Tribology

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Lecture No. # 19

Fluid Film Lubrication

Welcome to nineteenth lecture of video course on Tribology. Today's lecture, topic is fluid film lubrication, it involves mathematical analysis, it involves a merger or we say marriage of three equations. Equation number one is conservation of momentum, conservation of mass and conservation of energy. We need to marry this three equations to get overall fluid film lubrication equation often, emerge elastic deformation equation also along with this equation to get elasto hydro dynamic lubrication. Basic assumption in these equations is that gap between two metal surface or two material surfaces is infinitely small maybe in order of micron, it can go away with the nanometer or in mm, but overall size also will grow accordingly or reduce accordingly.

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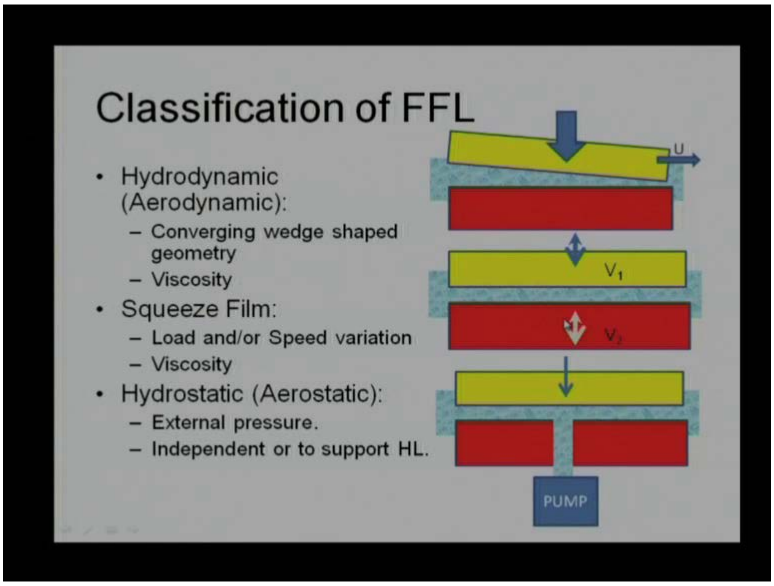

So, to start with that you can say there is a classification of FFL is a fluid film lubrication as I earlier mentioned, we can add fourth category that will be elasto hydro dynamic lubrication one more equation will come and give the right results for elastic deformation or consideration of elastic deformation. So, let us see first three which have been used a conventionally, first category says the hydro dynamic lubrication in that case it says that hydro dynamic lubrication. The concepts remain same only the governing lubrication media change in hydro dynamic lubrication, it will be the liquid which will be used as a lubricant while in a aero dynamic.

It will be the gas that is why to differentiate presence of liquid or gas we say hydro dynamic or aero dynamic both of this mechanism require two action, One is there should be some viscosity if viscosity is 0 than, it will not occur or we say solution will not be obtained using conventional equations. Another thing is a converging wedge shaped there should be a convergence, there should be some inclination there need to be a gradient which makes hydro dynamic or aero dynamic action mechanism. Second mechanism is a squeeze film this is a basically governed by speed variation and speed variation in direction of film thickness or changing the magnitude of the load or changing the direction of the load. There should be some sought of a variation which brings a squeeze film into action or this mechanism will be accounted only when, we have a load variation or speed variation, viscosity also plays a important role in this kind of mechanism.

Last one, in this category is a hydro static again in bracket is written aero static, hydro static for the liquid aero static for gasses here. The pressure is supplied or we need to provide external pressure to levitate one surface related to other surface as external pressure is supplied or applied, this will be a costly mechanism compared to squeeze film action compared to the hydro dynamic mechanism. So, often we avoid hydro static or we provide only partial hydro static mechanism unless, no possibility of developing hydro dynamic action, no possibility of developing a squeeze film action.

There are number of methods to develop hydro static lubrication mechanism, we will be discussing those in detail when we start covering the topic on a technology or we say applications of Tribology. To give a sketch of all these three mechanism I made a two blocks the yellow colour block and red colour block, I am assuming this is the material one material two and there is a liquid. So, it is a basically for hydro static and we are providing some relative velocity, relative velocity of a yellow block with respect to red block there is applied force normal load this may be a sulphate, may be imposed from outside or load to be supported you can see there is a wedge action, thickness on the left hand side or fluid film thickness on the left hand side is greater than thickness at the right hand side and this convergence is coming in the direction of velocity.

So, this full fills the hydro dynamic action there is a normal load there is a relative velocity there is a wedge action and we are assuming this liquid is viscous. Coming to the squeeze film action again that they are same block is a material one, material two yellow block and red colour block and there is lubricant at the interphase. Now, here we are saying the velocity is changing velocity of the second block is also changing, but necessary both the block have to have relative velocity or need to have a velocity zone if there is a velocity v 1 and there is no velocity v 2 and velocity v 1 itself, is a fluctuating even in that case this squeeze film action will occur or if there is a load applied is initially, may be say 100 Newton reduces 18 Newton naturally.

When load decreases this block will move up if load increases this block will come down or we say reduce the film thickness. So, indirectly even the load is going to induce the velocity of yellow block. So, whichever the situation is here squeeze film action will occur. If there is a relative movement of yellow block with respect to red block in the direction of film thickness, last one this is the hydro static, we say that we are feeding we are supplying pressure or supplying liquid with the pressure. So, there is a pump, which need to supply pressure to levitate this yellow block.

Yellow block is carrying the load right as I mentioned it will be the costly affair because using pump to lubricate requires, energy that will be consumption of energy as well as initial cost of the pump maintenance cost of the pump that is why? It can be say it is a least preferred compared to squeeze film action compared to the hydro dynamic action.

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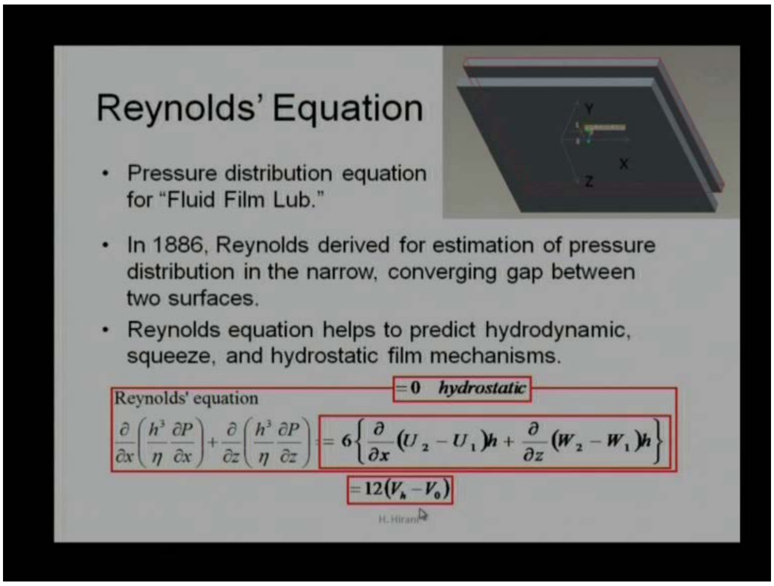

Mostly fluid film lubrication equation starts with the Reynolds equation. We discussed Tower's experiment in our earlier lecture; Tower was the person, who could figure out there is a some pressure generation, if there is a some inclination and viscosity of the liquid. And Reynolds after learning from experiments of the tower, try to give mathematical justification and that is known as a Reynolds equation.

You can see this is a Reynolds equation left hand side of the pressure terms and right hand side of the source terms. When we are talking about the source that means, there is a possibility of first term, which will make or which will induce the pressure between the or the interface may be the first term may be second term may be third term or may be all together, and this equation was derived long back may be say around 120 years back. The only probability of solving this equation was low, not much mathematical calculations were available or computers were available.

So, people used to think about simplification of this equation because this equation is the partial differential equation. So, there are number of estimation available either rejecting this term or rejecting this term rejecting one of this term not treating all terms together as we reject any term neglect any term that equation will simplify and solving that equation will be simpler.

So, we say there are there is a Reynolds equation and which can simulate all the behavior source term is 0. Suppose, the whole right hand side is equal to 0 that is possible in hydro static lubrication there is no velocity relative velocity there is no wedge action, there is no relative velocity between the two surfaces in film thickness direction. So, for hydro static this will be equal to 0. Coming to the hydro dynamic there is a need of relative velocity there is a need of wedge action may be in one direction or in both the direction. So, this may be in an x direction this in z direction we are neglecting y direction because y direction is a film thickness direction.

Last one is a squeeze film, where the source term is in y direction while here we were we were neglecting for the hydro dynamic while for squeeze film action we consider it. So, there is a possibility of only hydro static only hydro dynamic only a squeeze film action or may be all together a combination that is generally happens in engine bearing, where there is relative velocity there is a relative velocity in x direction, there is a relative velocity in film thickness direction also or because of the load, it will change its position shaft will change its position. So, will give some sought of relative velocity in film thickness direction.

We understand the Reynolds equation, but many times we feel how this equation was derived and what are the assumptions, which were considered, when Reynolds derived this equation develop this equation because if you want to modify any of term any of source term or any pressure term or viscosity term we need to know, what the assumptions are. So, that we can relax those assumptions and in our system, if those assumptions are valid we can keep same assumptions, if those assumptions are not valid we can relive those assumptions.

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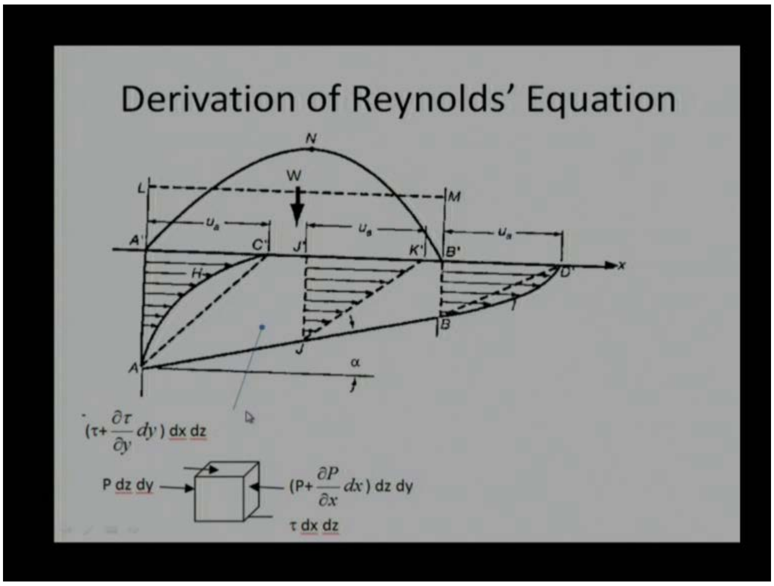

So, it is necessary for us to understand the derivation of this equation, this figure is a familiar in the which we discussed in one of the our lecture or earlier lecture. We say that if there is a some sought of convergence some sought of taper or one surface relate to the other surface, they are not parallel surface then velocity which is a liquid velocity, which comes between two surfaces because of the relative motion of one surface over other surface. Am assuming this surface is suppose, stationary and this surface is moving with a relative velocity of U a naturally liquid which comes into contact with the this moving plain will also start moving.

And that is shown over here at the shown over here at the center velocity profile is a linear while, velocity profile here is a non-linear velocity profile at the exit is a linear is a non-linear reason being, there is a pressure term which was added we say at the exit pressure term will boost the flow at the entrance it will retard the flow, but this was a this justification is from the physical point of view to understand it, but to quantify how exactly this profile will come we need to apply mathematical principles.

So, we can think about the mass conservation or sorry we can think about the force conservation at this start. Let us assume there is a some this element at this position as a liquid is a continuous, we can think about the continuum and top surface is moving at the relative velocity compared to the bottom surface. So, we are assuming the shared stress on the top surface of the element will be more compared to shared stress at the lower surface or we say that at the this surface share stress is more that is why it is been given at the tau plus some gradient of tau while, at the this surface this is no gradient this is only tau for the force we have to multiply with the area.

So, we are assuming that this is this area is a d x into d z that is an element coming to the pressure side pressure is developed in this side. So, what we can start pressure into area while here pressure plus pressure gradient, positive pressure gradient because the pressure is be more you can see over here pressure is increasing, may be quite possible at the we develop a system after satisfying all the equation will turn out to be negative at the latter part, but we do not want to prejudice about this we do not want to pre calculate we say, let us start with a mathematics let us start with a conservation laws and whatever comes out we will try to predict based on that.

So, there is a we are assuming there is a equilibrium. Now, you can see that we have not consider inner short term otherwise, if there is a element over here it will be having inner shaft and we should consider x relation term multiply with a mass of this element, but our assumption is that inner short terms are negligible x relation of this liquid will not be very high and mass is also low. So, inner short terms are neglect this is the first assumption in a Reynolds equation we do not consider inner short term.

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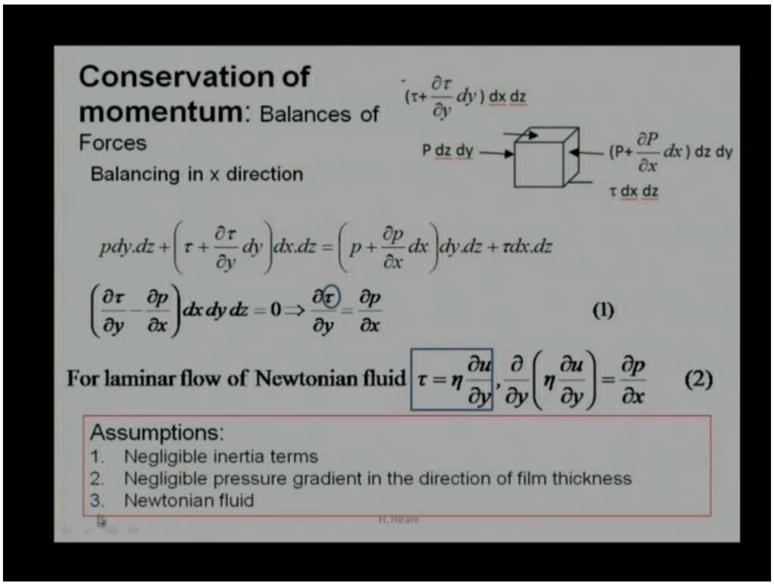

Now, continuing with this we say that we need to we are considering conservation of movement from and for time being, we are considering only the force balance and force balance in x direction can be given by this relation. The p into area tau plus gradient into area pressure plus gradient into d x into area plus tau into area, we know this tau d x and this tau d x will be cancelled p d y by d p d y d z and p d y d z it will be cancelled.

So, overall equilibrium comes with a gradient terms like this say am rearranging equation. The gradient of shared stress gradient of the pressure and interesting thing, they have a same unit pressure and shared stress will have a same unit x and y will be having same units and there is a volume of element, this can be written or rewritten something like this gradient of shared stress induce pressure or may increase pressure.

So, we are emphasizing here the in this tau is playing important role and in liquids, we know viscosity will play important role to induce this shared stress, the only question comes how to determine this tau. What is a mechanism on determining the tau? We already studied about the liquid lubricant you say, we can consider the Bingham liquid model. We can consider the non Newtonian model or Newtonian model a simplest one for Reynolds equation is a Newtonian model. What Newtonian model says tau shared stress in liquid is equal to viscosity into velocity gradient.

If I use this equation, in equation number one we can get pressure in terms of velocity and viscosity if any of this is 0 this viscosity is 0 and we know the pressure will not be generated or if there is no relative velocity or there is no velocity gradient, then again there will be 0 pressure. So, that is why there for hydro dynamic action we require both we require viscosity, we require velocity gradient, we require relative velocity. If there is a relative velocity, velocity gradient will come. So, we can list out down the assumptions what we have already made, we have not considered in a short term. So, we are saying it has a negligible inner shaft or fluid has a negligible inner shaft.

So, that is first assumption, second we have not consider the pressure gradient in direction of film thickness. You can see pressure gradient only in x direction may be later, we will consider pressure gradient in z direction. The film thickness direction this is a y direction, we are not considering any pressure gradient term. What we are assuming in that small space small gap, pressure will remain constant it is not going to change there will not be any gradient on pressure.

And last assumption in the what we consider here that was a Newtonian fluid or fluid is governed by Newtonian law or we say shared stress is proportional to velocity gradient and constant of the proportionality is viscosity, these are the three assumptions which we are making to derive equation number two.

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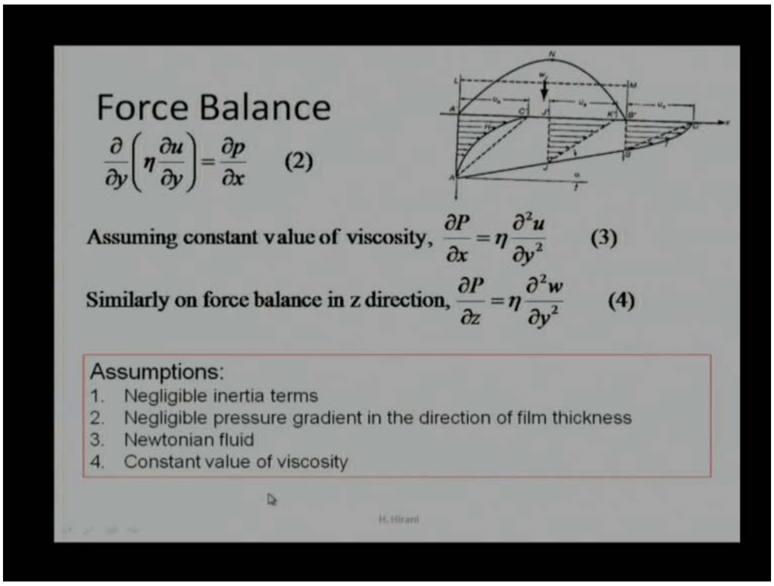

We continue it is a force same force balance equation has come. Now, we can make another assumption say that am assuming viscosity is not function of y that means, viscosity remain constant from bottom plate to the top plate, which is the slightly unrealistic assumption because we know there will be a thermal heating because of sharing and liquid is a strong for liquid has a strong function between viscosity and temperature.

So, viscosity will continuously change with the temperature, but we are not considering energy equation, we are not consider the temperature. So, there is no point to consider viscosity variation for Reynolds equation or to derive the Reynolds equation. So, that is why there is an assumption viscosity is constant it is not a function of y it remain constant throughout and off course, it is not a function of x and it is not a function of z we consider in the Reynolds equation.

So, with this modification we can say rearranging equation pressure gradient is equal to viscosity into velocity gradient of second order. Similarly, the way we have done in x direction similar force balance can be done in z direction also equation will be pressure gradient in a z direction is equal to viscosity, we are assuming there is no change in viscosity in x direction or z direction and velocity gradient of second order velocity gradient is coming over here right there is a velocity u in u x direction, velocity w in z direction and y is a direction of film thickness, where this velocity gradient is acting.

So, this is a fourth assumption what we made it is a constant value of viscosity, velocity is not a function of x, y, z we have a did first assumption was negligible in a short term second assumption no pressure gradient in y direction, third assumption Newtonian fluid and forth assumption is that constant value of viscosity, which is a unrealistic assumption, but for simplification to understand the physics of or to understand, the mathematics of F F L we need to account initially and latter we can modify this or we can relax this assumption.

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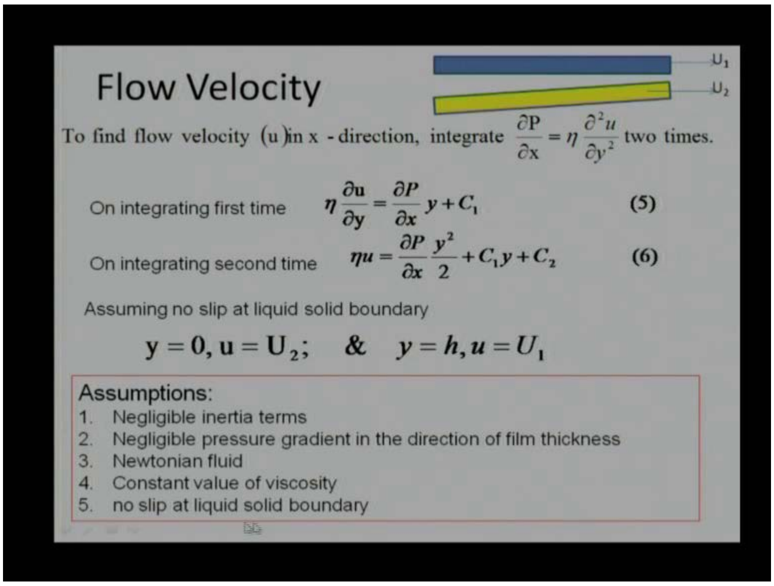

Now, we expressed pressure in terms of velocity and we know velocity is known to us. What will be the velocity of the bottom plate and what will be the velocity of top plate? So, it a easier for us to integrate velocity and get the pressure terms using boundary condition, which are already specified for this purpose we find to find the flow velocity or velocity of the liquid we integrate this term equation three and forth equation three was x direction equation four for the z direction.

Now, when we integrate the first time we are getting first integration constant, you can see n velocity gradient is your pressure gradient into y film thickness plus 1 integration constant. Now, we can integrate this equation five second time and we find this relation say eta into u, this pressure gradient is not depending on y. So, this y will turn out to be y square by 2 this C 1 is the constant it is not a function of y simply, this will be C y plus another integration constant C 2.

Now, we have a 2 velocity condition available to us we can substitute or we can use those boundary conditions to find out value of C 1 and C 2. What are the boundary condition given to us, we say the top plate has a velocity U 1 yellow plate has a velocity U 2 in earlier diagram, we showed a U a that was a relative velocity that means, U 1 minus U 2 and we are assuming over here that there is a slight tilt, but y is equal to 0 or our coordinate system is also along inclined along with this plate when y is zero that means, somewhere this point velocity is equal to U 2. When y is a film thickness equal to film thickness, it can be in any cross section y is equal to h that time velocity is U 1.

So, what are the conditions we are assuming there is a no slip at the liquid solid boundary condition or junction there is a no slippage whatever, the velocity of the plate same velocity is imparted to the liquid not a single or not a 0.1 percent variation, in the velocity same velocity is been imparted to the liquid when there is a interphases.

So, we can add this assumption also say initially we assume in shared terms are negligible second was a pressure gradient in direction of film thickness is negligible we are not accounting it or pressure gradient in x direction as well as z direction does not depend on the y coordinate. Then Newtonian fluid we assumed constant value of viscosity the viscosity is not function of x, y, z coordinates.

And last assumption what we made here is a there is no slip or the liquids solid boundary. If we use this kind of slip condition then expression will change and the for most of the time the liquid case this assumption is justified, but this will be a problem particularly gasses lubrication. In gasses lubrication there will be some sloppage which need to be considered many times, we say first order slips second order slip that can be considered, when we develop this equation for gear bearing or gas bearings.

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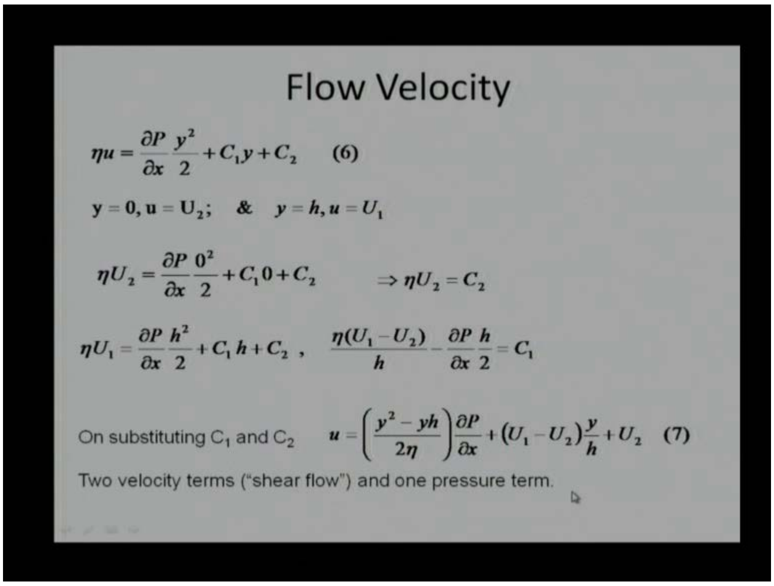

So, what we considered in previous slides this equation we derive this equation six, we consider two boundary conditions. Now, we substitute y is equal to 0 and u is equal to U 2. So, this pressure this term will be 0 as it is multiplied by 0 second term also will be 0 third term C 2 will be equal to eta into U 2. So, we got integration constant C 2 is equal to eta into U 2.

Now, time comes to find out the C 1 we can use another boundary condition see when y is equal to h, u is equal to u 1 as we already know what is a C 2, we can substitute C 2 value over here. So, eta into U 2 already there is a term eta this side. So, we can bring u to this side and say it will be U 1 minus U 2, this is a known term for time being to us and then we can determine C 1 or we say that C 1 is equal to velocity gradient pressure gradient and film thickness. We have C 1 we have C 2 and we can substitute in equation six to find out velocity profile on substitution we find this result.

Now, we can match this result with our earlier expression say when y is equal to 0, u is turning out to be U 2 this is the same boundary condition, we are not made any mistake when y is equal to h. So, this is a same again 0 this will be 1 by h, h by h is 1. So, U 1 minus U 2 plus U 2 so, this will be U 1. So, this shows we have done a right job, we have not committed mistake another thing is that maximum velocity is U 1, minimum velocity is U 2 in between the pressure is going to be accounted pressure gradient will be lower at the entrance side and will be positive on exit side.

So, we will be getting some pressure profile or velocity profile and another thing is that we have two velocity terms. If we say there is a U 2 is a 0 than only there will be one velocity term and one pressure term or whatever assumption or hypothesis was there because of inclination these is a boosting of the pressure or boosting of the flow at the exit and a retardation at entrance, it is a same thing is coming from mathematical treatment.

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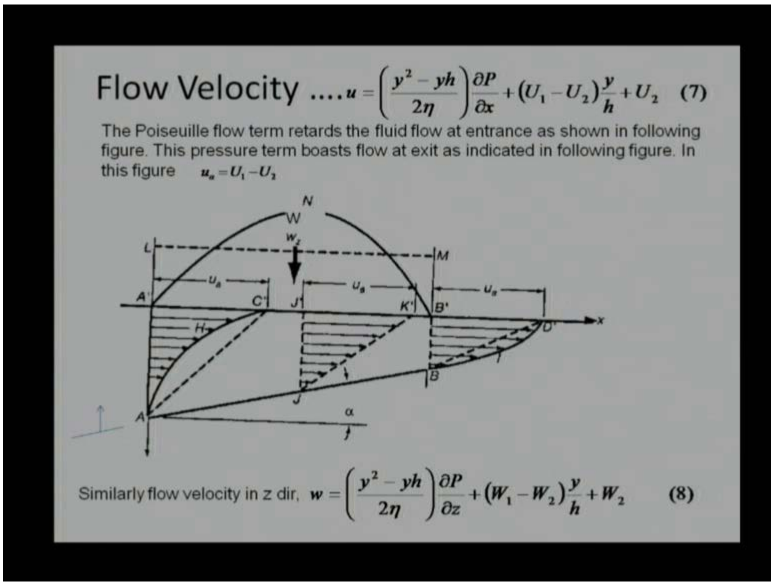

We can see what we have initially hypnotized this plate is inclined, this plate is a constant or is a horizontal because of this inclination at this when maximum pressure occurs, when velocity gradient is 0 velocity sorry pressure gradient is 0. We are finding the linear profile, you can just get the same result over here for maximum pressure we can substitute equal to 0 d p by d x when it is a 0, then we are going to get velocity profile simple profile.

We can find out at any h what will be the this term plus U 2 and we can finally, in a profile while when the there is a finite value of d p by d x either negative or positive, we can get this results this side results. So, this is indicated and in this diagram as I mentioned earlier this is a term U a, U a in this case is equal to U 1 minus U 2 there is a velocity term and in a when we use a relative velocity term naturally, we have to make other plate as a 0 velocity.

We transfer u and U a here and U 2 will be 0 that is a relative velocity, which we are considering we are considering only the difference in velocities, if we are not interested in absolute velocities right. So, what we can mention or what is mentioned particularly in this slide this is a Poiseuille flow terms retard the fluid flow at the entrance are shown in this figure, the pressure term boost the flow at the exit you can see that additional velocity term over here.

So, we can say that it is a conserving momentum as well as a there is a force mass balance given to this, same expression what we have done in x direction can be derived for the z direction, almost the same treatment there is a velocity of pressure gradient in x direction here pressure gradient in z direction. So, y square remains same Y h remain same 2 eta remain same here velocity is terms is coming. So, velocity in u in x direction here it will be the velocity in z direction.

Initial velocity of the plate or we say not initially the velocity of the lower plate in x direction, velocity of lower plate in the z direction. So, we have now the pressure term or sorry we have a velocity term in terms of the pressure, we can find out inverse of that we can find figure out what will be the pressure in terms of velocity. To do that we require equation or additional equation what we call as a conservation of mass or many we use the word continuity equation. So, that comes over here this conservation of mass we are making 1 assumption that density is not changing, it is not its incompressible liquid.

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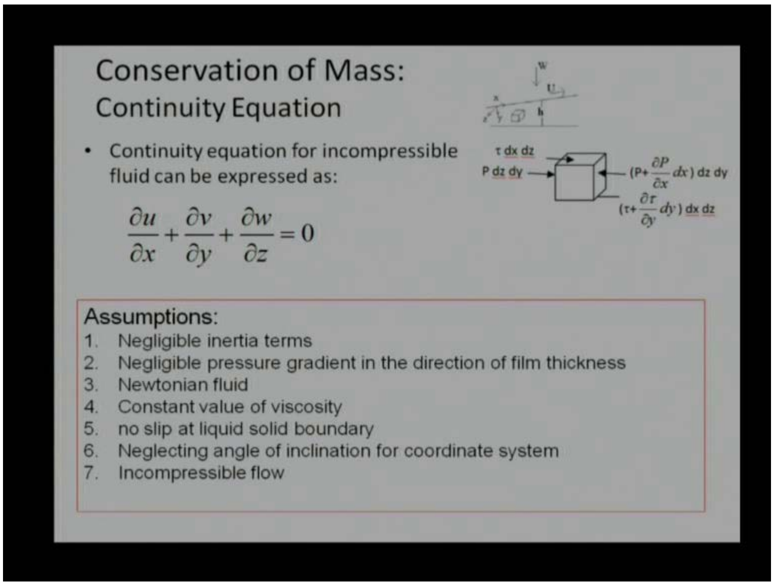

So, density will not depend on x direction, z direction or y direction with that kind of assumption we can simplify continuity equation or mass equation in this term velocity gradient in x direction, velocity gradient in y direction, z direction and this is given figure over here and there is a element over here and we are considering three dimensional modal z, direction x direction as well as y direction this will be is equal to 0.

In previous slide we have a expression or we figure out expression u in terms of pressure as well as we figure out w in terms of pressure, we can substitute these values those expressions and then find out other equation or merge momentum equation in this continuity equation. However, we have made one assumption over here that is a incompressive full fluid that is why? We want to add this.

Now, the assumption list is increasing first assumption was a negligible inertia term second assumption was a negligible pressure gradient, Newtonian fluid constant value of viscosity no slip at the liquid solid boundary neglecting angle of inclination for the coordinate system for time being, we can relax all these assumptions and final assumption in this case is incompressible fluid for air, for gasses we need to account viscosity density variation in coordinate system.

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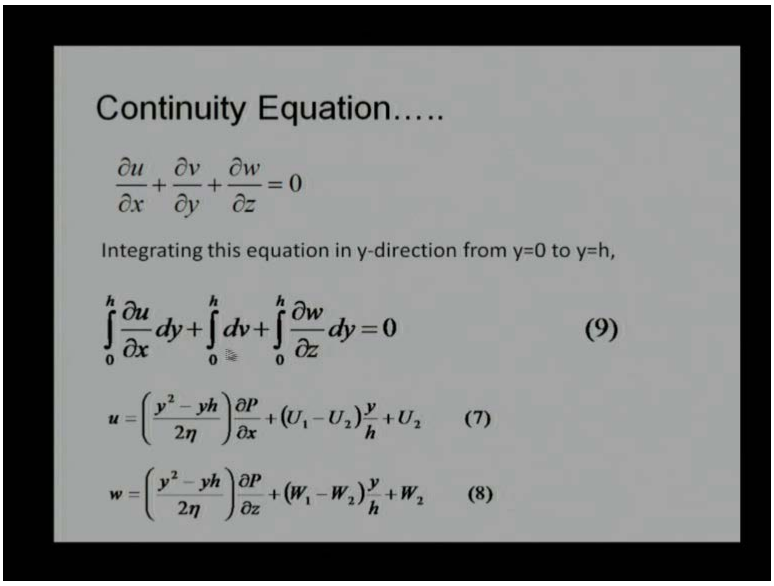

So, we need to account this equation and we can substitute expression of u expression of w, but there is a velocity gradient in y direction also. So, we need to integrate from 0 to h from for the complete film thickness 0. There is a lower coordinate system is placed at the lower plate and h, where the thickness is measured at the upper plate. So, there is h now, we are integrating with respect to y because the gap we are assuming is in y direction here when we when we integrate the d v by d y into d y, we will get velocity or say d v this expression will be relatively easier for us this will be slightly complex this will also be complex this expression is very simple.

We, what we earlier derived u in terms of y and in terms of x, whether assuming there is a some pressure, but coordinate system x and y and when we substitute here we are getting both the things we are getting x as well as y similarly, velocity in z direction is a coming in a relation with y and z here the z and y both are inwards. So, we need to use some sort of special treatment to integrate this equation, this term and integrate this term as I mentioned this integration of this term is simpler there will not be much problem for us.

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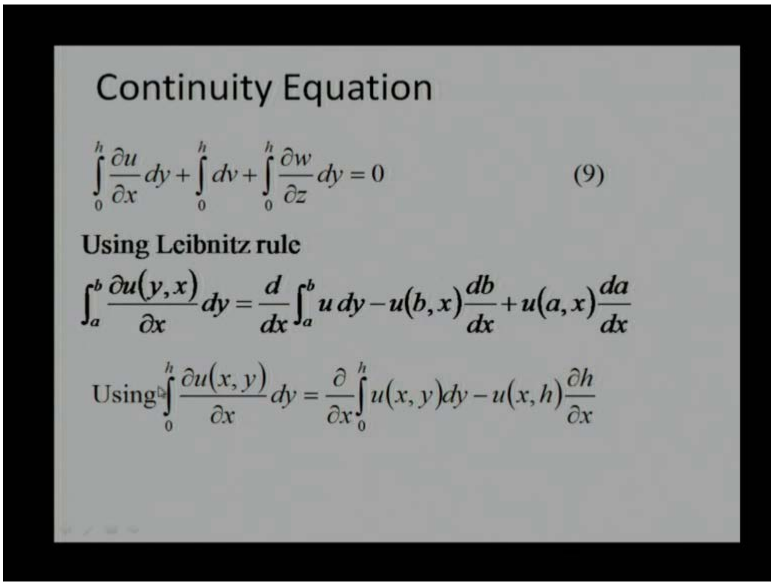

Now, we can use Leibnitz rule what you say that we can go ahead with partial integration or we say the step by step integration, when we know the u itself is a function of y, x and we need to differentiate as well as integrate it. So, we can take out this term out say assuming this term is u d y, but we need to subtract and add u at the b value. What is the value of the u at b and differentiation of b with respect to x similarly, what is the value of a value of u at a and differentiation of a with x.

So, this is a well established rule we are using it for our purpose, when we use this relation what we get this differential term is coming out there is u into d y minus because b is h in our case. So, it will be differentiation of h with respective x minus instead of a we have a 0 value over here. So, differentiation of the 0 will not be there this term will be neglected or will be plus 0 and then can be neglected can be removed.

So, what we say this term can be replaced with these two terms they can substitute back these terms and precede further integration. So, am just rearranging it we know u at the value of h is boundary condition that is a U 1, this is a gradient term and this is a integration term, we can substitute this and there was another term with integration of u with respect to y from 0 to h.

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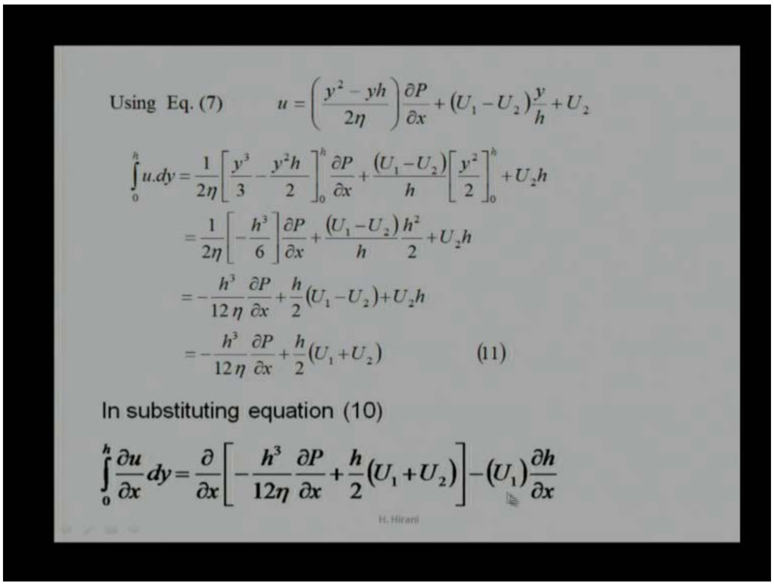

When we integrate this term, what we are getting? We are assuming d p by d x is not a function of y, U 1 and U 2 are not function of y and h is also there which we can integrate, we say that when we integrate u there will be y and integration limits of h and 0. So, it will turn out to be h minus 0 and that is why there is h will just repeat it say when we are integrating this, this is a y term this is y square. So, integration of y square will be y cube divided by 3. There is a term y, h is constant the integration of y will be y square by 2 and this 2 eta is taken out is a common pressure gradient does not depend on the y.

And there is a integration limits are h and 0 plus this velocity is not function of y, h is a in this case we are saying that is a constant it is not a function of y over here and y is coming over here. So, this y will be replaced with or integrated like y square by 2 there is a no term related to y, U 2 is constant. So, it will be U 2 y and as I say the integration limits are h and zero. So, h minus 0 will be equal to h (()) overall this term will turn out to be plus U 2 h.

Now, we can substitute values at h and values at 0 or value of 5 even is equal to the h or value of y when equal to be 0 or we say y is equal to h or y is equal to 0. When we substitute this what we get overall term from this is a minus h cube by 6 while from this it will turn out to be h square by 2 and this third term is U 2 into h, rearranging this equation in systematic manner we say that, this is a first term pressure term is a minus h cube by twelve eta pressure gradient plus term related to h and there is a relative velocity and last time is coming the absolute velocity of a lower plate.

Further, we can rearrange this term and what we get overall this number eleven that is summation of velocity and there is a pressure term. So, velocity term and pressure term. Now, we derive this equation and we have previous equation, equation number ten we have done this work we have integrated over here. So, that is available to us this is already constant or it is not going to be integrated.

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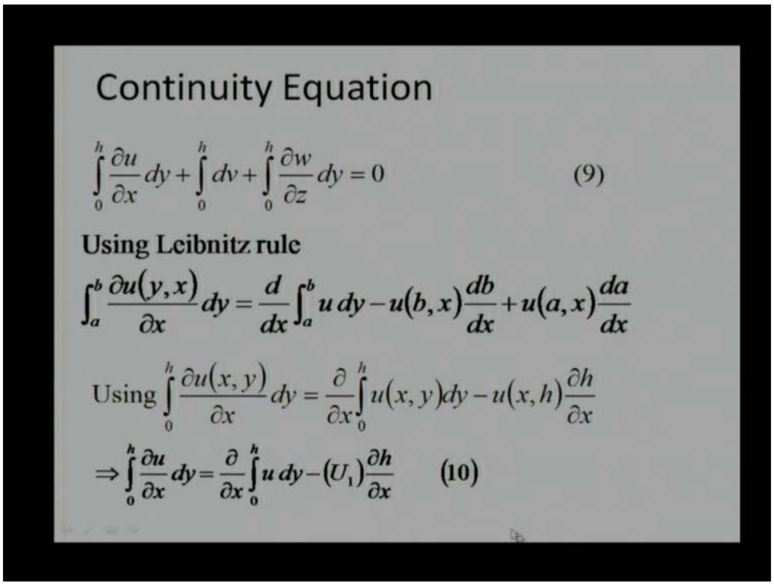

So, we have one term which needs to be differentiated. So, this term will be differentiated, when substitute here in a equation what we get over here is something like a differentiation and this is a same term minus U 1 wedge term or we say that gradient of film thickness. Now, we can differentiate as we know d p by d x depends on x, we have already made assumption eta does not depend on x, but h may be depending on x. So, we need to rearrange and some time we say that there is a possibility of U 1 and U 2 depends on h there is a possibility some typical examples are the rubber bearing. When there is a stretching action that is why we use a term for U 1 and U 2 as a stretching term.

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$$\int_0^h \frac{\partial u}{\partial x} dy = \frac{\partial}{\partial x}\left[-\frac{h^3}{12\eta}\frac{\partial P}{\partial x} + \frac{h}{2}(U_1 + U_2)\right] - (U_1)\frac{\partial h}{\partial x}$$

$$\int_0^h \frac{\partial u}{\partial x} dy = \frac{\partial}{\partial x}\left[-\frac{h^3}{12\eta}\frac{\partial P}{\partial x} + \frac{h}{2}(U_1 + U_2 - 2U_1)\right]$$

$$\int_0^h \frac{\partial u}{\partial x} dy = \frac{\partial}{\partial x}\left[-\frac{h^3}{12\eta}\frac{\partial P}{\partial x} + \frac{h}{2}(U_2 - U_1)\right] \qquad (12)$$

Alternative Method to find Eq. (12)

You say this is same term, which we written in previous slide and integrating or differentiating, we are first bringing this term inside because there is a common d y by d x that is taken as a common. We are bringing h into this bracket rearranging equation and then what we are getting something like this still we need to differentiate it, but there is a term pressure of gradient is remained same and velocity term due to this addition or due to this subtraction is getting changed, there is a relative velocity and that is negative relative velocity, we say in this case is the U 2 minus U 1, we know the U 1 is greater than U 2.

So, it will be a negative term or both will turn out to be negative, negative can be taken out now, but this equation was derived based on the Leibnitz theorem or Leibnitz law we can do alternative method also, we can use alternative method to get whatever we have assumed or solve the equation is a same or not we have alternative method to develop drive equation 12.

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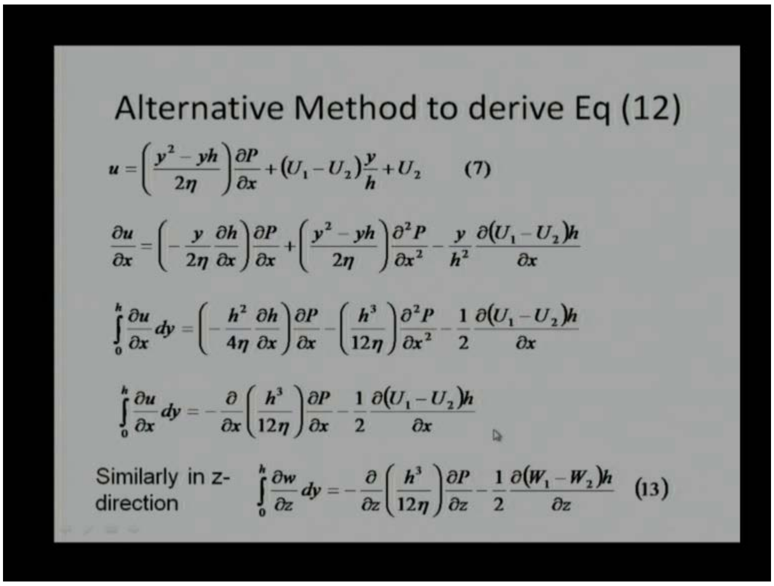

What that says we have velocity. Now, we need to differentiate this velocity right that differentiation here y and x there is no relation also they are independent coordinate. So, differentiating y with respect will not give any result. So, it is a 0 and differentiating h with respect to x gives a velocity gives a wedge term to us that is why this after differentiation will come like this, here we are taking the parts or in the first bracket was differentiated first, then second term need to be differentiated keeping first term as a constant.

The same term here for second differentiation comes second derivative of p with respect to x. When we are differentiating this term we have h term also and we can different h as this is the inverse. So, it will be minus sign will come and this expression will turn out to be y minus h square plus all multiple along with this. Now, we are saying that U 1 and U 2 also may be differentiated with respect to x.

When rearranging this equation because there is a pressure term here there is a pressure term here and a we need to we can rearrange later may be, but once we first in this case we have integrated when we have a velocity gradient it can be integrated with respect to y because there is a y term over here, when we integrate it what we get as such a d h by d x is not depending on y. So, there is a simple integration of y it will turn out to be y square by 2. So, this term is turning out to be minus h square by four eta because integration limits are 0 and h that is why? We say that this is overall minus h square divided by 4 eta this is same term this is not going to change with y.

Coming to the integration of these two terms, what we can say there is will be h cube divided by 3 minus y, h cube divided by 2. When we take common it will turn out to be minus h cube that is why the term is coming here minus h cube divided by 6, 6 into 2 will be 12 that is why? This is written here minus h cube by 12 and second gradient of the pressure. And this is again there is y we can integrate this y and it will turn out to be it will turn out to be h square by 2 here, h square will be cancelled out with h square it turn out to be overall minus 1 by 2 and this term will remain same.

Now, time to rearrange the equation what we are saying the rearranging the equation is that there is a second derivative of the pressure there is first derivative of pressure, we can rearrange in a such a manner that there is a 1 gradient terms are terms comes out and h and d p can be retained. What we are saying here in this case is I assume there is a term and now we want to integrate or want to differentiate by parts, when we differentiate first only that this term, what will be this 1 minus 3 h square divided by 12 eta 3 will be cancelled with 12. So, it will turn out to be h square by 4 eta into d h by d x this pressure term will remain same.

When you integrate second time, second term keeping minus h cube by 12 eta constant, that is over here and we will get a pressure term second derivative of pressure. So, these two terms can be rearranged in this term and this is a shame what we have obtained previously without with Leibnitz law, in the we do not use Leibnitz law we directly use simple our straight equations, we also get a same expressions. So, there is no problem in derivation of this.

Now, we have done in x direction similar expression comes in a will be we can be pointing out, we can get in z direction for same similar relation in z direction is something like this minus instead of x there will be z, it will be the gradient in z direction h will remain same h this term will remain same this term only the x will be replaced with the z and here x will be replaced with the z and U 1 will be replaced with the W 1, U 2 will be replaced with W 2 right; so got a term in x direction, term in z direction.

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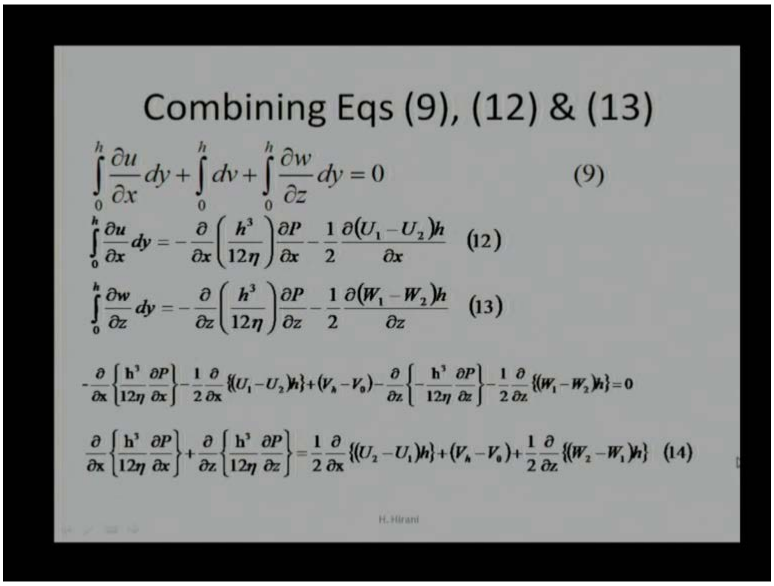

Now, we can substitute these relations in this continuity equation, which was integrated to get the results and we substitute this equation twelve and thirteen in this equation. What we get something like this after substitution, what we are getting the first pressure gradient, gradient with h cube twelve eta and d p by d x plus second term this term and this, this we know this expression is simple we know integrating v at the h and v at 0, lower limit is 0 and h is a upper limit; so, velocity at the velocity of upper plate and velocity of the lower plate in y direction.

Then comes this third term, third term is can be represented or can be written something like this is a same equation. Now, we can rearrange this equation there is a this term and this term has a lot of similarity and these are the both are the pressure terms that is why? We can take other side and we say pressure term in x direction, pressure term in z direction and these are the partial derivation of the second order while, this equation is on this side here the negative term is replaced or maybe we are taken inside negative term in minus U 1 plus U 2 that is written over here U 2 minus U 1 into h plus term v h minus v z 0.

Similarly, this negative term is been replaced with the positive term we are saying that half this and the W 2 minus W 1. If you remember this is same what we have done what we started as a Reynolds equation it is the same term. Now, we can give some names to these terms what we say this is a pressure term, which need to find out the pressure after solving this equation and these are the source terms this source term U 2 minus U 1. If considered this variation is a changing with respect to x then it will be called as a stretching term stretching action.

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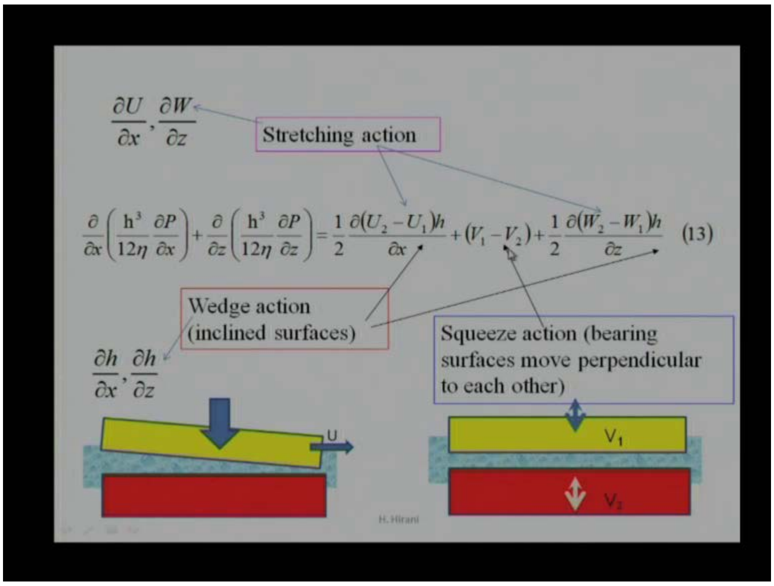

We say that U 2 and W 2 and minus W 1 this relative velocity is changing with z than, it will be stretching term and it happens in a micro motion at the rubber level. When the there is a elastic deformation and for two particles are even the same body they do not have a same velocity generally, for the solid motion the this gradient turn out to be 0 for steel for the material this will not be accounted to some extent.

When the there is a plastic flow or there is a elastic flow then this terms will be there or we say that velocity gradient also the z direction or along the x direction velocity, there is gradient along the z direction generally not possible, but will come if there is a elastic flow or plastic or a plastic flow may be say polymer bearings also polymer it can be accounted.

There is another term what we call the wedge term, there is a change in the film thickness along the x direction similarly, change in film thickness along the z direction. So, these are the wedge action or wedge terms need to be there is an essential need for these terms to develop hydro dynamic action or we say we require inclined surfaces to find the hydro dynamic action.

And in short it is been represented something like a tow h by tow x tow h by tow z. They need to be finite or either of this term need to be finite to develop hydro dynamic action last 1 is a squeeze action. We say that bearing surface move perpendicular to each other in this case or we say that film thickness h is continuously varying. There is a separation and because of the relative velocity along the film thickness direction, there will be change in film thickness right what we are talking this thickness will move up and down.

Now, as this diagram earlier we started this is a having a wedge action and then there is a relative velocity u with respect to the red block than, this is satisfying our requirement this hydro dynamic action. Similarly, this is squeeze action what we initially divided we say that there is a squeeze action because of this there is a change in film thickness. The change in film thickness may happen if there is relative velocity in moving up this plate is moving up and moving down.

There will be change in h that is going to change or that will be a source term to develop pressure because of this change because of the this squeeze action, there will be pressure generation or in other words whatever, the source term we choose there will be pressure generation this is a hydro dynamic action. Hydro dynamic action this is squeeze action and as I mentioned that because of the velocity, this is film thickness is changing. So, many times what we write V 1 minus V 2 as d h by d t t is a time h is a film thickness.

So, derivative of h with respect to time that brings viscosity variation with time, and that is a same; sorry film thickness variation with the time and that is a same thing, which is given over here. So, we will continue with a Reynolds equation some simplification of in Reynolds equation in our next lecture. Thanks for your attention.