will be easier compared to tackling one big zone.

That is why we divide the surface in sub number of nodes that is why in z direction and x direction. These lines are parallel lines and perpendicular lines, they are going to give us some nodes. And what we are saying that we are interested to find the pressure at some nodes, assume this is i j junction and we are taking the help of (( )) nodes (( )) nodes are going to help to find out the solutions.

And it is iterative procedure. I am going to explain how it is iterative procedure and how this (( )) nodes are going to help us? When we talk to divide this whole surface in a sub number of zones, we say that these are the nodes intersection points of the nodes. Where the pressure will be calculated and at any when a cannot this equation I need to give some nodes. I say I can vary from 1 2 number of nodes total number of nodes in x direction and j can vary from one number of node to number of node in z direction.

That is p bar i j and there is a pressure term, pressure gradient term that can be represent in final difference form something like this. If we are saying that, we are averaging term. We are making the average of this to find out the p i j gradient at the p i j, we are taking the help of 1 advance node in x direction and 1 node which is lesser than by 1 unit compared to p i j.

That is why the p i and plus 1 j and p i minus 1 j. If I know never nodes value, I can get the results and if we go for the second gradient, in that case we required to calculate p at the mid node, then there is somewhere here in between some pressure gradient at the mid node. Because, we know this can be after a midterm will not be there, mid node will not be there, when we implement this formula over here will be the only the nodes.

So, that is the some sort of accuracy requirement, we arrange it. We can explain this with a Taylor's series. We know, the Taylors series if x is define and there is some delta increment, that Taylors series can be expressed in terms of f x plus delta x. And first gradient of x, first gradient of x with plus to x second term. Similarly, when we take slight difference in negative sign, say x minus delta x delta that also can be represented as f x minus delta x. And prea this gradient differentiation of f with respect to x can be given and this will be the second ordered term. As if we assumption is a seca this delta x is very small then higher terms can be neglected.

Now, if is subtract second equation from the first equation. What I get f of x plus delta x minus f of x minus delta x is equal to 2 into del x and pressure gradient. And rearranging this, we can find the gradient of f in x direction as f x plus f of x plus delta x minus f of x minus delta x divide by 2 delta x. And there is the same thing which we have expressed over here i plus 1 pressure.

And whatever the I am assuming the at any point x is the i, whatever the next delta come, whatever the next difference comes that is the i plus 1. That means, the distance between the, these 2 points is equal to delta x so, p i plus 1 j and p i minus 1 j. That means, if I am calculating the p i j here, I am taking this term minus this term divide by distance between these 2 terms as a final difference term. We are calculating an accuracy is reasonable, if we assume the delta x is a smaller, it is not going to affect this solution accuracy.

So, based on that, we can really divide this equation in number of algebraic equations. And we know very well solution of the algebraic equation is much much faster compared to partial differential equations. Now we can go sequentially, we can go simultaneously. There are number of methods available for time, being I am just thinking about the sequentially to explain in this analogy of neglecting 1 term compared to other term using sequential steps.

And this equation after substituting value, what we have achieved from this over here we can rearrange and we can get this expressions that is it p i plus 1 j minus 2 p i j plus p i minus 1 j divide by del x square. So, we know how to rewrite this partial differential equation in terms of final difference, in terms of the nodal pressures. As nodal pressure can express in nodal pressure and we finally, want the nodal pressure to be evaluated. So, we require some sort of (( )) which we are going to explain in the next slide, how the equations going to help us?

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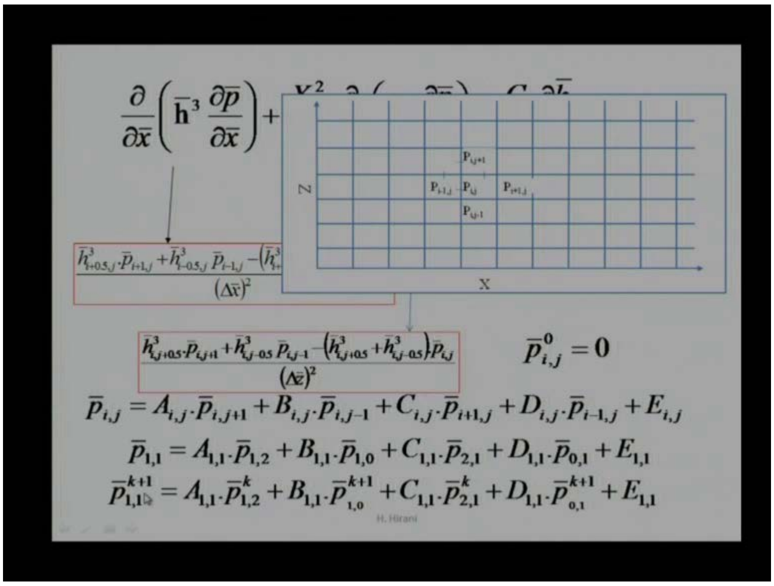

Now, this say that we have three terms, in Reynolds equation that is steady state. Reynolds equation with a number of assumption, the first term can be the present here h is the only 1 gradient. We will take the half node calculating the h and the half node is not a problem h is a geometry and which is well defined to us.

While, pressure is the second gradient that is why it is coming p i plus 1 j plus again film thickness at the half node, half node minus sorry i minus half node and pressure will be p i minus 1. Well third term is the submission of this in previous slide, we wrote it 2 while in this case there is a submission. So, if they have both h has same, it will turn out to be 2 and divided by h is square.

Because here this is a h velos h is turning term is coming, that is why we are written whole term like this. Same thing for the z h is coming we can whole term something like this h i here j instead of i plus 0.5 we are writing i c, but, here we are writing j plus point five while here it was only j. That means, variation in j direction and z direction was considered. While in this case variation in the I direction is not considered variation only in z direction is being considered, this is what we express using the final difference method.

Similarly, for third 1 we can simply write, final difference h i plus 1 h i minus 1 the pressure gradient sorry film thickness gradient is only in x direction. So, we can write the complete the Reynolds equation in terms of nodal pressures and geometric terms. We can rearrange in much more meaningful reasonable way that is like this. So, we want nodal pressure at any node i j that will be a function of nodal pressure at i j plus 1. Similarly, it will be a function of i j minus 1 function of i plus 1 j and function of the i minus 1 j.

You can see this is the centre point, we have 1 negative x direction, 1 positive x direction, 1 negative z direction and 1 positive z direction. That means, we are calculating the flow pressure around the nodal pressure, where we want to calculation and based on that we are evaluating the results. It is more like an averaging a scheme, I know pressure term nearby and try to figure out what will be pressure at the nodal point of that.

In addition, there will be a constant term if all these pressures are 0. Initially this term is going to decide what will be. This is a what we have expressed over here, if I, i need a p i j is somewhere here taking result of p i minus 1. I am taking results of p i plus 1, I am taking the result of p i j minus 1 and I am taking the result of p i j plus 1 to evaluate what will be p i j and these are the half node points. Whether we need to find out what will be the film thickness?

As I say the film thickness calculation is not a problem because, it is a geometry dependent and in a geometry is given a continuous function. We just need to substitute the value of x to find out what will be the h at that location? See it is not difficult, it can be figure out and once we find, we do not have to do any iterations to vary that results.

But, pressure we need to find out that is requiring it to the procedure. Just to give example, we can take a p 1 1. We will say assume the node point is somewhere here is going to be a function of 1 2 function of 1 0 function of 2 1 and 0 1 this is a generally boundary values. x in the initial point and the z in the initial point where the 0 0 term is expressed generally, these are the boundary values. If and 99.9 percent we know the results of the boundaries generally at the edges or the starting points, we will be knowing what will be the pressure value. So, we can say in this situation, we will be knowing what will be the 1 0 and what will be the 0 1 in most of the cases and what we need to find out the next one.

As this term is the 1 1 and we are trying to find out 1 2 and we are getting the value from 1 2 and 2 1 which are not known to us. We are expecting results from unexpected things. So, that is why we require iterations, we need sequence of iterations. So, that we can finally, come to 1 conversion point initially, we can assume pressures are 0, the (( )) and only this term is going to give me the results.

An interesting point is that, I have written i j b i j c i j d i j e i j. This is say that depend on the geometry, depend on the spacing one, how much spacing we require between two node based on that. Once it has been decided, this value remains fixed it will these values are not going to change with the iterations, with iteration only the pressure terms are going to change.

Now, this is what I explained and I mentioned about the iterate. It is scheme also we say that we want pressure at node i j or may be in this case we took example of 1 1. So, we need at a any iteration k plus 1 assuming the k iteration is already over. We evaluate pressure 1 1 at the k plus 1 iteration it will be function of 1 2, here the iteration is k. That means, 1 previous iteration whether the pressure known to us that is going to help us to find out pressure at an (( )) for the node i j. And here again 1 0 comes which we already have overcome that is a previous node compared to this node, which will be known to us.

That is why here we are writing k plus 1 iteration. So, whatever pressure is already known, we can directly utilize. While coming to this term here again 2 1 will not be known before hand and for k 1 iteration. So, we take a example, we take a value which was available from the previous iteration and coming to next term d 1 1 and p 0 1 here again that 0 is already over we already passed this to next node we are advancing. So, this value of this pressure will be known to us. So, it will be k plus 1 that is recent iteration.

That means, 2 terms are coming from previous iterations and 2 terms are coming at the present iteration. That is going to decide us, decide our pressure term while this term will remain constant or we say all the a b c d e will remain constant, cannot going to vary with iteration. So, iterations are only required for this purpose.

And for simplicity, when we start solution, we assume pressure at all the nodes is equal to 0 and we can assume that as a k equal to 0 the 0 iteration. We are iterated anything just beginning. So, the 0 iteration pressures are known that is equal to value equal to 0 that is known to us. Now, we can start putting all the 0 0 0 0 at this e 1 will give us, what will be the value of pressure at 1 1 node for the first iteration? So, that will give a sequence that will give a good result to us.

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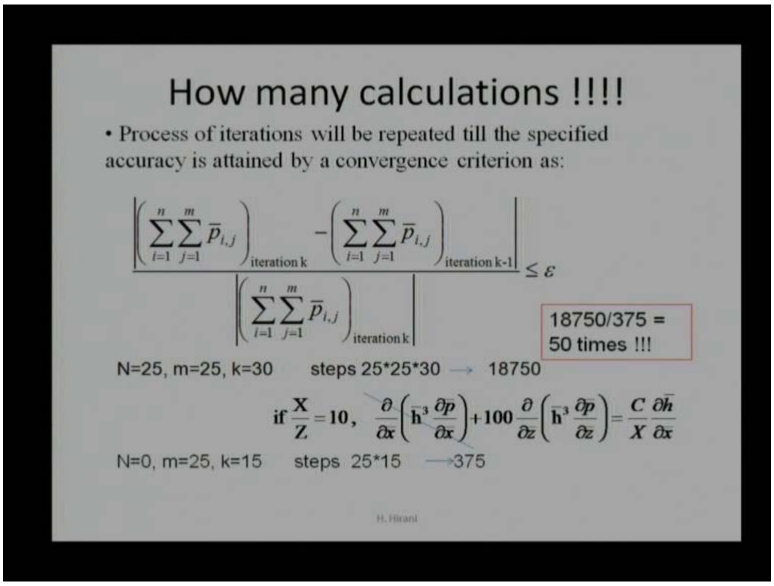

Now, the question comes how many calculations are really required, how many iterations are really required to find out; what will be the pressure? Say this is a repetitive process, we need to repeat till we get accuracy desirable accuracy. It may be 1 percent of the pressure, 0.1 percent of pressure, 0.0 1 percent of pressure, depends on. What is the order of analysis are we doing final design or we are doing the initial design, by initial design you may not require. So, many iterations we require some initial geometry.

So, that we can come up with the some results and then go with the compatibility with the other components. So, what we say pressure conversions term is based on the pressure, what is the meaning of this? We say that all the nodal pressures we are summing up. What is the value of all the nodal pressures?

For cavitation key, which is the present iteration suppose you are and this is the previous iteration and divide with the submission on absolute value. We are talking only the absolute value, may be sometime negative value also comes have to be go ahead with sort of cavitation with negative pressure is generated or you say pressure glow the atmospheric pressure is generated.

So, overall this percentage should be lesser than some epsilon value, it can be 0.01 it can be 0.001, 0.000 1 depends on the requirement. Now I am coming to the point, which we started, how non dimensionization is helping us to find which term is going to dominate?

And if I know which term is going to dominate, how we are going to save the calculation, how we are able to save the efforts, to illustrate that? We are giving this say, assuming I divide it plates in x direction in 25 nodes. Similarly, in z direction we divide it in 25 nodes and we require 30 iterations to come up with the right solution. It is just an assumption, just to example to convince what is the meaning of neglecting one term and how much benefit we are going to get. So, how many steps we are required 25 into 25 into 30, the assumptions are these all the steps are required to find out the pressure variation in tribal surface. That is going to this figure turn out to be 18,750.

Now, if I take this assumption, say dimension in x direction is 10 times dimension in z direction. What we are going to gain something like this, 100versus 1, this term is neglected. So, I had to consider only in z direction, what is the meaning of that I do not require any node in x direction. They should term in x directions are negligible.

Why do we require any node I do not have to divide that plate? Assume the pressure will be knowing to me. And pressure in twenty 25 same number of nodes we are keeping in z direction as we are not counting the pressure term in n direction. We can find pressure in, sorry, terms will or iterations will be almost half of the term iterations, which we require for the both the directions.

We say when the pressure term is unknown in z direction as well as x direction, we are requiring 30 iterations. When pressure is not calculated in x direction at all is calculated only in z direction and keeping the same number of nodes. There is a possibility we require only 50 percent of iterations compared to this and instead of 30 we required only 15 iterations. So, if I multiply 25 into 15 that is going to give me 375. You can come here 18750 was a 375 calculations. And we know the h steps and these are the steps and each steps there will be number of calculations, may be more than 1000 calculation of each step.

So, how many steps we are going to save from this? Is almost a 50 times or you say 187509 divide by 375 is going to give me 50 figure. I miss better understanding this system, we are able to save 50 times efforts or we assume that each effort is requiring 1000 calculation. That means, we are going to save 50000 calculations. Which we are may be a reasonable figure particularly, when we are starting the solution scheme or we are start learning the tribology subject. In those situations is going to really help us to a greater extent that is useful thing for us.

So, this is what we express, how we get the benefit of the non dimensionalization and when we non-dimensionalize this result, how we are going to get overall calculation benefits?

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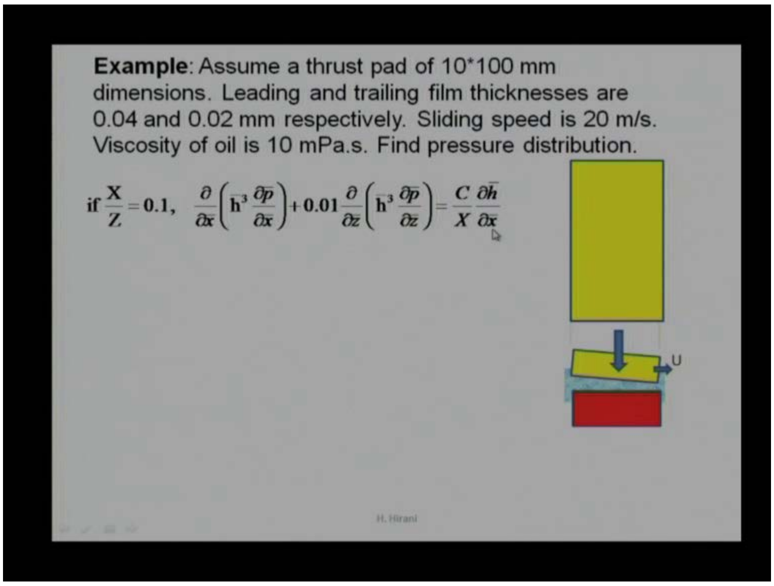

We can, I will this example, I will cover in next lecture, but, for the time being I can show what we expressed in previous slide 1 example related to that. I am assuming there is a first load or load perpendicular to the axis is coming over it. There is 1 plate with stationery plate, 1 sliding plate which has a velocity u and this velocity in the present example is given 20 meters per second, which is very, very high velocity.

And we are saying the film thickness of the two junctions are (( )) at the entrance. The film thickness is 40 micron and at the exit film thickness is 20 micron. This is the reasonable figure assuming that when the dimensions are 10 mm and 100 mm, this film thickness is reasonable 40 to 20.

And what viscosity of the oil which we use? We are using is something like a 10 millipascal. Second, now interesting point is that we are defined the dimension say the yellow plate dimension in x direction is 10 mm, while in z direction is a 100 mm. That means, x by z is going to turn out to be points 1 because, x is a only 10 unit and z is a 100 units. So, 10 by 100 will be turn out to be point one that means, if we multiply, with a x square by z square, next is the second term will be neglected.

Second term is a negligible, particularly in this situation. So, that will simplify our solution say x by z is point one. So, this term will be neglected and this term is going to be only useful for us. Based on that we can solve this, we can use a final difference to solve it. I will cover this in my next lecture. Thanks for your attention.